Models of an emergence of logic

澤，宏司

(Degree)
博士（理学）
(Date of Degree)
2010-03-25
(Date of Publication)
2011-08-02
(Resource Type)
doctoral thesis
(Report Number)
甲4916
(URL)
https://hdl.handle.net/20.500.14094/D1004916

Doctoral Dissertation

Models of an emergence of logic

January 2010

Graduate School of Science, Kobe University

Koji Sawa

Contents

Acknowledgements

I am deeply grateful to Prof. Yukio-Pegio Gunji. I have the good fortune to meet him.
I also thank Satoko Sawa and Izumi Sawa for their support.

1. Introduction

In this doctoral thesis, we propose some models of an emergence of logic.

What is logic? Here we regard logic as “a kind of calculator by which we can apply an experience in real life to the other situations”. In other words, logic is defined as formalized relations themselves extracted from causal, order, or inclusive relations between things of real world by discarding concrete property of the things.

Logic is often formalized from Aristotelian age to present day, and formal logic is a modern form of formalized logic (Troelstra and Schwichtenberg, 2000). Formal logic maintains the appearance of mathematics-like form. In fact, it is sometimes called mathematical logic or symbolic logic, and forms a component of foundation of mathematics. In general, the system of formal logic is composed by an axiomatic method, that is, logical connectives composing formal logic are given thetically. What is axiom? More precisely, where does the validity of setting of an axiom come from? The study of modern formal logic is not intended to answer these questions, but intended to observe the property of each axiomatic system.

Firstly we propose a model which explains that the transitive law of causal relations is derived only from local knowledge and local communication. The model, which is called Dialogue Model (I), is motivated by two arguments. The first thing is the assertion of Szabo (1978), which is an analysis about mathematics and logic in ancient Greece. Szabo advocated that “*mathematics was just a branch of dialectic*” in ancient Greece, which was an incunabular period of logic and mathematics. According to this assertion, we can consider that axioms in arbitrary axiomatic systems are mere minimal conventions for discussants. As mentioned above, axioms are given thetically in an ordinary system of formal logic. However here, instead of using axiomatic methods, we construct a model that simulates an emergence of an axiom of the transitive law, in the shape of the property which discussants obtain through dialogues.

The second is the embodied mathematics proposed by Lakoff and Núñez (2000). From this standpoint, the transitive law of causal relations is grounded in bodily experiences of each individual in real world. We deem it necessary to pluralize the subjective body which experiences causal relations in addition to their argument. We show that if there are multiple subjective bodies and appropriate dialogues between them, the transitive law can emerge even in the absence of a personal experience of the transitivity.

Implication is the logical connective that expresses causal relations in formal logic, and the transitive law is one of essences of the causality. In classical propositional logic which is the simplest formal logic, there are four logical connectives: negation, conjunction, disjunction and implication. As it is known well, conjunction and disjunction can be translated into formulas composed only of negation and implication. Therefore, classical propositional logic can be composed only of negation and implication. Thus, we can consider that Dialogue Model (I) is a representation of implication, more specifically, a model about logic itself from the standpoint of formal logic. Infomorphism, proposed by Barwise and Seligman (1997), can be regarded as a formalization of a dialogue between different logic. Dialogue Model (I) is the same as Infomorphism, while each body of local knowledge is regarded as local logic. However, Infomorphism is the concept of a dialogue between different logic that previously exist, in contrast, Dialogue Model (I) shows a formation itself of logic through dialogues.

While we consider out of context of the dialogues and the causality, Dialogue Model (I) can be regarded as mere transformations of graphs (cf. Paton, 2002). Dialogue Model (I) realizes dynamical interactions. In that sense, Dialogue Model (I) is similar to the influence model (Asavathiratham, 2000), for instance. The influence model represents interactions on networks; meanwhile, Dialogue Model (I) represents interactions of networks. Nowak et al. (1999, 2000) showed the evolution of language in manner of population dynamics, whereas we provide the model of shorter span of time. From the standpoint of the exchange of knowledge, Dialogue Model (I) has a resemblance to the model of Adamatzky (1998). Adamatzky argued the transitions of information under the condition that there is a two-level hierarchy of agents. Agents of one group can influence the agents of another group, and this is not commutative in his model. On the other hand, all individuals are equal with respect to dependence in Dialogue Model (I).

Section 2 is organized as follows: at the beginning, we schematize an individual who has knowledge about causal relations, and formulate interactions through dialogues between those individuals. And then, we check up on results of simulations. In the second place, considering the model described above as a basic model, we construct variant models with an additional concept called identification of objects. This concept invokes a denial of complete accord of each individual's perspective of things. The variant model is called Dialogue Model (II). We check up differences of behavior among the variant models. However, we observe that there is no difference from the standpoint of the emergence of the transitive law.

In Dialogue Model (I) and (II), the initial knowledge of each agent is given and arbitrary one in which there are no constraints. Each agent changes its own knowledge in manner of not obtaining new knowledge but losing invalid knowledge. In Section 3, autonomous change of knowledge of an agent on the basis of its knowledge itself is treated. This model is called Monologue Model. In addition, dialogical interaction among agents which act by Monologue Model is considered. The model of dialogical interaction with Monologue Model is called Dialogue Model (III). The result of Dialogue Model (III) is compared to the result of Dialogue Model (II).

Monologue Model does not have minimum objects in principle. All objects can vary their “size”, and their obviousness is deprived. In an ordinary formal system, especially in an axiomatic system, objects are irrefutable. However in Monologue Model, we regard the irrefutability as a mere assumption, and argue about that.

Gunji et al. (2006) also deal in variable objects that change according to the situation in the name of skeleton. These problem establishments are, in a manner, denial of reductionism. Ordinary reductionism inevitably requires components that are irreducible to other components, and the description of the relations of those components is the description of the system itself. Namely, components are inviolable and absolute. In contrast, the description of a system consisting of temporal components is that of the relations of temporal components that each observer can set up arbitrarily depending on standpoint and circumstances. Even if the size of a temporal component varies as time proceeds, the “inside” of the temporal component changes to neither inviolable nor “empty”. Each cannot observe the inside, and the hidden inside can be exposed afterward in some cases. An idea of the irrefutability is evolved in Dialogue Model (III).

We call this temporal component a soft object. We introduce some measures of the world in order to argue about the soft object. In Dialogue Model (I) and (II), the consistency of a system is ensured by the transitivity law of implication. In contrast, the consistency is ensured by at least two aspects in Dialogue Model (III). The first thing is the consistency in the light of relations between soft objects. The second is the consistency with respect to soft objects themselves. The former can be evaluated by the ratio of the part in which transitivity law is satisfied to the whole of the system, as well as the consistency of a system in Dialogue Model (I) and (II). The latter is measured by “softness”, which is defined by the number of directed edges in a soft object. These consistencies do not necessarily consist together. We show the difference of results induced by the choice of the consistencies.

In section 4, we propose a novel model of dynamical formal logic. Dynamical transition of formal logic was dealt by Gunji et al. (2004) in the context of Informorphism by Barwise and Seligman (1997). Gunji et al. (2006) also proposed another model based on lattice theory (Davey and Priestley, 2002). In Dialogue Model (I), (II) and (III), a method of multi-agent model is adopted. In Section 4, the method of multi-agent model itself is considered. We raise a problem with a multi-agent model as below.

A multi-agent model premises at least one agent by definition. What is an agent? As an answer to this question, firstly we assume that an agent is what is simply transformed in a system. If agents of a system are completely independent of, and external to the system, the behavior of the system can be attributed to the behaviors of agents. Thus, we must check up the property of agents in order to argue about the property of a multi-agent model. This may lead to infinite regress. Responding to this situation, instead of external agents, we introduce an agent which exists inside a system, in other words, is a part of a system. The model which we propose is an internal measurement model of formal logic, where internal measurement was proposed by Matsuno (1989). We call an agent which is inside a system completely an internal agent, and also call the model Internal Agent Model.

Another major characteristic of an agent is its autonomy. We define a guiding principle which is inherent in each agent and leads to the autonomous ability, and call it "purpose". Thus an agent in Internal Agent Model has two main characteristics: the internal and the autonomous.

As mentioned earlier, classical propositional logic can be composed only of negation and implication. Though here we treat only implication represented by a directed graph, it is sufficient because one directed edge between two nodes represents an implicational relation between them, and the absence of a directed edge represents the negation of the implicational relation. However, not every directed graph represents adequately formal logic. We observe the emergence of a directed graph which represents formal logic by the action of an internal agent. Gunji and Higashi (2001) also argued exactly about the relation between directed graphs and category theory (Mac Lane, 1998).

We here make the purpose of an internal agent as the origination of the transitive law of implication. Ordinarily the fundamental property of a logical system is given in the form of an axiom thetically, and the same applies to the transitive law of implication. Instead of this situation, we introduce the transitive law into the formal system as the purpose of an internal agent. This kind of introduction means differentiation or localization of the axiom. A formal system in which a law stands in the whole of the system simultaneously is the system without time for the law. In addition, this introduction enables the system to transform itself continuously, in contrast with the ordinary axiomatic systems which vary discontinuously according to which axioms are adopted.

In Dialogue Model (I), (II), and (III), there are multiple agents which are reciprocally influenced, whereas there is only one agent in Internal Agent Model. The agent in Internal Agent Model is influenced by the system in which the agent exists, instead of the reciprocal influence among agents in Dialogue Model (I), (II), and (III). The agent influences the system as mentioned above, thus the system and agent interact with each other in Internal Agent Model.

In addition, we also argue about logical objects in the process of observing the transition of a directed graph. The object in formal logic is obvious, for example, has the property of the reflexive law: X is X. In contrast with the obviousness, there is a critical problem such as Russell's paradox (Whitehead and Russell, 1925). We present an attempt to solve this problem by introducing the notion "softness" into logical objects. The object with softness is as "natural" as natural language, from the standpoint of the distribution. The object with softness is a soft object as mentioned earlier, and the directed edge (arrow) with softness is called a soft arrow.

While we regard the system as a mere graph out of context, the internal agent is nothing more than a subgraph. That is to say, the interaction between the system and the internal agent is the interaction between a graph and its subgraph. Moreover, from the definition of the purpose of the internal agent, we can regard the model as the independent applications of the transitive law to either the whole or the part of a system. In a similar way, the notion of softness of an object leads to the uncertainty of the reflexive law (the obviousness of the object). In short, we aim to observe the dynamical feature of formal logic in which the fundamental laws are either deprived or partially adopted.

Section 4 is organized as follows: firstly we define an internal agent inside a system. An internal agent differs from a part of the system only in that it has a purpose, that is, an internal agent is nothing more than a mere part of the system which has a purpose. Next, we schematize the purpose of an internal agent, and define the interaction between a system and an internal agent. We observe the emergence of a directed graph which represents formal logic adequately out of the interaction, and look into the results under some various conditions. We also observe some distinctive features of the emergent graph. In order to elaborate these features, we define the notion of softness of both an object and an arrow. And then we check up some results from particular cases in order to discuss the softness of both an object and an arrow, especially the influence of soft arrows on soft objects. At the last we sum up the difference of tendency among the values of some parameters, however in any case, all the emergent graphs can be regarded as formal logic.

In Section 5, we introduce a quantitative argument into formal logic. Quantity is usually represented as quantifiers in formal logic as well as in mathematics, and formal logic with quantifier is called predicate logic. Linear logic (Girard, 1987; Troelstra, 1992) also treats the quantity more directly in a way that the number of symbols represents the quantity. A universal proposition which is denoted by using universal quantifier is composed by induction, and deduction is the inference in the opposite direction. Peirce (1868) formulated deduction and induction in an unusual form. According to Peirce, there is the third inference, which is called “abduction”. We formulate deduction and induction, in addition, also represents abduction in a natural way. More precisely, the introduction of abduction at the same time as deduction and induction is inevitable from the standpoint of the proposed schemes.

In general, a concept generated by induction is on higher level than its components. For example, a pair of a set and its elements in set theory is a typical case. Though to treat things and a collective of the things on the same level leads to Russell’s paradox (Whitehead and Russell, 1925), we can treat both things on different levels without contradiction in real life. A system in which interaction among different levels is allowed is called heterarchical system

(McCulloch, 1945; Jen, 2003). The concept heterarchy is recently applied to some studies (Gunji and Kamiura, 2004; Kamiura and Gunji, 2006; Gunji et al., 2008; Sasai and Gunji, 2008). Heterarchical method is of course an effective tool for description of the non-hierarchical structure, however, we take another approach: we set up a system in which there is only one level and things and a collective of the things coexist on this level. This setting necessitates considering the extent of objects, such as the extent of a collective is "larger" than those of its elements. The extent of an object is set down as the size of a soft object in Monologue Model, and so on. Dynamical logic presented by Gunji et al. (2006) can be also interpreted as a process related to the extent of objects.

We represent this quantitative argument, the formulated inferences, and/or the extent of an object as the transformation of the transformation of a directed graph, which is "expansion" or "contraction" of objects or relations among objects. In general, an object and a relation among objects in a formal system are represented as a "point" and a "line", thus the interior of each component is ignored in principle. In classical propositional logic, the premise that there is an atomic formula which cannot be divided corresponds to the representation of an object by a point. We treat a model of formal logic without the premise of atomic formula by way of consideration of expansion and contraction of logical components. The model is called Mediation of Object-Relation Model. Let us return to the definition of logic described previously, that is, when we consider an experience in real life, is there an atom? This is a motivation for Mediation of Object-Relation Model.

Anticipatory systems (Dubois, 1998) makes it possible to concentrate the duration including the past, present, and future on the present. In other words, it enables us to treat information at multiple time instants as mixed information at one time instant. Meanwhile, consideration of an object with the spatial extent leads to the conservation of information inside the object. Thus Mediation of Object-Relation Model can become a representation of anticipatory systems, while the correspondence between the temporal extent and the spatial one is appropriately introduced.

In fact, the scheme of dialogical interaction of Dialogue Model (I), (II), and (III) can be interpreted in connection with three types of inferences of Peirce. Alternatively, different view of objects among agents in Dialogue Model (II) and (III) can be represented as an object which can expand and contract, introduced in Section 5. Moreover, a generated directed graph by Internal Agent Model in Section 4 is regarded as formal logic whose logical units are soft objects and soft arrows. Soft object is a weakened directed graph kin to complete subgraph. This corresponds to a weakened notion kin to equivalence law in Mathematics. Equivalence law is a fundamental notion in mathematics and a condition in order to treat a set as one unit. Thus in section 6, we discuss and summarize the models and notions proposed in this thesis in association with existing logical or mathematical notions.

Section 2, 3, and 5 are based on our previous papers (Sawa and Gunji, 2007, 2008, in press), respectively. As a whole, these studies are one of the very few applications of the method of complex systems to formal logic.

2. Dialogue

2.1. Dialogue Model (I)

2.1.1. Dialogical interaction

An agent represents an individual that perceives causal relations. Knowledge about causal relations, which each agent has, is represented by a set of arrows between objects corresponding things in real life. Needless to say, a source object of an arrow is an expression of cause; a target object is that of effect. An initial set of arrows is an expression of naive intuitions of each agent. It is independent of other agents, hence it may be an arbitrary set without any constraints.

We assign a real value between 0 and 1 to each arrow. The real value represents likelihood of the causal relation. Hereafter, we call this value intensity of an arrow. The intensity of an arrow is dependent on one agent, two objects and time instant, that is, the intensity of an arrow between given two objects may be different in each agent, and may vary dependent on time.

The intensity of an arrow fluctuates through dialogues between agents. A dialogue occurs only on a one-on-one basis, however, all combinations of two agents are chosen in every units of time. If two agents in a dialogue have a same arrow, each intensity of the arrow of both agents ticks up a little bit. This is the simplest effect through a dialogue. In addition to this, we introduce a scheme of interaction of two agents. The fundamental concept is as follows (see Fig. 1):

> If an intuition of agent A infills a blank in some intuitions of agent B, and a new sequence consists of intuitions of A and B explains another sequence of intuitions of agent B, the arrow of agent A is reinforced by the arrows of agent B.

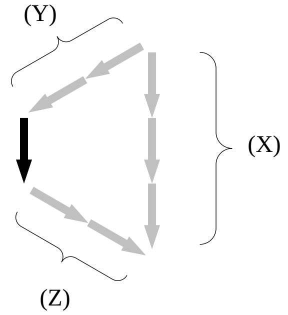


Fig. 1: Scheme of interaction through a dialogue. Only the intensity of the black arrow (the arrow of agent A) increases, the gray arrows (the arrows of agent B) do not vary due to this scheme.

The black arrow that represents an intuition of agent A (the agent influenced through this dialogue) is exactly one arrow in the scheme. On the other hand, the number of the gray arrows of agent B (the influence agent) might be one or more in the part (X), zero or more in the parts (Y) or (Z) respectively. If the number of the arrows is one in (X), and zero in both (Y) and (Z), the scheme accurately corresponds to the simplest effect. If zero in both (Y) and (Z), an arrow of agent A explains a sequence of arrows of agent B. Seen in this light the scheme can be regarded as an extension of the simplest effect through a dialogue.

Thus, each agent makes a judgment about the intuition of others on the basis of only its own intuitions. There is no collective knowledge shared among agents. And agents are reciprocally influenced.

2.1.2. Details of the simulation

We define a model called Dialogue Model (I) as follows. Hereafter, we represent a set of agents by Γ and the discrete time by t. We can see naturally a set of arrows that represents all knowledge of an agent, as a directed graph (Harary, 1969). Each node represents an object. In general, a directed graph can be represented by an adjacency matrix, as a jk element corresponds to a directed edge from a node j to a node k. We express a directed graph of an agent A_m at t as $G(A_m,t)$. And we also express an adjacency matrix representing $G(A_m,t)$ as $I(A_m,t)=\left(i_{jk}(A_m,t)\right)$, where $i_{jk}(A_m,t)$ expresses the intensity of an arrow from a node j to a node k, if an arrow exists. Otherwise, we assign 0 to $i_{jk}(A_m,t)$. We do not argue about an arrow from an object to itself in this model, therefore, all the values of the main diagonal are 0.

Next, we introduce a transformation D of an adjacency matrix, in order to shorten expressions of the multiple dialogues among agents as described above. For a given matrix $I(A_m,t)$, D gives a new matrix $D(I(A_m,t))=\left(d_{jk}(I(A_m,t))\right)$ that represents the potentiality of A_m, for reinforcement of arrows of other agents at t. That is, for each ordered pair of nodes j and k in $G(A_m,t)$,

$$d_{jk}(I(A_m,t))=\begin{cases} p & \text{(if } j\neq k \text{ and two nodes } j' \text{ and } k' \ (j'\neq k') \text{ satisfying the conditions (1),} \\ & \quad \text{(2) and (3) below exist in } G(A_m,t)) \\ 0 & \text{(if } j=k) \\ q & \text{(otherwise),} \end{cases}$$

where p and q are given positive and negative constants, respectively. The conditions are as follows:

(1) A sequence of directed edges from j' to k' exists in $G(A_m,t)$.
(2) A sequence of directed edges from j' to j exists in $G(A_m,t)$, or $j = j'$.
(3) A sequence of directed edges from k to k' exists in $G(A_m,t)$, or $k = k'$.

These conditions correspond to the parts (X), (Y) and (Z) of the scheme of Fig. 1, respectively. Especially, if $j = j'$ and $k = k'$ on the conditions (2), (3), we express the transformation as D' for later analysis. Figure 2 shows examples.

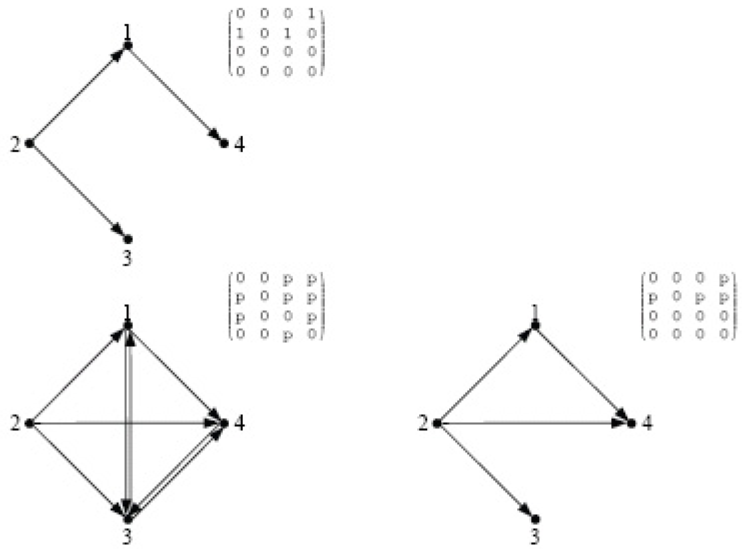


Fig. 2: Examples of the transformation D and D' of $I(A_m,t)$ (upper). $D(I(A_m,t))$ (lower left) and $D'(I(A_m,t))$ (lower right). Here $q = 0$ for viewability.

Since we assign a negative value to $d_{jk}(I(A_m,t))$ if such two nodes j', k' do not exist, the arrows that are not reinforced by other agents have a tendency to decrease their intensities. Thus, a time transition from $I(A_m,t)$ to $I(A_m,t+1)$ is expressed as

$$i_{jk}(A_m,t+1) = \begin{cases} r(i_{jk}(A_m,t) + \sum_{A_{m'} \in \Gamma, m' \neq m} d_{jk}(I(A_{m'},t))) & (i_{jk}(A_m,t) \neq 0) \\ 0 & (i_{jk}(A_m,t) = 0), \end{cases}$$

where

$$r(x) = \begin{cases} 1 & (x \geq 1) \\ x & (0 < x < 1) \\ 0 & (x \leq 0). \end{cases}$$

Figure 3 shows an example of the time transitions of $I(A_m, t)$ of ten agents.

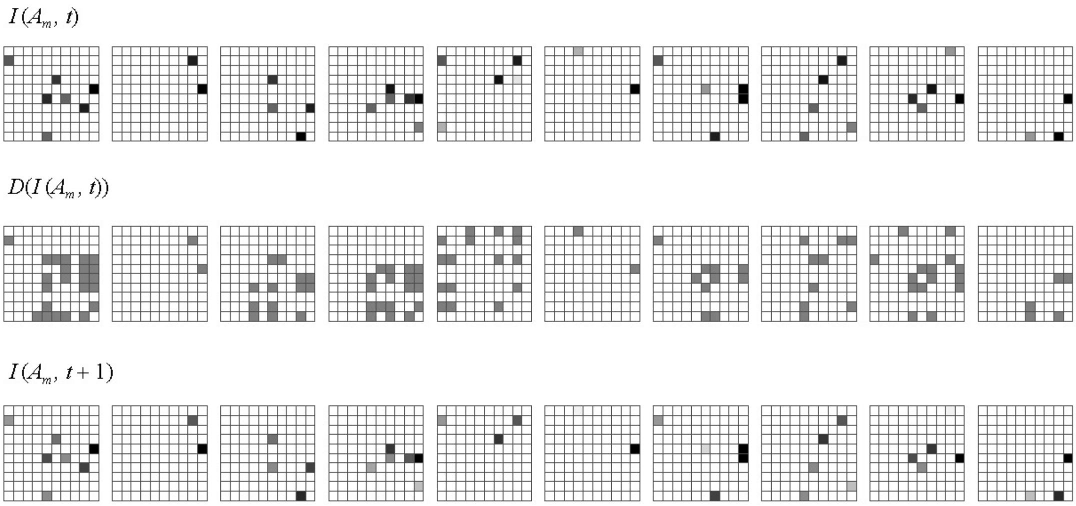


Fig. 3: Dialogues among 10 agents. Each block corresponds to one matrix, and the nonnegative values are represented by grayscale. The smaller values are shown lighter. The negative values are not indicated in $D(I(A_m, t))$ for viewability. Each $D(I(A_m, t))$ is determined only by $I(A_m, t)$. Each $I(A_m, t+1)$ is calculated from $I(A_m, t)$, and all $D(I(A_{m'}, t))$ where $m' \neq m$. The intensities of arrows fluctuate through dialogues, and as a consequence, the arrow from ninth node to first node of fifth agent disappears for instance.

Finally, we define an adjacency matrix of Γ at t as

$$I(\Gamma, t) = \sum_{A_m \in \Gamma} I(A_m, t),$$

and a directed graph represented by $I(\Gamma, t)$ as $G(\Gamma, t)$.

2.1.3. Evaluation method and results

As mentioned previously, we are concerned with the transitive law of the causality:

$$j \to k \text{ and } k \to l \text{ imply } j \to l.$$

Since a directed graph of each agent $G(A_m,t)$ represents merely its intuitions, it is not necessarily the case that the transitive law holds in $G(A_m,t)$. The same applies to a graph $G(\Gamma,t)$. Hence we introduce TR (transitivity rate) of a directed graph G for an index to show the emergence of the transitive law, as follows:

Definition 2.1.1 (*Transitivity rate*). Given a directed graph *G*, TR is defined as

$$\text{TR} := |G| \ / \ |D'(G)| \ ,$$

where $|G|$ is the number of directed edges in G, $D'(G)$ expresses a graph that is transformed from G with D'.

As is clear from Fig. 2, D' transforms a former directed graph into a directed graph in which the transitive law holds completely, by adding requisite arrows. Therefore, it is obvious by definition that the value of TR is 1 if the transitive law holds in the whole of the directed graph. And as the value of TR approaches 1, the part in which the transitive law is satisfied broadens in the directed graph. Figure 4 shows the relation between TR and the number of directed edges distributed randomly in a directed graph that consists of 10 nodes. The probability that two directed edges are allocated as they are adjacent (to be accurately, adjacent as an initial node of one is a terminal node of another) decreases, therefore, TR approaches 1 at the left part of the graph. Similarly at the right, because the numerator $|G|$ approaches the maximum value of 90 and the denominator $|D'(G)|$ constantly takes the value of 90 when a directed graph contains the certain number of directed edges.

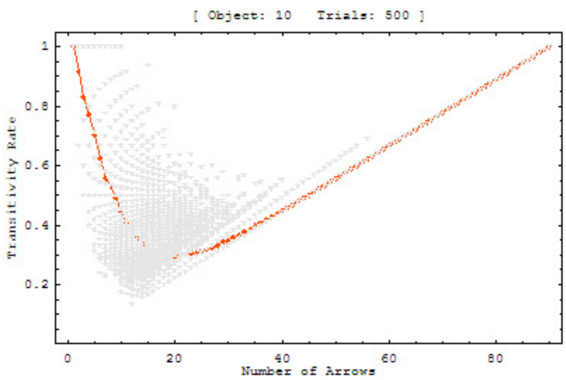


Fig. 4: Relation between TR and the number of directed edges. The gray dots are the values of each trial. The curve line represents the average.

If we observe that some directed graph has a relatively high value of TR, we consider that the transitive law of the causality emerges in what the directed graph represents.

Figure 5 and Table 1 show the results of 100 trials under the conditions that are as follows: 10 agents, 10 objects, the number of iterations of dialogues $t_{\omega}=10$, the initial intensity of arrows 0.5, and the probability of the emergence of an initial arrow 0.1. The values of p, q are given variously as indicated below.

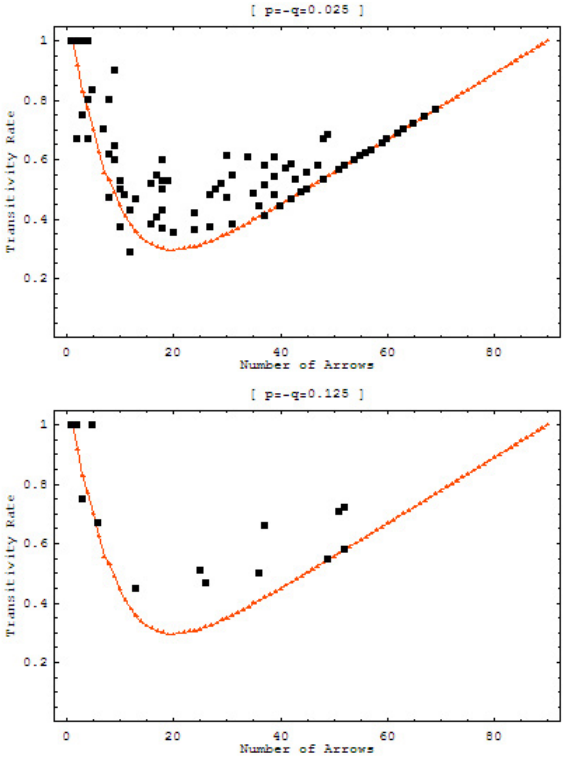


Fig. 5: TR and the number of directed edges of $G(\Gamma, t_{\omega})$. $p=-q=0.025$ (upper) and $p=-q=0.125$ (lower). Most of the trials under both conditions demonstrate higher values of TR.

Table 1: The relation between p, q and the distribution

p $(=-q)$	The number of trials that do not collapse	The number of trials whose TR is smaller than the average
0.025	98	6
0.050	65	7
0.075	52	5
0.100	32	1
0.125	23	1

If the intensities of all arrows of all agents are 0 at t_ω, namely, $I(\Gamma,t_\omega)$ is zero matrix in a trial, we say that this trial collapses. We regard a collapsing trial as dialogues that agents cannot draw a conclusion from. Inevitably the collapsing trials are not plotted in Fig. 5. Many trials that do not collapse demonstrate higher values of TR than the average regardless of the values of p, q. It can be concluded from these results that the transitive law emerges while agents can draw a conclusion from dialogues. $G(\Gamma,0)$, that represents the initial state of knowledge of a set of agents, is simple union graph of each $G(A_m,0)$. Therefore, there are many directed edges (58.9 on average) in $G(\Gamma,0)$, and all TRs of them are on the curve line of the average. In sum, TRs are initially on the right part of the curve line, however, deviate upwards from the curve line as time proceeds, by virtue of the interactions.

2.2. Identification of objects

2.2.1. Object and arrow

We observed that knowledge of agents is rooted with the transitive law through only dialogues among agents in the simulations based on Dialogue Model (I). Dialogue Model (I) however, is slightly inappropriate for dialogues in real life, as follows. In Dialogue Model (I), all agents have exactly the same perspective of objects, so that means, for example, the number of objects that the agents are concerned with, or the perception of objects. That is to say, all the agents view a dog as a dog, a cat as a cat. Compared to this, we consider that we have a dialogue without clear definitions of things in real life. In addition, there is a case that we cannot supply a definition for objects. Or, we might talk to each other with discrepancy in perception. For example, one can distinguish between a dog and a cat, though another cannot distinguish and views both as nothing more than a “four-footed animal chummy with human”. Even if we are in these circumstances, we reach a conclusion in some cases. Or rather, the dialogues that we have in real life do not premise exactly the same perspective.

Moreover, in this study, considering only in the model premised on exactly the same perspective is close to petitio principii. To give exactly the same perspective to agents is nearly to premise the complete description of the world. And the observation of the transitive law of the causality may not be so difficult if we can completely describe the world. Our aim is the observation that dialogues are the field of the generation, or the generators itself of the transitive law. It is difficult to distinguish between a model on too strong premises and a mere axiomatic system.

Based on the above understanding, we add a new concept to Dialogue Model (I). Here, we call this new concept identification of objects. If there is a cycle of directed edges in each $G(A_m,t)$, all the objects corresponding to the nodes in the cycle are identified from the standpoint of causal relation. What that means is that, if there are a cycle and a directed edge whose initial (terminal) node is in the cycle, we add new directed edges from every node in the cycle to the terminal node of the former directed edge (from initial node of the former directed edge to every node in the cycle). In other words, a cycle of arrows itself is regarded as an object. Figure 6 shows a transformation of a directed graph by identification of objects.

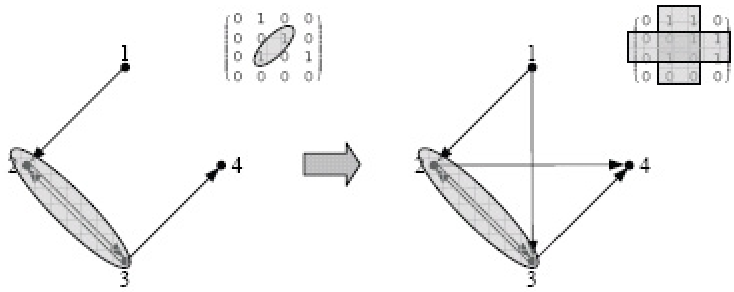


Fig. 6: Due to identifying the second object (left) with the third object (lower), the second row (column) and the third row (column) become equivalent except the main diagonal.

If a cycle is dissolved due to disappearance of arrows as time passes, it follows that the objects in the former cycle are not identified from then on. With that, the objects and the arrows hidden inside the cycle make an appearance. So to speak, concepts are reconsidered, and obviousness is deprived.

2.2.2. Speaker and listener

Dialogues obviously need two individuals: a speaker and a listener (cf. Nowak et al., 1999; Oliphant, 1996). Accordingly, we must consider two applications of the identification of objects to speakers or listeners, respectively. The applications in the simulations are illustrated by an example in Fig. 7.

As is clear from Fig. 7, an application to a speaker reduces the number of directed edges in the graph represented by $D(I(A_m,t))$. As a consequence, the chances of the reinforcement decrease. In contrast, an application to a listener increases the chances of the reinforcement. We regard Dialogue Model (I) with the identification of objects as a variant model. We can choose whether or not to apply the identification of objects, and also between to speakers and to listeners, therefore, there are four variant models including Dialogue Model (I). We call variant models Dialogue Models (II), and compare the results of Dialogue Models (II) in the next section.

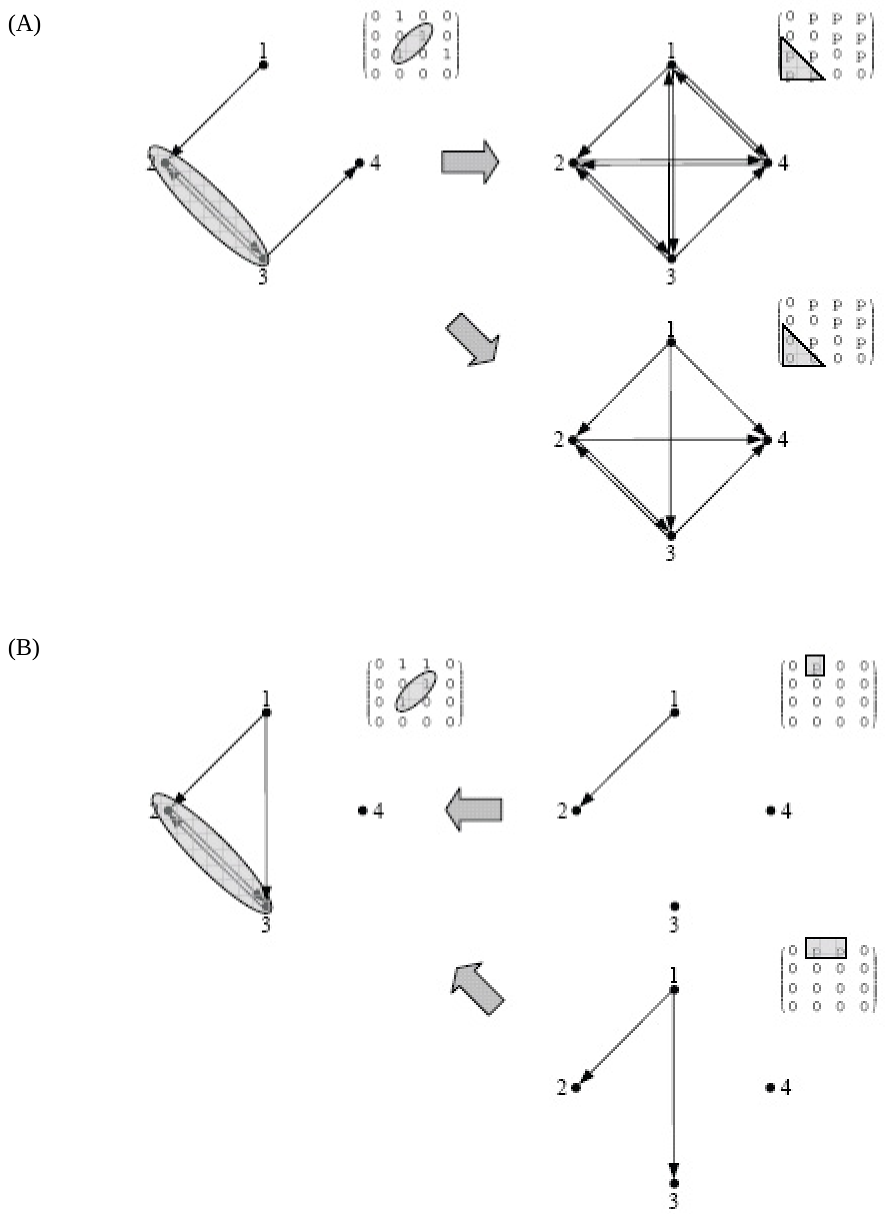


Fig. 7: Influence of the identification of objects. $q = 0$. (A) Application to a speaker. For $I(A_m, t)$ (left), $D(I(A_m, t))$ with the identification (upper right) and $D(I(A_m, t))$ without the identification (lower right). The part indicated by the gray triangle is the difference. (B) Application to a listener. The agent A_m that has the knowledge depicted by left diagram, interprets $D(I(A_{m'}, t))$ depicted by upper right diagram as lower right diagram due to its own identification.

2.3. Dialogue Model (II)

Figure 8 shows the differences of the results among Dialogue Models (II). All models begin with same allocations of initial arrows.

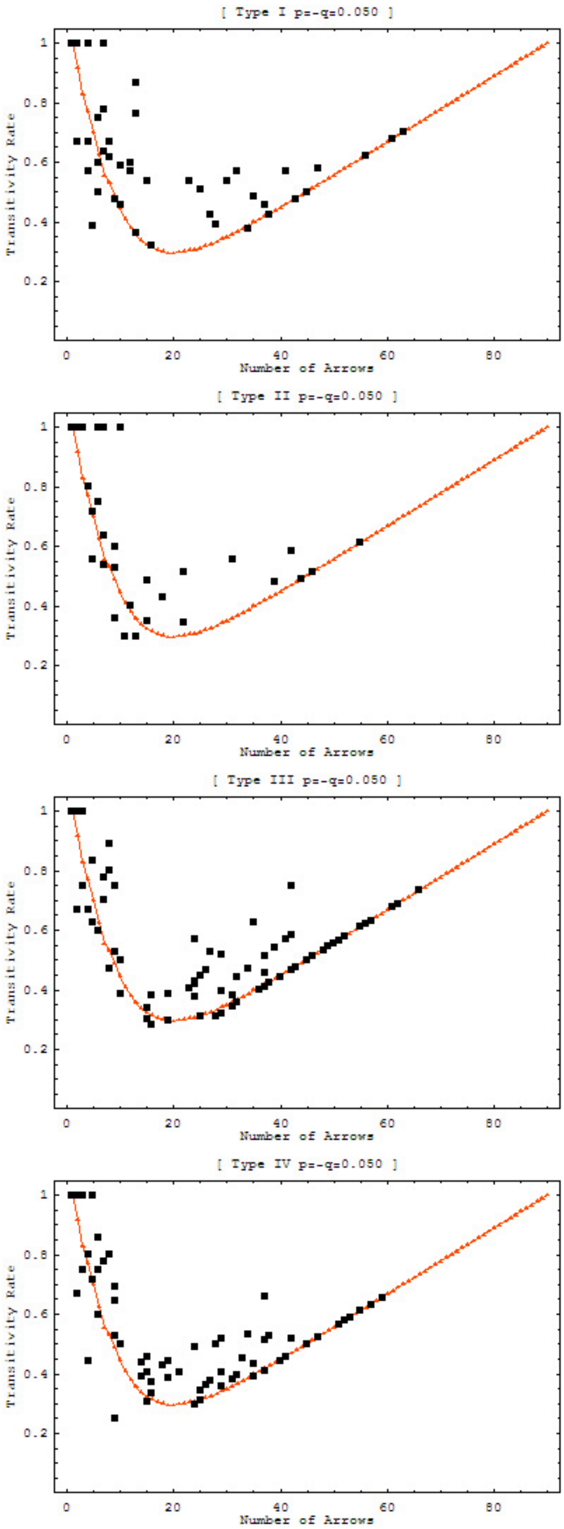


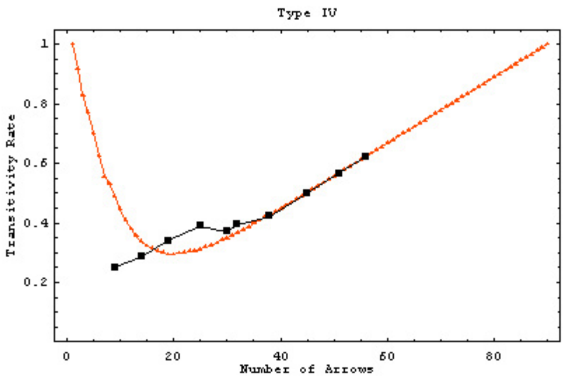
Fig. 8: 10 agents, 10 objects, $p = 0.05$, $q = -0.05$, $t_{\omega} = 10$, the initial intensity of arrows 0.5, and the probability of the emergence of an initial arrow 0.1. 100 trials. The numbers of the trials that do not collapse are 61, 48, 85 and 80 in order.

The model without the applications of the identification of objects to both speakers and listeners (Type I) is indeed equivalent to Dialogue Model (I). As shown in the preceding section, the applications to speakers or listeners lead to decrease or increase of the chances of the reinforcement, respectively. It follows that the number of directed edges in $G(\Gamma, t)$ decreases rapidly or slowly as time proceeds. Therefore, the trials of the model with the application only to speakers (Type II) are distributed leftward and upward in comparison with Type I. And there are more collapsing trials in Type II than in Type I. On the contrary, the trials are distributed rightward in the model with the application to only listeners (Type III). The model with the application to both (Type IV) shows an intermediate tendency between Type II and Type III. And the values of TR remain lower in Type IV than in Type I. At any rate, most importantly, many trials demonstrate higher values of TR, hence the identification of objects cannot have an influence as it disturbs the emergence of the transitive law in every model.

Next, as a peculiar case, we show a trial in Type IV, that has a low value of TR. Figure 9 shows the shift of TR and the time transitions of each $I(A_m, t)$.

It is clear from the time transitions of Fig. 9, that most arrows of $I(\Gamma, t_\omega)$ are the arrows of one agent A_7. In $G(A_7, t)$, five objects are identified at every time instant. This trial is, so to speak, a case in which, the agent that has a peculiar view of the world construes the opinions of others just its style, and retains its own opinion, while the opinions themselves of the others disappear. This is an exceptional case however. Indeed, there are several trials in which five objects are identified, and the values of TR are not low.

(A)



(B)

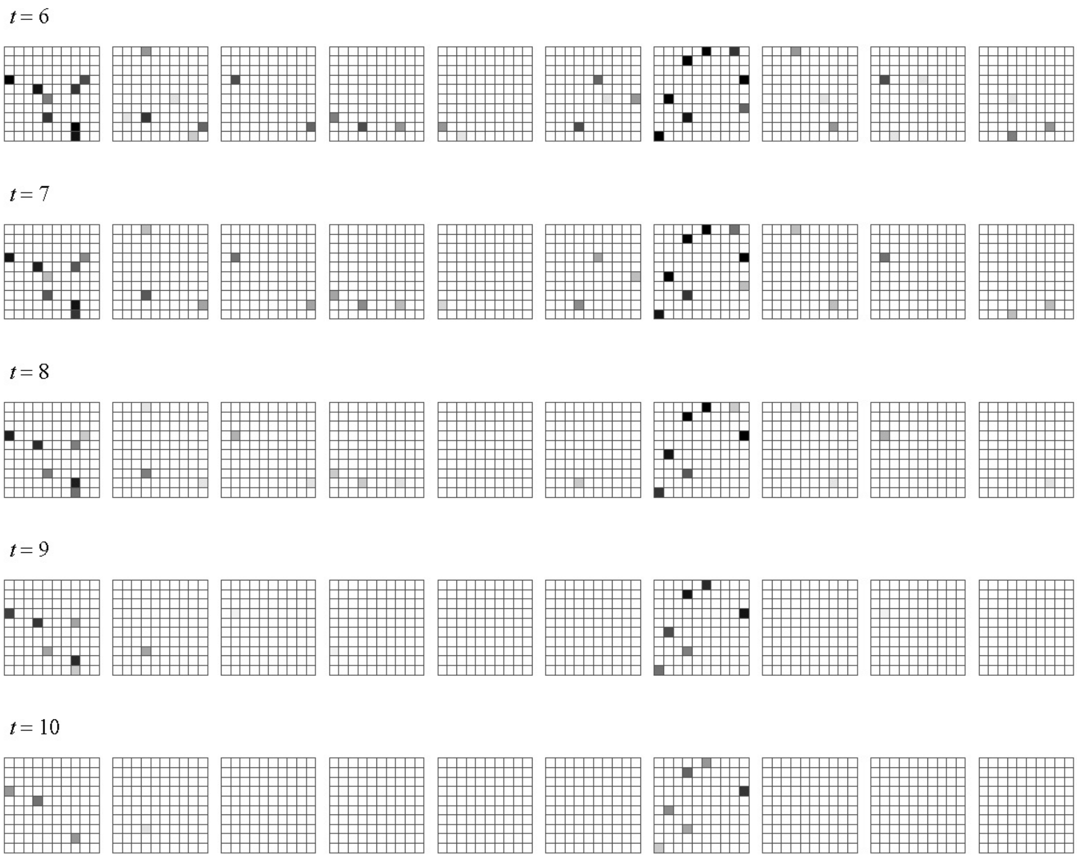


Fig. 9: (A) The shift of TR. The number of arrows monotonously decreases in every trial, hence the dot moves from right to left as time passes. (B) The time transitions of each $I(A_m, t)$.

3. Monologue and Dialogue

3.1. Soft object, and measures of the consistency of the world

In order to introduce the variable object as described in Section 1 into the scheme of causal relations, we prepare and define as below.

In an ordinary formal system, an object has the property of the identity, i.e. $X \to X$. If a cycle of causal relations (e.g. $X \to Y$, $Y \to Z$, and $Z \to X$) exists, there are causal relations between two arbitrary objects in the cycle. Therefore, we can consider that those objects are identified, in other words, the cycle itself can be regarded as one object. We call the set regarded as one unit, a soft object (see Fig. 10).

Definition 3.1.1 (*Soft object*). In a given directed graph, we call a set of nodes which has the following property a soft object: the set consists of at least 2 nodes, and there is at least one sequence of directed edges in the same direction between every ordered pair of two nodes of the set. Moreover, a node which is not the component of any soft objects composed of multiple nodes (i.e. a “singleton”) is also called a soft object.

In a soft object consisting of *n* nodes, there are at least *n* directed edges, $n(n-1)$ at a maximum. We regard the density of directed edges as softness, since the transitive law does not necessarily hold in the scheme, and the number of ordered pair of nodes that are connected directly increases in proportion to the density.

(A) (B)

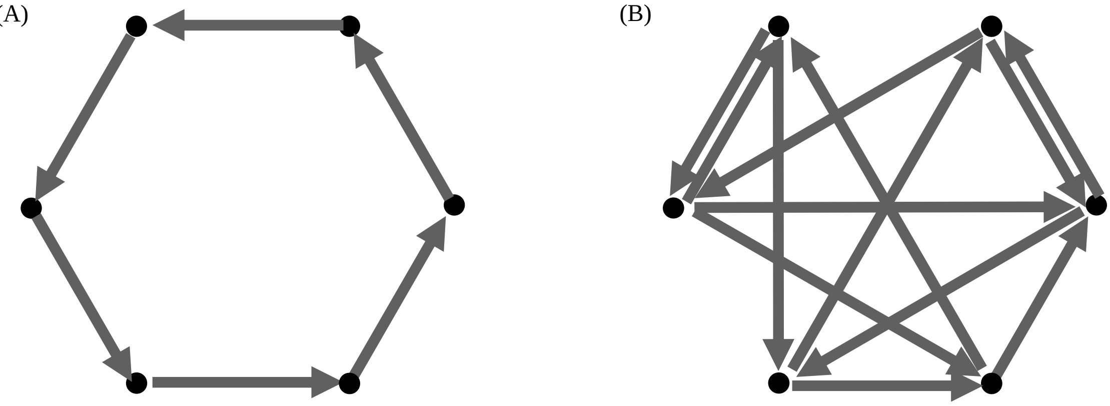


Fig. 10: Examples of soft object. (A) If one arbitrary arrow is removed, the soft object breaks into 6 soft objects (singletons), and 5 arrows among them. (B) Removing an arbitrary arrow cannot break up the soft object into smaller ones. Soft object of (A) is “softer” than one of (B).

Definition 3.1.2 (*Softness rate of a soft object*). Given a soft object *s* consisting of n_s nodes, the softness of the soft object is defined as

$$\mathrm{SR}(s) \coloneqq \begin{cases} 1 & (n_s = 1) \\ |s| \;/\; (n_s(n_s - 1)) & (n_s \geq 2) \end{cases} ,$$

where |*s*| is the number of directed edges in *s*.

Definition 3.1.3 (*Softness of a directed graph*). Given a directed graph *G* consisting of *n* nodes, the softness of the whole graph is defined as

$$\mathrm{SR} \coloneqq \sum_{s \in G} n_s \mathrm{SR}(s) \;/\; n ,$$

where n_s is the number of nodes of soft object *s*.

Note that the denominator of SR(*s*) is the number of directed edges in a complete graph consisting of n_s nodes. SR is weighted average of all SR(*s*), using the number of nodes in each soft object as the weighting factor.

SR is a measure of the consistency of the world in the light of respective things themselves. On the other hand, TR (transitivity rate) defined in Section 2 is a measure of the consistency from the standpoint of relations among things. Moreover in this section, we define a measure of the transitivity among soft objects.

Definition 3.1.4 (*Graph induced by soft object*). Given a directed graph *G*, we define a new graph G_s as follows: nodes, all soft objects in *G*; directed edges, ordered pairs of soft objects such that there exist one or more directed edges from nodes of one soft object to those of another in *G*.

Definition 3.1.5 (*Transitivity rate among soft objects*). Given a directed graph *G*, STR is defined as

$$\mathrm{STR} \coloneqq |G_s| \;/\; |G_s'| ,$$

where $|G_s|$ is the number of directed edges in G_s, and G_s' is the graph transformed from G_s, in which the transitive law holds completely by adding requisite directed edges.

For instance, the value of TR, SR, and STR of the directed graph in Fig. 11 are 0.79, 0.39, and 0.80, respectively.

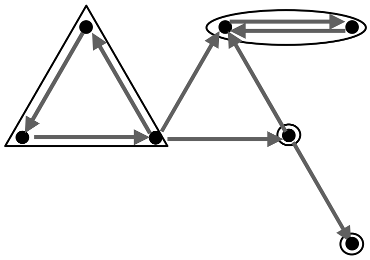


Fig. 11: An example. There are four soft objects, which are framed by a triangle, an oval, and circles respectively.

Definition 3.1.6 (*Inner-arrow and Inter-arrow*). Given a directed graph, we call an arrow (directed edge) in a soft object, an inner-arrow. Meanwhile, an arrow between two soft objects is called an inter-arrow.

If an agent intends to improve the consistency of the description in soft objects, the agent is faced with the choice between such two consistencies (SR and STR) that do not necessarily consist together. Roughly speaking, adding an inner-arrow is related to the increase in SR; adding an inter-arrow invokes the increase in STR. In this regard, however, addition of an inter-arrow makes a new soft object in some cases, and it leads to the reconsideration of the view of the world. Figure 12 shows an instance.

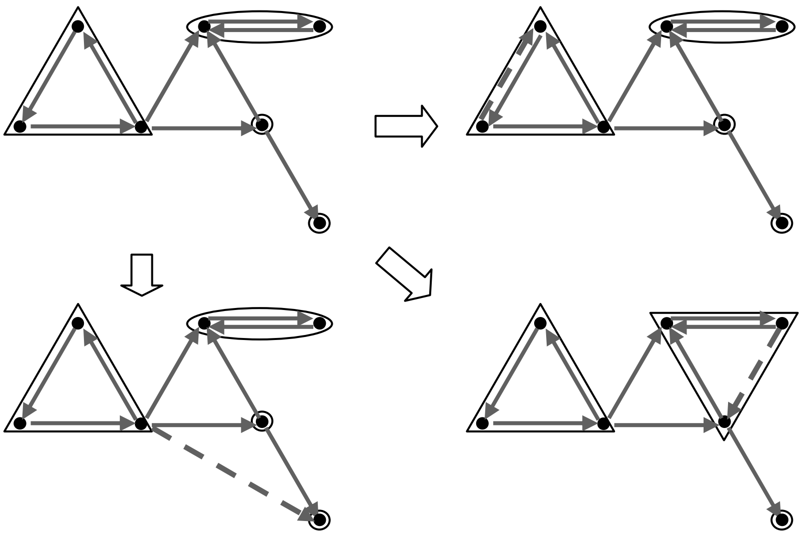


Fig. 12: Examples of adding an inner- or inter-arrow (dashed line). For a given directed graph (upper left), if an inner-arrow is added, SR and TR increase (upper right). Adding an inter-arrow, STR and TR increase (lower left). On the contrary, a new soft object is composed and the values of all measures decrease (lower right).

3.2. Monologue Model

Due to two partially incompatible consistencies, the improvement of knowledge described in soft objects is in diverse ways, as seen in the example of Fig. 12. Thus we classify the way of the improvement of knowledge represented by a directed graph as follows.

Definition 3.2.1 (*Classification of improvement of a directed graph*). When we transform a directed graph as its TR increases or does not change, we call this way of transformation TR-oriented way. Same applies to SR, or STR.

We regard the transition of a directed graph as the monologue of an agent, and call the model Monologue Model. Here we compare monologues caused by the various ways of improvement. A directed graph, in which some soft objects consisting of multiple nodes exist, is added one directed edge, and then one directed edge which is randomly selected is removed in every units of time. The number of consecutive transitions is 200. See Table 2 and Fig. 13.

In a monologue caused by the SR-oriented way, a soft object tends to split into smaller ones. Therefore the number of soft objects increases. The tendency toward the split can be explained from the number of a complete graph, which is the denominator of SR(*s*). Similarly, the number of soft objects increases, if TR-oriented way is adopted. A cycle of arrows need more arrows than a sequence of the same number of arrows in order that the value of TR is 1. As it were, the “cost” for keeping a cycle is high, hence the cycle tends to be dissolved.

In contrast, a soft object has a tendency to be enlarged by the STR-oriented way. Once a new soft object is made synthetically from two soft objects due to adding an arrow between them, thereafter inner-arrows can be added inside the newly made soft object as STR does not change. Hence it is sometimes difficult that the new soft object split apart again. The cost for keeping soft objects may be kept to a minimum, therefore, the relations between soft objects can be described adequately, even if the total number of arrows stays constant.

Table 2: Results by the various ways of improvement, starting with a same directed graph consisting of 20 nodes. Each value is the average. Random way makes one soft object consisting of all 20 nodes, hence the value of STR is indeterminate.

	Random	TR-oriented	SR-oriented	STR-oriented
TR	0.32	0.86	0.54	0.33
SR	0.77	0.99	1.00	0.77
STR	Indeterminate	0.87	0.65	0.90
Number of soft objects	14.02	19.62	18.43	14.13

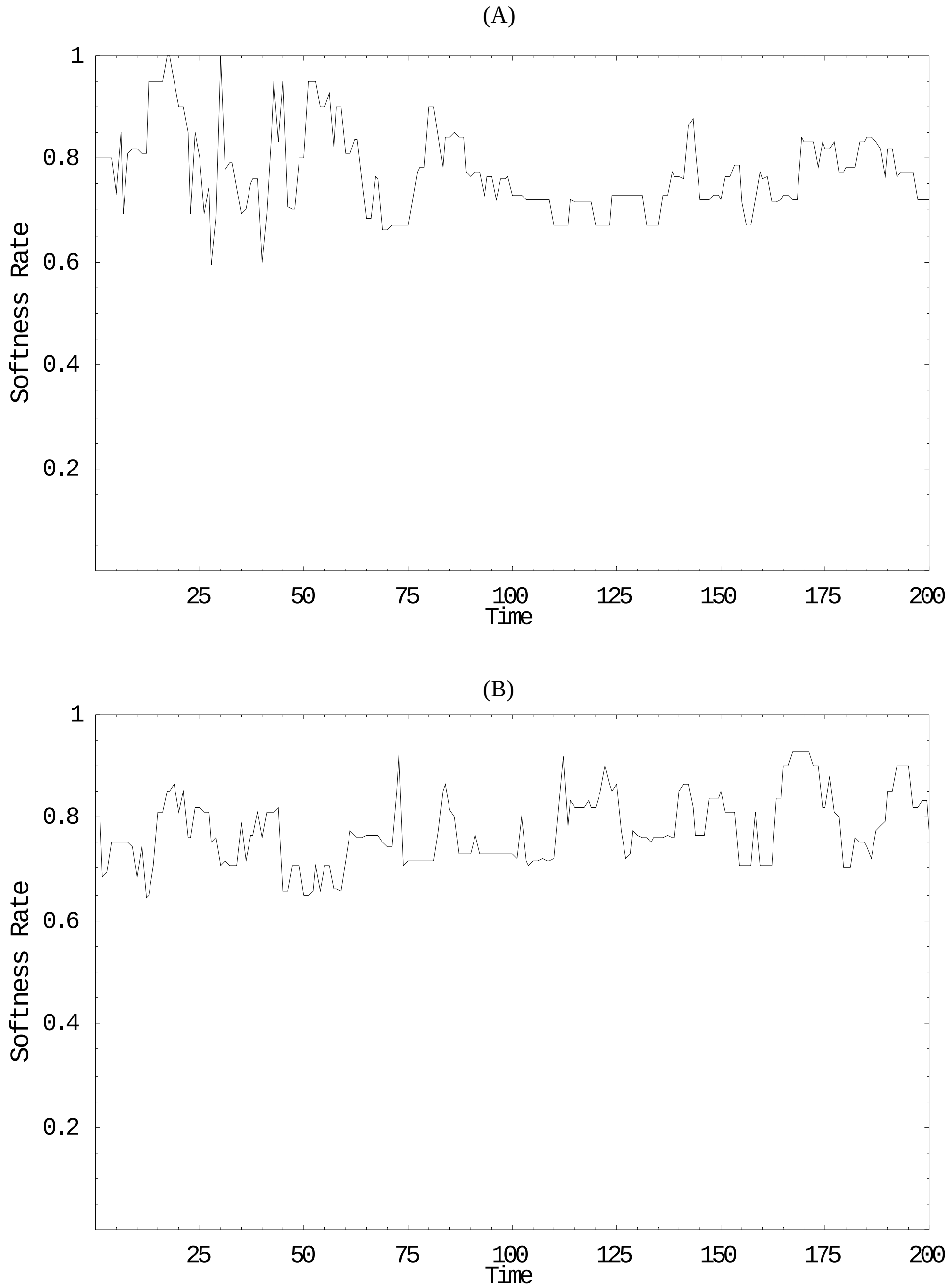


Fig. 13: Time transitions of SR by Random way (A), by the STR-oriented way (B). Monologue by the STR-oriented way can also keep average value of SR.

Thus if we adopt the STR-oriented way, soft objects can keep their size moderately by the assistance of random disappearance of arrows. Consequently arrows accumulated inside soft objects can be exposed afterward in quality of hidden knowledge. In addition, given a directed graph, there are more occasions that increase the value of STR than TR. The value of TR can increase if the added arrow is a requisite arrow (the arrow from an initial node to a terminal node of a sequence of arrows). On the other hand, STR can also increase by virtue of the appearance of a new soft object, corresponding to reconsideration of the view of the world.

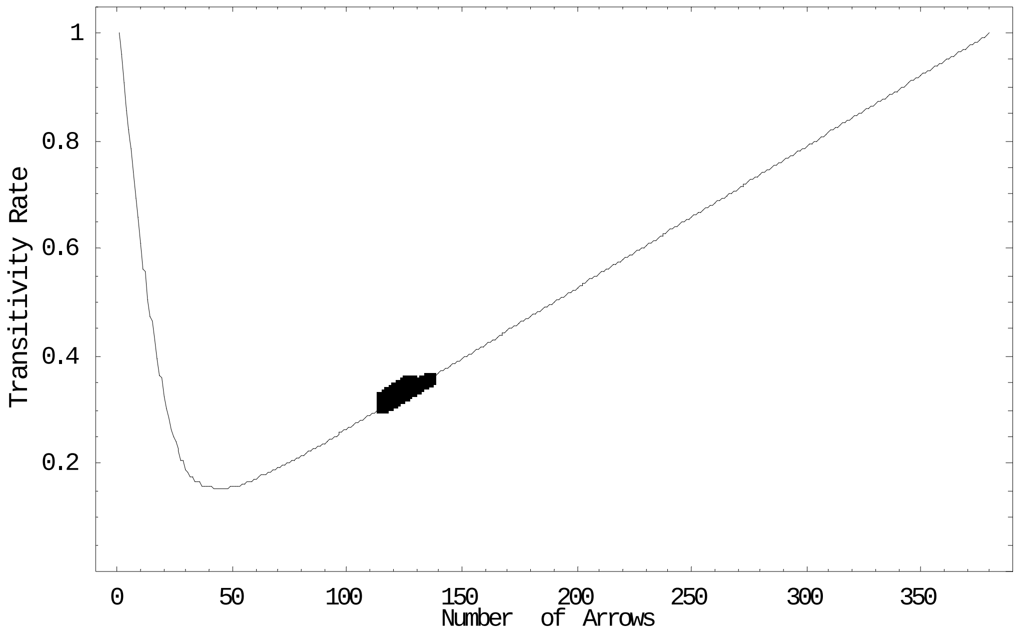
3.3. Dialogue Model (III)

Here we consider about the influence of soft object upon dialogue. We implement a concept of soft object on Dialogue Model (II). We call the model Dialogue Model (III). In essence, it is impossible for agents to share exactly the same view of the world without previous agreement. Hence it is natural that we introduce the soft object when we consider about dialogue among agents. In the model, the arrow of one agent which is not reinforced by the arrows of the other agents disappears. We here show the difference of results by the TR-oriented and the STR-oriented way, respectively. See Fig. 14.

Each dot represents the values of the union of graphs of all agents at each time instant. The curve line represents the average. By virtue of STR-oriented improvement of each agent, TR deviates upwards from the curve line as time proceeds. This indicates an emergence of the consistent description of the world, if STR-oriented way is adopted.

This is caused by the difference of the number of occasions between TR-oriented and STR-oriented way, as stated in Section 3.2. In these simulations, if there are no occasions that increase the value of each measure, a randomly selected arrow is added in order to hold the number of arrows constant. There are fewer choices by the TR-oriented way than the STR-oriented way, therefore, the dialogue by the TR-oriented way tends to fall into the average.

(A)



(B)

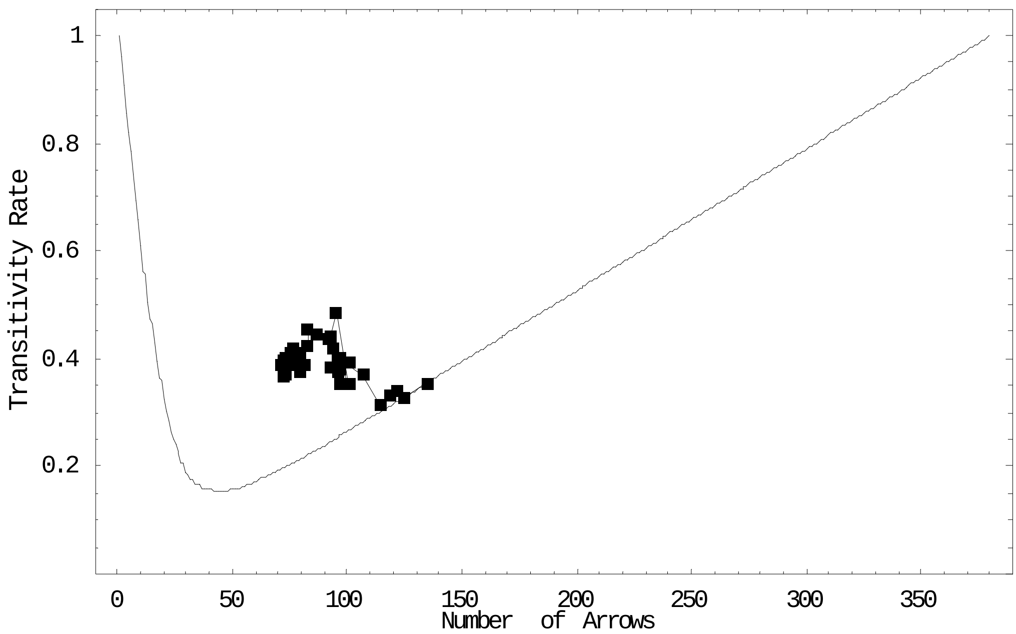


Fig. 14: (A) Results by the TR-oriented way. (B) By the STR-oriented way. The initial directed graphs which represent knowledge of agents are same in both trials, however, the consistent description emerges due to the STR-oriented improvement of each agent.

4. Internal agent

4.1. Internal Agent Model

4.1.1. System and internal agent

In an ordinary multi-agent model, an agent exists independently outside of the world which is represented by the whole of the system. That is, the agent is an observer and the world is the observable. There is a rigid distinction between them. However, we consider that the externality of an agent is a mere postulate. The agent obviously requires things of the world, which it thinks about or treats. The knowledge which the agent has consists of the components of the world, hence we can regard an aggregate of the components as an agent itself. Thus we set out an agent inside the world. For instance, when the world is represented by a directed graph, we regard a particular subgraph as an agent. Figure 15 shows an example. Due to this setting, we can treat an agent and objects which are observable things of the world on the same level. We are in state of denial of discrimination of an agent from a system in order to describe completely independent transitions of a system. In addition, an agent becomes nothing more than an object which can observe from the standpoint of internal measurement (Matsuno, 1989). An agent as an object can be naturally influenced by a system. Therefore, there may be interaction between an agent and a system. Now we call such a part of a system an internal agent. We sometimes abbreviate internal agent to agent hereafter.

Another main characteristic of an agent is its autonomy. In general, agents are treated as if agents consider autonomously in a system. The autonomous behavior of an agent requires a guiding principle which is inherent in the agent and can vary according to circumstances, though it may not be seen. We call the guiding principle a purpose. The system which is the outside of the agent cannot concern the purpose of the agent by definition. Indeed we give a purpose to a part of a system and regard the part as an internal agent, and the purpose is independent of the system.

Based on the above understanding, we define an internal agent as an object of world which has purpose.

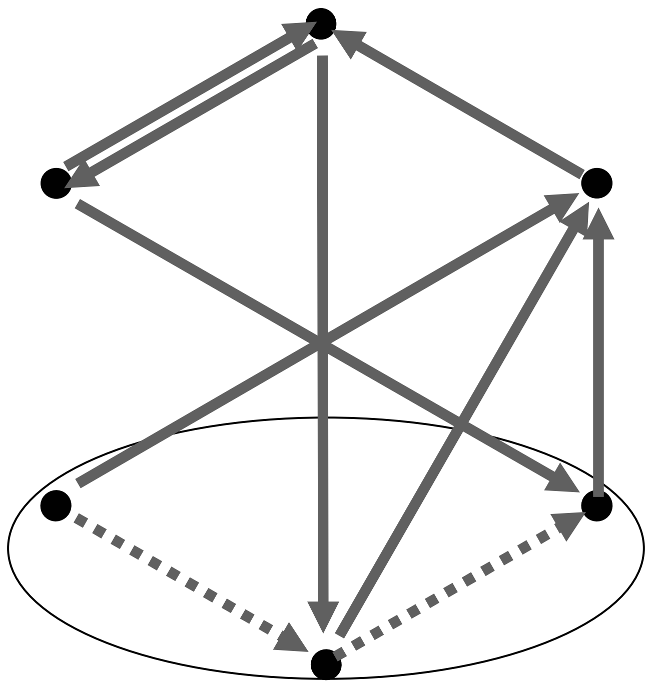


Fig. 15: An example of a system and an internal agent. While the whole directed graph represents a system, an internal agent is the part of the graph represented by dashed arrows.

4.1.2. Purpose of agent

As mentioned previously, an arbitrary directed graph does not necessarily hold all the properties of formal logic. Hence not every directed graph represents adequately formal logic. We pay notice to the transitive law in this section as well as in previous sections. Note that transitivity rate (TR) is one of measures of reliability of a directed graph as formal logic. Here, the increase of TR is defined as the purpose of an agent.

4.1.3. Interaction between system and internal agent

The internal agent influences the system through pursuit of its purpose. To be more precise, the arrow satisfying the conditions as below can be added to the system. Note that the arrow is added not to the internal agent but to the system. The conditions for adding a new arrow at certain time are set up as follows:

(i) The arrow can increase TR of the agent if it exists.
(ii) It does not exist in the system at the time (by definition, it inevitably does not exist in the agent).
(iii) It shares at least one node with arrows of the agent.

On the other hand, the agent is a mere object in the system, hence there may also be the influence of a system on an agent as well as ordinary objects. We also set up the influence of the system on the agent. The detail conditions are similar to the influence in the opposite direction:

(i’) The arrow can increase TR of the system if it exists.
(ii’) It does not exist in the agent at the time.
(iii’) It shares at least one node with arrows of the agent.

In this way, we introduce interaction between the system and agent in the model. We call this interaction S-IA interaction. The system and agent influence each other alternately, and a couple of influences in both directions conduct at each time instant. Figure 16 shows an example of the transitions by S-IA interaction. The added arrow is randomly chosen in each case. If the finite number of searches of an arrow satisfying the conditions is conducted though the arrow is not found, the influence in that direction is skipped so that no arrows are added. Thus the number of arrows in the system increases monotonically as time proceeds, and the same applies to the agent. The maximum number of arrows is obviously n (n-1) in the directed graph which consists of n nodes, actually the transitions is halted at lower number of arrows in almost every case as we present in Section 4.2. We call this model Internal Agent Model.

4.2. Results

The initial graph of the system is given at random, and the arrows of the graph of the agent are picked up from the graph of the system at random likewise. Note that the graph of the agent is a subgraph of the system. Figure 17 and 18 show the results under the conditions that are as follows: 50 nodes, the number of iterations of interactions 1500, the rate of the number of arrows of the initial system to the number of all possible arrows 0.02, and the rate of the numbers of arrows of the initial agent to the number of arrows of the initial system 0.5.

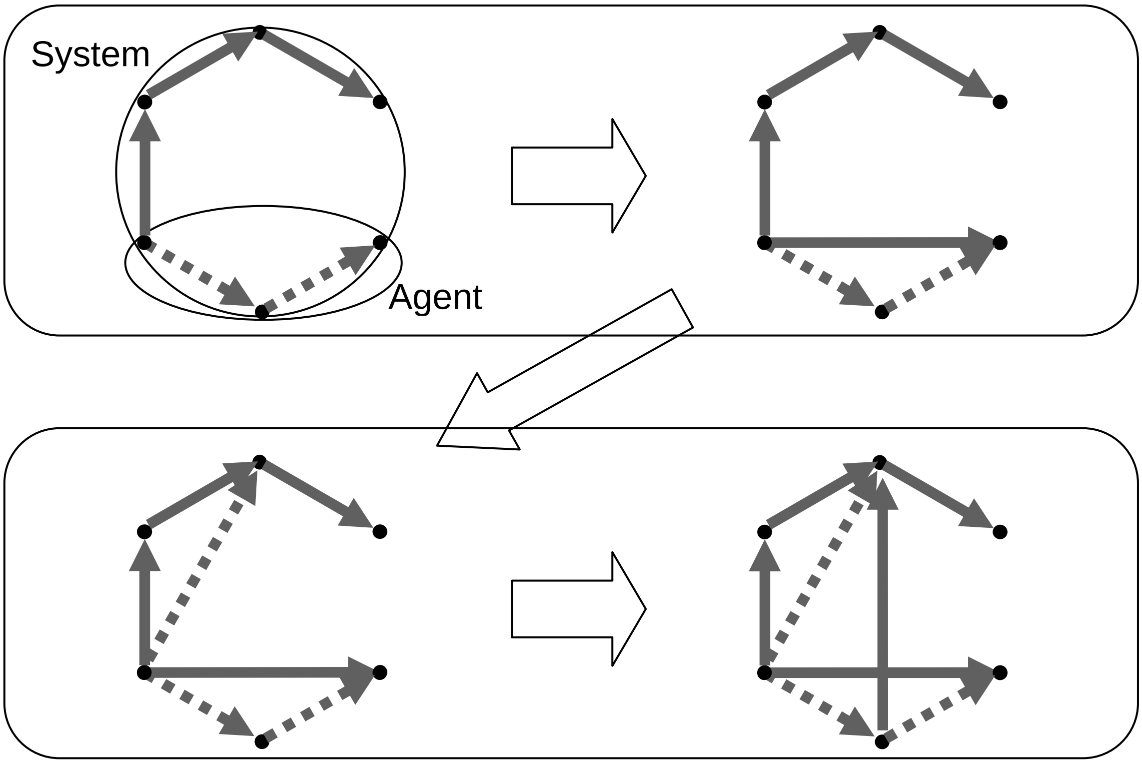


Fig. 16: An example of time transitions by S-IA interaction. Dashed arrows represent the agent and all arrows (solid and dashed arrows) represent the system.

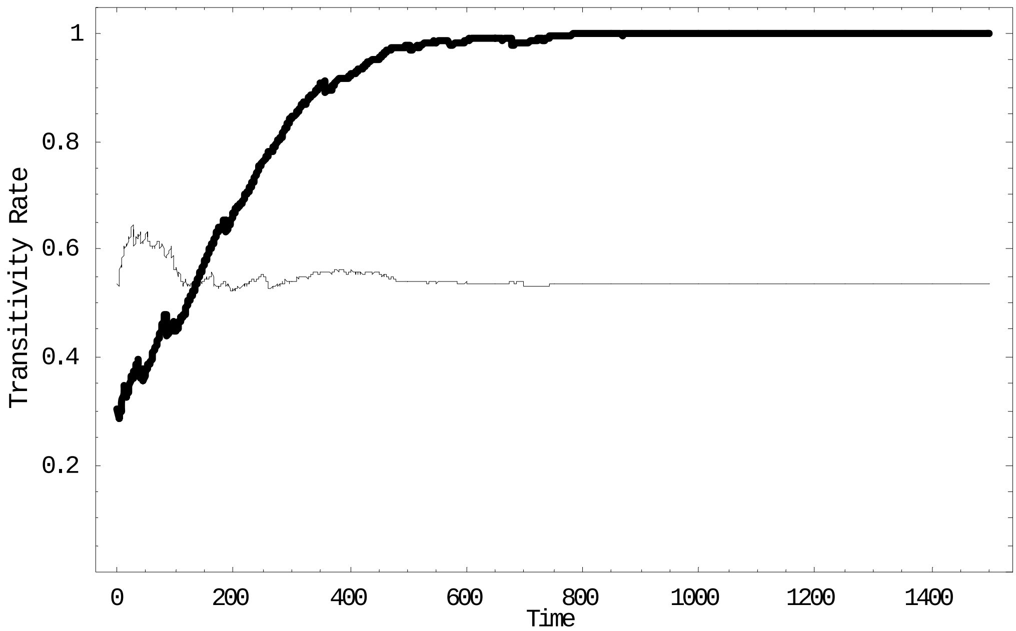


Fig. 17: Time transitions of TRs of a system (bold line) and an agent (solid line). Only TR of the system converges at 1.

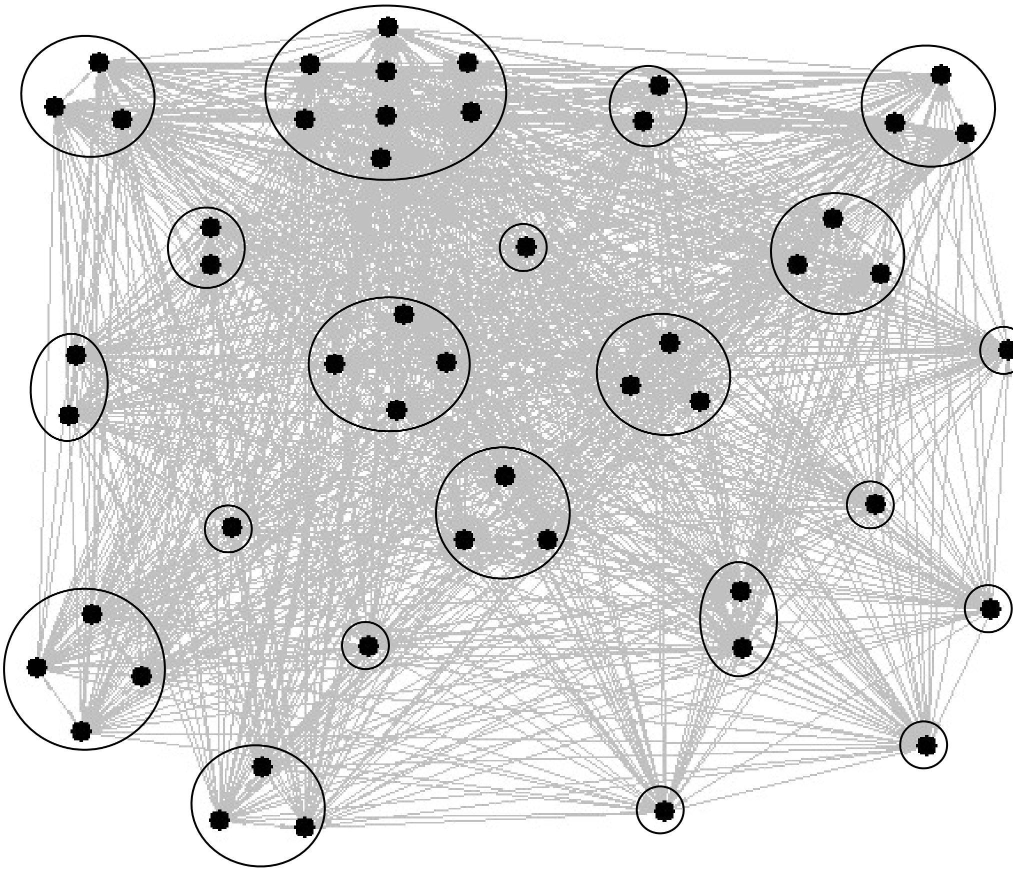


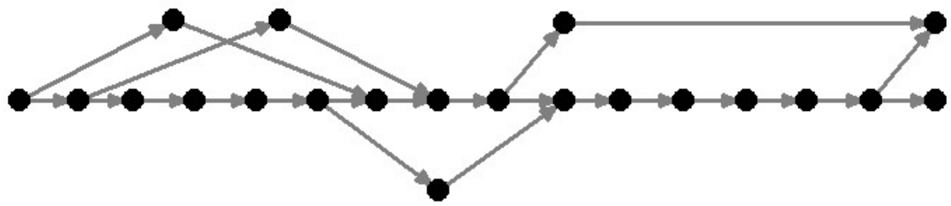
Fig. 18: Final state of the graph of a system. Complete subgraphs (including ones composed only of one node) are framed by ovals. There also are “complete” arrows among the complete subgraphs. These complete arrows have the property of the transitive law.

It is clear from Fig. 17 that TR of the system converges at 1. This means an emergence of reliable formal logic in which the transitive law holds completely, however, it may be trivial by definition of S-IA interaction.

There are 1150 arrows in the final state of the graph of the system as shown in Fig. 18. More importantly, 21 complete subgraphs are generated. These complete subgraphs do not overlap, and include 8 complete subgraphs composed only of one node (8 singletons). At the same time, the arrows between two arbitrary complete subgraphs are also “complete” ones. That is, there is maximum number of arrows in the same direction between two distinct complete subgraphs. For instance, given two complete subgraphs composed of *m* and *n* nodes respectively, there are *mn* arrows between the two complete subraphs. The transitive law holds among all complete arrows as evidenced by the result that the value of TR is 1.

Thus the distribution of the arrows becomes non-uniform and characteristic one in the final state. In short, the graph is divided into two parts: in which the arrows go in cycles (complete subgraph); in which the arrows flow uniformly (complete arrow). These complete subgraphs and arrows can be regarded as the “hardest” ones as ordinary logical components, as discussed previously. A detailed account of this reason will be given again in a later section, and here we turn our attention to the transformed graph in which the complete subgraph and arrow are compressed into one node and one arrow respectively.

(A)



(B)

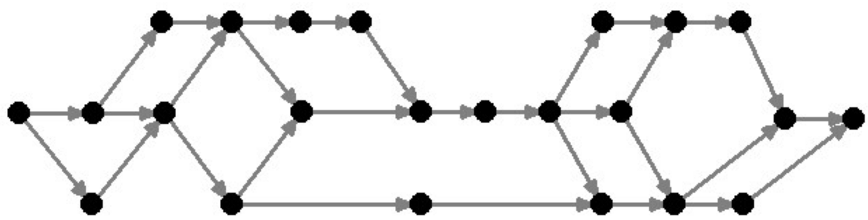


Fig. 19: The transformed graphs in which complete subgraphs are compressed into one node. The compression of complete subgraphs naturally involves the compression of complete arrows between complete subgraphs. The transitive law holds in both graphs, therefore only minimum number of arrows is depicted in order to facilitate visualization. (A) The graph which is transformed from the graph of Fig. 18 by compressing of 21 complete subgraphs and corresponding complete arrows. (B) The graph from the other conditions: the rate of the numbers of arrows of the initial agent to the number of arrows of the initial system 0.75, and the others are same as the first trial. It exhibits a more complex structure than the graph of (A).

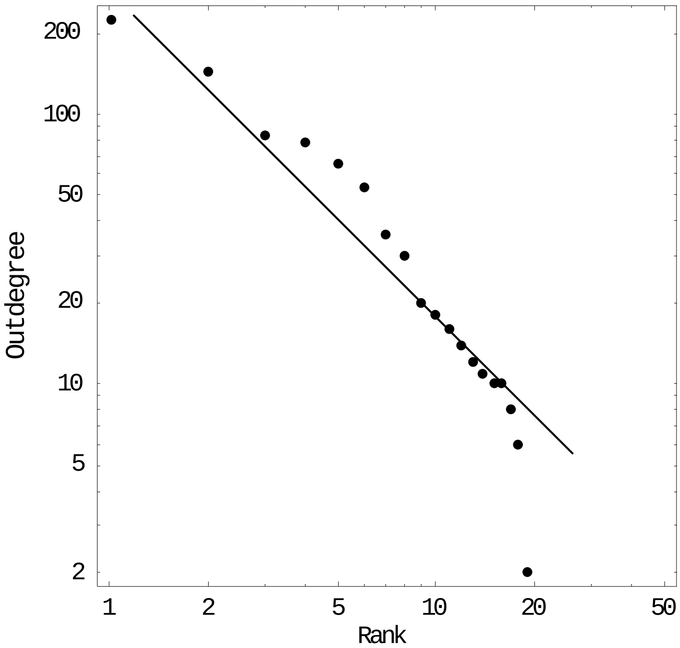
Figure 19 (A) shows the graph which is transformed from the graph shown in Fig. 18. Each complete subgraph including singletons is represented by one node. The arrows are distributed mostly in a line, so that the meaning as formal logic is not rich. That is, the structure of the directed graph is hardly a conjunctive and disjunctive structure, and there are hardly complements of each object. However, the other trial from different conditions can generate a richer structure. See Fig. 19 (B).

Figure 20 shows the distribution of the product of the number of arrows and the degree of each complete subgraph of the first trial, which presents a power law. In this regard however, there are no arrows in a singleton hence the number of arrows of a singleton was valued at 1. By comparison of two graphs, it is natural to regard a complete graph as a unit.

The interactional relation between the system and agent is fundamentally linked to the results. In fact, the control experiments show the following results. If the agent is influenced by the agent itself instead of by the system, TR of the system converges at 0.41 and only two cycles of arrows including big one composed of 35 nodes emerge. If the system is influenced by the system itself, TR of the system converges at 1, however, only 3 small complete subgraphs emerge. While the agent is fixed on the initial state and only the system is influenced by the agent, TR of the system converges at 0.29 and only 5 cycles of arrows including a big cycle emerge. In any case, we cannot observe the emergence of a directed graph, which is appropriate to be called formal logic.

In summary, we conclude that (1) S-IA interaction: the interaction between the system and agent inside the system itself yields formal logic; (2) the complete subgraph and arrow, which can be regarded as logical components, are inevitably induced by the emergence of formal logic.

(A)



(B)

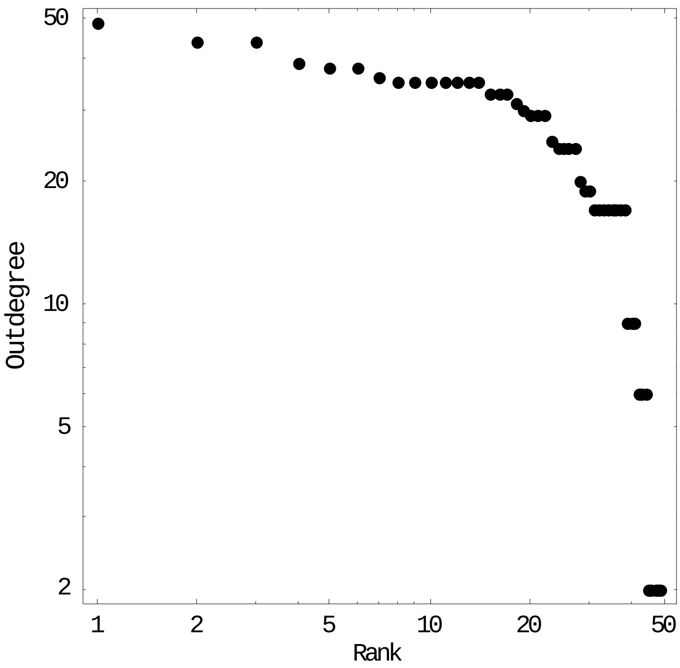


Fig. 20: (A) Relation between the product of the number of arrows and the out degree (vertical axis), and the rank ordered by the product (horizontal axis) of each complete subgraph. It presents a power law in the most part. The slope of the line is -1. (B) Relation between the out degree (vertical axis) and the rank ordered by the out degree (horizontal axis) of each node. It exhibits an exponential distribution. Both graphs are double logarithmic plots.

4.3. Softness of logical component

4.3.1. Why softness is needed?

In connection with the completeness of logical components above mentioned, we develop an argument about the inside of logical components. The complete component can be construed as the “hardest” one, while the inside of components is considered and “softness” is introduced into components as described later.

In an ordinary formal logic, the validness of an object is forced to be alternative of 1 or 0, that is, an object either exists or never exists. The intermediate state is unconsidered and not represented. The same applies to an arrow. For example in predicate calculus LK (Troelstra and Schwichtenberg, 2000), there are inevitably sequents such as $X \vdash X$ at the tops of the derivation, in which X denotes an atomic formula. This means that only atomic formulas and formulas composed of atomic formulas are valid, and the other cannot exist in LK. There is a clear distinction between the existence and the nonexistence, and no intermediate states. An atomic formula is a minimum unit in LK. In spite of the arbitrariness of an atomic formula, there is no doubt about the obviousness of an atomic formula. However in our opinion, an atomic formula must be a temporal minimum one for a superior argument. It is realized through the inspection of the inside of an object. For instance, “a dog” is absolutely “a dog”, however, the thing “a dog” splits into “a brown dog”, “a big dog” and so on, when we show our preference.

Furthermore, this setting of LK realizes that an infinite decomposition of a formula is not permitted, while the infinite composition is permitted. That is to say, a one-way infinity is permitted and there is asymmetry of the decomposition and composition of a formula. In addition, a set of objects is sharply distinguished from objects themselves in terms of logical status (Whitehead and Russell, 1925). In our opinion, such a property of objects also is a mere postulate.

Instead of minimum objects and their hierarchical structure, we here present the argument about the inside of each component in the form of the introduction of softness. Due to this introduction, the validness of each component is permitted to become the intermediate value between 1 and 0. The existing component of an ordinary formal logic corresponds to the complete one in Section 4.2, and we can consider an intermediate state. It follows that the complete component becomes the “hardest” one.

Moreover, we can address the disconnection of a transition of formal logic. It is sometimes difficult that we represent the emergence of a component consecutively. The reason of the emergence never exists inside the system itself. We cannot help regarding the system as what is either dependent on some other thing, or random. In this way, the system implicitly comes to require its outside. The preceding state of the system is not intrinsically related to the following state when we regard the system as an independent one. We consider that this disconnection between the preceding and the following is caused by composing of components whose interior is ignored. While we consider the inside and the softness of components, the transition of the system becomes a kind of continuous one and the outside of the system may not be required.

4.3.2. Soft object and soft arrow

A soft object, which is already defined in Section 3.1, is one of the representations of an object with softness. Now we again argue about the soft object. In predicate logic, each formula has the property of the reflexive law: $X \rightarrow X$, where "$\rightarrow$" denotes implication. If a cycle of implication (e.g. $X \rightarrow Y$, $Y \rightarrow Z$, and $Z \rightarrow X$) exists, there are implicational relations between two arbitrary objects in the cycle under the transitive law. It follows that every object in the cycle can be assigned to both sides of logical connective "$\rightarrow$", for example, $X \rightarrow Z$, $Y \rightarrow Y$, and so on. This assignability enables the cycle itself to be regarded as one extended object. Indeed, there is a bundle of arrows in the same direction from the new extended object to itself, and this situation is similar to the reflexive law. Figure 21 clearly illustrates the similarity by a diagram. Thus we define the set of objects regarded as one unit, as a soft object.

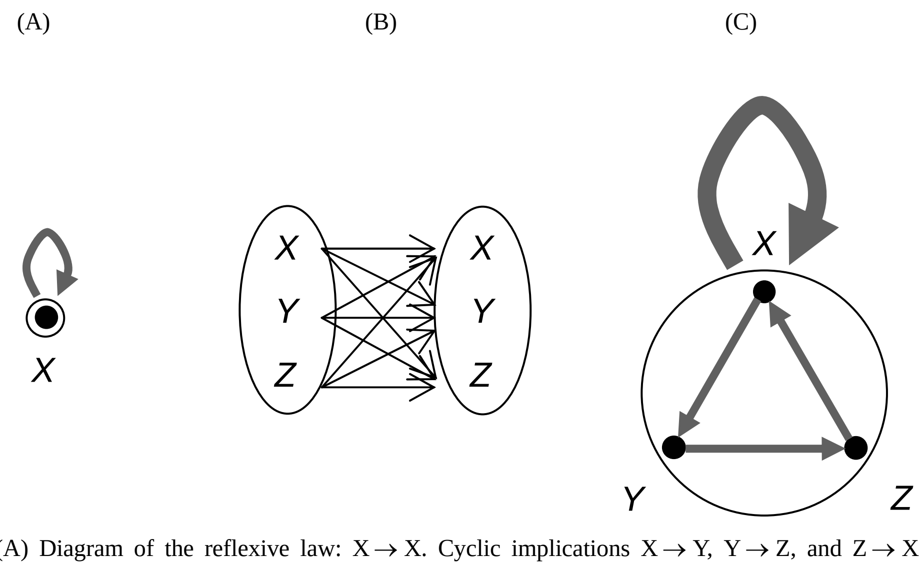


Fig. 21: (A) Diagram of the reflexive law: $X \rightarrow X$. Cyclic implications $X \rightarrow Y$, $Y \rightarrow Z$, and $Z \rightarrow X$ lead to the situation shown by the diagram of (B) under the transitive law. This can be depicted in the diagram of (C) while the bundle of arrows from the set {X, Y, Z} to itself is represented by one arrow. The left and right diagrams are similar hence we regard the set composed of three nodes as one unit, and call it a soft object.

As mentioned in Section 3.1, soft objects differ in fragility according to the number of arrows in each soft object. A soft object composed of many arrows is more difficult to break than one composed of fewer arrows. By definition a complete graph is a soft object, moreover, is actually the “hardest” one.

The introduction of softness enables objects to be divided or united in a nonhierarchical way, and also resolves the asymmetry of the decomposition and composition of a formula in theory. Each soft object has its size, which can increase or decrease.

In a similar way, we define a bundle of arrows in the same direction as a soft arrow.

Definition 4.3.1 (*Soft arrow*). In a given directed graph, and given two non-empty sets of nodes, we call a bundle of arrows in the same direction from the set of nodes to another one a soft arrow.

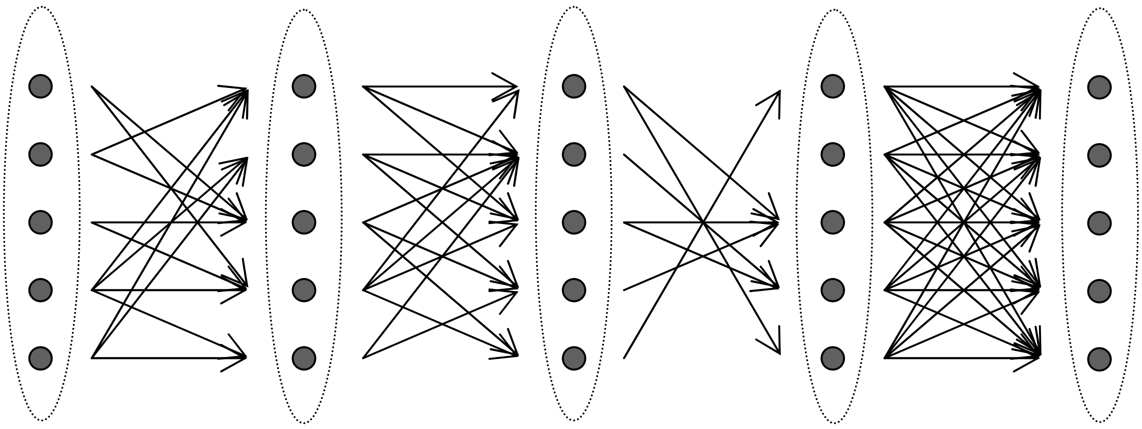
Note that a soft arrow is not between two soft objects but between two sets of nodes. We can also consider the softness of a soft arrow in the same manner of a soft object. The number of arrows in a soft arrow represents the softness: a soft arrow becomes harder in proportion to the increase of arrows in the soft arrow. The maximum number of arrows in a soft arrow is mn, if the numbers of nodes of two sets are m and n respectively.

From the standpoint of soft object and arrow, the results in Section 4.2 showed the emergence of the soft objects and soft arrows among soft objects, which are in the hardest state. These results reinforce the validness of formal logic induced by S-IA interaction.

4.3.3. Transition of formal logic induced by soft arrows

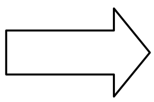
We conduct a following experiment in order to observe the relation between the softness of objects and arrows. As an initial graph, we give a directed graph composed of 25 nodes. The nodes are divided into 5 sets of 5 nodes. The sets of nodes are linearly-arranged and adjacent two sets are linked by a soft arrow. We emphasize that there are no soft arrows between unadjacent sets, and also no arrows inside a set of nodes. Figure 22 clearly illustrates an example of the directed graph and its adjacent matrix.

(A)



(B)

$$\begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$



$$\left(\begin{array}{ccccc|ccccc|ccccc|ccccc|ccccc}
0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&1&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&1&1&1&0&1&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0&0&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0\\
\hline
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&1&1&1\\
\hline
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{array}\right)$$

Fig. 22: (A) An example of the initial directed graph. There are 5 sets of 5 nodes framed by ovals, and 4 soft arrows among the sets of nodes. The softness of soft arrows are various, for example, the number of the rightmost soft arrows is 25 so that the hardest one. (B) The adjacent matrix of the above directed graph (right). This matrix is formed out of the left matrix, by replacements of 0 by a zero submatrix, 1 by a submatrix including one or more 1. These replacements imply the inspection of the inside of each object.

The initial directed graph is formed considering the inside of each object and arrow of the graph composed of 5 objects and 4 arrows. Each set of nodes represents a “latent” object. We mean by “latent” that each set of nodes is not a soft object, however can be regarded as one unit only by force of soft arrows. Soft arrows obviously represent arrows of the former graph. There are soft arrows only between adjacent sets (latent objects) hence the soft arrows do not hold the transitive law.

The graph is consecutively transformed by S-IA interaction, where an initial system is composed of all given arrows and an initial agent is a part of them. The arrows of the initial system are randomly chosen from the graph composed of 4 hardest soft arrows. Note that the "hardest" means that there are 25 arrows between 2 latent objects composed of 5 nodes. The arrows of the initial agent are also randomly chosen from the arrows of the initial system. The rate of the number of arrows of the initial system to the graph composed of 4 hardest soft arrows (p), and the rate of the initial agent to the initial system (q) are given as initial conditions. After a sufficient number of transitions (at most about 200 transitions), the graph converges in the form of reliable formal logic. That is to say, soft objects and soft arrows emerge as the hardest ones; the transitive law is satisfied among soft arrows; and TR of system is naturally 1. However, the emergence of soft objects from the latent objects involves some errors: some latent objects are divided into smaller soft objects; some soft arrows are also divided into smaller soft arrows and new soft arrows emerge inside of a latent object; new soft objects which are composed of nodes of different latent objects emerge. It is clear from the difference between two results in Fig. 23 that the frequency of the error increases in proportion to the softness of soft arrows of the initial graph of the system (p), and also of the agent (q). However in either case, the emergent graph represents formal logic adequately, and the logical structure expected from the former graph (the size of each object, and the direction of each arrow and so on) is roughly retained. See also Table 3.

This is a time development derived by the softness of arrows, in other words, a transition of formal logic induced by the inspection of the inside of logical components. The candidate of an object (latent object) which is not defined clearly is transformed into a valid object by the relation between the candidates. We note for comparison that a transformation by S-IA interaction from the graph composed of 5 nodes cannot lead to the similar result. It leads to a simpler logic, for example, in which only two soft objects exist. Thus we consider that the inspection of the inside of logical components is fundamental to transition of formal logic.

While we consider that the result in Section 4.2 represents the first emergence of logic, the result in this section represents the second transformation of logic which has been already established. That is to say, if we remove all arrows of each soft object in the emergent graph, and conduct time transitions by S-IA interaction once again, the graph representing formal logic must be newly transformed. This following experiment in this section is a simple and partial example of this second transformation.

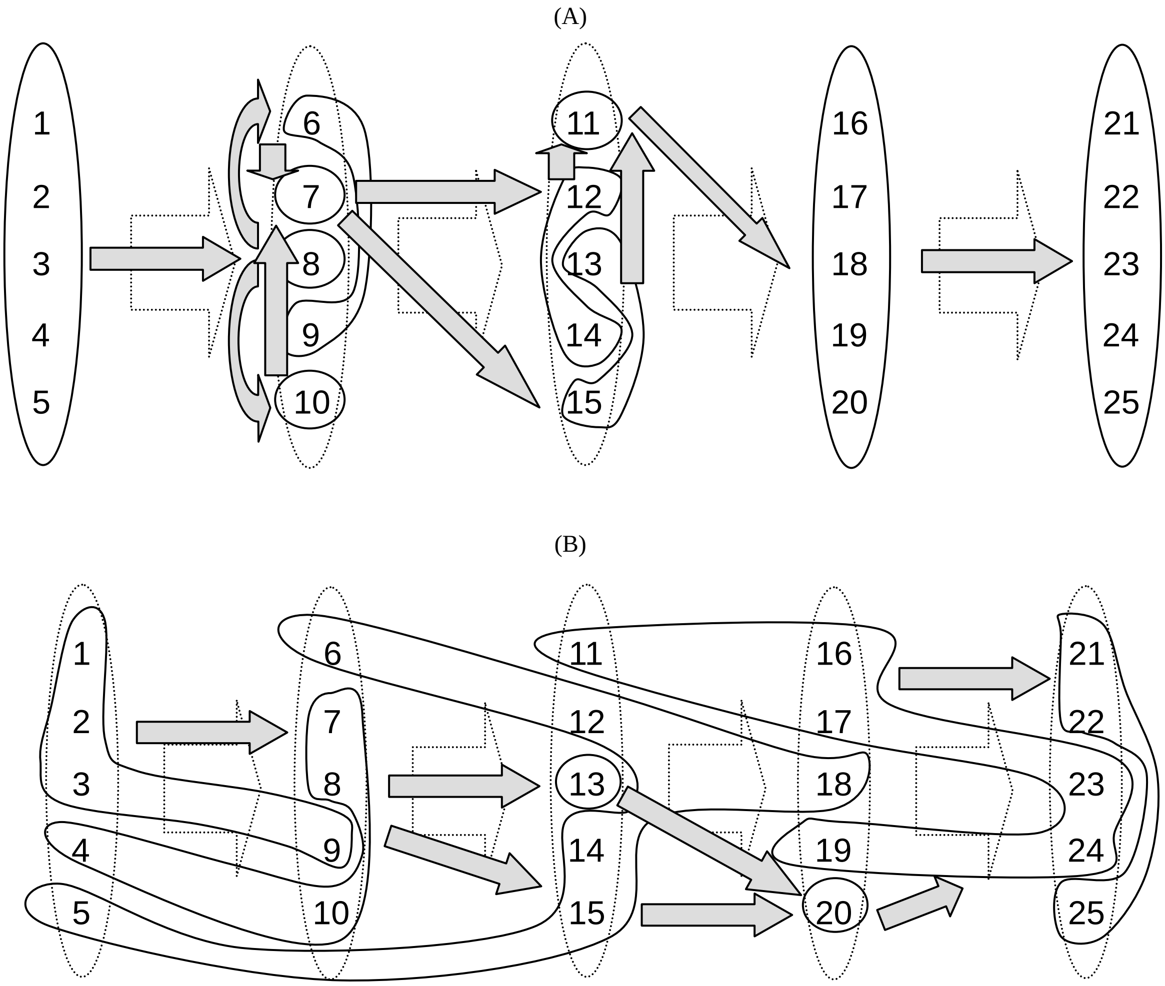


Fig. 23: Final states of graphs transformed from different initial graphs. Each number represents a node. Soft objects are framed by curved lines. The minimum number of arrows is depicted by heavy arrows. (A) The rates p (for the initial system) and q (for the initial agent) are 1.0 and 0.75 respectively. Though the second and third sets of nodes (latent objects) break into some soft objects, the other ones become soft objects expected from soft arrows. (B) The rates p and q are both 0.5. Soft objects emerge in the different forms than expected ones, however the whole graph represents formal logic.

Table 3: Results by the various pairs of *p* and *q*. For each pair of *p* and *q*, three trials ((1), (2), and (3)) are conducted. The value of (A) is the number of soft objects consisting of multiple nodes in the emergent graph. The value of (B) is the number of singletons (soft objects consisting of only one node) in the emergent graph. The value of (C) is equal to (A) + (B). The value of (D) is the number of soft objects which are composed of nodes of different latent objects in the emergent graph. The average value of (D)/(C) generally decreases in proportion to *p*, and also *q*. The value of (D)/(C) is one of the indexes of the frequency of the error.

q		1.00					0.75					0.50					0.25				
p		(A)	(B)	(C)	(D)	(D)/(C)	(A)	(B)	(C)	(D)	(D)/(C)	(A)	(B)	(C)	(D)	(D)/(C)	(A)	(B)	(C)	(D)	(D)/(C)
1.00	(1)	9	0	9	1	0.11	8	4	12	0	0.00	9	7	16	2	0.13	5	0	5	4	0.80
	(2)	8	1	9	2	0.22	6	4	10	0	0.00	6	0	6	3	0.50	4	2	6	3	0.50
	(3)	8	0	8	0	0.00	8	1	9	2	0.22	6	1	7	3	0.43	4	0	4	3	0.75
	Ave.					0.11					0.07					0.35					0.68
0.75	(1)	7	0	7	2	0.29	8	3	11	5	0.45	6	2	8	3	0.38	5	0	5	4	0.80
	(2)	9	3	12	3	0.25	9	3	12	5	0.42	6	0	6	4	0.67	5	0	5	4	0.80
	(3)	8	2	10	4	0.40	8	2	10	3	0.30	2	0	2	1	0.50	4	0	4	2	0.50
	Ave.					0.31					0.39					0.51					0.70
0.50	(1)	11	1	12	4	0.33	6	5	11	5	0.45	10	3	13	5	0.38	6	1	7	4	0.57
	(2)	7	9	16	1	0.06	11	0	11	4	0.36	5	2	7	4	0.57	3	1	4	3	0.75
	(3)	9	2	11	5	0.45	7	4	11	3	0.27	6	5	11	4	0.36	3	3	6	2	0.33
	Ave.					0.28					0.36					0.44					0.55
0.25	(1)	9	5	14	4	0.29	8	1	9	8	0.89	5	3	8	4	0.50	8	0	8	6	0.75
	(2)	8	4	12	7	0.58	11	2	13	6	0.46	9	2	11	6	0.55	7	1	8	6	0.75
	(3)	10	1	11	7	0.64	8	5	13	6	0.46	9	5	14	9	0.64	9	1	10	8	0.80
	Ave.					0.50					0.60					0.56					0.77

5. Mediation between object and relation

5.1. Deduction, Induction, and Abduction

5.1.1. Deduction vs. Induction

Firstly, we confirm the well-known concepts, deduction and induction. Deduction is the inference which derives a special case from a general principle. Contrariwise, induction is a formulation of a general principle by collecting special cases. Mathematical induction is a case of induction which contributes our easy understanding of induction. That is, a general proposition on $\mathbb{N}$, $\forall n F(n)$ is formulated through observing each special case $F(m)$ for a natural number m. On the other hand, deduction is an application of $\forall n F(n)$ to a natural number m, and consequently a proposition $F(m)$ is realized. In this way, deduction and induction yields knowledge along the line between generalization and specialization.

5.1.2. Three types of inferences classified by Peirce

Peirce (1868) clearly classified inference into three types including deduction and induction. He called the third one abduction (sometimes called hypothesis or retroduction). The classification can be interpreted by using triangular diagrams. See Fig. 24. Deduction is the inference which brings a new proposition from two consecutive propositions. In other words, using the terminology of Aristotelian logic, deduction brings conclusion from major premise and minor premise. Deduction is nothing more than an application of the transitive law which is treated in previous sections. Note that major premise, minor premises, and conclusion correspond to “rule”, “case”, and “result” of terminology of Peirce, respectively. According to Peirce, induction is the inference which brings rule from result and case.

There are of course three arrows in a triangular diagram, and the last combination of the three propositions is abduction. That is, abduction brings case from result and rule. Considering from the diagrams, there is not much difference between rule and case. Hence seen in this light, induction and abduction are the similar inferences.

It may be difficult to maintain consistency with the association between the triangular diagrams and the ordinary understanding of deduction and induction. The consistency is maintained while deduction is regarded as the transformation of rule into result; induction is regarded as the transformation of result into rule; and both deduction and induction are conditional on case.

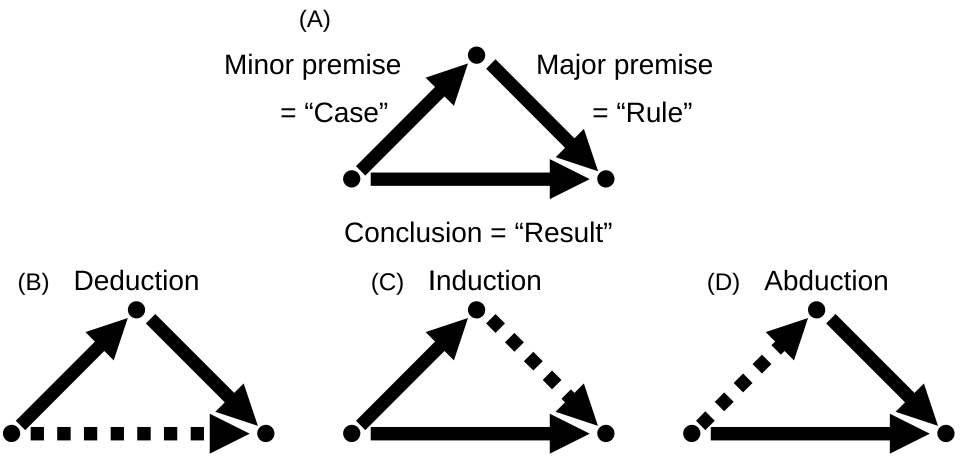


Fig. 24: (A) Names of each relation represented by an arrow. (B) - (D) Three types of inference classified by Peirce. The directed graphs which represent deduction, induction, and abduction. Dashed arrow is inferred from two other arrows in each diagram.

5.2. Mediation of Object-Relation Model

On the basis of such understanding of inference, we compose a following model of formal logic. In common with other models in this thesis, here we represent a formal logic composed only of implication as a directed graph (Harary, 1969). Of course a node and an arrow (directed edge) represent an object and a non-commutative relation between two objects. The fact “A implies B” is represented as $A \to B$. The presence of an arrow between two nodes represents an implicational relation between them, and the absence of an arrow represents the negation of an implicational relation. Classical propositional logic can be composed only of negation and implication, therefore, a directed graph represents propositional logic sufficiently.

5.2.1. Representation of the inferences by a directed graph

First, we formalize the three types of inferences schematically.

Definition 5.2.1 (*Inductive and abductive transformations*). We call a transformation of a set of arrows $A_1 \to B$, $A_2 \to B$, ... , $A_n \to B$ into an arrow $A_i \to B$ where $1 \le i \le n$ *inductive* transformation. In a similar way, we call a transformation of a set of arrows $A \to B_1$, $A \to B_2$, ... , $A \to B_n$ into an arrow $A \to B_i$ where $1 \le i \le n$ *abductive* transformation.

Definition 5.2.2 (*Deductive transformations*). We call a transformation of an arrow $A_i \to B$ into a set of arrows $A_1 \to B$, $A_2 \to B$, ... , $A_n \to B$ where $1 \le i \le n$ *deductive* [I] transformation. Alternatively, we call a transformation of an arrow $A \to B_i$ into a set of arrows $A \to B_1$, $A \to B_2$, ... , $A \to B_n$ where $1 \le i \le n$ *deductive* [II] transformation.

Deductive [I] and [II] transformations are inverse ones of inductive and abductive ones, respectively. We call all types of the transformations inferential transformation, as collective name. Figure 25 shows clearly the inferential transformations.

Note that, as shown in Section 5.1.2, induction classified by Peirce is a transformation of result into rule. Result and rule have the right vertex in common. In addition, it is a formulation by collecting some cases. These are the reason why we define the inductive transformation in such a way. The other transformations are also defined in a similar way.

The inductive and abductive transformations make some arrows into one arrow. On the other hand, deductive [I] and [II] transformations make one arrow into some arrows. In this way, inductive and abductive transformations are contracting transformations; deductive [I] and [II] are expanding ones. The contraction or expansion of arrows can be interpreted as the contraction or expansion of the nodes which are either sources or targets of arrows. Furthermore, the contraction of nodes represents an emergence of a new comprehensive object, and the expansion of nodes represents an emergence of respective objects which satisfy the conditions. Thus, the inferential transformations represent re-formation of logical objects caused by inference.

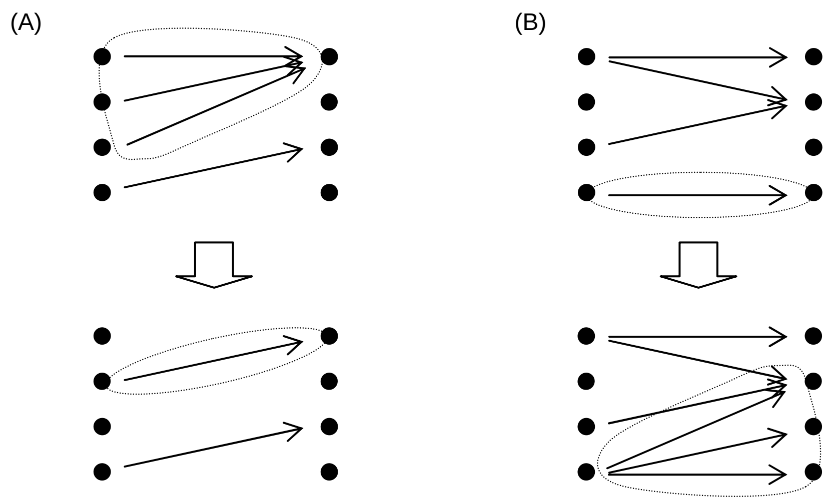


Fig. 25: (A) An example of inductive transformation. The set of three arrows framed by curved line (upper) is transformed into an arrow framed by an oval (lower). (B) An example of deductive transformation [II]. An arrow is expanded to a set of three arrows.

5.2.2. Model

We define a model called Mediation of Object-Relation Model as follows. As an initial graph, we give two sets of nodes and some arrows from nodes of one set of nodes to nodes of another set in the same direction. The pairs of nodes between which arrows exist are randomly chosen. It is permitted that there are arrows from one node to multiple nodes, and vice versa. However, there is only one arrow between a pair of nodes. The unit composed of nodes and arrows can be naturally expressed by a matrix which has m rows and n columns, where m and n are the numbers of nodes in the sets of nodes.

The directed graph is consecutively transformed by inferential transformations. In inductive or abductive transformations, the number of arrows transformed into one arrow is randomly given more than 2. In deductive [I] or [II] transformations, an arrow is expanded to arrows, whose number is also randomly given more than 2. The type of inference is also chosen randomly from inferential transformations, and a set of arrows (inductive or abductive transformations) or an arrow (deductive [I] or [II] transformations) are also chosen randomly.

Regarding the two sets of nodes and a set of arrows between them as one unit, we arrange multiple units linearly in the way that two neighbour units share the set of nodes. A unit is influenced by the transformation of the neighbour units as follows:

(1) If the inductive (abductive) transformation occurs in a right-hand (left-hand) unit, each

arrow whose target (source) is in the pre-contracted set of nodes is replaced by an arrow from the original source node to the contracted node (from the contracted node to the original target node);

(2) If the deductive [I] ([II]) transformation occurs in a right-hand (left-hand) unit, each arrow whose target (source) is the pre-expanded node is replaced by an arrow from the original source node to a node randomly chosen from the expanded set of nodes (from a node randomly chosen from the expanded set of nodes to the original target node).

The rightmost or leftmost units are influenced only by the left-side or right-side unit, respectively. In addition, the arrows of the transformed unit itself are also influenced in a similar manner. Figure 26 shows an example of transformations. As stated previously, the contraction or expansion of nodes imply the re-formation of objects. The influence on the neighbour unit is intended to the consistency of objects shared by two units.

This consistency of objects is not complete one. The definition of setting (2) that one node of a replacing arrow is randomly chosen makes the consistency incomplete. However, this incompleteness itself invokes the indefinite behavior of the model.

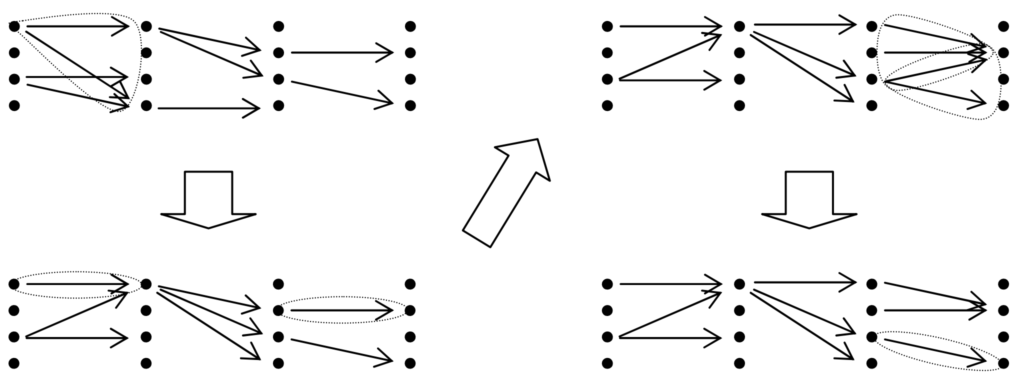


Fig. 26: An example of consecutive transformations of a directed graph composed of 3 units. First, the directed graph is transformed by abductive transformation at first unit (from upper left to lower left, flamed by curved lines). The arrow from fourth node to fourth node of second unit is replaced by the arrow from first to fourth by the influence of the transformation at first unit. In addition, an arrow of first unit itself is influenced and moved. The second and third transformations are deductive [I] and abductive ones both at third unit. The second transformation influences replacement of an arrow of second unit. The third transformation does not influence the units at all.

5.2.3. Results

We conduct an experiment under the following conditions: there are 5 units, which are composed of two sets of 20 nodes. Arrows are allocated randomly in each initial unit. The rate of the number of arrows to the number of all possible arrows is 0.05, hence there are about 20 arrows. The upper bound of the number of contracted or expanded arrows is 5 in an inferential transformation. The number of iterations of inferential transformations is 1000.

The number of arrows fluctuates under about 30 in each unit. In order to observe the distribution of arrows, we define an index as follows.

Definition 5.2.3 (*Existing object of thought*). Given a unit, we define a node which is a source or a target of at least one arrow as an existing object of thought, or simply, an object. Furthermore, there are of course two sets of nodes: sources or targets of arrows. We call an object in the set of sources an antecedent object. Similarly, an object in the set of targets is called a consequent object. We denote the sets of antecedent objects and consequent objects by O_1 and O_2, respectively.

Definition 5.2.4 (*Distribution of arrows relative to objects*). Given a unit, the distribution of arrows relative to the number of objects is defined as

$$\mathrm{DA} := |A| \ / \ (|O_1| \ |O_2|),$$

where $|A|$ is the number of arrows, and each $|O_i|$ is the number of objects in the unit.

The value DA indicates unevenness of the distribution of arrows. That is, if DA is 1, there are arrows between every pair of antecedent and consequent objects, therefore, this is the case of the most uniform distribution. As DA falls, the arrows are distributed more unevenly. When there are *m* arrows where $|O_1| = m$, $|O_2| = n$, and $m > n$, DA reaches a minimum value.

Unevenness of the distribution of arrows implies the degree of differentiation of objects. That is to say, while DA demonstrates higher value, antecedent objects cannot be distinguished from one another by consequent ones. The same applies to the indistinguishableness of the consequent by the antecedent. In contrast, antecedent objects are characterized by consequent ones, and vice versa, while DA is lower value. In other words, the set of most arrows itself must be treated as one unified arrow if DA is higher; it can be treated as a collection of similar but distinctive arrows if DA is lower. DA is an index which represents a state of arrows.

Figure 27 shows an example of time transitions of DA. It also fluctuates as well as the number of arrows. Time intervals while DA is smaller than given border usually exhibit the exponential law. However, the central unit of the sequence sometimes tends to exhibit the power law but the exponential law. See Fig. 28.

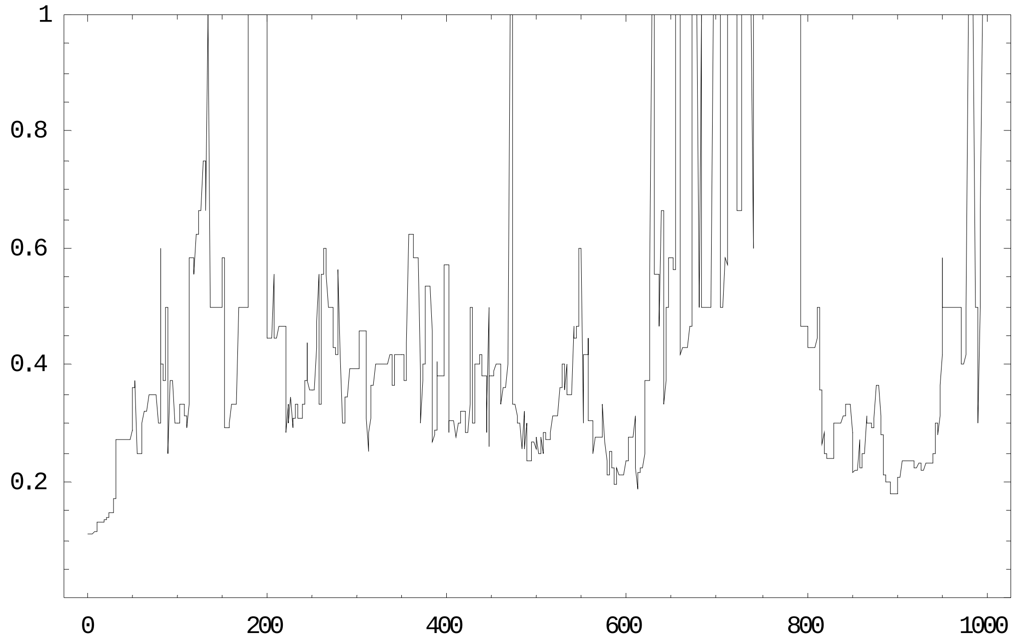


Fig. 27: An example of time transitions of DA of a unit, which is the central unit in a trial.

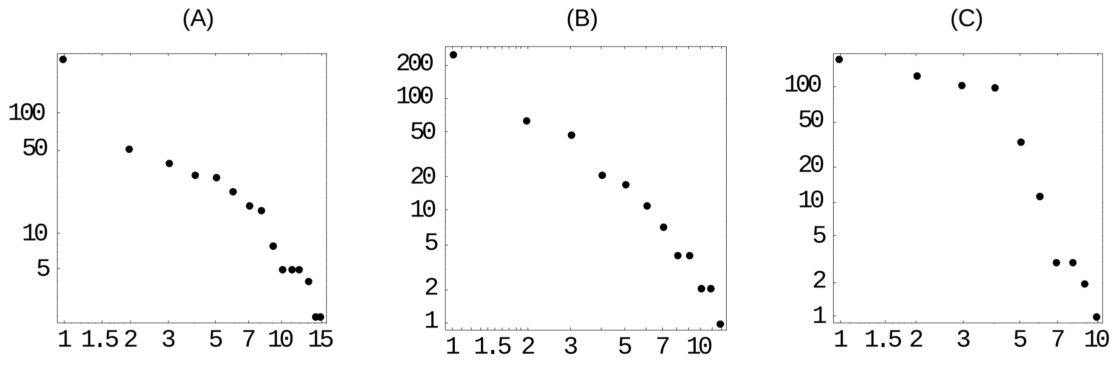


Fig. 28: The difference due to the location in the sequential units in a trial. Each graph shows the relation between the length of each time interval (vertical axis) and the rank ordered by the length (horizontal axis). All graphs are double logarithmic plots, and the borders which form the time intervals at respective units are all 0.3. (A) The graph of the leftmost unit. (B) The graph of the central unit. (C) The graph of the rightmost unit.

6. Discussions

In section 2, we propose a framework representing transformations of the directed graphs of the agents and the set of all agents with mutual dependence. Indeed, directed graphs of each agent are subgraphs of the directed graph of the set of all agents, and a latter graph of a given agent is subgraph of the former graph of the agent as time passes. In addition, if directed graphs are regarded as pseudo-categories, this study is connected with category theory (Mac Lane, 1998). Gunji and Higashi (2001) argued exactly about the relation between category theory and directed graphs.

Classical propositional logic can be composed only of negation and implication as mentioned previously, whereas we concentrated our thoughts only on implication here. Actually, however, the value of q plays a role in negation in Dialogue Model (I). In the model that we considered at first, instead of q, every arrow is given a uniform negative value at each time instant. We consider that, while the uniform negative value is regarded as "global negation", the negative value of q given for each agent that does not corroborate can be regarded as "local negation". Gunji and Higashi (2001) also deal in negation. In addition, we note that there are little differences in the distributions between the model with uniform negative value and Dialogue Model (I). In a similar way, we can regard each $G(A_m, t)$ as "local implication" and $G(\Gamma, t)$ as "global implication", respectively.

According to embodied mathematics, the transitivity of implication is inevitably grounded in the transitive experience of the causality. On the other hand in Dialogue Model (I), the transitivity of implication comes into being even if each individual never experiences the transitivity. More specifically, in the absence of the plurality of the subjective body, the experience which many individuals have similarly is generalized down to the experience which all individuals must have identically if an opportunity occurs. This bold generalization can be regarded as the transformation from discrete experiences of each individual into a conceptualized experience. Meanwhile in Dialogue Model (I), an experience of an individual influences not only experiences of others between exactly the same two things, but also adjacent experiences of others, that the individual never experiences. Put simply, experiences penetrate into unexperienced things. There is no bold generalization in Dialogue Model (I). This difference is depicted in Fig. 29.

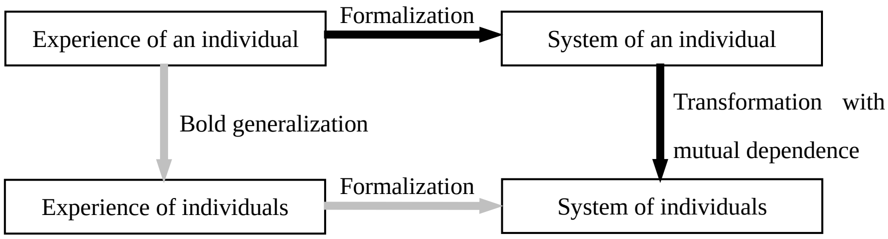


Fig. 29: The difference between embodied mathematics and Dialogue Model (I). Embodied mathematics formalizes a discrete experience of each individual (upper left) in a way that is indicated by the gray arrows. The way of Dialogue Model (I) is indicated by the black arrows.

The scheme of interaction of two agents in Dialogue Model (I), (II), and (III), which is introduced in Section 2.1.1, can be interpreted in connection with three types of inferences of Peirce. Let us compare Fig. 24 (B), (C), and (D) with Fig. 1. If the number of the arrows is two in (X) and zero in both (Y) and (Z) in the scheme of interaction in Fig. 1, the scheme accurately corresponds to deduction of Peirce depicted in Fig. 24 (B). If zero in (Z) and one in both (X) and (Y), the scheme corresponds to induction. In a similar way, if zero in (Y) and one in both (X) and (Z), we can see abduction.

In Dialogue Model (II) in which a cycle of arrows itself is regarded as an object, if an arrow that is a component of a cycle disappear, the number of objects increases as the cycle itself (equal to one object) changes to the objects inside the cycle (multiple objects). In this way, the decrease of arrows correlates directly with the increase of objects. However, the change in the opposite direction, the increase of arrows and the decrease of objects cannot be represented in Dialogue Model (I) and (II). This is mainly caused by the construction of Dialogue Model (I) and (II) without the emergence of arrows. We described nothing about intuitions nor their sources, and treat them as given in Dialogue Model (I) and (II). Dialogue Model (III) is a model in which the emergence of arrows is realized, hence we can change the size of the object in Dialogue Model (III) in principle.

While we regard an object as a thing, it is a kind of conceptualization to regard multiple objects as one object. To treat things and their conceptualized thing on the same level leads to Russell's paradox (Whitehead and Russell, 1925). A way to circumvention of the paradox is introduction of the logical types, which corresponds to discrimination between objects and sets of identified objects. In the whole of this thesis, especially in Section 3 (Monologue Model), we do not dare to circumvent the paradox. We aim to compare the system based on soft objects with the ordinary logical system premised on logical types. Only the STR-oriented way can make a new set of identified objects in Monologue Model. The

SR-oriented way is scarcely, and the TR-oriented way is never able to make a new set. It can be concluded that conceptualization can be realized only on the STR-oriented way. Both the TR-oriented and the STR-oriented way can only confirm an existing state, from the standpoint of conceptualization.

As stated in Section 1, Monologue Model is a kind of denial of reductionism. In general, the alternative of reductionism is holism, however, we do not side with the holism. We do not consider that a whole is indivisible into parts, but assert that a whole is divisible into temporal parts that can change depending on the situation. Monologue Model is intermediate between reductionism and holism, and is, as it were, pseudo-reductionism.

A dialogue model consisting of ordinary objects such as Dialogue Model (I) expresses an open and aboveboard world, in which one can see transparently what others say or consider. The transparency conduces to the consistent description of the world: the transitivity law. In contrast, there is not such transparency in Dialogue Model (III) which is a dialogue model composed of soft objects. Instead, the softness of soft object plays a role as mediator between inter-agent relation and inter-object relation, and consequently yields the consistency. STR-oriented way has a resemblance to Euclidean geometry which is composed of points that have no size, and lines that are infinitely-thin, in the sense that the interior of each component is ignored. Soft objects can accumulate information inside themselves and the information can be exposed afterward. This aspect is never represented in an ordinary system in which minimum objects exist.

In short, Internal Agent Model is driven by considering the reflexive and transitive laws, which are the fundamental properties of formal logic. S-IA interaction is indeed the succession of the applications of the transitive law to two parts: the system and agent. When we regard formal logic as what is already formed and rigid one, the transitive law is nothing more than a consequence. However in our opinion, it is also a cause of the transition of logic as well as being a consequence. If the transitive law has such a double meaning, it can impel formal logic to become a dynamical one which responds to diverse situations. And at the same time, ordinary formal logic becomes a snapshot of the dynamical formal logic.

On the other hand, the transition of logic from latent objects as observed in Section 4.3.3 is a transition due to an invalidation of the reflexive law. A soft object except the hardest one (a complete graph) is an object in which the reflexive law is partially invalidated, and a latent object is the most completely invalidated one. All nodes are directly connected to all nodes except themselves without the transitive law in the hardest soft object. Though the fragility of a soft object correlates with a lack of arrows, it can be obtained by the application of the transitive law. Thus the transitive law realizes the satisfaction of the reflexive law in Internal Agent Model.

A fundamental concept of mathematics, equivalence law consists of reflexive law, transitive laws and symmetric law ($A \to B$ implies $B \to A$). Equivalence law is the condition that a set is treated as one unit. Shinohara et al. (2007) pay notice to symmetrical bias of human cognition (also see Takahashi et al., 2010). While we associate the symmetrical bias with symmetric law, Internal Agent Model and other models in this thesis are related to these studies with a central focus on equivalence law.

While an object in formal logic represents a concept in the world which formal logic represents, introducing the notion of soft object enables us to represent the conceptualization of objects. Some concepts are united into one new concept, which is treated in the same manner as the constitutive concepts.

Moreover, in formal logic, all formulas are homogeneous, especially from the standpoint of the relations among formulas. That is to say, there are no cases that one formula is associated to many other formulas, and the other is associated to the fewer. Formal logic originally deals with the relations of concepts hence it is natural that formulas are deprived of their individual characteristics. However, while we consider that formal logic is derived from our natural linguistic behavior, we may deal with the individual characteristics in the early stages of the emergence of formal logic. We obtain the transitive law as an axiom in the whole of formal logic and lose the individual characteristic of each formula in process of changing the view of the “natural” formal logic to mere ordinary formal logic.

In Internal Agent Model, due to the simple definition of TR, a new arrow can appear not only at requisite places which are from a source to a target of a sequence of arrows in the same direction, but also at the other diverse places in a directed graph. This positional diversity leads to the emergence of soft objects. The purpose of both the agent and system, which is the guiding principle to transformation, was the increase of TR. We introduce another guiding principle to transformation as substitute for TR as a further experiment, however, the obtained result is not similar to the result by TR, that is, the soft object does not emerge and the distribution is not remarkable. The new guiding principle enables a directed graph to satisfy the transitive law microscopically. That is, as it were, the minimum agents which are unevenly distributed and have no memory. From this result and the results of the control experiments mentioned in Section 4.2, we conclude that it is necessary for the emergence of formal logic that the agent is sufficiently large in comparison with the size of the system, and can retain an appropriate memory.

In Internal Agent Model, both a soft object and an agent are mere subgraphs of whole of a system. A soft object is an alternative to an ordinary object: a nonhierarchical, divisible, and incorporable object. Meanwhile, an agent as a mere subgraph at each time instant, however, has purpose when the progress of time is taken into consideration. The agent in Internal Agent

Model purposes the adequacy of the system as formal logic. As shown in Section 4.2, we have the result that soft objects emerge assuming the purpose of an agent. Moreover, we consider that we can treat of the opposite direction: the emergence of the purpose of an agent assuming soft objects, by the argument of the positional relation or inclusive relation among soft objects. That is to say, the purpose of an agent is the temporal property of an object, and the soft object is the spatial property of an object.

In Mediation of Object-Relation Model, the inferential transformation influences the neighbour unit. A series of influences enables the object to be "natural" one. Both ends of a sequence of units are influenced only by one side, therefore they are not natural. The units next to the end units are influenced by both unnatural one and more natural one. In this way, the central part of the sequential units is more natural than any other part. It helps us easily understand this aspect of Mediation of Object-Relation Model that we regard an arrow in the model as a subject-predicate relation. In fact, an object, which corresponds to a name in subject-predicate relations, can be treated as a subject or a predicate. As objects are reciprocally defined, an object can become a definite one as a notion and the network of meaning is formed. The result of Mediation of Object-Relation Model coincides with subject-predicate relations.

The influence on the neighbour unit realizes the consistency of objects together with the fact that inferential transformation is represented as contraction or expansion of objects. However, the consistency of objects is not complete one, caused by the setting (2). If the setting (2) is replaced to a rule such as:

(2') If the deductive [I] ([II]) transformation occurs in a right-hand (left-hand) unit, each arrow whose target (source) is the pre-expanded node is replaced by a set of arrows from the original source node to *all* nodes of the expanded set of nodes (from *all* nodes of the expanded set of nodes to the original target node),

the consistency of objects is retained as complete one. However, as a consequence of the replacement of (2) by (2'), DA of every unit comes to 1 immediately and never decreases. Namely, the differentiation does not occur.

In Mediation of Object-Relation Model, the difference between induction and abduction is presented as the difference of the side of arrows in which the contraction of objects occurs. In a general sense, the difference between induction and abduction is not considered in such a simple way. In our opinion, this is caused by the asymmetry of subject-predicate relations. As compared with Mediation of Object-Relation Model, induction is the manipulation of subjects. On the other hand, abduction is the manipulation of predicates. In general, as it were, "subject-oriented manner" is adopted, hence abduction is dismissed. This aspect can be observed in mathematical formalization, as mentioned in Section 5.1.1. That is to say, "$\forall n F(n)$" is a regular expression, however "$\forall F F(n)$" is not allowed in first-order predicate logic, where "n" and "F" are regarded as a subject and a predicate in a proposition "$F(n)$".

In predicate logic, quantifier of course takes on quantitative problem, i.e. "all" and "exists" are denoted by "$\forall$" and "$\exists$", respectively. Induction inevitably yields the proposition with the quantity "all", however, there are neither quantifiers nor like things in Mediation of Object-Relation Model as well as in propositional logic. The quantitative problem, especially "all", is dealt with by the potentiality of contraction/expansion of objects. In addition, this potentiality of contraction/expansion addresses the irrefutability of object, which we treat by use of soft objects in Monologue Model and so on. The expansion of an object corresponds to the inspection of the inside of an object, hence it realizes a representation of refutable objects.

7. Conclusions

7.1. Characteristics of proposed models

We proposed some models of an emergence of logic. The characteristic of each model is as follows.

Dialogue Model (I): This model simulates dialogues among agents. The initial intuitions of each agent are arbitrary, and do not necessarily hold the transitive law. The agents are influenced not by collective knowledge of agents, which is on a higher order, but rather by only one another through dialogues. In spite of the absence of global knowledge, the mere union of knowledge of the agents obtains the transitive law after multiple dialogues. The transitive law is one of essences of the logical connective, implication.

Dialogue Model (II): This model is a variant model of Dialogue Model (I). The model does not premise the common view of the world among agents, hence, are closer to real and vague dialogues than Dialogue Model (I). The dialogue with this vagueness also conduces to the transitive law.

Monologue Model: This model is a kind of transitions of knowledge which depends on various measures of the world. The transitions represent autonomous change of knowledge of an agent on the basis of its knowledge itself.

Dialogue Model (III): This model is a variant model of Dialogue Model (I), which consists of agents acting by Monologue Model. The lack of transparency among agents is realized.

Internal Agent Model: This model is a model of interaction between a system and a part of the system, which is called an internal agent. The system is influenced only by the internal agent, hence the transition of the system is completely autonomous one in contrast with an ordinary multi-agent model which premises outside of a system, by the name of agent.

Mediation of Object-Relation Model: This model represents expansion and contraction of objects and relations among objects. By definition, the model can also represent objects and arrows with the spatial extent, consequently, information can be conserved inside each object and arrow. The model implies two fundamental logical inferences, deduction and induction in the form of classification of Peirce. In addition, it also implies the third inference of Peirce, abduction, which is usually disregarded.

7.2. Principal proposed concepts

Soft object: We define a cycle of arrows in a directed graph as a soft object. A soft object is a weakened directed graph kin to complete subgraph. This concept is intended to represent an object which can expand and contract, in contrast with the fact that an object is generally represented by a point without the spatial extent.

Soft arrow: We define a bundle of arrows in the same direction in a directed graph as a soft arrow. This concept is also intended to represent an arrow which can expand and contract, in contrast with the fact that an arrow is generally represented by a line which is infinitely-thin.

Internal agent: In a given system, we define a part of the system as an internal agent. An internal agent has a purpose which is independent from the system.

Inferential transformation: We represent the three types of inference: deduction, induction, and abduction classified by Peirce as transformations of a directed graph. We call these transformations of a directed graph inferential transformation, as collective name.

7.3. Future studies related to proposed models

Formulation with category theory: All models are represented by directed graphs, hence can be formulated in the context of category theory.

Representation of purpose of an agent: As mentioned in Section 6, we have the result that soft objects emerge assuming the purpose of an agent in Internal Agent Model. The opposite direction, that is, the emergence of the purpose of an agent assuming soft objects is an issue in the future.

Further experiment of Mediation of Object-Relation Model: Mediation of Object-Relation Model demonstrates dynamical feature in which the exponential law is observed. The exponential law sometimes tends to transform into the power law in central part of the allocation. The detailed condition that the power law is observed is a task for future studies.

Application of Mediation of Object-Relation Model to an unformed directed graph: In mediation of Object-Relation Model the units are linearly arranged, and the number of nodes in a set of a unit is fixed. In this way, inferential transformations are applied to a formed directed graph. In order to treat general logical inferences we ought to apply inferential transformations to an unformed directed graph.

Application of softness of components: Soft object and soft arrow can become a tool for description of non-hierarchical, divisible, and incorporable things. For example, cell motility as sol-gel transformation may be represented by soft objects and soft arrows.

References

Adamatzky, A., 1998. Master-slave algebra: On the binary compositions of distributed beliefs. Applied Mathematics and Computation 95, 173-180.

Asavathiratham, C., 2000. The Influence Model: A Tractable Representation for the Dynamics of Networked Markov Chains. In Dept. of EECS, MIT, Cambridge.

Barwise, J., Seligman, J., 1997. Information flow: the logic of distributed systems. Cambridge University Press, New York.

Davey, B. A., Priestley, H. A., 2002. Introduction to Lattices and Order (2nd ed.). Cambridge Univ. Press, Cambridge.

Dubois, D.M., 1998. Introduction to Computing Anticipatory Systems, International Journal of Computing Anticipatory Systems 2, 3–14.

Girard, J.-Y., 1987. Linear logic. Theoretical Computer Science 50:1, 1–102.

Gunji, Y.-P., Haruna, T., Sawa, K., 2006. Principles of biological organization: Local–global negotiation based on “material cause”. Physica D 219, 152-167.

Gunji, Y.-P., Higashi, H., 2001. The origin of universality: making and invalidating a free category. Physica D 156, 283–313.

Gunji, Y.-P., Kamiura, M., 2004. Observational heterarchy enhancing active coupling. Physica D 198, 74–105.

Gunji, Y.-P., Sasai, K., Wakisaka, S., 2008. Abstract heterarchy: time/state-scale next term re-entrant form. BioSystems 91, 13–33.

Gunji, Y.-P., Takahashi, T., Aono, M., 2004. Dynamical infomorphism: form of endo-perspective. Chaos Solitons Fractals 22 1077-1101.

Harary, F., 1969. Graph theory. Addison-Wesley, London.

Jen, E., 2003. Stable or robust? What’s the difference? Complexity 8 (3), 12–18.

Kamiura, M., Gunji, Y.-P., 2006. Robust and ubiquitous on-off intermittency in active coupling. Physica D 218, 122–130.

Lakoff, G., Núñez, R. E., 2000. Where mathematics comes from: how the embodied mind brings mathematics into being. Basic Books, New York.

Mac Lane, S., 1998. Categories for the working mathematician (2nd ed.). Springer, New York.

Matsuno, K., 1989. Protobiology: Physical Basis of Biology, CRC Press, Boca Raton, FL.

McCulloch, W.S., 1945. A heterarchy of values determined by the topology of nervous nets. Bull. Math. Biol. 7 (2), 89–93.

Nowak, M. A., Plotkin, J. B., Krakauer, D. C., 1999. The Evolutionary Language Game. Journal of Theoretical Biology 200, 147-162.

Nowak, M. A., Plotkin, J. B., Jansen, V. A. A., 2000. The evolution of syntactic communication. Nature 404, 495-498.

Oliphant, M., 1996. The dilemma of Saussurean communication. BioSystems 37, 31-38.

Paton, R. C., 2002. Diagrammatic representations for modeling biological knowledge. BioSystems 66, 43-53.

Peirce, C. S., 1868. Some Consequences of Four Incapacities, Collected papers of Charles Sanders Peirce, edited by Hartshorne C. and Weiss P., 1960. Harvard University Press.

Sasai, K., Gunji, Y.-P., 2008. Heterarchy in biological systems: A logic-based dynamical model of abstract biological network derived from time-state-scale re-entrant form. BioSystems 92, 182–188.

Sawa, K., Gunji, Y.-P., 2007. Dialogue and causality: Global description from local observations and vague communications. BioSystems 90, 783-791.

Sawa, K., Gunji, Y.-P., 2008. Monologue and Dialogue Under Soft Object. International Journal of Computing Anticipatory Systems 21, 45–54.

Sawa, K., Gunji, Y.-P., Dynamical Logic Driven by Classified Inferences Including Abduction. AIP Conference Proceedings on Computing Anticipatory Systems. (in press).

Shinohara, S., Taguchi, R., Katsurada, K., Nitta, T., 2007. A Model of Belief Formation Based on Causality and Application to N-armed Bandit Problem. JSAI (The Japanese Society for Artificial Intelligence).

Szabo, A., 1978. The beginnings of Greek mathematics. D. Reidel, Dordrecht.

Takahashi, T., Nakano, M., and Shinohara, S., 2010. Cognitive Symmetry: Illogical But Rational Biases Symmetry: Culture and Science 21, 1–4.

Troelstra, A. S., 1992. Lectures on Linear Logic (2nd Ed.). Cambridge Univ. Press.

Troelstra, A. S., Schwichtenberg, H., 2000. Basic Proof Theory (2nd Ed.). Cambridge Univ. Press.

Whitehead, A. N., Russell, B., 1925. Principia Mathematica I. 2nd ed. Cambridge Univ. Press.

Wooldridge, M., 2009. An Introduction to MultiAgent Systems (2nd Ed.). Wiley.