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Chapter 1

Introduction

Voting is one of the most basic procedures by which a group of people with diverse judgments makes a decision. Voting is employed in diverse processes in the society. In politics, people select their representatives (president, members of Parliament) by voting in democratic countries. In judiciary, defendants are judged guilty or innocent by verdicts of juries in some countries. In management, stockholders decide important matters of their companies by voting based on their stockholdings in stockholder meetings. Executive officers decide some business policies of their companies by voting in boards of directors. Various forms of social communities (faculty meetings in universities, communities of residents) decide by voting.

The researchers have studied voting in various fields of social science; political science, economics, business administration, sociology and so on. The common issues in these studies are how people behave in group decision making processes and what property decisions of groups have.

The important function of voting is the aggregation. It aggregates preferences of voters. It also aggregates information of voters. Voting aggregates both preferences and information simultaneously in some occasions and it aggregates only one of them in other occasions. The issue of information aggregation matters in the following situation. Imagine decision-making in committees as follows. Suppose that there are two alternatives and that a committee of $2n + 1$ members must decide to choose one of the alternatives. One is better than the other for the committee, but no member of the committee knows which one is better. However, each committee member has their own information about which alternative is the better alternative. Based on this information, he votes and the committee decision is made by the decision rule. When a member votes for one of the alternatives, the information which the member holds is transformed into a vote which he casts. Then, each vote is summed into the total number of votes. Thus, the information which is scattered at hands of committee members is aggregated into a single decision of the committee.

The committee can not make a correct decision by voting for sure because no member knows which is the better alternative. Then, the decision is more efficiently when the probability that the committee chooses the better alternative is higher. This efficiency of decision depends on voting behavior of committee members, decision rule, and committee size. The important issues of information aggregation by voting is to understand the voting behavior of committee members and the effects of decision rule and committee size on the voting behavior. Then, based on these understandings, how to design the committees so as to make a decision as much efficiently as possible also becomes an important issue.

There is a well-known classical result on the efficiency of voting, called the Condorcet jury theorem (CJT), established by Condorcet (1785). This theorem states that the committees can decide more efficient under the simple majority rule as the committee size becomes larger (monotonicity CJT) and that the efficiency approaches to the perfect as the committee size becomes unboundedly large (Limit CJT). The CJT is regarded as the fundamental achievement of our understanding of democracy, because it provides a firm rationale for the reasoning that the wisdom of many people is better than an idea of any genius.

During the last twenty years, some doubt has been cast upon the validity of the assumptions which the CJT makes. Theoretical economists have began to study whether the CJT holds or not under an alternative assumption of strategic voting.

In this dissertation, we study strategic voting and the efficiency of decision in committees in a new perspective to which the research of these two decades has not paid attention. We introduce an uncertainty of the degree of information precision, that is, the degrees of information precision are not the same among the members and the degree of information precision is his private information. We study two cases, a case of homogenous committee in terms of members' information precision and a case of heterogenous committee. In the case of homogenous committee, we show that the the committees can decide more efficient as the committee size becomes larger as long as the committee size is sufficiently large (the monotonicity CJT holds for sufficiently large committees). We also show that there exists a case in which the monotonicity CJT does not hold for small committees. In the case of heterogenous committee, we show that the Limit CJT holds.

This dissertation is organized as follows. We provide an overview of the literature in Chapter 2. In Chapter 3, we study the CJT in the homogenous committees. We study the CJT in the heterogenous committees in Chapters 4 and 5. In Chapter 4, we focus on a case in which the heterogeneity is small. In Chapter 5, we focus on a case in which the heterogeneity is large.

Chapter 2

Review of the Literature

2.1 Introduction

In this chapter, we review the literature on strategic voting and the Condorcet jury theorem.

The issue in this literature is the efficiency of information aggregation by voting. The efficiency of decision is measured by the probability that the committee chooses the better alternative. The efficiency depends on voting behavior of committee members, decision rule, and committee size. Using the framework of strategic voting, economists have studied the voting behavior in committees and how to design the committees to decide efficiently.

For this study, a prototype model of decision-making in committees is as follows. Suppose that there are two alternatives and that a committee of $2n + 1$ members must decide to choose one of the alternatives. One is better than the other for the committee, but no member of the committee knows which one is better. However, each committee member receives some private information that conveys which alternative is the better alternative. After receiving the information, they vote and the committee decision is made by the decision rule.

2.2 Classic Condorcet Jury Theorem

The issue of information aggregation by voting in committees was argued by Condorcet (1785). (McLean and Hewitt (1994).) Today, his results is called the classic Condorcet jury theorem. The classic Condorcet jury theorem (CJT) states as follows. Assume that the probability that a member votes for better alternatives are exogenously given and larger than 1/2. Assume also that each member's voting probabilities are independent. Then, the probabilities that the committee decides to choose better alternatives under the simple majority rule are increasing in n (monotonicity theorem). Moreover, under the same assumptions, the probabilities go to one as the committee size goes to infinity (limit theorem). This implies that large committee can decide more efficiently than any single person decision.

In spite of the importance of the CJT in our understanding of our society, the claim of Condorcet (1785) has not been noticed by economists until recently, partly because he wrote in French and partly because it was said that his contribution was not understood appropriately and was not appreciated by later sociologists in France. When economists began to develop the political economy by using mathematical models in the middle of the 20th century, Black (1958) discovered the claim of Condorcet (1785). Then, economists came to pay attentions to the CJT. However, even though Condorcet (1785) proved his claims mathematically (pp.3-9), his proof of the theorem has not been widely noted to economists. Several economists rather tried to prove the CJT themselves. The monotonicity CJT is proved by applying the properties of binomial distribution and the Limit CJT is an application of the law of large numbers. (See Nitzan and Paroush (1985).)

In the classic CJT, it is studied how the efficiency of decision depends on decision rule and the committee size when it is assumed that the voting behavior is exogenously given. However, in the view of game theory, the assumption of the voting behavior is suspicious because the voting behavior is endogenously decided by the player in the strategic situation. Then, it is also doubtful whether the CJT holds or not. In fact, it is well known today that the assumptions made in the classic CJT may not hold under strategic voting.

In 1990's, economists began to study voting behaviors in the framework of strategic voting; a member considers voting behaviors of the other committee members and votes to maximize his utility from a committee decision that is expected under his vote. These studies constitute the literature of strategic voting.

2.3 Basic Model of Strategic Voting in Committees and the CJT

In this section, we describe the basic model of strategic voting in committees and state the earliest and most fundamental results based on this model.

2.3.1 The Basic Model

It is Austen-Smith and Banks (1996) that first analyzed the issue of the equilibrium of Bayesian voting games. There are two states of the world, A and B. State A is a state in which alternative A is better than alternative B for all the members. State B is the opposite. No member knows the true state. Before each member votes, he receives a signal $s_i \in \{a, b\} = S$ that conveys information about the state of the world. Member i receives a signal $s_i = a$ with probability t or $s_i = b$ with probability $1 - t$ when the state is $w = A$. Similarly, member i receives a signal $s_i = b$ with probability t or $s_i = a$ with probability $1 - t$ when the state is $w = B$. The probability t is interpreted as the degree of information precision of member i. It is assumed that the degree of information precision t is the same for all the members and $t > 1/2$. After receiving the signal, the committee members vote for alternative A or B simultaneously. A decision $d \in \{A, B\}$ is made by the simple majority rule. After the decision, each member receives payoff depending on the decision and the state. It is assumed that the utility from choosing the better alternative is the same between the states A and B and normalized to 1, and the utility from choosing the worse alternative is the same between states and normalized to 0. Then, the utility function is as follows;

$$u(d = A|w = A) = u(d = B|w = B) = 1$$

and

$$u(d = B|w = A) = u(d = A|w = B) = 0.$$

2.3.2 Voting Behavior in Equilibrium and the CJT

Two natural voting behaviors are conceivable in this voting game. The first is the sincere voting. This behavior means that a committee member votes for A if and only if his expected utility from committee decision A under the posterior belief given his signal is larger than his utility from committee decision B. The sincere voting is intuitive because it is optimal to choose an alternative based on his own information when he decides for himself.

The second is the informative voting. It is a voting behavior as follows. A committee member votes for A if he receives the signal $s_i = a$ and he votes for B if he receives the signal $s_i = b$. Under the simple majority rule, it is natural to aggregate the signals received by the committee members directly. Moreover, if a committee member takes the informative voting behavior, the probabilities that he votes for the better alternatives on each states are larger than 1/2. Then, if all committee members take the informative voting, the Limit CJT holds.

Austen-Smith and Banks (1996) examined whether these two behaviors constitute an equilibrium. They showed that the informative voting is the sincere voting and it constitute an equilibrium in the voting game only under some restricted condition. In generic cases, the sincere voting is neither an informative voting nor an equilibrium behavior. This is because in addition to his signal, a committee member uses in equilibrium the information that he is pivotal, that is, the n members vote for A and the other n member votes for B. His vote affects the committee decision if and only if he is pivotal. In this situation, the committee decision is $d = A$ if he votes for A and $d = B$ if he votes for B. On the other hand, his vote does not change the committee decision if he is not pivotal. When he is pivotal, this additional information updates his posterior. Then, the sincere voting will not be a best response generically.

Austen-Smith and Banks (1996) did not show what is the equilibrium voting behavior. Feddersen and Pesendorfer (1998) explicitly stated the condition for an equilibrium in mixed strategies in the model of Austen-Smith and Banks (1996). They showed by considering this condition that the Limit CJT holds under the simple majority rule. They also showed that the CJT holds under other majority rules and the CJT does not hold under the unanimity rule in this model. This is the earliest and most fundamental results on the CJT under strategic voting.

2.4 Extended Models

In the basic model, three important assumptions were made. They are the assumptions about information structure, procedures of decision-making, and common preferences. In this section, we review studies that followed Feddersen and Pesendorfer (1998) and examined whether the CJT holds or not without these assumptions.

2.4.1 Information Structure

In the basic model, it is assumed that each member receives binary signal with common information precision. There are some extended models about information structures. One is a continuous signal model. It is an extended model in which the signal space is continuous. Another extension is a costly information acquisition model. While an information system about the true state is given in the basic model, this model describe the process by which a committee member is endowed with the information about the state.

(1-a) Continuous Signal

Duggan and Martinelli (2001) and Meirowitz (2002) studied continuous signal models. They assume that each member receives a signal realized in a compact interval according to a continuous density. They showed that the Limit CJT also holds in these extended signal models and that the Limit CJT holds not only in the majority rules but also the unanimity rule if the likelihood ratio of the signal distribution is unbounded. Their result means that the Feddersen and Pesendorfer's result on majority rules is robust with respect to signal space. On the other hand, their negative result on the unanimity rule depends on the specific property of the binary signal model that it does not satisfy the unboundedness of the likelihood ratio of the signal distribution.

(1-b) Costly Information Acquisition

In the costly information acquisition model, each member chooses to acquire the information or not. If he acquires the information, he pays the cost and receives a signal. He has no information about the state when he does not acquire the information. Mukhopadhaya (2003) considered then simple majority rule and showed a case in which large committees can not decide more efficiently than small committees for some parameters, that is, the monotonicity theorem does not hold. The members in the large committees has incentive to free-ride on the other members' information acquisition. Persico (2004) analyzed the optimal voting rule in terms of the fraction of the necessary votes in the total number of voters under costly information acquisition. Koriyama and Szentes (2009) analyzed the optimal committee size under costly information acquisition. They showed that the optimal committee size n^* is bounded, and that any oversized committee beyond n^* is inferior than the optimal committee only slightly in the sense that it can decide more efficiently than the size of $n^* - 2$.

2.4.2 Procedures of Decision-making

In the basic model, three particular features in a procedure for decision-making are assumed. First, a member is not allowed to abstain. Second, a member is not allowed to communicate. Third, all the members must vote simultaneously. In this section, we review studies that considered other procedures of decision-making in committees: strategic abstention, communication before voting, and sequential voting.

(2-a) Strategic Abstention

In the basic model, it is assumed that the degrees of information precision are the same for all members. Feddersen and Pesendorfer (1996) studied strategic abstention. They introduced the uncertainty of the degrees of information precision in an election model. Let t_i denote member i's degree of information precision. In their model, there are two types of the degree of information precision. One is that the degree of information precision is perfect, that is, $t_i = 1$. The other one is that the degree of information precision is null, that is, $t_i = 1/2$. It is assumed that the degree of the information precision is his private information. They showed that the perfect information type members take the informative voting and the null information type members choose abstain.

McMurray (2013) extended their model. He studied a model in which the degree of information precision is continuously distributed on the interval from 1/2 to 1. He showed that a member takes the informative voting when his information precision is above the cutoff and a member chooses abstain when his information precision is below the cutoff. In his model, the information of the less informed members is ignored. Then, the committee with more members can decide more efficiently than or at least as efficiently as the committee with less members. That is, the monotonicity CJT holds.

(2-b) Communication

Coughlan (2000) studied the role of communication before voting. He considered a two stage voting in which all the members simultaneously vote in the preliminary voting, they observe the result of the preliminary voting, and they vote to decide the committee decision. He showed that the sincere revelation strategy constitutes an equilibrium in the preliminary voting stage under some conditions. Coughlan (2000) showed that the committee can decide efficiently under the unanimity rule. This result contrast with the negative result of Feddersen and Pesendorfer (1998) that the CJT does not hold under the unanimity rule.

(2-c) Sequential Voting

In the basic model, each member votes simultaneously. When the voting is sequential, the later voter can use the information of earlier voting. Dekel and Piccione (2001) studied sequential voting. In their model, the order of voters is exogenously given. They showed that there exists an equilibrium in the sequential voting that supports the outcome of the equilibrium of simultaneous voting game. In this equilibrium, each member votes according to his private information and previous votes under the condition that he is pivotal. This voting behavior is consistent with the voting behavior in the simultaneous voting that depends on his private information under the condition that he is pivotal. Their result means that the efficiency of decision under sequential voting is at least as good as that under the simultaneous voting.

In contrast with Dekel and Piccione (2001), Rokas and Tripathi (2008) studied endogenous sequential voting. In the endogenous sequential voting model, each member can choose the timing of his vote. Rokas and Tripathi (2008) showed that the committee can achieve the ex-post optimal decision if there are sufficiently many chances to vote. This implies that both of the monotonicity theorem and the limit theorem hold under the endogenous sequential voting.

Rokas and Tripathi (2008) considered a particular algorithm to achieve the ex-post optimal decision in the endogenous sequential voting. Nagaoka and Suehiro (2013) showed that the ex-post optimal decision can be supported in a wider class of algorithms.

2.4.3 Conflicts of Interests

In the basic model, it is assumed that each member has the same preferences. That is, there are no conflict among the members. There are some alternative approaches to relax this assumption. One is to allow the conflicting preferences over the decision. Another one is to introduce the reputation concern preferences.

(3-a) Conflicting Preferences

Feddesen and Pesendorfer (1997) consider a case in which each member may have different preferences over the decision than the other members. They analyzed a voting game in which each member's payoff depends on the state, the committee's decision and his preference type. They showed that the fraction of voters who take informative action goes to zero as the committee size goes to infinity. In the equilibrium, the full information equivalence occurs, that is, the decision would not change with almost probability one even if all the private information were common knowledge under some assumptions.

(3-b) Reputation Concern

A member is said to have reputation concern preferences when he would like to show that his degree of information precision is high. It is easily imagined that reputation concern preferences may distort the information aggregation by voting in committees.

Levy (2004) studied the decision-making by an individual with reputation concern preferences. She showed that the reputation concerned individual has an incentive to deviate from the sincere decision. Levy (2007) considered the simultaneous voting by reputation concerned members in which the evaluator of the voters observe each vote and the true state. The equilibrium behavior in this voting game becomes identical with the one in Levy (2004).

Ottaviani and Sørensen (2001) analyzed the sequential voting under the assumption of the reputation concerns. They showed that the later voter has an incentive to take a herding behavior. They also analyzed the optimal order of voting.

2.5 The Relationship between This Study and the Literature

We study two issues on the strategic voting and the CJT in this dissertation.

The first is whether the monotonicity theorem holds or not under the strategic voting. The previous studies on the strategic voting focused on the limit theorem and they did not argue the monotonicity theorem. Our study is the first attempt to study the monotonicity theorem under the strategic voting. We show two results on this issue. One is that the monotonicity theorem holds under strategic voting for sufficiently large committees. The other one is that there exists a case in which the monotonicity theorem does not hold for a small committee.

The second issue in our studies is whether the CJT holds or not under the heterogeneity of the committee members. In the previous studies, it was assumed that the information system of each member is homogeneous, that is, each member is identical in his ability to receive his information about the state of the world. However, it is common in the real world that members' information systems are heterogeneous. For example, some members are professional and some members are amateur on the problem for the committee to decide. Under the heterogeneity of information systems, it is naturally expected that the voting behaviors are complexed under the strategic voting. Therefore, it remains unclear whether this complexity disturbs the claims of the CJT.

We study the strategic voting in a committee with the members who have heterogeneity of information systems. We consider two extreme cases in which heterogeneity of information systems prevails among the committee members.

One is a case in which the heterogeneity is small in the sense that almost all members are identical in their information systems and only one member differs from the rest of the members. We show that in this case, the voting behavior in equilibrium converges to informative voting. When there are no heterogeneity, the voting behavior in equilibrium under the simple majority rule converges to the informative voting. The result for the committee with small heterogeneity is consistent with the fact for the homogenous committee. Then, the limit CJT holds for the committee with small heterogeneity.

The other one is a case in which the heterogeneity is large in the sense that there are two large sub-groups in the committee and the information systems of the members in one sub-group are different from those in the other sub-group, while members in the same sub-group are identical in their information systems. We show that in this case, the voting behavior in equilibrium may not converge to the informative voting. However, we also show that the limit CJT still holds.

To study these issues, we consider the uncertainty about member's information precision. That is, a member's degree of information precision is a realization of a continuous random variable and a realized degree of information precision is his private information. Our model is regarded as an extension of the basic model of Austen-Smith and Banks (1996) and Feddersen and Pesendorfer (1998) with binary signals to the model with continuous signals.

Duggan and Martinelli (2001) and Meirowitz (2002) also studied the strategic voting by using continuous signal models. Their models are similar to our model. Our model can be transformed to their continuous signal models. Our model has several advantages over them in the study of the CJT. One is that we can understand the voting behavior in equilibrium and its properties more intuitively in our signal models than in their abstract signal models. Another advantage is that we can define the heterogeneity of information systems among the members by natural way in our signal model. In the above mentioned real world situation, we can describe the professional members' distribution and the amateur members' distribution by using stochastic ordering of distributions.

Chapter 3

Strategic Voting and Condorcet Jury Theorem under Uncertainty about Player's Information Precision

3.1 Introduction

Voting is one of the most basic procedures by which a group of people with diverse judgments makes a decision. The question is how efficiently voting aggregates individual judgments. There is a well-known classical result, called the Condorcet jury theorem (CJT), established by Condorcet (1785). This theorem states the following. Suppose that there are two alternatives and that a committee of $2n + 1$ members must decide to choose one of the alternatives. One is better than the other for the committee, but no member of the committee knows which one is better. Assume that the probability that a member votes for better alternatives are exogenously given and larger than 1/2. Then, the probabilities that the committee decides to choose better alternatives under the simple majority rule are increasing in n.[1] The CJT provides an important implication for the committee design problem. When a group of individuals organizes a committee by selecting its members from the group and lets the committee make a decision for the group, the best is achieved if all the individuals of the group join the committee.

Today, it is well known that the assumptions made in the CJT may not hold under strategic voting; a member votes to maximize his utility from a committee decision that is expected under his vote. The purpose of this chapter is to reexamine the issue of the relation between the expected utility and the number of committee members when they vote strategically.

The voting game that we analyze is as follows. There are two states that represent which is the better alternative. Each committee member receives a signal. After receiving the signal, they vote simultaneously. The committee decision is made by the simple majority rule. In this model, we assume that a member receives a signal with some precision. The information precision of a voter is determined randomly. Then, the voter receives a binary signal about the better alternative with his information precision. We assume that the information precision and the signal become his private information.

We study the symmetric responsive equilibrium in this Bayesian voting game. We establish three results. First, we provide a sufficient condition for welfare monotonicity. Second, we give an example in which adding members

[1] Under the same assumptions, Condorcet also showed that the probabilities go to one as the committee size goes to infinity.

to the committee does not improve welfare in a small committee. The effect of adding the members to the committee depends on a probability distribution of information precision and the parameters of the game. Finally, we show that the CJT holds for sufficiently large committees, that is, the probability that the committee selects the better alternative under the simple majority rule is increasing in n for all n large enough.

The rest of this chapter is as follows. In Section 2, we explain our model. In Section 3, we show that there exists a symmetric responsive equilibrium and it is characterized by a cutoff strategy. Section 4 presents the welfare analysis. Section 5 discusses the welfare in the limit as n goes to infinity. Section 6 provides some concluding remarks.

3.1.1 Related Literature

There is another claim known as the Condorcet jury theorem. It is the Limit Condorcet jury theorem (Limit CJT). It claims that the probability that the committee decides to choose the better alternative goes to one as the committee size goes to infinity. Feddersen and Pesendorfer (1998) studied whether the Limit CJT holds under strategic voting. They analyzed a voting game in which each voter receives a binary signal. They calculated the symmetric mixed strategy equilibrium of this game and showed that the Limit CJT holds for any voting rule except unanimity rules.[2] They assumed that the degree of information precision is exogenously given and the same for all voters and that the degree of precision is common knowledge. We introduce

[2]The basic result by Feddersen and Pesendorfer (1998) was followed by many studies that examine strategic voting and the Limit CJT. Gerling et al. (2005) and Li and Suen (2009) provide an overview of this literature.

an uncertainty about the degree of information precision. Our model allows a situation in which some members may have more precise information than other members. The cutoff strategy equilibrium in our model is related to the mixed strategy equilibrium in the Feddersen and Pesendorfer model. In the Feddersen and Pesendorfer model, a voter mixes his vote between two alternatives when he receives a particular signal. In our model, given a particular signal, a voter votes for an alternative when his information precision is above a certain cutoff point and votes for the other alternative when the information precision is below the cutoff point. We can interpret this cutoff strategy in our model as a purification of the mixed strategy in the Feddersen and Pesendorfer model.

This chapter is closely related to Duggan and Martinelli (2001) and Meirowitz (2002). They constructed a continuous signal model. They showed that there exists a symmetric cutoff equilibrium and that the Limit CJT holds for majority rules. They also showed that the Limit CJT holds for unanimity rules if and only if there exists a strong signal.

Meirowitz (2002) also studied a similar continuous signal model and showed the Limit CJT. Then, he pointed out that this Limit CJT implies that the decision-making by n-members committee is more efficient than a single person decision-making for sufficiently large n. Furthermore, Meirowitz (2002) claimed to show an example in which the efficiency of decision-making by three-member committee is less efficient than a single person decision-making.

In contrast with them, we consider the relation between the expected utility and the number of committee members. Furthermore, we point out that the example of Meirowitz (2002) seems to have a flaw. We provide an example in which the decision-making by a larger committee is not necessarily more efficient than the decision-making by a smaller committee.

McMurray (2013) is an important study related to this chapter. His model is a continuous model of strategic abstention of Feddersen and Pesendorfer (1996). He constructed a continuous information precision model with an option that the voter can choose abstention. He showed that a voter who is uninformed chooses to abstain and a voter who has precise information votes according to his signal. This model can be interpreted as an election model. In contrast, we consider a committee model in which the committee members are not allowed to abstain.

3.2 Model

We consider a $2n+1$ member committee. The committee chooses between alternatives A and B. The members have the same preferences over the alternatives, depending on the state of the world. There are two states of the world, A and B. State A is a state in which alternative A is better than alternative B for all the members. State B is the opposite. Let $d \in \{A, B\}$ denote a committee's decision and $w \in \{A, B\}$ denote a state of the world. Then, the utility function is as follows: $u(d = A|w = A) = u(d = B|w = B) = 0, u(d = A|w = B) = -c, u(d = B|w = A) = -(1-c)$. We assume that the utility from choosing the better alternative is the same between state A and state B and normalized to 0. The parameter c represents a utility loss from choosing an alternative A when the state is B. Similarly, the parameter $1-c$ represents a utility loss from choosing an alternative B when the state is A.

The committee decision is made by voting by the committee members. The voting rule is the simple majority rule. Each member votes for alternative A or B.[3] The committee decision is $d = A$ if and only if at least $n + 1$ members vote for A.

No member knows the true state. Before each member votes, however, he receives a signal $s \in \{a, b\} = S$ that conveys information about the state of the world. Member i receives a signal $s_i = a$ with probability t_i or $s_i = b$ with probability $1 - t_i$ when the state is $w = A$. Similarly, member i receives a signal $s_i = b$ with probability t_i or $s_i = a$ with probability $1 - t_i$ when the state is $w = B$. The probability t_i is interpreted as the degree of information precision of member i. The degree of information precision t_i is private information of member i. The degree of information precision t_i is distributed on $[1/2, 1] = T$ with a full support density f for all i. Thus, each member has two kinds of private information, the signal s_i and the degree of information precision t_i. The signal s_i is realized independently across the members given the state. The degree of information precision t_i is realized independently across the members and the states.

The timing of the game is as follows. A state $w \in \{A, B\}$ is realized with probability $\Pr(w)$. We denote $\Pr(B) = q > 1/2$. Each committee member i is endowed with his information precision $t_i \in [1/2, 1]$ according to the density f. Then, each committee member i receives a signal $s_i \in \{a, b\}$. Then, the committee members vote for alternative A or B simultaneously. A decision $d \in \{A, B\}$ is made by the simple majority rule. The members receive utilities depending on the decision made and the state realized.

[3] The members are not allowed to choose abstention.

3.3 Equilibrium Analysis

A committee member has private information $(s, t) \in S \times T$. A (pure) strategy of member i is a function $m_i : S \times T \to \{A, B\}$. We consider strategies such that the member changes his vote according to his information. We call them responsive strategies. They are formally defined as follows.

Definition 3.1 (responsive strategy)**.** *The strategy is responsive if there exist* (s, t) *and* (s', t') *such that* $m_i(s, t) \neq m_i(s', t')$ *and the sets* $\{(s, t)|m_i(s, t) = A\}$ *and* $\{(s, t)|m_i(s, t) = B\}$ *have positive measures.*

Let $\gamma_A = \Pr(m = A|w = A)$ and $\gamma_B = \Pr(m = B|w = B)$ denote the voting probability that a member votes for the better alternative given the state. Then, the voting probabilities γ_A and γ_B under a responsive strategy are $0 < \gamma_A, \gamma_B < 1$. To see this, let $\Pr(s, t|A)$ and $\Pr(s, t|B)$ denote the joint probability distribution of (s, t) given the state. Note that $\Pr(s, t|A)$ and $\Pr(s, t|B)$ are (almost everywhere) full support because f is full support. Then, the voting probabilities for each alternative are

$$\begin{aligned}
\Pr(m = A) &= \int_{(s,t)\in\{(s,t)|m_i(s,t)=A\}} \Pr(A)\Pr(s, t|A)dsdt \\
&\quad + \int_{(s,t)\in\{(s,t)|m_i(s,t)=A\}} \Pr(B)\Pr(s, t|B)dsdt \\
&= \Pr(A)\gamma_A + \Pr(B)(1 - \gamma_B)
\end{aligned}$$

$$\begin{aligned}
\Pr(m = B) &= \int_{(s,t)\in\{(s,t)|m_i(s,t)=B\}} \Pr(A)\Pr(s, t|A)dsdt \\
&\quad + \int_{(s,t)\in\{(s,t)|m_i(s,t)=B\}} \Pr(B)\Pr(s, t|B)dsdt \\
&= \Pr(A)(1 - \gamma_A) + \Pr(B)\gamma_B.
\end{aligned}$$

Suppose $\gamma_A = 0$, that is

$$\int_{(s,t)\in\{(s,t)|m_i(s,t)=A\}} \Pr(s,t|A)dsdt = 0.$$

This implies that the set $\{(s,t)|m_i(s,t) = A\}$ has a zero measure because $\Pr(s,t|A)$ is full support. Then,

$$\int_{(s,t)\in\{(s,t)|m_i(s,t)=A\}} \Pr(s,t|B)dsdt = 0.$$

This implies $\Pr(m = A) = 0$. This is a contradiction to the definition of a responsive strategy and, therefore $\gamma_A > 0$. Similarly, $\gamma_B > 0$ and $\gamma_A, \gamma_B < 1$.

We analyze the symmetric equilibrium in responsive strategies, in which all the members use the same responsive strategy. First, we consider the best response to responsive strategies. The condition by which a voter who has private information (s,t) weakly prefers voting for A to voting for B is that the expected utility from voting for A is more than or equal to the expected utility from voting for B. That is,

$$\begin{aligned}
\Pr(A)\Pr(s,t|A) &\times [\{\Pr(\text{more than } n+1 \text{ voters vote for } A|A) \\
&\quad + \Pr(n \text{ voters vote for } A|A)\}u(A|A) \\
&\quad + \{\Pr(\text{fewer than } n-1 \text{ voters vote for } A|A)\}u(B|A)] \\
+\Pr(B)\Pr(s,t|B) &\times [\{\Pr(\text{more than } n+1 \text{ voters vote for } A|B) \\
&\quad + \Pr(n \text{ voters vote for } A|B)\}u(A|B) \\
&\quad + \{\Pr(\text{fewer than } n-1 \text{ voters vote for } A|B)\}u(B|B)] \\
\geq \Pr(A)\Pr(s,t|A) &\times [\{\Pr(\text{more than } n+1 \text{ voters vote for } A|A)\}u(A|A) \\
&\quad + \{\Pr(n \text{ voters vote for } A|A) \\
&\quad + \Pr(\text{fewer than } n-1 \text{ voters vote for } A|A)\}u(B|A)] \\
+\Pr(B)\Pr(s,t|B) &\times [\{\Pr(\text{more than } n+1 \text{ voters vote for } A|B)\}u(A|B) \\
&\quad + \{\Pr(n \text{ voters vote for } A|B) \\
&\quad + \Pr(\text{fewer than } n-1 \text{ voters vote for } A|B)\}u(B|B)].
\end{aligned}$$

Let *piv* denote an event in which a member is pivotal, that is, n voters vote for A. Then, the condition is arranged as follows.

$$\frac{\Pr(A)\Pr(s,t|A)\Pr(piv|A)}{\Pr(B)\Pr(s,t|B)\Pr(piv|B)} \geq \frac{u(A|B)}{u(B|A)} \tag{3.1}$$

In particular, the indifference condition is

$$\frac{\Pr(w=A|s,t)}{\Pr(w=B|s,t)}\frac{\Pr(piv|A)}{\Pr(piv|B)} = \frac{u(A|B)}{u(B|A)}. \tag{3.2}$$

We show that the best response is a cutoff strategy if the other members use responsive strategies.

Definition 3.2 (cutoff strategy)**.** *A strategy is a cutoff strategy if there exists a cutoff point* (s^*, t^*) *such that* $m(s,t) = A$ *if and only if* $\Pr(w = A|s,t) \geq \Pr(w = A|s^*, t^*)$.

Lemma 3.1. *A best response to responsive strategies must be a cutoff strategy.*

Proof. The indifference condition (3.2) is rearranged to

$$\frac{\Pr(w = A|s,t)}{\Pr(w = B|s,t)} = \frac{\Pr(piv|B)u(A|B)}{\Pr(piv|A)u(B|A)}. \tag{3.3}$$

The right-hand side of (3.3) is finite and positive because we consider responsive strategies. The left-hand side of (3.3) is the ratio of the posterior belief given his information (s,t), which is

$$\frac{\Pr(w = A|s,t)}{\Pr(w = B|s,t)} = \begin{cases} \frac{(1-q)}{q}\frac{t}{(1-t)} & \text{for } s = a \\ \frac{(1-q)}{q}\frac{(1-t)}{t} & \text{for } s = b \end{cases}. \tag{3.4}$$

For $s = a$, this ratio is increasing in t and goes to infinity as t goes to 1. For $s = b$, this ratio is decreasing in t and goes to 0 as t goes to 1. Note that the ratio is $(1-q)/q$ at $t = 1/2$ for $s = a$ and $s = b$. Therefore, the ratio is continuous. Hence, there exists a unique (s^*, t^*) such that equation (3.3) holds. Then,

$$\frac{\Pr(w = A|s,t)}{\Pr(w = B|s,t)} \geq \frac{\Pr(piv|B)u(A|B)}{\Pr(piv|A)u(B|A)}$$

for (s,t) such that $\Pr(w = A|s,t) \geq \Pr(w = A|s^*, t^*)$. This means that the best response must be a cutoff strategy. $\square$

By Lemma 3.1, we focus on the symmetric equilibrium in cutoff strategies. Let (s^*, t^*) be a cutoff point in a cutoff strategy. Then, the voting probabilities given the states by the cutoff strategy is as follows. If the cutoff point signal is $s^* = a$, a member votes for A if and only if $s = a$ and $t \geq t^*$. Therefore,

$$\begin{aligned} \gamma_A &= \int_{t^*}^{1} t f(t) dt \\ \gamma_B &= 1 - \int_{t^*}^{1} (1-t) f(t) dt. \end{aligned}$$

Similarly, if the cutoff point signal is $s^* = b$,

$$\begin{aligned}\gamma_A &= 1 - \int_{t^*}^{1} (1-t) f(t) dt \\ \gamma_B &= \int_{t^*}^{1} t f(t) dt.\end{aligned}$$

With these probabilities, the indifference condition (3.2) is written as follows.

$$\frac{\Pr(w = A|s,t)}{\Pr(w = B|s,t)} \frac{\gamma_A^n}{(1-\gamma_B)^n} \frac{(1-\gamma_A)^n}{\gamma_B^n} = \frac{u(d = A|w = B)}{u(d = B|w = A)} \tag{3.5}$$

In the next theorem, we prove the unique existence of cut point (s^*, t^*) that satisfies condition (3.5).

Theorem 3.1. *There exists a unique symmetric equilibrium in responsive strategies characterized by a cutoff strategy with cutoff point (s^*, t^*).*

Proof. To consider symmetric equilibrium, let $\tau = t = t^*$ on the left-hand side of (3.5). The first term on the left-hand side of (3.5) is the ratio of posterior belief (3.4). We have already shown in the proof of Lemma 3.1 that it is continuous and monotonic in τ, it goes to infinity as τ goes to 1 for $s = a$, and it goes to 0 as τ goes to 1 for $s = b$. We show that the second term

$$\left(\frac{\gamma_A}{1-\gamma_B}\right)^n \left(\frac{1-\gamma_A}{\gamma_B}\right)^n$$

on the left-hand side of (3.5) has the same properties.

Consider the case of $s = a$. Then,

$$\frac{\gamma_A}{1-\gamma_B}$$

is obviously continuous in τ. It is increasing in τ because

$$\frac{d}{d\tau}\frac{\int_\tau^1 t f(t)dt}{\int_\tau^1 (1-t) f(t) dt} = \frac{f(\tau)\int_\tau^1 (t-\tau) f(t)dt}{\left(\int_\tau^1 (1-t) f(t)dt\right)^2} > 0.$$

Finally, by L'Hospital's rule, we have

$$\lim_{\tau\to 1}\frac{\int_\tau^1 tf(t)dt}{\int_\tau^1(1-t)f(t)dt}=\lim_{\tau\to 1}\frac{-\tau f(\tau)}{-(1-\tau)f(\tau)}=\infty.$$

Similarly,

$$\frac{1-\gamma_A}{\gamma_B}$$

is continuous in τ. It is increasing in τ because

$$\frac{d}{d\tau}\frac{1-\int_\tau^1 tf(t)dt}{1-\int_\tau^1(1-t)f(t)dt} \quad = \quad \frac{(2\tau-1)f(\tau)+f(\tau)\int_\tau^1(t-\tau)f(t)dt}{\left(1-\int_\tau^1(1-t)f(t)dt\right)^2}>0.$$

Finally, we have

$$\lim_{\tau\to 1}\frac{1-\int_\tau^1 tf(t)dt}{1-\int_\tau^1(1-t)f(t)dt}=1.$$

Hence, the second term on the left-hand side of (3.5), that is,

$$\left(\frac{\gamma_A}{1-\gamma_B}\right)^n\left(\frac{1-\gamma_A}{\gamma_B}\right)^n$$

is continuous and increasing in τ, and it goes to infinity as τ goes to 1 for $s=a$.

For the case of $s=b$, an analogous argument establishes that the second term of (3.5) is continuous and decreasing in τ, and it goes to 0 as τ goes to 1.

Note that $\gamma_A=\gamma_B=E[t]$ at $\tau=1/2$ for $s=a$ and $s=b$. Therefore, the second term of (3.5) is 1 at $\tau=1/2$ for $s=a$ and $s=b$.

Hence, the left-hand side of (3.5) is continuous and monotone in τ, and it goes to ∞ as τ goes to 1 for $s=a$ and goes to 0 as τ goes to 1 for $s=b$. Therefore, there exists a unique (s^*,t^*) such that (3.5) holds. $\square$

The intuition of Theorem 3.1 is as follows. Suppose that every member uses the same cutoff strategy. If the cutoff point signal is $s^* = a$ and t^* is large enough, a member who has the threshold degree of precision t^* should not be indifferent between voting for A and voting for B, because he believes with a high probability that the state is $w = A$. First, the received signal $s^* = a$ indicates that the state is $w = A$ with a high probability, because he receives the signal under the high precision t^*. Second, the event that he is pivotal conveys information from which he believes that the state is $w = A$. The members vote for A when they receive the signal $s = a$ and t is large enough. Therefore, when the state is $w = B$, the probability that a member votes for A is very small, because a member with a large t receives the signal $s = b$ with a high probability. Hence, when the state is $w = B$, the probability that he is pivotal is small. From this information, he believes with a high probability that the state is $w = A$, and he votes for A. Therefore, a large t^* is not an equilibrium. Similarly, if the cutoff point signal is $s^* = b$ and t^* is large enough, a member who has the threshold degree of precision t^* should not be indifferent between voting for A and voting for B. Both the ratio of posterior beliefs and the ratio of probabilities that he is pivotal given the states are continuous and monotonic. Therefore, there exists a unique equilibrium.

3.4 Welfare Analysis

In this section, we analyze the relationship between the committee size and the ex ante expected utility from the symmetric equilibrium in responsive strategies.

The classical result is the CJT. Suppose that the voting probabilities for the better alternative given the state are exogenously given and fixed. Then, if the voting probabilities are larger than 1/2, the following holds about the probabilities that the committee chooses better alternatives given the states.

Proposition 3.1 (CJT). *Suppose that γ_A and γ_B are exogenously given and fixed for n. If $\gamma_A, \gamma_B > 1/2$, then*

1. $\Pr(d = A|w = A)$ *and* $\Pr(d = B|w = B)$ *are monotonically increasing in n,*

2. $\Pr(d = A|w = A)$ *and* $\Pr(d = B|w = B)$ *go to* 1 *as n goes to infinity.*

CJT implies that the expected utility

$$E_{2n+1}[u] = \Pr(A)\Pr(d = B|A)u(B|A) + \Pr(B)\Pr(d = A|B)u(A|B)$$

is increasing in n and goes to 0 as $n \to \infty$ if γ_A and γ_B are exogenously given and larger than 1/2.

3.4.1 A Sufficiency Theorem for Welfare Monotonicity

To provide a sufficient condition for welfare monotonicity, we first consider a relation between the committee size and the cutoff point in equilibrium.

Lemma 3.2. *t^* is decreasing in n.*

Proof. We have shown that the left-hand side of (3.5) is increasing in t^* for $s^* = a$ and decreasing in t^* for $s^* = b$. The left-hand side of (3.5) is increasing in n for $s^* = a$ and decreasing in n for $s^* = b$ because

$$\frac{\gamma_A(1-\gamma_A)}{(1-\gamma_B)\gamma_B}$$

is 1 when $t^* = 1/2$ and increasing (decreasing) in t^* for $s^* = a$ ($s^* = b$). Therefore, t^* is decreasing in n. $\square$

The indifference condition (3.5) is arranged as follows.

$$\frac{\Pr(s|w=A,t)}{\Pr(s|w=B,t)}\frac{\Pr(piv|w=A)}{\Pr(piv|w=B)} = \frac{q}{1-q}\frac{c}{1-c}. \tag{3.6}$$

Let $\lambda = \frac{q}{1-q}\frac{c}{1-c}$. The next lemma states a relation between λ and the cutoff point (s^*, t^*).

Lemma 3.3. *The cutoff point (s^*, t^*) satisfies the following properties.*

- *If $\lambda > 1$, the cutoff point (s^*, t^*) satisfies $s^* = a$ and $t^* \in (1/2, 1)$. Furthermore, t^* is increasing in λ and goes to 1 as λ goes to infinity.*
- *If $\lambda < 1$, the cutoff point (s^*, t^*) satisfies $s^* = b$ and $t^* \in (1/2, 1)$. Furthermore, t^* is decreasing in λ and goes to 1 as λ goes to 0.*
- *If $\lambda = 1$, the cutoff point (s^*, t^*) satisfies $s^* = a = b$ and $t^* = 1/2$.*

Proof. The first term of the left-hand side of (3.6) is

$$\frac{\Pr(s|w=A,t)}{\Pr(s|w=B,t)} = \begin{cases} \frac{t}{1-t} & \text{if } s = a \\ \frac{1-t}{t} & \text{if } s = b \end{cases}.$$

This is increasing in t if $s = a$ and decreasing in t if $s = b$, and equal to 1 if $t = 1/2$. We have shown in the proof of Theorem 3.1 that the second term of the left-hand side of (3.6),

$$\frac{\Pr(piv|w=A)}{\Pr(piv|w=B)} = \left[\frac{\gamma_A(1-\gamma_A)}{(1-\gamma_B)\gamma_B}\right]^n,$$

is increasing in t if $s = a$ and decreasing in t if $s = b$, and equal to 1 if $t = 1/2$. Therefore, if $\lambda > 1$, $s^* = a$ and t^* is increasing in λ. If $\lambda < 1$, $s^* = b$ and t^* is decreasing in λ. If $\lambda = 1$, $t^* = 1/2$.

Finally, we have shown in the proof of Theorem 3.1 that the left-hand side of (3.6) goes to infinity as t goes to 1 for $s = a$ and goes to 0 as t goes to 1 for $s = b$. Therefore, t goes to 1 as λ goes to infinity and t goes to 1 as λ goes to 0. $\square$

With this lemma, we can establish the following relation between λ and the voting probabilities in equilibrium.

Lemma 3.4. *The voting probability γ_A is decreasing in λ, $\lim_{\lambda\to 0}\gamma_A = 1$, $\lim_{\lambda\to\infty}\gamma_A = 0$, and $\gamma_A = E[t]$ at $\lambda = 1$. The voting probability γ_B is increasing in λ, $\lim_{\lambda\to 0}\gamma_B = 0$, $\lim_{\lambda\to\infty}\gamma_B = 1$, and $\gamma_B = E[t]$ at $\lambda = 1$.*

Proof. We consider a case of $\lambda > 1$. From Lemma 3.3, $s^* = a$. Furthermore, γ_A is decreasing in λ because t^* is increasing in λ and γ_A is decreasing in t^* for $s^* = a$. We have shown in Lemma 3.3 that t^* goes to 1 as λ goes to infinity. Obviously, γ_A goes to 0 as t^* goes to 1 for $s^* = a$. Therefore, γ_A goes to 0 as λ goes to infinity. The rest of Lemma 3.4 is proved similarly. $\square$

By this lemma, there exists a unique λ for which $\gamma_A = 1/2$. We call it $\overline{\lambda}$. Similarly, $\underline{\lambda}$ is a unique λ for which $\gamma_B = 1/2$. We know from Lemma 3.4 that $\underline{\lambda} < 1 < \overline{\lambda}$. These λ are illustrated in Figure 3.1.

These thresholds $\underline{\lambda}$ and $\overline{\lambda}$ depend on n, because t^* depends on n in equation (3.6) and, hence the voting probabilities γ_A and γ_B depend on n. To be explicit about this fact, let us denote the thresholds as $\underline{\lambda}(n)$ and $\overline{\lambda}(n)$. In the next theorem, we show the sufficient condition under which the expected utilities from equilibrium strategic voting in committees with different sizes can be compared.

Theorem 3.2 (a sufficient condition for welfare monotonicity)**.** *Take any n and n' such that $n < n'$. If $\lambda \in [\underline{\lambda}(n), \overline{\lambda}(n)]$, then $E_{2n+1}[u] < E_{2n'+1}[u]$.*

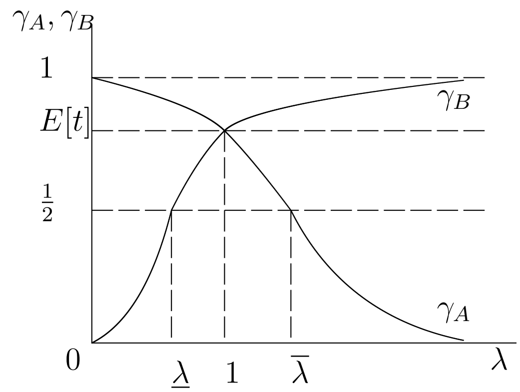

Figure 3.1: $\underline{\lambda}$ and $\overline{\lambda}$

Proof. Suppose $\lambda \in [\underline{\lambda}(n), \overline{\lambda}(n)]$. Let $\gamma_A(n)$ and $\gamma_B(n)$ denote the voting probabilities in the equilibrium in a committee with size $2n+1$. Then, Lemma 3.4 guarantees that $\gamma_A(n) \geq 1/2$ and $\gamma_B(n) \geq 1/2$ hold where at least one inequality must be strict. By CJT, we know that if these voting probabilities $\gamma_A(n)$ and $\gamma_B(n)$ apply in a committee of size $2n' + 1$, then the expected utilities for the committee are higher than for a committee of size $2n + 1$. Then, we compare the expected utility under the voting probabilities $\gamma_A(n)$ and $\gamma_B(n)$ with the expected utility from equilibrium strategic voting in a committee with size $2n' + 1$. In this voting game, every committee member has an identical utility function. Then, a symmetric responsive strategy that maximizes the expected utility is a symmetric responsive equilibrium (see McLenan (1998)). By Theorem 3.1, the symmetric equilibrium in responsive strategies is unique. Therefore, a symmetric strategy that maximizes the expected utility is unique and that is the unique symmetric responsive equilibrium. Hence, the expected utility in equilibrium is weakly larger than an expected utility from any symmetric responsive strategy. The voting probabilities $\gamma_A(n)$ and $\gamma_B(n)$ are generated by a symmetric responsive strategy. Hence, the expected utility from equilibrium strategic voting is weakly larger than the expected utility under the voting probabilities $\gamma_A(n)$ and $\gamma_B(n)$ in a committee of size $2n' + 1$. This establishes $E_{2n+1}[u] < E_{2n'+1}[u]$. $\square$

3.4.2 Small Committee

We consider the case in which $\lambda \notin [\underline{\lambda}(n), \overline{\lambda}(n)]$. In this case, the effect of adding members to the committee is not clear. We present two examples that show its effect in opposite directions.

Uniform Distribution Model

Let f be a uniform distribution and suppose $c = 1 - c = 1/2$. We consider a one-member committee and a three-member committee. Let $t^*(1)$ and $t^*(3)$ be the cutoff points in equilibrium in each committee. Note that $s^* = a$ and $t^*(1) = q$. For sufficiently large q, $\gamma_A(1) < 1/2$, so that we can not apply Theorem 3.2 with $n = 0$ and $n' = 1$. However, we can check that $E_3[u] - E_1[u]$ is positive for any $q \in [1/2, 1]$.[4]

[4]Meirowitz (2002) analyzed the efficiency of decision between the one-member committee and the three-member committee. He considered the continuous signal model. Each member receives the signal $x_i \in [0, 1]$ with conditional probability density functions $g(x_i|A)$ and $g(x_i|B)$. He considered an example in which the prior is $q = 0.7$ and the signal distributions are $g(x_i|A) = 2x_i$ and $g(x_i|B) = 2(1 - x_i)$. He claimed that in this example, the efficiency of decision-making by three-member committee is less efficient than a single person decision-making.

However, it seems that this claim is not true. To see it, note that our uniform distribution model corresponds to the example of Meirowitz (2002) through the following

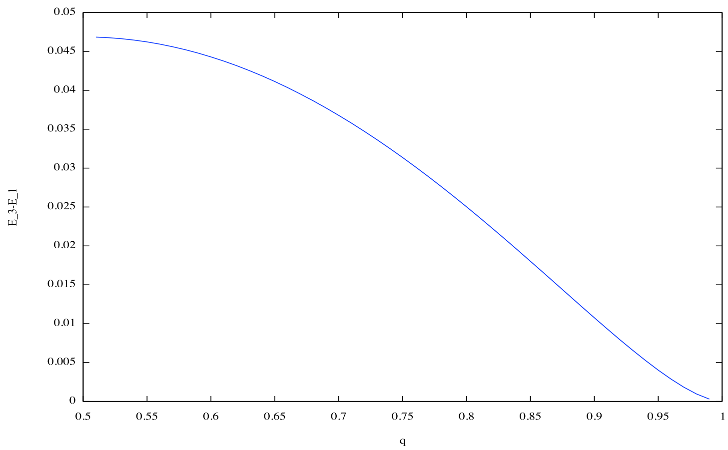

Figure 3.2: $E_3[u] - E_1[u]$ for $q \in [1/2, 1]$ under the uniform distribution

Bimodal Distribution Model

In this example, we consider a bimodal distribution in which probability is concentrated around $t = 1/2$ and $t = 1$. Consider the following distri-

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transformation. Consider a function $\lambda : (s_i, t_i) \to [0, 1]$ such that

$$\lambda(s_i, t_i) = \begin{cases} 1 - t_i & \text{if } s_i = a \\ t_i & \text{if } s_i = b \end{cases}$$

and let $x_i = \lambda(s_i, t_i)$. When the probability density function of t_i is uniform distribution, the probability density function of x_i induced by the transformation λ coincides with the above example.

In our model, we showed in Figure 2 that $E_3[u] - E_1[u]$ is positive for any $q \in [1/2, 1]$. Particularly, at $q = 0.7$, the efficiency of decision-making by three-member committee is more efficient than a single person decision-making. Meirowitz (2002) seems to have a flaw.

In the following subsection, we present a bimodal distribution model and show that in this model, the efficiency of decision-making by three-member committee is less efficient than a single person decision-making.

bution model as an extreme case of such a distribution. Suppose that the distribution of t is discrete and $\Pr(t = 1) = p$ and $\Pr(t = 1/2) = 1 - p$. Moreover, suppose $c = 1 - c = 1/2$. The equilibrium in this discrete model is as follows. A member who has $t = 1$ votes according to his signal. A member who has $t = 1/2$ votes for A with probability σ. This probability is determined by

$$\frac{\Pr(A)}{\Pr(B)} \frac{\Pr(piv|A)}{\Pr(piv|B)} = 1.$$

This equation corresponds to the indifference condition (3.6) of the continuous model.

The next graph shows $E_3[u] - E_1[u]$ with $p = 1/10$. We see $E_3[u] - E_1[u] < 0$ for large q.

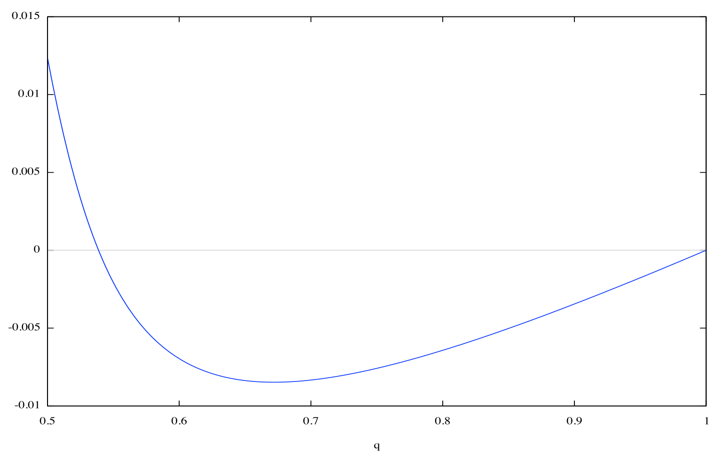

Figure 3.3: $E_3[u] - E_1[u]$ for $q \in [1/2, 1]$ under a discrete distribution

The intuition of this example is as follows. Consider a one-member committee. When the member has $t = 1/2$, adding two members to the committee brings him a higher expected utility, because the added members may have perfect information ($t = 1$) about the true state with positive probabilities and the information is reflected in the committee decision in equilibrium. On the other hand, when the member has $t = 1$, adding two members brings him a lower expected utility. He chooses a better alternative with probability 1 in the one-member committee. However, the three-member committee can not choose a better alternative with probability 1, because the added members may have no information ($t = 1/2$) about the true state and vote for an inferior alternative with positive probabilities. The net effect of adding two members depends on the distribution of the degree of information precision t. For the case of $p = 1/10$, the second effect is dominant for large q in which a member with $t = 1/2$ votes for B with a high probability in the three-member committee.

Finally, let us examine the behavior of the expected utility for a wider range of committee sizes. Let us choose the prior $q = 3/4$ in the above discrete model. The following table (Table 1) presents the results from $2n + 1 = 1$ to $2n + 1 = 61$. The next graph (Figure 4) illustrates the relation between the committee size and the expected utility in the equilibrium. It shows that the expected utility is decreased not only between 1 and 3 (in Figure 3) but also between 3 and 5. However, it is monotonically increasing for $2n + 1 > 5$.

The figure (Figure 4) also illustrates Theorem 2. The table shows that γ_A is monotonically increasing and γ_A is larger than $1/2$ for $2n+1 \geq 55$. According to Theorem 2, the expected utility must be monotonically increasing for $2n + 1 \geq 55$.

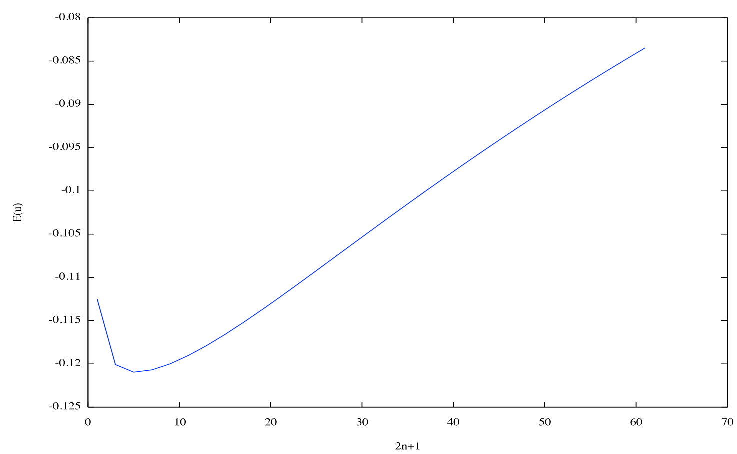

Figure 3.4: committee size and expected utility

3.4.3 Welfare Monotonicity for Large Committee

In the previous section, we showed that the expected utility may not be increasing in n. However, it turns out that the decreasing expected utility occurs only in small committees. We can show that the expected utility is increasing for sufficiently large n.

First, we show that the cutoff point goes to 1/2 as n goes to infinity.

Lemma 3.5. $t^* \to 1/2$ *as* $n \to \infty$.

Proof. The indifference condition (3.5) is arranged as follows.

$$\frac{\Pr(A)}{\Pr(B)} \frac{\Pr(s|A,t)}{\Pr(s|B,t)} \left[\frac{\gamma_A(1-\gamma_A)}{(1-\gamma_B)\gamma_B}\right]^n = \frac{u(B|A)}{u(A|B)} \tag{3.7}$$

For this equation to hold for all n, a limit of the ratio

$$\frac{\gamma_A(1-\gamma_A)}{(1-\gamma_B)\gamma_B} \tag{3.8}$$

must exist and it must be 1. This implies that t^* goes to 1/2. □

The intuition is as follows. The ratio (3.8) is the likelihood ratio of the voting split among two voters in state $w = A$ and state $w = B$. The ratio of probabilities that he is pivotal given the states is the likelihood ratio to the power of n. In the sequence of equilibria as n goes to infinity, the likelihood ratio must go to one. This implies that t^* must go to 1/2.

In the next theorem, we show that the interval $[\underline{\lambda}(n), \overline{\lambda}(n)]$ is monotonic in n and covers the whole range of λ for sufficiently large n.

Theorem 3.3. *$\underline{\lambda}(n)$ is decreasing in n and $\overline{\lambda}(n)$ is increasing in n. Moreover, $\underline{\lambda}(n)$ goes to 0 and $\overline{\lambda}(n)$ goes to infinity as n goes to infinity.*

Proof. First, we show that $\underline{\lambda}$ is decreasing in n. Recall that $\underline{\lambda}$ is defined as a unique λ for which $\gamma_B = 1/2$. Then, by Lemma 3.4, it must be that $\underline{\lambda} < 1$. By Lemma 3.3, this implies that $s^* = b$. Recall from Lemma 3.2 that t^* is decreasing in n. When $s^* = b$, this means that γ_B is increasing in n. On the other hand, by Lemma 3.4, γ_B is increasing in λ. This implies that $\underline{\lambda}(n)$ is decreasing in n.

Second, we show that $\underline{\lambda}(n)$ goes to 0 as n goes to infinity. Take any $\epsilon > 0$. By Lemma 3.5, γ_B goes to $E[t] > 1/2$. Therefore, when we take $\lambda = \epsilon$, there exists N such that $\gamma_B(n, \lambda) > 1/2$ for any $n \geq N$. By Lemma 3.4, $\underline{\lambda}(n) < \epsilon$ holds for any $n \geq N$. Therefore, $\underline{\lambda}(n)$ goes to 0 as n goes to infinity.

Similarly, $\overline{\lambda}(n)$ is increasing in n and goes to infinity as n goes to infinity.

□

Theorem 3.3 guarantees that, for any λ, there exists n^* such that $\lambda \in [\underline{\lambda}(n), \overline{\lambda}(n)]$ for $n \geq n^*$. Then, together with Theorem 3.2, the expected utility is increasing for sufficiently large n. Formally, the following statement holds.

Corollary 3.1. *There exists n^* such that $E_{2n+1}[u] < E_{2n'+1}[u]$ holds for any $n' > n \geq n^*$.*

3.4.4 Optimal Committee Design

We consider the problem of optimal committee design. Suppose there are N committee member candidates. The committee design problem is to choose the number n of committee members so as to maximize the expected utility from the decision by the committee. Theorem 3.3 and Theorem 3.4 establishes the following result.

Proposition 3.2 (optimal committee design)**.** *There exist N^* such that the optimal committee size is $n = N$ for any $N \geq N^*$.*

Proof. Take n^* in Corollary 1. Then, consider $\max_{n \leq n^*} E_{2n+1}[u]$. By Theorem 3.4, there exist N^* such that $E_{2N^*+1}[u] > \max_{n \leq n^*} E_{2n+1}[u]$. Note that $N^* > n^*$ by definition. Take any $N \geq N^*$. Then, $E_{2N+1}[u] > \max_{N > n > n^*} E_{2n+1}[u]$ by Corollary 1, because $N > n^*$. Hence, $E_{2N+1}[u] > \max_{n<N} E_{2n+1}[u]$. $\square$

3.5 Limit CJT under Strategic Voting

In this section, we discuss the convergence of cutoff point in the cutoff strategy and we prove that the limit property of the correct decision in the CJT holds under strategic voting.

Lemma 3.5 states that as n goes to infinity, the cutoff point t^* converges to 1/2. This means that in the limit, a voter who receives signal $s = a$ votes for A and a voter who receives signal $s = b$ votes for B, that is, a voter votes in such a way that his vote conveys the information about his signal directly.

This voting behavior is called informative voting. Under the informative voting according to an informative signal, the voting probabilities γ_A and γ_B are larger than 1/2, so that the assumption of CJT is satisfied. However, Austen-Smith and Banks (1996) showed that the informative voting may not constitute equilibrium. Nevertheless, our result shows that the informative voting is an equilibrium in the limit. To be explicit, the following holds.

Proposition 3.3 (Informative voting). *The voting behavior in equilibrium converges to informative voting when $n \to \infty$.*

Proposition 3.3 implies that the probabilities that the committee chooses the better alternative given the state go to 1 as n goes to infinity.

Theorem 3.4 (Limit CJT under strategic voting). $\Pr(d = A|A) \to 1$ *and* $\Pr(d = B|B) \to 1$ *as* $n \to \infty$.

Proof. In the limit, $\gamma_A = \gamma_B = E[t] > 1/2$. Then we can apply the law of large numbers. □

Theorem 3.4 implies that the expected utility $E_{2n+1}[u]$ goes to 0 as n goes to infinity.

3.6 Concluding Remarks

In this chapter, we studied the symmetric responsive equilibrium in the Bayesian voting game. We focused on the issue of how the welfare from the committee decision depends on the number of committee members. We showed that when the number of committee members is large enough, the welfare is increased monotonically by adding other members to the committee. This implies that if there are sufficiently many committee member candidates, the optimal committee design is that everyone participates in the committee.

We proved the welfare monotonicity result for the committee whose members are homogenous in their distributions of the degree of information precision. The distribution is interpreted as describing the uncertainty about the degree of information precision which is realized for a member. It is often the case in many committee decisions that the uncertainty is different among the members. For example, an expert of a decision problem is more likely to have a higher degree of information precision than a person who is not an expert. For these committee decisions, the committee design becomes a more complex problem of how to organize the committee by choosing from candidates who are heterogeneous in their distributions of information precision. This is an important issue that should be studied in the future research.

$2n+1$	σ	γ_A	γ_B	$\Pr(d=A\|w=A)$	$\Pr(d=B\|w=B)$	$E_{2n+1}[u]$
1	0	0.1	1	0.1	1	-0.1125
3	0.047283802	0.142555422	0.957444578	0.055172108	0.994721241	-0.120083021
5	0.116960143	0.205264128	0.894735872	0.06204271	0.990100299	-0.120957049
7	0.174594176	0.257134758	0.842865242	0.07732943	0.985702529	-0.120695373
9	0.220593492	0.298534143	0.801465857	0.097022674	0.98101715	-0.119990734
11	0.257163855	0.331447469	0.768552531	0.119272119	0.976178822	-0.119023927
13	0.286417882	0.357776094	0.742223906	0.14281619	0.971392611	-0.117875747
15	0.310068208	0.379061387	0.720938613	0.166774964	0.966818828	-0.116596069
17	0.329424446	0.396482002	0.703517998	0.190563787	0.962557034	-0.115220639
19	0.345466122	0.410919509	0.689080491	0.213816746	0.958656953	-0.113776549
21	0.358922205	0.423029985	0.676970015	0.23632065	0.955133867	-0.112284719
23	0.370337272	0.433303545	0.666696455	0.257964171	0.951981581	-0.110761386
25	0.380121555	0.442109399	0.657890601	0.278701607	0.949181658	-0.109219177
27	0.388587362	0.449728626	0.650271374	0.298528244	0.946709485	-0.107667913
29	0.395975182	0.456377664	0.643622336	0.317464072	0.944538043	-0.106115225
31	0.402472349	0.462225114	0.637774886	0.335543252	0.942640138	-0.104567042
33	0.408226448	0.467403804	0.632596196	0.352807372	0.940989648	-0.10302796
35	0.413355013	0.472019511	0.627980489	0.369301223	0.939562174	-0.101501532
37	0.417952599	0.476157339	0.623842661	0.385070174	0.938335307	-0.099990488
39	0.422096015	0.479886413	0.620113587	0.400158611	0.937288694	-0.098496913
41	0.425848209	0.483263388	0.616736612	0.414609027	0.936403977	-0.09702238
43	0.429261206	0.486335086	0.613664914	0.428461553	0.935664665	-0.095568056
45	0.432378341	0.489140507	0.610859493	0.44175375	0.935055988	-0.094134786
47	0.435235969	0.491712372	0.608287628	0.454520565	0.934564729	-0.092723156
49	0.437864802	0.494078322	0.605921678	0.466794395	0.934179068	-0.09133355
51	0.440290946	0.496261851	0.603738149	0.478605199	0.933888438	-0.089966186
53	0.442536724	0.498283051	0.601716949	0.489980654	0.933683377	-0.088621152
55	0.444621339	0.500159205	0.599840795	0.500946349	0.933555398	-0.087298432
57	0.446561388	0.501905249	0.598094751	0.511525905	0.933496895	-0.085997926
59	0.448371196	0.503534076	0.596465924	0.521740645	0.9335012	-0.08471947
61	0.450063696	0.505057326	0.594942674	0.531612701	0.933561515	-0.083462844

Table 3.1: expected utility in the discrete model

Chapter 4

Strategic Voting and Condorcet Jury Theorem under Uncertainty about Player's Information Precision with Heterogeneity in Distributions of Information Precision (I)

4.1 Introduction

In this chapter, we analyze a heterogenous model, that is, the diverse judgments of people are realized by different probability distributions. We introduce a heterogeneity as follows. We assume that most of the members receive their degrees of information precision according to a particular distribution and the remaining minority of members receive their degrees of information precision according to a different distribution. As an extreme case, we analyzed the case in which the only one member is minority. That is, $2n$ members receive their degrees of information precision according to a particular distribution and one member receives his degree of information precision according to a different distribution.

We study the distribution-wise symmetric equilibrium in this Bayesian voting game. In this class of equilibrium, there may exist an asymmetric equilibrium in which the equilibrium behavior for the players in the majority group is not identical to the equilibrium behavior for the minority player. Our main result is the following. In spite of the possibility of the asymmetric equilibrium, when the degree of heterogeneity is small, that is, only one-member has different distribution, voting behavior of all the members must converge to informative voting under simple majority rule. This result coincides with the case of the unique distribution of the degree of information precision. In the limit, informative voting behavior implies that the CJT holds.

The rest of this chapter is as follows. In Section 2, we explain our model. In Section 3, we show that the voting behavior in the equilibrium is characterized by a cutoff strategy. In Section 4, we show that the voting behavior in the equilibrium converges to informative voting.

4.2 Model

The voting game that we analyze in this chapter is similar to the model in Chapter 3. We introduce the heterogeneity as follows. The degree of information precision t_i is distributed on $[1/2, 1] = T$ with a full support density. For the $2n$ members, the degree of information precision t_i is distributed according to a full support density f. For the only one member, the degree of information precision t_i is distributed according to a full support density g. Thus, each member has three kinds of private information, the signal s_i, the degree of information precision t_i, and his distribution of the information precision.[1]

Moreover, to simplify the analysis, we consider following utility function. We assume that the utility from choosing the better alternative is the same between state A and state B and and normalized to 1. We also assume that the utility from choosing the worse alternative is the same between state A and state B and and normalized to 0. Then, the utility function is as follows;

$$u(d = A|w = A) = u(d = B|w = B) = 1$$

and

$$u(d = B|w = A) = u(d = A|w = B) = 0.$$

Then, the timing of the game is as follows. A state $w \in \{A, B\}$ is realized with probability $\Pr(w)$. We denote $\Pr(B) = q > 1/2$. Each committee member i is endowed with his information precision $t_i \in [1/2, 1]$ according to the densities f or g. Then, each committee member i receives a signal

[1]The result of this chapter also holds even if the probability distribution of t_i is common knowledge.

$s_i \in \{a, b\}$. Then, the committee members vote for alternative A or B simultaneously. A decision $d \in \{A, B\}$ is made by the simple majority rule. The members receive utilities depending on the decision made and the state realized.

4.3 Equilibrium Analysis

A committee member has private information $(s,t) \in S \times T$. A (pure) strategy of member i is a function $m_i : S \times T \times \{f, g\} \to \{A, B\}$. We analyze the distribution-wise symmetric equilibrium, in which all the f-type members use the same strategy. Moreover, we consider strategies such that the f-type member changes his vote according to his information. We call them responsive strategies. They are formally defined as follows.

Definition 4.1 (responsive strategy)**.** *The strategy is responsive if there exist (s,t) and (s',t') such that $m_i(s,t) \neq m_i(s',t')$ and the sets $\{(s,t)|m_i(s,t) = A\}$ and $\{(s,t)|m_i(s,t) = B\}$ have positive measures.*

The responsive strategy for the f-type implies that the voting probabilities for better alternatives are $0 < \Pr(m_i = A|w = A, f), \Pr(m_i = B|w = B, f) < 1$.

We consider the best response to responsive strategies. The condition by which a voter who has private information (s,t) weakly prefers voting for A to voting for B is that the expected utility from voting for A is more than or equal to the expected utility from voting for B. That is,

$$\frac{\Pr(w = A|s_i, t_i, piv, f)}{\Pr(w = B|s_i, t_i, piv, f)} \geq 1$$

for the f-type, and

$$\frac{\Pr(w=A|s_i,t_i,piv,g)}{\Pr(w=B|s_i,t_i,piv,g)} \geq 1$$

for the g-type. In the left-hand side of these conditions, piv means that the voter is pivotal; n members vote for A and n members vote for B. When the member i is pivotal, if he votes for $A(B)$ then the committee decision is $A(B)$. When the member i is not pivotal, his vote does not change the committee decision. Thus, the optimality of his voting behavior matters only when he is pivotal. Then, he chooses his vote depending on his private information $(s_i, t_i, \cdot)$ and information that he is pivotal. By the Bayes rule, these conditions are arranged to

$$\frac{\Pr(w=A|s_f,t_f,f)}{\Pr(w=B|s_f,t_f,f)}\frac{\Pr(piv|w=A,f)}{\Pr(piv|w=B,f)} \geq 1$$

and

$$\frac{\Pr(w=A|s_g,t_g,g)}{\Pr(w=B|s_g,t_g,g)}\frac{\Pr(piv|w=A,g)}{\Pr(piv|w=B,g)} \geq 1.$$

Then, the indifference conditions for f-type and g-type are

$$\frac{\Pr(w=A|s_f,t_f,f)}{\Pr(w=B|s_f,t_f,f)}\frac{\Pr(piv|w=A,f)}{\Pr(piv|w=B,f)} = 1 \tag{4.1}$$

and

$$\frac{\Pr(w=A|s_g,t_g,g)}{\Pr(w=B|s_g,t_g,g)}\frac{\Pr(piv|w=A,g)}{\Pr(piv|w=B,g)} = 1. \tag{4.2}$$

Given the responsive strategy, the ratio of probabilities that he is pivotal has finite value. The first term (the ratio of posterior probabilities given his private information) of these conditions is

$$\frac{\Pr(w=A|s,t,\cdot)}{\Pr(w=B|s,t,\cdot)} = \begin{cases} \frac{1-q}{q}\frac{t}{1-t} & \text{if} \quad s=a \\ \frac{1-q}{q}\frac{1-t}{t} & \text{if} \quad s=b \end{cases}.$$

For $s = a$, this ratio is increasing in t and goes to infinity as t goes to 1. For $s = b$, this ratio is decreasing in t and goes to 0 as t goes to 1. This implies that the best response is a cutoff strategy if the other f-type members use responsive strategies.

Definition 4.2 (cutoff strategy)**.** *A strategy is a cutoff strategy if there exists a cutoff point* (s^*, t^*) *such that* $m_i(s, t) = A$ *if and only if* $\Pr(w = A|s, t) \geq \Pr(w = A|s^*, t^*)$.

Theorem 4.1. *A best response to responsive strategies must be a cutoff strategy.*

4.4 Limit Theorem

4.4.1 Limit Theorem on Voting Behavior

In this section, we show that the equilibrium cutoff points (s_f^*, t_f^*) and (s_g^*, t_g^*) converge to $1/2$ as n goes to infinity. This means that in the limit, a voter who receives signal $s = a$ votes for A and a voter who receives signal $s = b$ votes for B, that is, a voter votes in such a way that his vote conveys the information about his signal directly. This voting behavior is called informative voting.

Let us denote that the voting probabilities for better alternatives by the cutoff points (s_f^*, t_f^*) and (s_g^*, t_g^*) as

$$\begin{aligned}
\gamma_{A,f} &= \Pr(m = A|s_f^*, t_f^*, f) \\
\gamma_{B,f} &= \Pr(m = B|s_f^*, t_f^*, f) \\
\gamma_{A,g} &= \Pr(m = A|s_g^*, t_g^*, g) \\
\gamma_{B,g} &= \Pr(m = B|s_g^*, t_g^*, g).
\end{aligned}$$

These probabilities are

$$\begin{aligned}
\gamma_{A,f} &= \int_{t_f^*}^{1} tf(t)dt \\
\gamma_{B,f} &= 1 - \int_{t_f^*}^{1} (1-t)f(t)dt \\
\gamma_{A,g} &= \int_{t_g^*}^{1} tf(t)dt \\
\gamma_{B,g} &= 1 - \int_{t_g^*}^{1} (1-t)f(t)dt
\end{aligned}$$

for the case of $s_f^* = a$ and $s_g^* = a$, and

$$\begin{aligned}
\gamma_{A,f} &= 1 - \int_{t_f^*}^{1} (1-t)f(t)dt \\
\gamma_{B,f} &= \int_{t_f^*}^{1} tf(t)dt \\
\gamma_{A,g} &= 1 - \int_{t_g^*}^{1} (1-t)f(t)dt \\
\gamma_{B,g} &= \int_{t_g^*}^{1} tf(t)dt
\end{aligned}$$

for the case of $s_f^* = b$ and $s_g^* = b$. By Theorem 4.1, the indifference conditions are rearranged to

$$\frac{\Pr(w = A|s_f^*, t_f^*, f)}{\Pr(w = B|s_f^*, t_f^*, f)} \left[\frac{\gamma_{A,f}^n (1-\gamma_{A,f})^{n-1}(1-\gamma_{A,g})\binom{2n-1}{n} + \gamma_{A,f}^{n-1}(1-\gamma_{A,f})^n \gamma_{A,g}\binom{2n-1}{n-1}}{\gamma_{B,f}^n (1-\gamma_{B,f})^{n-1}(1-\gamma_{B,g})\binom{2n-1}{n} + \gamma_{B,f}^{n-1}(1-\gamma_{B,f})^n \gamma_{B,g}\binom{2n-1}{n-1}} \right] = 1 \tag{4.3}$$

or

$$\frac{\Pr(w=A|s_f^*,t_f^*,f)}{\Pr(w=B|s_f^*,t_f^*,f)}\left[\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})}\right]^{n-1}\left[\frac{\gamma_{A,f}(1-\gamma_{A,g})+(1-\gamma_{A,f})\gamma_{A,g}}{\gamma_{B,f}(1-\gamma_{B,g})+(1-\gamma_{B,f})\gamma_{B,g}}\right]=1 \tag{4.4}$$

for f-type, and

$$\frac{\Pr(w=A|s_g^*,t_g^*,g)}{\Pr(w=B|s_g^*,t_g^*,g)}\left[\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})}\right]^{n}=1 \tag{4.5}$$

for g-type.

First, we show that the cutoff point of f-type goes to $1/2$ as n goes to infinity.

Lemma 4.1. $t_f^* \to 1/2$ *as* $n \to \infty$.

Proof. Suppose that t_f^* does not converge to $1/2$. Since T is compact, we can take a convergent subsequence with limit greater than $1/2$. There exists $\epsilon > 0$ such that for sufficiently large n, it holds that

$$\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})} > 1+\epsilon.$$

This implies

$$\left[\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})}\right]^{n} \to \infty.$$

This is a contradiction. Then, it must hold

$$\lim_{n\to\infty}\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})} = 1.$$

This implies that t_f^* goes to $1/2$. $\square$

Second, we show that the cutoff point of g-type also goes to $1/2$ as n goes to infinity.

Lemma 4.2. $t_g^* \to 1/2$ *as* $n \to \infty$.

Proof. By the indifference condition of g-type,

$$\frac{\Pr(w=A|s_g,t_g,g)}{\Pr(w=B|s_g,t_g,g)}\left[\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})}\right]=1/\left[\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})}\right]^{n-1}.$$

Then, the indifference condition of f-type is

$$\frac{\Pr(w=A|s_f,t_f,f)}{\Pr(w=B|s_f,t_f,f)}\left[\frac{\gamma_{A,f}(1-\gamma_{A,g})+(1-\gamma_{A,f})\gamma_{A,g}}{\gamma_{B,f}(1-\gamma_{B,g})+(1-\gamma_{B,f})\gamma_{B,g}}\right]=\frac{\Pr(w=A|s_g,t_g,g)}{\Pr(w=B|s_g,t_g,g)}\left[\frac{\gamma_{A,f}(1-\gamma_{A,f})}{\gamma_{B,f}(1-\gamma_{B,f})}\right].$$

As $t_f\to 1/2$, it must be

$$\frac{1-q}{q}\left[\frac{E_f[t](1-\gamma_{A,g})+(1-E_f[t])\gamma_{A,g}}{E_f[t](1-\gamma_{B,g})+(1-E_f[t])\gamma_{B,g}}\right]=\frac{\Pr(w=A|s_g,t_g,g)}{\Pr(w=B|s_g,t_g,g)}.$$

This equation holds for $t_g=1/2$. □

By Lemma 4.1 and Lemma 4.2, Theorem 4.2 holds.

Theorem 4.2. *The voting behavior in equilibrium converges to informative voting when $n\to\infty$.*

4.4.2 Limit CJT

Under the informative voting according to an informative signal, the voting probabilities $\gamma_{A,f},\gamma_{B,f},\gamma_{A,g}$ and $\gamma_{B,g}$ are larger than 1/2. Therefore, the Limit CJT holds.

Proposition 4.1 (Limit CJT)**.** *Both of* $\Pr(d=A|w=A)$ *and* $\Pr(d=B|w=B)$ *go to one.*

This proposition implies that even if there exists a minority group of people who receive their degrees of information precision according to different distribution than the majority of people, the voting by all the people including the minority leads to a correct decision with almost probability one. Thus, the efficiency of the simple majority rule is robust with respect to heterogeneity of people as long as the heterogeneity is limited.

Chapter 5

Strategic Voting and Condorcet Jury Theorem under Uncertainty about Player's Information Precision with Heterogeneity in Distributions of Information Precision (II)

5.1 Introduction

In this chapter, we analyze another heterogenous model. We introduce a heterogeneity as follows. We assume that the population consist of two major groups. The members in one group receive their degrees of information precision according to a particular distribution. The members in the other group receive their degrees of information precision according to a different distribution. We consider a case in which two groups are similar size. As an extreme case, one group is composed of $n+1$ members and the other group is composed of n members. In this case, the population ratio is $1/2$.

We study the efficiency of decisions in the distribution-wise symmetric equilibrium when the committee size becomes large. Our main result is the following. First, the asymmetric equilibrium does not converge to informative equilibrium when the committee size becomes large. Second, however, the voting probabilities that the committee chooses the better alternatives converge to one.

The rest of this chapter is as follows. In Section 2, we explain our model. In Section 3, we study the symmetric equilibrium. In Section 4, we show that there exists an asymmetric equilibrium. In Section 5, we show the limit properties of the asymmetric equilibrium.

5.2 Model

The voting game that we analyzed in this chapter is similar to the model in Chapter 4. We introduce a heterogeneity as follows. The degree of information precision t_i is distributed on $[1/2, 1] = T$. For the $n+1$ members, the degree of information precision is distributed uniformly, that is, its density f is $f(t_i) = 2$. For the other n members, the degree of information precision is distributed according to a particular discrete distribution g such that $\Pr(t_i = 1) = \Pr(t_i = 1/2) = 1/2$. Thus, each member has three kinds of private information, the signal s_i, the degree of information precision t_i, and his distribution of the degree of information precision.

The utility function is the same in Chapter 4;

$$u(d = A|w = A) = u(d = B|w = B) = 1$$

and

$$u(d = B|w = A) = u(d = A|w = B) = 0.$$

To simplify the analysis, we assume that the prior probability satisfies $\Pr(w = A) = \Pr(w = B)$. Then, the timing of the game is as follows. A state $w \in \{A, B\}$ is realized with the same probability between A and B, that is, $\Pr(w = A) = \Pr(w = B) = 1/2$. Each committee member i is endowed with his information precision $t_i \in [1/2, 1]$ according to the distribution f or g. Then, each committee member i receives a signal $s_i \in \{a, b\}$. Then, the committee members vote for alternative A or B simultaneously. A decision $d \in \{A, B\}$ is made by the simple majority rule. The members receive utilities depending on the decision made and the state realized.

5.3 Symmetric Equilibrium

We analyze the distribution-wise symmetric equilibrium, in which all the f-type members use the same strategy and all the g-type members use the same strategy. A committee member has private information $(s, t) \in S \times T$. Then, a strategy of member i who is f-type is a function $m_i : S \times T \to \{A, B\}$. Moreover, we consider strategies such that the f-type member changes his vote according to his information. We call them responsive strategies. They are formally defined as follows.

Definition 5.1 (responsive strategy)**.** *The strategy is responsive if there exist* (s, t) *and* (s', t') *such that* $m_i(s, t) \neq m_i(s', t')$ *and the sets* $\{(s, t)|m_i(s, t) = A\}$ *and* $\{(s, t)|m_i(s, t) = B\}$ *have positive measures.*

The responsive strategy for the f-type implies that the voting probabilities for better alternatives are $0 < \Pr(m_i = A|w = A, f), \Pr(m_i = B|w = B, f) < 1$.

Moreover, we restrict the strategy of f-type. We assume that the f-type members use cutoff strategy.

Definition 5.2 (cutoff strategy)**.** *A strategy is a cutoff strategy if there exists a cutoff point* (s^*, t^*) *such that* $m_i(s, t) = A$ *if and only if* $\Pr(w = A|s, t) \geq \Pr(w = A|s^*, t^*)$*.*

On the other hand, a strategy of member i who is g-type is as follows. When $t_i = 1$, his optimal voting behavior is that he votes for A if and only if he receives $s_i = a$. When $t_i = 1/2$, his signal has no information. Then, we consider a mixed action of g-type with $t_i = 1/2$, that is, he votes for A with probability σ when $t_i = 1/2$.

The indifference condition of f-type is

$$\frac{\Pr(w = A|s_f, t_f, f, piv)}{\Pr(w = B|s_f, t_f, f, piv)} = 1.$$

In the left-hand side of the indifference condition, piv means that he is pivotal. This is arranged to

$$\frac{\Pr(w = A|s, t)}{\Pr(w = B|s, t)} \frac{\Pr(piv|w = A, f)}{\Pr(piv|w = B, f)} = 1. \tag{5.1}$$

Similarly, the indifference condition of g-type with $t_i = 1/2$ is

$$\frac{\Pr(piv|w = A, g)}{\Pr(piv|w = B, g)} = 1. \tag{5.2}$$

If the left-hand side of this equation is larger than one, he votes for A with $\sigma = 1$.

Let $(\gamma_{A,f}, \gamma_{B,f})$ denote the voting probabilities of f-type for better alternatives in each states. Similarly, let $(\gamma_{A,g}, \gamma_{B,g})$ denote the voting probabilities of g-type for better alternatives in each states. With cutoff point (s_f^*, t_f^*) and σ^*, these probabilities are

$$\begin{aligned}
\gamma_{A,f} &= \int_{t^*}^{1} t f(t) dt \\
\gamma_{B,f} &= 1 - \int_{t^*}^{1} (1-t) f(t) dt \\
\gamma_{A,g} &= \frac{1}{2} + \frac{1}{2}\sigma^* \\
\gamma_{B,g} &= \frac{1}{2} + \frac{1}{2}(1-\sigma^*)
\end{aligned}$$

for $s_f^* = a$. Then, the indifference conditions of f-type and g-type are rearranged to

$$\frac{t}{1-t} \frac{\sum_{k=0}^{n} (\gamma_{A,f})^k (1-\gamma_{A,f})^{n-k} (\gamma_{A,g})^{n-k} (1-\gamma_{A,f})^k \binom{n}{k}\binom{n}{k}}{\sum_{k=0}^{n} (\gamma_{B,f})^k (1-\gamma_{B,f})^{n-k} (\gamma_{B,g})^{n-k} (1-\gamma_{B,g})^k \binom{n}{k}\binom{n}{k}} = 1 \tag{5.3}$$

and

$$\frac{\sum_{k=1}^{n} (\gamma_{A,f})^k (1-\gamma_{A,f})^{n+1-k} (\gamma_{A,g})^{n-k} (1-\gamma_{A,f})^{k-1} \binom{n+1}{k}\binom{n-1}{k-1}}{\sum_{k=1}^{n} (\gamma_{B,f})^k (1-\gamma_{B,f})^{n+1-k} (\gamma_{B,g})^{n-k} (1-\gamma_{B,g})^{k-1} \binom{n+1}{k}\binom{n-1}{k-1}} = 1. \tag{5.4}$$

5.3.1 Informative Voting and Symmetric Equilibrium

In this section, we show that there exists an informative voting equilibrium; every $2n+1$ members use informative voting strategy.

Definition 5.3 (informative voting). *A voting strategy is informative voting if the voter votes for A when his signal is $s_i = a$ and votes for B when his signal is $s_i = b$.*

Proposition 5.1 (informative voting equilibrium)**.** *For any n, there exists an equilibrium in which both of f-type and g-type use informative voting strategy.*

Proof. Consider informative voting strategy, that is, $t^* = 1/2$ and $\sigma^* = 1/2$. Under the informative voting, the voting probabilities for better alternatives are

$$\gamma_{A,f} = \gamma_{B,f} = \gamma_{A,g} = \gamma_{B,g} = 3/4.$$

The indifference condition of f-type is

$$\frac{t}{1-t}\left[\frac{\frac{3}{4}\frac{1}{4}}{\frac{3}{4}\frac{1}{4}}\right]^n \frac{\sum_{k=0}^{n}\binom{n}{k}\binom{n}{k}}{\sum_{k=0}^{n}\binom{n}{k}\binom{n}{k}} = 1.$$

This means that the presumed cutoff $t^* = 1/2$ is a cutoff under best response to other players' strategies $t^* = 1/2$ and $\sigma^* = 1/2$.

The indifference condition of g-type is

$$\left[\frac{\frac{3}{4}\frac{1}{4}}{\frac{3}{4}\frac{1}{4}}\right]^n \frac{\sum_{k=1}^{n}\binom{n+1}{k}\binom{n-1}{k-1}}{\sum_{k=1}^{n}\binom{n+1}{k}\binom{n-1}{k-1}} = 1.$$

This means that the presumed mixed strategy $\sigma^* = 1/2$ is a best response for $t_i = 1/2$ of g-type against other players' strategies $t^* = 1/2$ and $\sigma^* = 1/2$. $\square$

5.4 Asymmetric Equilibrium

In the previous section, we show that there exists a "symmetric" equilibrium; both of f-type and g-type use informative voting strategy. In this section, we show that there exists an "asymmetric" equilibrium; f-type's voting behavior is biased for B and g-type's voting behavior is biased for A. We also show that the Limit Condorcet jury theorem holds for the asymmetric equilibrium.

5.4.1 Existence of Asymmetric Equilibrium

We show that there exists an "asymmetric" equilibrium, in which $s = a, t > 1/2$ and $\sigma = 1$ are satisfied. In this equilibrium, f-type's voting behavior is biased for B and g-type's voting behavior is biased for A. The indifference condition (5.3) for f-type of this equilibrium is reduced to

$$\frac{t}{1-t}\frac{(1-\gamma_{A,f})^n}{\sum_{k=0}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}} = 1. \tag{5.5}$$

The condition for $\sigma = 1$ of g-type is

$$\frac{(\gamma_{A,f})(1-\gamma_{A,f})^n\binom{n+1}{1}}{\sum_{k=1}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n+1-k}(1/2)^{n-1}\binom{n+1}{k}\binom{n-1}{k-1}} \geq 1. \tag{5.6}$$

In contrast with the equation (5.4), the inequality (5.6) accommodates the case in which the optimal behavior for $t_i = 1/2$ of g-type becomes a corner solution of σ over the interval $[0, 1]$.

We show that there exists t^* which satisfies these conditions.

Theorem 5.1. *There exists an equilibrium with $s^* = a, t^* > 1/2$ and $\sigma^* = 1$. Moreover, t^* is a unique cutoff.*

Proof. First, we show that there exists t^*. The indifference condition (5.5) of f-type is rearranged to

$$\frac{t}{1-t}\frac{1}{\sum_{k=0}^{n}(\frac{\gamma_{B,f}}{1-\gamma_{A,f}})^k(\frac{1-\gamma_{B,f}}{1-\gamma_{A,f}})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}} = 1. \tag{5.7}$$

Since $s^* = a$,

$$\frac{\gamma_{B,f}}{1-\gamma_{A,f}} = \frac{1-(1-t)^2}{t^2} = \frac{t(2-t)}{t^2} = \frac{(2-t)}{t}$$

and

$$\frac{1-\gamma_{B,f}}{1-\gamma_{A,f}} = \frac{(1-t)^2}{t^2} = \left(\frac{1-t}{t}\right)^2.$$

Then the second term of (5.7) is increasing in t. Hence, the left-hand side of (5.7) is increasing in t. For $t = 1$, the voting probabilities for the better alternatives are $\gamma_{A,f} = 0$ and $\gamma_{B,f} = 1$. The ratio of the probabilities that he is pivotal is

$$\begin{aligned}
& \frac{1}{\sum_{k=0}^{n}(\frac{\gamma_{B,f}}{1-\gamma_{A,f}})^k(\frac{1-\gamma_{B,f}}{1-\gamma_{A,f}})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}} \\
= & \frac{1}{(1/2)^n} \\
= & 2^n.
\end{aligned}$$

Then, the left-hand side of (5.7) is larger than one for sufficiently large t.

On the other hand, for $t = 1/2$, the left-hand side of (5.7) is

$$\begin{aligned}
\frac{\Pr(piv|w = A, f)}{\Pr(piv|w = B, f)} &= \frac{1}{\sum_{k=0}^{n}(3)^k(1/2)^n\binom{n}{k}\binom{n}{k}} \\
&< \frac{1}{\sum_{k=0}^{n}(1/2)^n\binom{n}{k}\binom{n}{k}} \\
&= \frac{2^n}{\sum_{k=0}^{n}(\binom{n}{k})^2} \\
&= \frac{2^n}{\binom{2n}{n}} \\
&= \frac{2^n n!}{(2n)!/n!} \\
&= \frac{\prod_{l=0}^{n-1}2(n-l)}{\prod_{l=0}^{n-1}(2n-l)} \\
&\leq 1.
\end{aligned}$$

This implies that there exists an unique t^* such that the condition of f-type is satisfied.

Second, we show that t^* also satisfies the condition of g-type. The left-hand side of (5.6) is rearranged to

$$\begin{aligned}
&\frac{(\gamma_{A,f})(1-\gamma_{A,f})^n\binom{n+1}{1}}{\sum_{k=1}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n+1-k}(1/2)^{n-1}\binom{n+1}{k}\binom{n-1}{k-1}} \\
= \;& \frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{(1-\gamma_{A,f})^n\binom{n+1}{1}}{\sum_{k=1}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n\binom{n+1}{k}\binom{n-1}{k-1}} \\
= \;& \frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{(1-\gamma_{A,f})^n\binom{n+1}{1}}{\sum_{k=1}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n[\frac{n+1}{n+1-k}\frac{k}{n}]\binom{n}{k}\binom{n}{k}} \\
\geq \;& \frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{(1-\gamma_{A,f})^n\binom{n+1}{1}}{\sum_{k=1}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n[n+1]\binom{n}{k}\binom{n}{k}} \\
= \;& \frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{(1-\gamma_{A,f})^n}{\sum_{k=1}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}} \\
\geq \;& \frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{(1-\gamma_{A,f})^n}{\sum_{k=0}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}}.
\end{aligned}$$

By the condition (5.5) of f-type,

$$\begin{aligned}
&\frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{(1-\gamma_{A,f})^n}{\sum_{k=0}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}} \\
= \;& \frac{1}{2}\frac{\gamma_{A,f}}{(1-\gamma_{B,f})}\frac{1-t}{t} \\
= \;& \frac{1}{2}\frac{1-t^2}{(1-t)^2}\frac{1-t}{t} \\
= \;& \frac{1+t}{2t} \\
\geq \;& 1.
\end{aligned}$$

Then, (5.6) also holds. □

5.4.2 Limit Property of Asymmetric Equilibrium

In this section, we show that the Condorcet jury theorem holds in the asymmetric equilibrium. We consider the asymmetric equilibrium which we studied in the previous section. We show that both of the probabilities that the committee chooses better alternatives ($\Pr(d = A|w = A)$ and $\Pr(d = B|w = B)$) in this asymmetric equilibrium go to one as n goes to infinity.

First, we consider the case of $w = B$. In the asymmetric equilibrium, the f-type member uses cutoff strategy with $s^* = a$ and the g-type member votes for A when his degree of information precision is $t = 1/2$ and votes for B when his degree of information precision is $t = 1$. Then, the probability that the f-type member votes for the better alternative B is larger than $1/2$, and the probability that the g-type member votes for the better alternative B is equal to $1/2$. The assumption of the classical CJT is satisfied. Then, by the law of large numbers, the CJT holds.

Second, we consider the case of $w = A$. For $w = A$, the g-type member votes for A with probability one since $\sigma = 1$. Then, the decision of the committee is B if and only if every $n+1$ f-type member votes for B. Then, the Condorcet jury theorem holds if

$$\lim_{n\to\infty} t^* < 1. \tag{5.8}$$

Lemma 5.1 provides a sufficient condition for (5.8).

Lemma 5.1. *There exist $\bar{t}$ such that for any $t \in (\bar{t}, 1)$, $\lim_{n\to\infty} \Pi(t, n) > 1$ where $\Pi(t, n) = \frac{\Pr(piv|w=A,f)}{\Pr(piv|w=B,f)}$ with $\sigma = 1$.*

Proof. We prove Lemma 5.1 in four steps.

Step 1 The ratio of the probabilities that the voter is pivotal is

$$\begin{aligned}
\Pi(t,n) &= \frac{(1-\gamma_{A,f})^n}{\sum_{k=0}^{n}(\gamma_{B,f})^k(1-\gamma_{B,f})^{n-k}(1/2)^n\binom{n}{k}\binom{n}{k}} \\
&= \frac{(1-\gamma_{A,f})^n}{\sum_{k=0}^{n}(\gamma_{B,f})^{n-k}(1-\gamma_{B,f})^{k}(1/2)^n\binom{n}{k}\binom{n}{k}} \\
&= \left[\frac{\sqrt{2}(1-\gamma_{A,f})}{\gamma_{B,f}}\right]^n \frac{1}{\sum_{k=0}^{n}\left[\frac{1-\gamma_{B,f}}{\gamma_{B,f}}\right]^k (1/\sqrt{2})^n\binom{n}{k}^2}.
\end{aligned}$$

For sufficiently large t, the first term satisfies

$$\frac{\sqrt{2}(1-\gamma_{A,f})}{\gamma_{B,f}} = \frac{\sqrt{2}t^2}{t(2-t)} > 1.$$

Then, we show

$$\lim_{n\to\infty}\frac{1}{\sum_{k=0}^{n}\left[\frac{1-\gamma_{B,f}}{\gamma_{B,f}}\right]^k (1/\sqrt{2})^n\binom{n}{k}^2} \neq 0$$

for sufficiently large t. This condition holds if

$$\lim_{n\to\infty}\frac{\sum_{k=0}^{n+1}\left[\frac{1-\gamma_{B,f}}{\gamma_{B,f}}\right]^k (1/\sqrt{2})^{n+1}\binom{n+1}{k}^2}{\sum_{k=0}^{n}\left[\frac{1-\gamma_{B,f}}{\gamma_{B,f}}\right]^k (1/\sqrt{2})^n\binom{n}{k}^2} < 1$$

for sufficiently large t. Let $z = \frac{1-\gamma_{B,f}}{\gamma_{B,f}}$, this condition is rearranged to

$$\lim_{n\to\infty}\frac{\sum_{k=0}^{n+1}(1/\sqrt{2})\binom{n+1}{k}^2 z^k}{\sum_{k=0}^{n}\binom{n}{k}^2 z^k} < 1$$

for sufficiently small z.

Step 2 We approximate

$$\lim_{n\to\infty}\frac{\sum_{k=0}^{n+1}(1/\sqrt{2})\binom{n+1}{k}^2 z^k}{\sum_{k=0}^{n}\binom{n}{k}^2 z^k}. \tag{5.9}$$

Step 2-1 We approximate the denominator of (5.9) as following. Here, we define a step function ϕ_n as follows;

$$\phi_n(x) = \begin{cases} \binom{n}{k}^2 z^k & \text{for } \frac{k}{n} \leq x < \frac{k+1}{n}, k = 0, ..., n \\ 0 & \text{otherwise.} \end{cases}$$

By the definition of ϕ_n,

$$\int_{\frac{k}{n}}^{\frac{k+1}{n}} \phi_n(x)dx = \frac{1}{n}\binom{n}{k}^2 z^k$$

and

$$\sum_{k=0}^{n} \int_{\frac{k}{n}}^{\frac{k+1}{n}} \phi_n(x)dx = \frac{1}{n}\sum_{k=0}^{n}\binom{n}{k}^2 z^k.$$

Let $\phi(x) = \lim_n \phi_n(x)$, we get

$$\int_0^1 \phi(x)dx = \lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n}\binom{n}{k}^2 z^k.$$

We apply the Stirling's approximation[1] to approximate ϕ,

$$\begin{aligned} \phi(x) &= \lim_{n\to\infty}\phi_n(x) \\ &= \lim_{n\to\infty}\binom{n}{k}^2 z^k \\ &\approx \lim_{n\to\infty}\binom{n}{xn}^2 z^{xn} \\ &= \left(\frac{n!}{(xn)!((1-x)n)!}\right)^2 z^{xn} \\ &\approx \left(\frac{\sqrt{2\pi n}(\frac{n}{e})^n}{\sqrt{2\pi xn}(\frac{xn}{e})^{xn}\sqrt{2\pi(1-x)n}(\frac{(1-x)n}{e})^{(1-x)n}}\right)^2 z^{xn} \\ &= \frac{1}{n}\frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n. \end{aligned}$$

[1]Stirling's approximation:

$$n! \approx \sqrt{2\pi n}(\frac{n}{e})^n$$

Hence, we approximate the denominator of (5.9) as

$$\lim_{n\to\infty}\sum_{k=0}^{n}\binom{n}{k}^2 z^k \approx \int_0^1 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx.$$

Step 2-2 Similarly, we approximate the numerator of (5.9)

$$\begin{aligned}
\lim_{n\to\infty}\sum_{k=0}^{n+1}\binom{n+1}{k}^2 z^k &= \lim_{n\to\infty}\sum_{k=0}^{n+1}\left(\frac{n+1}{n+1-k}\right)^2\binom{n}{k}^2 z^k \\
&= \lim_{n\to\infty}\sum_{k=0}^{n+1}\left(\frac{1}{1-\frac{k}{n+1}}\right)^2\binom{n}{k}^2 z^k \\
&= \lim_{n\to\infty}\sum_{k=0}^{n+1}\left(\frac{1}{1-\frac{k}{n}\frac{n}{n+1}}\right)^2\binom{n}{k}^2 z^k \\
&\approx \int_0^1 \left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx.
\end{aligned}$$

Step 2-3 From Step 2-1 and Step 2-2, we approximate (5.9) as

$$\lim_{n\to\infty}\frac{\sum_{k=0}^{n+1}(1/\sqrt{2})\binom{n+1}{k}^2 z^k}{\sum_{k=0}^{n}\binom{n}{k}^2 z^k} \approx \frac{\int_0^1 \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^1 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}. \tag{5.10}$$

Step 3 We state some properties of the right-hand side of (5.10).

Step3-1 We define

$$\rho(x) = \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2. \tag{5.11}$$

It is easy to check that $\rho(x)$ is increasing in x. Moreover, $\rho(x) = 1/\sqrt{2}$ at $x = 0$ and $\rho(x) \to \infty$ as $x \to 1$. Then, $\rho(x) \le 1$ if and only if

$$x \le 1 - \left(\frac{1}{\sqrt{2}}\right)^2 \ (< 1). \tag{5.12}$$

Step3-2 We define

$$\psi(x,z) = \frac{z^x}{x^{2x}(1-x)^{2(1-x)}}. \tag{5.13}$$

$\psi(x,z)$ is increasing in x for $x < \frac{\sqrt{z}}{1-\sqrt{z}}$ and decreasing in x for $x > \frac{\sqrt{z}}{1-\sqrt{z}}$. Moreover, $\psi(x,z) \to 1$ as $x \to 0$ and $\psi(x,z) \to z$ as $x \to 1$. Then, there exists $x_z^* \in (0,1)$ such that $\psi(x_z^*, z) = 1$ where $x_z^* > \frac{\sqrt{z}}{1-\sqrt{z}}$. Note that x_z^* is strictly increasin in z.

Step4 We show that threr exist $\bar{z}$ such that

$$\lim_{n\to\infty} \frac{\sum_{k=0}^{n+1}(1/\sqrt{2})\binom{n+1}{k}^2 z^k}{\sum_{k=0}^{n}\binom{n}{k}^2 z^k} < 1$$

for any $z \in (0, \bar{z})$.

We define $\bar{z}$ as z which satisfies $x_{\bar{z}}^* = 1 - (1/\sqrt{2})^2$ Then, $\psi(x,z) < 1$ for

x where $x > x_z^*$. Note that $x_z^* < x_{\bar{z}}^*$ since $z < \bar{z}$. Then,

$$
\begin{aligned}
& \lim_{n\to\infty} \frac{\sum_{k=0}^{n+1}(1/\sqrt{2})\binom{n+1}{k}^2 z^k}{\sum_{k=0}^{n}\binom{n}{k}^2 z^k} \\
\approx\; & \frac{\int_0^1 \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^1 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx} \\
=\; & \frac{\int_0^{x_z^*} \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx + \int_{x_z^*}^1 \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^{x_z^*} \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx + \int_{x_z^*}^1 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx} \\
=\; & \frac{\frac{\int_0^{x_z^*} \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^{x_z^*} \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx} + \frac{\int_{x_z^*}^1 \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^{x_z^*} \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}}{1 + \frac{\int_{x_z^*}^1 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^{x_z^*} \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}} \\
\approx\; & \frac{\int_0^{x_z^*} \frac{1}{\sqrt{2}}\left(\frac{1}{1-x}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^{x_z^*} \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx} \\
\leq\; & \frac{\int_0^{x_z^*} \frac{1}{\sqrt{2}}\left(\frac{1}{1-x_z^*}\right)^2 \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx}{\int_0^{x_z^*} \frac{1}{2\pi x(1-x)}\left(\frac{z^x}{x^{2x}(1-x)^{2(1-x)}}\right)^n dx} \\
=\; & \frac{1}{\sqrt{2}}\left(\frac{1}{1-x_z^*}\right)^2 \\
<\; & \frac{1}{\sqrt{2}}\left(\frac{1}{1-x_{\bar{z}}^*}\right)^2 \\
=\; & 1.
\end{aligned}
$$

□

Lemma 5.1 implies (5.8), and the Condorcet jury theorem holds in the asymmetric equilibrium.

Theorem 5.2 (CJT in the asymmetric equilibrium)**.** *The Condorcet jury theorem holds.*

It may hold that the asymmetric equilibrium remains when the sizes of two groups are the same. We showed that the CJT holds in the asymmetric equilibrium for particular distributions. The implication of this result is that the CJT is robust for the heterogeneity of distributions. However, the generality of this result will be studied in the future research.

Appendix

An Alternative Proof of Lemma 5.1

In this appendix, we provide an alternative proof of Lemma 5.1. The probability that some members of f-type vote for A and the other members of f-type vote for B on the state $w = A$ has binomial distribution with parameter n and γ_A since the voting probabilities for each members are independent. Let random variable $Y_{A,f}$ denote the numbers which f-type members vote for A on the state $w = A$. For large n, we apply the central limit theorem, $Y_{A,f}$ is asymptotically distributed normal distribution with mean $n\gamma_{A,f}$ and variance $n\gamma_{A,f}(1-\gamma_{A,f})$. Then, $\bar{Y}_{A,f} \overset{d}{\sim} N(\gamma_{A,f}, \gamma_{A,f}(1-\gamma_{A,f})/n)$ where $\bar{Y}_{A,f} = Y_{A,f}/n$. On the other hand, let random variable $Y_{B,f}$ denote the numbers which f-type members vote for A on the state $w = B$ and $\bar{Y}_{B,f}/n \overset{d}{\sim} N(1-\gamma_{B,f}, \gamma_{B,f}(1-\gamma_{B,f})/n)$. Similarly, $\bar{Y}_{A,g}$ and $\bar{Y}_{B,g}$ are defined. Note that $\bar{Y}_{\cdot,f}$ and $\bar{Y}_{\cdot,g}$ are independent since (s_i, t_i) and (s_j, t_j) are independent given the state.

We define $\bar{Z}_A$ and $\bar{Z}_B$ as

$$\bar{Z}_A = \bar{Y}_{A,f} + \bar{Y}_{A,g} \tag{5.14}$$

and

$$\bar{Z}_B = \bar{Y}_{B,f} + \bar{Y}_{B,g}. \tag{5.15}$$

For sufficiently large n,

$$\bar{Z}_A \overset{d}{\sim} N(\gamma_{A,f} + \gamma_{B,f}, \frac{\gamma_{A,f}(1-\gamma_{A,f}) + \gamma_{A,g}(1-\gamma_{A,g})}{n})$$

and

$$\bar{Z}_B \overset{d}{\sim} N(2 - \gamma_{B,f} - \gamma_{B,g}, \frac{\gamma_{B,f}(1-\gamma_{B,f}) + \gamma_{B,g}(1-\gamma_{B,g})}{n}).$$

Then, the probability density function of $\bar{Z}_A$ is

$$f_A(z_A) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{\gamma_{A,f}(1-\gamma_{A,f})+\gamma_{A,g}(1-\gamma_{A,g})}{n}}} \exp\left\{-\frac{1}{2}\frac{(z_A - \gamma_{A,f} - \gamma_{A,g})^2}{\frac{\gamma_{A,f}(1-\gamma_{A,f})+\gamma_{A,g}(1-\gamma_{A,g})}{n}}\right\}.$$

Similarly, the probability density function of $\bar{Z}_B$ is

$$f_B(z_B) = \frac{1}{\sqrt{2\pi}\sqrt{\frac{\gamma_{B,f}(1-\gamma_{B,f})+\gamma_{B,g}(1-\gamma_{B,g})}{n}}} \exp\left\{-\frac{1}{2}\frac{(z_B - 2 + \gamma_{B,f} + \gamma_{B,g})^2}{\frac{\gamma_{B,f}(1-\gamma_{B,f})+\gamma_{B,g}(1-\gamma_{B,g})}{n}}\right\}.$$

The ratio of the densities at $z_A = z_B = 1$ is

$$\begin{aligned} L(t) &= \sqrt{\frac{\gamma_{B,f}(1-\gamma_{B,f}) + \gamma_{B,g}(1-\gamma_{B,g})}{\gamma_{A,f}(1-\gamma_{A,f}) + \gamma_{A,g}(1-\gamma_{A,g})}} \\ &\quad \times \exp\left\{-\frac{n}{2}\left(\frac{(1-\gamma_{A,f}-\gamma_{A,g})^2}{\gamma_{A,f}(1-\gamma_{A,f}) + \gamma_{A,g}(1-\gamma_{A,g})} - \frac{(1-\gamma_{B,f}-\gamma_{B,g})^2}{\gamma_{B,f}(1-\gamma_{B,f}) + \gamma_{B,g}(1-\gamma_{B,g})}\right)\right\}. \end{aligned}$$

This ratio goes to infinity if

$$\frac{(1-\gamma_{A,f}-\gamma_{A,g})^2}{\gamma_{A,f}(1-\gamma_{A,f}) + \gamma_{A,g}(1-\gamma_{A,g})} - \frac{(1-\gamma_{B,f}-\gamma_{B,g})^2}{\gamma_{B,f}(1-\gamma_{B,f}) + \gamma_{B,g}(1-\gamma_{B,g})} < 0.$$

We define a function

$$l(t) = \frac{(1-\gamma_{A,f}-\gamma_{A,g})^2}{\gamma_{A,f}(1-\gamma_{A,f})+\gamma_{A,g}(1-\gamma_{A,g})} - \frac{(1-\gamma_{B,f}-\gamma_{B,g})^2}{\gamma_{B,f}(1-\gamma_{B,f})+\gamma_{B,g}(1-\gamma_{B,g})}.$$

We consider the asymmetric equilibrium, then

$$\begin{aligned}
\gamma_{A,f} &= 1-t^2 \\
\gamma_{B,f} &= 1-(1-t)^2 \\
\gamma_{A,g} &= 1 \\
\gamma_{B,g} &= 1/2.
\end{aligned}$$

The function is

$$\begin{aligned}
l(t) &= \frac{1-t^2}{t^2} - \frac{((1-t)^2-1/2)^2}{(1-(1-t)^2)(1-t)^2+1/4} \\
&= \frac{1-t^2}{t^2} - \frac{(1-t)^4-(1-t)^2+1/4}{-(1-t)^4+(1-t)^2+1/4}
\end{aligned}$$

and it is easy to check that $l(1) < 0$. By the continuity of t, there exists $\bar{t} < 1$ such that $l(t) < 0$ for any $t > \bar{t}$. Then, Lemma 5.1 holds.

Bibliography

Austen-Smith, D. and J.S. Banks, "Information Aggregation, Rationality, and the Condorcet Jury Theorem," *American Political Science Review*, 1996, *90* (1), 34–45.

Black, D., *The Theory of Committees and Elections*, Cambridge University Press, 1958.

Condorcet, M., *Essay sur l'application de l'analyse de la probabilité des decisions: Redues et pluralité des voix*, l'Imprimerie royale, 1785.

Coughlan, P.J., "In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting," *American Political Science Review*, 2000, *94* (2), 375–393.

Dekel, E. and M. Piccione, "Sequential Voting Procedures in Symmetric Binary Elections," *Journal of Political Economy*, 2000, *108* (1), 34–55.

Duggan, J. and C. Martinelli, "A Bayesian Model of Voting in Juries," *Games and Economic Behavior*, 2001, *37* (2), 259–294.

Feddersen, T. and W. Pesendorfer, "The Swing Voter's Curse," *American Economic Review*, 1996, *86* (3), 408–424.

——— **and** ——— , “Voting Behavior and Information Aggregation in Elections with Private Information,” *Econometrica*, 1997, *65* (5), 1029–1058.

——— **and** ——— , “Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting,” *American Political Science Review*, 1998, *92* (1), 23–35.

Gerling, K., H.P. Gruner, A. Kiel, and E. Schulte, “Information Acquisition and Decision Making in Committees: a Survey,” *European Journal of Political Economy*, 2005, *21* (3), 563–597.

Koriyama, Y. and B. Szentes, “A Resurrection of the Condorcet Jury Theorem,” *Theoretical Economics*, 2009, *4* (2), 227–252.

Levy, G., “Anti-herding and Strategic Consultation,” *European Economic Review*, 2004, *48* (3), 503–525.

——— , “Decision Making in Committees: Transparency, Reputation, and Voting Rules,” *American Economic Review*, 2007, *97* (1), 150–168.

Li, H. and W. Suen, “Decision-making in Committees,” *Canadian Journal of Economics*, 2009, *42* (2), 359–392.

McLean, I. and F. Hewitt, *Condorcet: Foundations of Social Choice and Political Theory*, Edward Elgar Publishing, 1994.

McLennan, A., “Consequences of the Condorcet Jury Theorem for Beneficial Information Aggregation by Rational Agents,” *American Political Science Review*, 1998, *92* (2), 413–418.

McMurray, J.C., "Aggregating Information by Voting: the Wisdom of the Experts Versus the Wisdom of the Masses," *Review of Economic Studies*, 2013, *80* (1), 277–312.

Meirowitz, A., "Informative Voting and Condorcet Jury Theorems with a Continuum of Types," *Social Choice and Welfare*, 2002, *19* (1), 219–236.

Mukhopadhaya, K., "Jury Size and the Free Rider Problem," *Journal of Law, Economics, and Organization*, 2003, *19* (1), 24–44.

Nagaoka, N. and H. Suehiro, "Sequential Voting and Efficiency: An Example," *The Kokumin-keizai zasshi*, 2013, *208* (3), 21–60. (in Japanese).

Nitzan, S. and J. Paroush, *Collective Decision Making: an Economic Outlook*, Cambridge University Press, 1985.

Ottaviani, M. and P. Sørensen, "Information Aggregation in Debate: Who Should Speak First?," *Journal of Public Economics*, 2001, *81* (3), 393–421.

Persico, N., "Committee Design with Endogenous Information," *Review of Economic Studies*, 2004, *71* (1), 165–191.

Rokas, K.N. and V. Tripathi, "Information Aggregation in Voting with Endogenous Timing," *Unpublished*, 2007.