



Anomalous Higgs Interactions in Gauge-Higgs Unification with Deconstructing 5D Scalar QED

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Doctoral Dissertation

**Anomalous Higgs Interactions in
Gauge-Higgs Unification with
Deconstructing 5D Scalar QED**

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Abstract

In this doctoral thesis, we discuss anomalous Higgs interactions, namely the interactions which deviate from those predicted by the standard model, in two candidates of physics beyond the standard model, gauge-Higgs unification and dimensional deconstruction. As the characteristic feature, these scenarios predict the presence of non-linear Higgs interactions at the classical level with matter fields. Also the predicted Yukawa coupling or corresponding coupling with scalar matter of Higgs is not a constant and depends on the vacuum expectation value of the Higgs field, in clear contrast to the case of the standard model. After the discovery of the Higgs, main focus will be on the precision tests of Higgs interactions and we expect that such analysis concerning anomalous Higgs interaction makes sense. We also clarify what are essential causes for the anomalous Higgs interaction and point out that there are two: periodicity of physical observables in the Higgs field and the violation of translational invariance along the extra dimension. These two ingredients are argued to exist in both scenarios of gauge-Higgs unification and dimensional deconstruction.

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1 Introduction

1.1 Particle physics after the discovery of the Higgs

It was a great achievement of the LHC experiments [1], [2] to have discovered the Higgs particle.

We, however, should note that at this stage, we do not know whether the discovered scalar particle is really what the standard model predicts or a particle some theory of physics beyond the standard model (BSM) has in its low energy effective theory. In other words, we still do not have any conclusive argument of the origin of the Higgs itself. It is worth noticing that many of the theories of BSM actually have been proposed in order to solve the theoretical problems held by the Higgs particle, such as the hierarchy problem.

Thus, we may be able to claim that the Higgs physics is now just at the starting point and that it is the right time to think about which direction we should pursue among possible candidates of BSM theories with the valuable information given by the LHC experiments in our hands.

From such a point of view it is quite interesting to note that the observed Higgs mass $M_H = 126$ GeV by the LHC experiments is comparable to the weak scale M_W , roughly speaking. Namely the Higgs has turned out to be “light” and only the theories predicting a light Higgs is favored among possible BSM theories.

This fact of light Higgs also suggests that the Higgs self-coupling λ to determine the Higgs mass-squared is of $\mathcal{O}(g^2)$ (g : gauge coupling constant) and that it is handled by gauge interactions.

It is interesting to note that such self-coupling is realized in MSSM (Minimal Supersymmetric Standard Model). MSSM is a model constructed by making the standard model supersymmetric in order to solve the famous problem in the Higgs sector of the standard model, i.e. “hierarchy problem”. The hierarchy problem at the quantum level can be stated as a question of how to avoid the quadratically divergent quantum correction to the Higgs mass. In MSSM the quadratic divergence is cancelled by the presence of “super-partners” (supersymmetric partners of ordinary particles). In order to preserve the supersymmetry of the theory, Higgs sector is extended: two Higgs doublets are introduced to the model.

In fact, in this scenario λ is really of the order of g^2 , since the self-coupling gets the contribution only from “D-term”, which leads to the relation $M_H \leq \cos 2\beta M_Z$ at the classical level, though it is remarkably modified at the quantum level for larger SUSY breaking mass scales.

There is another candidate of BSM scenario which predicts light Higgs in general: Gauge-Higgs Unification (GHU) [3], [4], [5]. In this scenario, Higgs field is identified with the extra-space component of the higher-dimensional gauge field: in the simplest 5D U(1) GHU model, the higher-dimensional gauge field is decom-

posed as

$$A_M = (A_\mu, A_y) \quad (\mu = 0, 1, 2, 3), \quad (1.1)$$

where A_μ just behaves as ordinary 4D gauge field, while (the Kaluza-Klein (KK) zero mode of) A_y is understood as the Higgs field.

The fact that the Higgs is originally a gauge boson in GHU provides a new type of solution for the hierarchy problem relying on higher dimensional gauge symmetry, thus opening a new avenue for BSM theories [6]. Let us recall that photon never gets a mass even at the quantum level. To be more precise, by summing up the contribution of all Kaluza-Klein (KK) modes in the virtual state, the quadratic divergence cancels out, by the presence of the KK partners of the ordinary particles.

In this scenario of GHU, the Higgs self-coupling is handled by gauge principle, just because Higgs is originally a gauge field. Thus, light Higgs is natural consequence of this scenario. For instance, in 5D GHU models the Higgs potential does not exist at the classical level and the Higgs mass just vanishes, though it gets a non-vanishing contribution at the quantum level. One comment here is that in the GHU models in the presence of multiple extra dimensions the situations changes and in some cases it is possible to get non-vanishing Higgs mass even at the classical level. For instance, in 6-dimensional (6D) space-time, GHU model, with an orbifold T^2/Z_3 as its extra space, predicts $M_H = 2M_W$ at the classical level [7], similarly to the prediction $M_H \sim M_Z$ of MSSM.

It is worth noticing that both of MSSM and the idea of GHU have been proposed aiming to solve the hierarchy problem; the problem of quadratic divergence. There also exist other types of well-discussed BSM theories formulated in 4D space-time, also for the purpose of solving the problem of quadratic divergence. Interestingly, some of these theories have close relationship with GHU scenario mentioned above. Namely, both of

- (i) Dimensional deconstruction (DD) [8]
- (ii) Little Higgs (LH) [9]

are known to be closely related to the GHU scenario. Basically, in both of these scenarios Higgs is regarded as (pseudo) Nambu-Goldstone (NG) boson and therefore is non-linearly realized in a form $e^{i\frac{\phi}{f}}$, where ϕ denotes the NG boson treated as the Higgs field and f corresponds to the “decay constant” of the NG boson. Let us note that this leads to an important consequence that physical observable are periodic in the Higgs field ϕ . Similar periodicity exists, as we will discuss later, in the scenario of GHU too. The periodicity will be argued to be (one of) the key ingredient to get the “anomalous Higgs” interactions, whose meaning is explained below. As long as Global symmetry is exact the Higgs mass should vanish, similarly to the case of 5D GHU. Thus these scenarios naturally predict a light Higgs.

As another circumstantial evidence of the mutual relation between GHU and DD or LH, both have shift symmetries, i.e. the invariance under the transformations

$$A_y \rightarrow A_y + \partial_y \lambda \quad (\text{for GHU}), \quad \phi \rightarrow \phi + c \quad (\text{for DD, LH}), \quad (1.2)$$

where λ is a gauge parameter in the simplest U(1) 5D GHU model, e.g., and c is a constant denoting the parameter of some global transformation like a phase in the case of U(1) symmetry. These shift symmetries are nothing but the local gauge symmetry in the case of GHU and the global symmetry in the case of DD or LH. These symmetries in turn strictly forbid the presence of (local) operators responsible for the masses of A_y and ϕ : $m^2 A_y^2$ and $m^2 \phi^2$, respectively.

Such mutual relation becomes manifest and more solid, once we realize that the DD scenario may be regarded as the “latticed” version of GHU scenario, where the extra space is latticed to several lattice points and $e^{i\frac{\phi}{f}}$ is regarded as the (product of) “link variables” (or Wilson-loop) in a lattice gauge theory, as we will see later. Also note that the scenario of the LH was originally proposed being inspired by the DD scenario.

1.2 The purpose of the thesis- “anomalous Higgs interactions”

The purpose of this thesis is to discuss “anomalous Higgs interactions” in GHU and in its closely related scenario, DD. Namely we show that in these two types of BSM theories the couplings of Higgs interactions such as Yukawa coupling deviates from what the standard model (SM) predicts. This is what we mean by the anomalous interactions.

After the discovery of the Higgs main focus is now to check whether its interactions are what the SM predict or not. In fact, the LHC experiments have reported some results on the Yukawa couplings with τ and some results may be released on the coupling with μ . If some deviation from the predictions of the SM is found by precision tests of the Higgs interactions it will clearly signal the presence of new physics. The precision tests of the Higgs interactions should be one of the main purposes of the proposed ILC experiment.

In addition to GHU and DD, MSSM is also known to predict anomalous Yukawa coupling as we will see below. It is quite interesting to note that all of GHU, DD and MSSM were proposed in order to solve the hierarchy problem and that all predict anomalous interactions, while another BSM scenario, i.e. universal extra dimension (UED) predicts “normal” Higgs interactions, just because it was constructed just making the SM higher dimensional.

The fact that the scenarios aiming to solve the hierarchy problem shows anomalous Higgs interactions may be a natural consequence. All of these scenarios extend

the Higgs sector by introducing the partners of Higgs, the super-partner and/or another scalar form additional Higgs doublet in the case of MSSM, or the KK partners in the case of GHU and DD. Also in the case of GHU and DD, the Higgs has its own specific origin, i.e. as a gauge field and a NG boson, respectively. Thus it may not be surprising that these scenarios all predict anomalous Higgs interactions.

Thus, Higgs interactions are divided into two categories: anomalous versus normal, and to see which categories the Nature choose is quite important in order to see whether physics beyond the standard model is realized and also to see which type of BSM is selected in Nature.

Concerning the anomalous Higgs interactions, now we will make tow comments. First, by taking Yukawa coupling as an example, the anomalous Higgs interactions of MSSM and that of GHU or DD are qualitatively different.

In MSSM, the couplings of the Higgs interactions generally depend on the two parameters β and α , denoting by which ratio the VEV of the Higgs and the mass eigenstate of CP even scalar are divided into the two Higgs doublets: e.g. $\tan \beta = \frac{v_u}{v_d}$. For instance, the ratios of the Yukawa coupling of the SM-like Higgs to that of the SM for top and bottom quarks are given as

$$\frac{f_t^{(MSSM)}}{f_t^{(SM)}} = \frac{\cos \alpha}{\sin \beta}, \quad (1.3)$$

$$\frac{f_b^{(MSSM)}}{f_b^{(SM)}} = -\frac{\sin \alpha}{\cos \beta}. \quad (1.4)$$

These ratios do just depend on the two angles α and β , and are generation-independent, in clear contrast to the case of GHU where the deviation of Yukawa coupling appears only for light quarks, as we will discuss later.

The second comment we should make is that such deviation goes away in the extreme limit of the scale of new physics goes infinity. Namely when the scale of new physics, such as the SUSY breaking mass scale M_{SUSY} in MSSM and the compactification mass scale $M_c = \frac{1}{R}$, with R being the size of the extra space, in GHU, is much larger than the weak scale M_W , the SM is recovered. This is a very reasonable result, since in this limit all new particles of BSM models get much heavier than the weak scale and are expected to decouple from the low energy effective theory.

In fact, at the limit of $M_{SUSY} \rightarrow \infty$, $\alpha \rightarrow \beta - \frac{\pi}{2}$, and we easily confirm that the ratios in (1.4) just reduce to the unity.

Probably one of the most important observation in this thesis is that the anomalous Higgs interactions in the scenarios of GHU and DD are the inevitable consequence of the following two properties of these theories:

(a) The periodicity of the physical observables in the Higgs field

(b) The violation of the translational invariance along the extra space

The property (or condition) (b) is due to the presence of “ Z_2 -odd” bulk mass of the fermion in the case of GHU and the fact that the extra space is latticed in the case of DD, as we will discuss later.

Before discussing how the anomalous Higgs interaction is obtained by realizing above two properties, we briefly summarize the ideas of the scenarios of GHU and DD, successively below.

1.3 Gauge-Higgs unification

As was mentioned above, in the scenario of gauge-Higgs unification (GHU), the Higgs is originally a gauge field A_y in the simplest 5D models. Thus the local operator, responsible for the mass-squared of the Higgs, $m^2 A_y A^y$ is strictly forbidden by the local gauge symmetry and we naively expect that the Higgs never gets a mass even at the quantum level, leading to the fact that the quantum correction to the Higgs mass does not suffer from the quadratic divergence, thus solving the hierarchy problem. As has been already mentioned above, we believe that photon never gets a mass so that photon can propagate at the speed of light.

The concrete calculation for the Higgs mass in 5D QED, however, has shown that the quantum correction to the Higgs mass is (UV-)finite, as was expected, but non-vanishing [6] ! A remaining logical possibility is that non-local but gauge-invariant operator, i.e. Wilson-loop

$$W = e^{ie \oint A_y dy} = e^{ie A_y^{(0)} (2\pi R)}, \quad (1.5)$$

is allowed and is responsible for the Higgs mass. In (1.5) e is the electric charge and R is the radius of the circle S^1 , the extra space. By performing Fourier series expansion of A_y in the line integral along the circle, only KK zero mode $A_y^{(0)}$ survives. Let us note that (1.5) does not contain any derivative of $A_y^{(0)}$, thus resulting quadratic term $(A_y^{(0)})^2$ behaves as the Higgs mass-squared term.

If the $A_y^{(0)}$ is treated as the Higgs field, it should have a constant VEV (vacuum expectation value) in order to realize the spontaneous symmetry breaking [5]. Then a natural question to ask is whether such a constant gauge field ever has some physical meaning. It provides vanishing field strength (vanishing “electromagnetic” fields) and therefore seems to have no physical meaning: i.e. it is just a “pure gauge” configuration. Actually, however, we can argue that the constant $A_y^{(0)}$ really is physically meaningful: it may be understood as a sort of Aharonov-Bohm (AB) phase when the compactified extra space is non-simply-connected space such as a circle S^1 . Namely it may be regarded as a AB phase when magnetic flux penetrates inside the circle, which is nothing but the phase in (1.5). In fact, by

use of Stokes' theorem the phase can be written as

$$e \oint A_y dy = e A_y^{(0)} (2\pi R) = e\Phi, \quad A_y^{(0)} = \frac{\Phi}{2\pi R}, \quad (1.6)$$

where Φ is the magnetic flux. Thus $A_y^{(0)}$ cannot be “gauged away”.

The fact that the Higgs field h or equivalently $A_y^{(0)}$ appears only through a phase leads to an important consequence that physical observables are periodic in h , which is one of the key ingredients to get the anomalous Higgs interaction in GHU, as was already mentioned.

1.4 Dimensional deconstruction

In this subsection, we briefly discuss the idea of dimensional deconstruction scenario (DD) [8], emphasizing its close relation with the scenario of GHU.

The theory of DD is a gauge theory realized in ordinary 4D space-time and therefore renormalizable, in contrast to the case of GHU formulated in higher dimensional space-time. In this scenario, the hierarchy problem, i.e. the problem of quadratic divergence is claimed to be solved in a certain condition, without relying on supersymmetry.

The scenario is characterized by the following features:

There is a repetition of gauge symmetries N -times: $(G \times G_s)^N$ ($G = \text{SU}(m)$, $G_s = \text{SU}(n)$).

Higgs is a pseudo NG boson, a bound state of fermions, just as pions in QCD.

Interestingly, it can be shown that the quantum correction to the Higgs mass becomes finite for $N \geq 3$. We thus know that the essential ingredient to solve the hierarchy problem is the enough repetition of 4D gauge symmetries. This reminds us the situation in GHU that the quadratic divergence disappears when the sum over all KK modes is made [6]. Namely, in the DD scenario, we have N “K-K” modes, instead of the infinite number of KK modes in GHU.

In the model N pairs of Weyl fermions with the same chirality, belonging to the bi-fundamental repr.s of $\text{SU}(m) \times \text{SU}(n)$, are introduced:

$$\begin{aligned} (m, \bar{n}), & \quad (\text{SU}_i(m), \text{SU}_i(n)) \\ (\bar{m}, n) & \quad (\text{SU}_{i+1}(m), \text{SU}_i(n)) \quad (i = 1 - N), \end{aligned} \quad (1.7)$$

where periodic boundary conditions $\text{SU}_{N+1}(m) = \text{SU}_1(m)$ etc. are imposed.

The theory is schematically represented by “moose diagram” shown in Fig.1, where each oriented line corresponds to each Weyl fermion with either $(m, \bar{n})$ or $(\bar{m}, n)$ repr., depending on the blob denoting the gauge group G_s is at the right or left of the line.

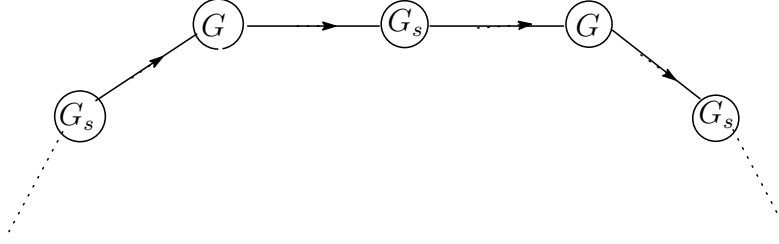


Fig. 1

Figure 1: moose diagram

Both of $SU(m)$ and $SU(n)$ are asymptotically free gauge symmetries and their typical mass scales are Λ , Λ_s , respectively. Assuming $\Lambda \ll \Lambda_s$, the $SU(n)$ interaction is treated as a strong force, which forms a bound state from a pair of Weyl fermions, just as the hadrons in QCD. At lower energies, $E \leq \Lambda_s$, the effective low energy theory is described by pseudo scalars corresponding to pion π_i as the (pseudo) NG bosons due to the spontaneous breaking of the chiral symmetry $SU_i(m) \times SU_{i+1}(m)$:

$$U_i = e^{i \frac{\pi_i^a T_a}{f}} \quad (T_a : \text{generators of } SU(m)), \quad (1.8)$$

where f corresponds to the pion decay constant. The non-linear realization U_i behaves as a bi-fundamental repr. of the chiral symmetry: i.e. $(m, \bar{m})$ under $(SU_i(m), SU_{i+1}(m))$.

The effective action for U_i and the gauge fields of “weak interaction” $SU_i(m)$ is given as

$$S = \int d^4x \left(-\frac{1}{2g^2} \sum_{j=1}^N \text{tr}(F_j^{\mu\nu})^2 + f^2 \sum_{j=1}^N \text{tr}[(D_\mu U_j)^\dagger (D^\mu U_j)] \right), \quad (1.9)$$

with the covariant derivative

$$D_\mu U_j = \partial_\mu U_j - i A_\mu^j U_j + i U_j A_\mu^{j+1}, \quad (1.10)$$

where A_μ^j are gauge bosons of $SU(m)$ and $F_j^{\mu\nu}$ are their field strengths.

This action can be regarded as the one for a 5D $SU(m)$ pure non-Abelian gauge theory, where the extra space is “latticed”. Namely, U_j corresponds to the link variable in the lattice gauge theory. From such point of view U_j may be regarded as a Wilson line along the extra dimension. In fact, in the simplified case of Abelian gauge theory

$$D_\mu U_j \rightarrow i[\partial_\mu \pi - gf(A_\mu^{j+1} - A_\mu^j)]U_j, \quad (1.11)$$

and identifying the lattice spacing a of the extra dimension as

$$a = \frac{1}{gf}, \quad (1.12)$$

the covariant derivative just corresponds to the field strength $F_{\mu y}$ in the GHU:

$$D_\mu U_j \rightarrow \partial_\mu A_y - \partial_y A_\mu = F_{\mu y} \quad (\pi \equiv A_y). \quad (1.13)$$

Thus, the extra dimension has been constructed by the dynamics in 4D space-time: The circle of the moose diagram thus may be understood to represent the circle of the extra space.

The gauge symmetry $(\text{SU}(m))^N$ is spontaneously broken into a single $\text{SU}(m)$:

$$\langle U_i \rangle = 1 \rightarrow g_i \langle U_i \rangle g_{i+1}^\dagger = \langle U_i \rangle, \quad \text{only when } g_i = g_{i+1}, \quad (1.14)$$

where g_i is an element of $\text{SU}_i(m)$. Thus among N NG bosons U_i ($i = 1, 2, \dots, N$), $N-1$ pieces are “eaten” to massive gauge bosons by the Higgs mechanism, resulting in only one physical (pseudo) NG boson, which is identified as the Higgs field. The remaining Higgs is described by

$$\text{Tr}(U_1 U_2 \dots U_N), \quad (1.15)$$

which just corresponds to the product of Wilson-lines, i.e. Wilson-loop (1.5) in GHU.

In Abelian case, $U_1 U_2 \dots U_N = e^{\frac{i}{f}(\pi_1 + \dots + \pi_N)}$ and defining $\phi = \frac{\pi_1 + \pi_2 + \dots + \pi_N}{\sqrt{N}}$, the field ϕ just corresponds to the zero-mode of A_y , i.e. the Higgs field in GHU.

Part I

Anomalous Higgs Interaction in gauge-Higgs Unification

2 Introduction for Part I

The purpose of this first part of the thesis is to discuss the anomalous Higgs interactions in the scenario of gauge-Higgs unification (GHU). The discussion here will be of great help in understanding the arguments extended in the second part of the thesis concerning the anomalous Higgs interactions in the scenario of dimensional deconstruction (DD), which is closely related to GHU.

As was pointed out in the introduction as the whole of the thesis, in the scenario of GHU, the Higgs field is originally gauge field and as the characteristic feature of the scenario, Higgs field may be interpreted as a sort of Aharonov-Bohm (AB) phase or Wilson loop along the direction of the extra space (, when the extra space is non-simply-connected space like a circle S^1). This was why the VEV of the Higgs, even though it is a constant gauge field configuration, is not a pure gauge configuration but has a physical meaning (as a sort of magnetic flux penetrating the non-simply-connected space).

The fact that Higgs field appears basically only through the Wilson loop phase implies that in this scenario physical observables are functions of the Wilson loop and therefore has a periodicity in the Higgs field with a period

$$\frac{2}{g_4 R}, \quad (2.1)$$

where g_4 is the 4D gauge coupling and R is the radius of S^1 . The typical example to show the periodicity is the effective Higgs potential induced at the quantum level:

$$V(v) \propto \frac{3}{4\pi^2} \frac{1}{(2\pi R)^4} \sum_{n=1}^{\infty} \frac{\cos(ng_4\pi Rv)}{n^5}, \quad (2.2)$$

where v is the VEV of the Higgs and n is a “winding number”, denoting how many times the loop of the Feynman diagram is wrapped around S^1 , which appears after the usage of Poisson resummation technique.

We now argue that this periodicity of the observables in the Higgs field is (one of) essential ingredients to have anomalous Higgs interactions. First, we all know that typical periodic functions are trigonometric functions like the cos function in (2.2). The trigonometric function also appears in the mass spectrum of the matter fermions. We will see below that 4D mass eigenvalue of the Kaluza-Klein (KK) zero mode of light quarks, say quarks of 1st or 2nd generation, is well approximated by sine-function of the Higgs VEV v :

$$m(v) \propto \sin\left(\frac{g_4}{2}\pi Rv\right), \quad (2.3)$$

which is a non-linear function of v , in clear contrast to the case of the standard model (SM), where $m(v)$ is linear in the VEV v . This strongly suggests that we have anomalous Yukawa coupling in this scenario. Here we will see briefly how the anomalous Yukawa coupling is obtained from such non-linear mass eigenvalue. Since physical Higgs particle is identified with the shift of the Higgs scalar field from its VEV. It, therefore, seems to be reasonable to expect that Higgs interaction with each quark is obtained by replacing v by $v + h$ in the mass term $m(v)\bar{\psi}\psi$ (ψ : KK zero mode):

$$m(v + h)\bar{\psi}\psi \propto \sin\left(\frac{g_4}{2}\pi R(v + h)\right)\bar{\psi}\psi, \quad (2.4)$$

which clearly contain non-linear interactions of the Higgs with light quarks, such as $h^2\bar{\psi}\psi$ at the classical (tree) level, which does not happen in the SM.

By performing Taylor expansion we obtain the Yukawa coupling constant f of Yukawa interaction $h\bar{\psi}\psi$:

$$f \propto \cos\left(\frac{g_4}{2}\pi Rv\right), \quad (2.5)$$

which is not a constant as in the SM but behaves as cos function, which even vanishes for a specific value of the VEV, satisfying

$$v = \frac{1}{g_4 R}. \quad (2.6)$$

Namely, in this case Yukawa coupling vanishes and may behave as “dark matter”, while Higgs provides quarks with their masses, as was first pointed out by Y. Hosotani et al [10, 11, 12, 13].

Though such drastic possibility has been ruled out by the recent discovery of the Higgs at LHC, the scenario generally predicts Yukawa coupling, which deviates from what we expect in the SM depending on the VEV v : “anomalous Higgs interaction”. We will give a general formula for the deviation and will see that GHU generally gives Yukawa coupling, always smaller than the SM prediction, though it approaches to the SM prediction for $M_W \sim g_4 v \ll \frac{1}{R}$, namely when the weak scale is much smaller than the compactification scale $M_c \equiv \frac{1}{R}$ of the extra space, as we reasonably expect since in this limit non-zero KK modes are expected to decouple from low energy effective theory and the SM is recovered.

To be precise, as we will see below in this scenario of GHU the Yukawa coupling shows another unusual property. Namely, the Yukawa coupling written as a form of a matrix in the base of all KK modes is non-diagonal. Namely Higgs has “off-diagonal” couplings with different KK modes of quarks. It will be argued that this mixing is understood as the cause of the non-linear Higgs interaction. Let us note that the Yukawa coupling in the lagrangian to start with is linear in the Higgs field and the non-linear Higgs interactions with zero mode quarks (of light generations) shown in (2.4) are those which are obtained by “integration out” the heavy quarks with non-zero KK modes. In this understanding the Yukawa coupling of (2.5) is known to coincide with the diagonal element of the matrix for zero mode quark.

Argument above seems to suggest that the periodicity of physical observables in the Higgs field is only essential ingredient to lead to the anomalous Higgs interactions. As the matter of fact as we will discuss later, there is another key ingredient to get the anomalous interactions, i.e. the violation of translational invariance along the direction of extra space. In our model the violation of the translational invariance is realized by the introduction of so-called Z_2 -odd bulk mass for quarks introduced for light quarks to realize the smallness of zero mode quark masses. This a little unusual mass term is needed in order to guarantee the

discrete Z_2 symmetry of the orbi-fold S^1/Z_2 , which we adopt as the extra space in our model.

So, here we briefly review what is orbi-fold and why we need it from physical point of view. First, let us note that in general starting from higher dimensional theories when the theory is reduced to our 4D space-time, we always get non-chiral theory, if we take ordinary manifold like circle or sphere S^n ($n = 1, 2, \dots$) as the extra space. By non-chiral we mean that the theory contains both right- and left-handed chiralities of 4D fermions with exactly the same quantum numbers and the repr. of gauge group and “chiral theory” such as the SM with different gauge interactions depending on the chirality (parity violation in the sector of weak interaction) cannot be accommodated. In fact, for odd space-time dimension, such as 5D, chiral operator does not exist and Weyl spinor with definite chirality cannot be defined. When space-time dimension is even, such as 6D, we have higher-dimensional chiral operator such as $\Gamma_7 = i\Gamma^0\Gamma^1\cdots\Gamma^6$ so that Weyl fermion with definite eigenvalue of the operator can be defined. But the problem is such higher dimensional Weyl fermion contains a pair of 4D Weyl fermions with both R and L chiralities: a 6D Weyl fermion is equivalent to a 4D Dirac fermion.

Orbi-fold is obtained by “dividing” manifold by its discrete symmetry. S^1/Z_2 orbi-fold is obtained by dividing circle S^1 by Z_2 symmetry, i.e. the symmetry under

$$Z_2: y \rightarrow -y, \quad (x^\mu \rightarrow x^\mu), \quad (2.7)$$

where y is the extra space coordinate along the circle while x^μ is ordinary 4D space-time coordinates. Z_2 has some similarity to the parity symmetry and the circle clearly is invariant under the transformation (2.7). S^1/Z_2 is obtained by identifying the points of coordinates y and $-y$. S^1/Z_2 orbi-fold has two fixed points, the points with coordinates $y = 0, \pm\pi R$, which are invariant under the Z_2 transformation ($y = \pi R$ and $y = -\pi R$ denote the same point).

Roughly speaking, by dividing the original S^1 by Z_2 , the number of chirality is also divided by 2. More precisely, we will see that only Weyl fermion with either R or L chirality survives as the KK zero mode. The situation is similar to the case of quantum mechanical system with parity symmetry, where because of the parity symmetry either even- or odd-function of space coordinates is allowed for wave functions. Similarly, in our case either even- or odd-function of y is allowed for mode functions, eigenfunctions with definite 4D mass-eigenvalues, and concerning the KK zero mode only the even function is allowed to exist (recall that $\sin 0 = 0$). Since Z_2 transformation behaves as a sort of chiral transformation for fermion, $\psi \rightarrow \gamma_5\psi$, its eigenvalue is just opposite for R and L chiralities. Thus concerning the KK zero mode, either R or L (not both) Weyl fermion is allowed to exist and therefore the low energy effective theory becomes a chiral theory (, though in the sector of non-zero KK modes we still have both chiralities and theory

is non-chiral).

Since Z_2 transformation is a sort of chiral transformation, ordinary bulk mass term of the form $M\bar{\psi}\psi$ (M : bulk mass) is not Z_2 invariant, even though it is allowed by gauge symmetry. However, a specific sort of mass term, i.e. “ Z_2 -odd” bulk mass term, i.e. the bulk mass, which is odd function of y ,

$$\epsilon(y)M\bar{\psi}\psi \quad (2.8)$$

is Z_2 invariant and is allowed as far as it is also gauge invariant, where $\epsilon(y)$ is sign function: $\epsilon(y) = 1$ or -1 for $y > 0$ or $y < 0$.

The Z_2 -odd bulk mass $\epsilon(y)M$ mimics a soliton-like kink solution of some scalar field and its presence clearly violates translational invariance along the extra space. The kink-like solution also causes the localization of the mode function of zero mode Weyl fermion at different fixed points depending on its chirality. Thus the Yukawa coupling, which is obtained by overlap integral of mode functions of different chiralities gets exponential suppression factor, behaving as e^{-MR} , roughly speaking. Thus we will introduce the bulk mass term only for light quarks.

We will argue that the violation of translational invariance due to the presence of the Z_2 -odd bulk mass, together with the periodicity in the Higgs field, leads to the anomalous Yukawa coupling for light quarks as mentioned above. In fact, for heavy quark such as top quark, the bulk mass is not necessary and the obtained mass spectrum of KK modes is linear in the VEV v :

$$m_n = \frac{n}{R} \pm g_4 v, \quad (2.9)$$

and obtained Yukawa coupling is a constant just as in the SM. (Actually in the case of top quark, whose mass is larger than M_W , we need a mechanism to enhance the Yukawa coupling.)

3 5D SU(3) gauge-Higgs unification

3.1 The structure of the model

The model we adopt is 5D GHU model with SU(3) gauge symmetry. The SU(3) symmetry is for electro-weak interaction and SU(3)_c interaction has not been included, though it can be included straightforwardly if we wish. Let us first discuss why the electro-weak gauge symmetry is not SU(2)×U(1) but SU(3).

In the scenario of GHU, as its characteristic feature, the Higgs is originally gauge field and therefore belongs to the adjoint repr. of the gauge group. On the other hand in the SM with gauge group SU(2)×U(1), Higgs belongs to the fundamental repr. of SU(2). Thus we cannot use SU(2)×U(1) for the gauge group

and we need to extend the gauge group a little. Since the rank of $SU(2) \times U(1)$ group is 2, the simplest possibility to include the electro-weak sector of the SM is $SU(3)$. This is the reason why we chose 5D $SU(3)$ GHU model [14], [15] with orbifold extra space S^1/Z_2 as our model to study the anomalous Higgs interactions.

One of the reason to choose the orbifold S^1/Z_2 is to realize a chiral theory, as we have already seen in the introduction of part I of this thesis. Another reason is by such “orbifolding” the extended gauge group $SU(3)$ can be broken to the gauge group of the SM, i.e. $SU(2) \times U(1)$. The method is to assign different non-trivial “ Z_2 -parity”, i.e. the eigenvalue under the Z_2 transformation for each member of irreducible repr. of gauge group [16]. As we will see below, in this way among 4D $SU(3)$ gauge bosons A_μ , only the gauge bosons of the SM, i.e. the gauge bosons of $SU(2) \times U(1)$ have even Z_2 -parity with non-vanishing KK zero modes. Hence in the effective low energy theory remaining gauge symmetry is just that of the SM. On the other hand, concerning the 4D scalar A_y , the situation is just opposite and only the A_y bosons belonging to the broken generators in the breakdown $SU(3) \rightarrow SU(2) \times U(1)$ (the off-diagonal part of the 3×3 matrix) have even Z_2 -parity with non-vanishing KK zero modes. The broken generators behaves as a complex repr. of $SU(2)$ doublet, which is just what we need for the Higgs doublet of the SM. The reason why the Z_2 -parity assignment is just opposite for A_μ and A_y is that only A_y gets overall minus sign, $A_y \rightarrow -A_y$ under the Z_2 transformation, in order to coincide with the transformation of the extra space coordinate y , $y \rightarrow -y$. Let us note that (A_μ, A_y) behaves as 5D vector just as (x^μ, y) does. Let us note that the adjoint repr. of $SU(3)$, the octet 8, decomposes into $3 + 2 \times 2 + 1$ under the $SU(2)$. So, what is happening is that 4D gauge boson (3+1) and 4D Higgs scalar (2×2) just have been prepared in order to form the adjoint repr. of $SU(3)$ together ! This may be just a coincidence, but interesting.

Concerning strong interaction, we just introduce $SU(3)_c$ as a direct product and make trivial assignment of Z_2 parity, say giving even parity for all members of color triplet, so that we can get 8 gluon fields and no 4D scalar.

We also make a brief comment on the Weinberg angle. Since the electro-weak interaction is unified in a framework of $SU(3)$, the model predicts the value of Weinberg angle: $\sin^2 \theta_W = \frac{3}{4}$ at the tree level. Though this value may be modified by quantum corrections as in the case of GUT, the value is too large for the model to be realistic. We will discuss how this problem may be resolved in a separated section below.

After explaining the basic structure of our model we now move to the concrete description of the model: 5D $SU(3)$ GHU model for electro-weak interaction.

As the matter field we introduce quarks belonging to SU(3) triplet:

$$\Psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \end{bmatrix}. \quad (3.1)$$

The assignment of the electric charges of elements of the triplet is known to be $(\frac{2}{3}, -\frac{1}{3}, -\frac{1}{3})$ and the members are assigned to (u, d, d) quarks.

Since we want to break SU(3) into SU(2)×U(1) we make an assignment of Z_2 -parity for the fundamental triplet repr. as follows:

$$\begin{pmatrix} + \\ + \\ - \end{pmatrix} \quad (3.2)$$

where $+$, $-$ denotes even and odd Z_2 -parity, respectively. Correspondingly, the Z_2 -parity of quark fields are fixed as follows:

$$\Psi(-y) = -\gamma^5 P \Psi(y) \quad , \quad P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}. \quad (3.3)$$

The matrix P is an operator for the Z_2 parity assignment and we know Z_2 transformation contains a chiral transformation due to $-\gamma^5$. This is because by this chiral transformation the bi-linear form of spinors just coincides with the transformation of space-time coordinates (x^μ, y) shown in the (2.7):

$$\bar{\Psi}' i \gamma^5 \Psi' = -\bar{\Psi} i \gamma^5 \Psi, \quad \bar{\Psi}' \gamma^\mu \Psi' = \bar{\Psi} \gamma^\mu \Psi \quad (\Psi' = -\gamma^5 \Psi). \quad (3.4)$$

Since R and L Weyl fermions have opposite eigenvalues of γ^5 : $\gamma^5 \psi_R = \psi_R$, $\gamma^5 \psi_L = -\psi_L$, either R or L Weyl fermion is chosen to have KK zero mode for each component of SU(3) triplet. Thus the zero mode sector of the triplet is identified with the following chiral quarks (to be precise, when the VEV v can be ignored),

$$\Psi^{(0)} = \begin{bmatrix} u_L \\ d_L \\ d_R \end{bmatrix}. \quad (3.5)$$

Namely the upper two components of the triplet is nothing but the SU(2) doublet of left-handed quarks while the component at the bottom is identified with right-handed singlet of d-type quark.

Once the Z_2 -parity assignment is made for the fundamental repr., the parity assignments for 4D gauge boson A_μ and 4D scalar A_y are fixed accordingly as follows:

$$A_\mu(-y) = P A_\mu(y) P^{-1}, \quad A_y(-y) = -P A_y(y) P^{-1}. \quad (3.6)$$

Namely each component of these bosons has the following Z_2 -parities:

$$A_\mu = \begin{pmatrix} + & + & - \\ + & + & - \\ - & - & + \end{pmatrix}, \quad A_y = \begin{pmatrix} - & - & + \\ - & - & + \\ + & + & - \end{pmatrix}. \quad (3.7)$$

Thus the sector of KK zero mode for these bosons read as

$$A_\mu = \frac{1}{2} \begin{pmatrix} W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & \sqrt{2}W_\mu^+ & 0 \\ \sqrt{2}W_\mu^- & -W_\mu^3 + \frac{B_\mu}{\sqrt{3}} & 0 \\ 0 & 0 & -\frac{2}{\sqrt{3}}B_\mu \end{pmatrix}, \quad A_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 0 & \phi^+ \\ 0 & 0 & \phi^0 \\ \phi^- & \phi^{o*} & 0 \end{pmatrix}. \quad (3.8)$$

As we have already mentioned above the remaining 4D gauge bosons of A_μ are just for the SM and therefore $SU(3)$ symmetry has been broken into $SU(2) \times U(1)$ by orbi-folding, while the remaining 4D scalars just form Higgs doublet of the SM, without introducing any unnecessary exotic states.

A comment on the Weinberg angle is in order. For a unified theory with simple gauge group, the Weinberg angle is readily calculated by use of a convenient formula:

$$\sin^2 \theta_W = \frac{\text{Tr}(I_3)^2}{\text{Tr}Q^2} = \frac{3}{4}, \quad (3.9)$$

where Q is the charge operator and I_3 is the operator for the 3rd component of weak isospin, and the Tr is taken for the $SU(3)$ triplet (3.5). The obtained Weinberg angle is too large, although it may get quantum correction. Possible way out of this problem will be discussed in a section below.

3.2 Mass eigenvalues and mode functions for fermions

Now we discuss the 4-dimensional (4D) mass eigenvalues and corresponding mode functions of y for KK modes of quarks. In the following subsection we discuss the anomalous Yukawa couplings (for light quarks). We thus pick up the relevant part of the lagrangian for that purpose:

$$\mathcal{L} = \bar{\Psi} \left\{ \Gamma^5 \left(i\partial_y + \frac{g_5}{2} A_y^{6(0)} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \right) - M\epsilon(y) \right\} \Psi \quad \left(\Gamma^5 = i\gamma^5 \right), \quad (3.10)$$

where $\epsilon(y)$ is the sign function

$$\epsilon(y) = \begin{cases} +1 & \text{for } y > 0 \\ -1 & \text{for } y < 0 \end{cases} \quad (3.11)$$

and $A_y^{6(0)}$ denotes the zero mode of A_y^6 , which is identified with the neutral component of the Higgs doublet:

$$A_y^{6(0)} = v_5 + H . \quad (3.12)$$

Note that $g_5 v_5 = g_4 v$ (g_5, g_4 : 5D & 4D gauge couplings, $g_4 = \frac{g_5}{\sqrt{2\pi R}}$) with v being 4D VEV of Higgs and h defined by $h = \sqrt{2\pi R}H$ is nothing but our Higgs field.

In this model only d -quark has interaction with the Higgs field and gets its mass through spontaneous symmetry breaking, we focus on the two component “subspace” of Ψ ,

$$\psi = \begin{bmatrix} \psi_2 \\ \psi_3 \end{bmatrix} , \quad (3.13)$$

whose free lagrangian (with the contribution of the VEV) is read off from (3.10) as

$$\mathcal{L}_{\text{free}} = \bar{\psi} \left\{ -\gamma^5 \left(\partial_y - i \frac{g_4}{2} v \sigma_1 \right) - M \epsilon(y) \right\} \psi , \quad (3.14)$$

where σ_1 is a Pauli matrix.

According to the assigned Z_2 -parity each chirality of ψ_2 and ψ_3 satisfies

$$\psi_{2L}(x, -y) = +\psi_{2L}(x, y) \quad , \quad \psi_{2R}(x, -y) = -\psi_{2R}(x, y) , \quad (3.15a)$$

$$\psi_{3L}(x, -y) = -\psi_{3L}(x, y) \quad , \quad \psi_{3R}(x, -y) = +\psi_{3R}(x, y) . \quad (3.15b)$$

They also satisfy the following periodic boundary condition (b.c.):

$$\psi_{2L}(x, -\pi R) = \psi_{2L}(x, \pi R) \quad , \quad \psi_{2R}(x, -\pi R) = \psi_{2R}(x, \pi R) \quad , \quad \text{etc.} \quad (3.16)$$

The equation of motion for $\psi_{2,3}$ is given by

$$\left\{ i \not{\partial} - \gamma^5 \left(\partial_y - i \frac{g_4}{2} v \sigma_1 \right) - M \epsilon(y) \right\} \psi = 0 . \quad (3.17)$$

The presence of σ_1 term makes the equation of motion coupled equations. So, in order to simplify the analysis we perform a sort of local gauge transformation where the gauge parameter is linear in the extra space coordinate y : define $\hat{\psi}$ so that

$$\psi = \exp \left\{ i \frac{g_4}{2} v y \sigma_1 \right\} \hat{\psi} . \quad (3.18)$$

The local gauge transformation causes the shift of the Higgs field: $h \rightarrow h - v$, so that the VEV of the Higgs is erased. Then, in terms of ψ , the term proportional to σ_1 disappears in its equation of motion:

$$\left\{ i\not{\partial} - \gamma^5 \partial_y - M\epsilon(y) \right\} \hat{\psi} = 0 . \quad (3.19)$$

On the other hand, such local gauge transformation causes the change of the b.c. for quark fields. In particular $\hat{\psi}$ is no longer periodic: from (3.16) and (3.18),

$$\hat{\psi}(x, -\pi R) = \langle W \rangle \hat{\psi}(x, \pi R) \quad \text{where} \quad \langle W \rangle \equiv e^{i\pi R g_4 v \sigma_1} , \quad (3.20)$$

where $\langle W \rangle$ is nothing but the VEV of the Wilson loop (or AB phase factor).

The 4D mass eigenvalues of the n -th KK mode and corresponding mode functions of y is obtained by solving the equation of motion (3.19) under the b.c. given in (3.20). We will skip all the details of this procedure and will list the final results. For the details of the derivation, refer to [17].

First, the 4D mass eigenvalue for n -th KK mode m_n is implicitly given by a relation

$$\sin^2\left(\frac{g_4}{2} v \pi R\right) = \frac{m_n^2}{m_n^2 - M^2} \sin^2\left(\sqrt{m_n^2 - M^2} \pi R\right) . \quad (3.21)$$

It should be noticed that there exist two kinds of mass eigenvalues for $n \neq 0$, say $m_n^{(\pm)}$, which are defined by

$$\frac{m_n^{(\pm)}}{\sqrt{m_n^{(\pm)2} - M^2}} \sin\left(\sqrt{m_n^{(\pm)2} - M^2} \pi R\right) = \pm (-1)^n \sin\left(\frac{g_4}{2} v \pi R\right) . \quad (3.22)$$

Note that $m_n^{(\pm)}$ get degenerate for $v = 0$.

Although (3.22) cannot be solved analytically in general, in the specific case of $M = 0$ the solution is easily obtained to be

$$m_n^{(\pm)} = \frac{n}{R} \pm \frac{g_4}{2} v . \quad (3.23)$$

Next, we discuss the KK mode expansion of the quark fields. $\hat{\psi}(x, y)$ is expanded in terms of KK mode functions as follows:

$$\hat{\psi}(x, y) = \sum_n \left[\begin{array}{l} A_n f_{Le}^{(n)}(y) \hat{\psi}_L^{(n)}(x) \mp i A_n f_{o}^{(n)}(y) \hat{\psi}_R^{(n)}(x) \\ \pm i B_n f_{o}^{(n)}(y) \hat{\psi}_L^{(n)}(x) + B_n f_{Re}^{(n)}(y) \hat{\psi}_R^{(n)}(x) \end{array} \right] \quad (3.24)$$

where

$$A_n^{(\pm)} \equiv \sqrt{\frac{\cos(\varphi_n^{(\pm)} - \alpha_n^{(\pm)})}{2 |\cos \varphi_n^{(\pm)} \cos \alpha_n^{(\pm)}|}} \quad , \quad B_n^{(\pm)} \equiv \sqrt{\frac{\cos(\varphi_n^{(\pm)} + \alpha_n^{(\pm)})}{2 |\cos \varphi_n^{(\pm)} \cos \alpha_n^{(\pm)}|}} \quad . \quad (3.25)$$

and

$$\tan \alpha_n \equiv \frac{M}{\sqrt{m_n^2 - M^2}}, \quad \varphi_n = \sqrt{m_n^2 - M^2} \pi R. \quad (3.26)$$

Each mode function is defines as

$$f_o^{(\pm,n)}(y) = \frac{1}{\sqrt{\pi R N_n^{(\pm)}}} \sin\left(\sqrt{m_n^{(\pm)2} - M^2} y\right) \quad , \quad (3.27a)$$

$$f_{Le}^{(\pm,n)}(y) = -\frac{1}{\sqrt{\pi R N_n^{(\pm)}}} \cos\left(\sqrt{m_n^{(\pm)2} - M^2} |y| + \alpha_n^{(\pm)}\right) \quad , \quad (3.27b)$$

$$f_{Re}^{(\pm,n)}(y) = \frac{1}{\sqrt{\pi R N_n^{(\pm)}}} \cos\left(\sqrt{m_n^{(\pm)2} - M^2} |y| - \alpha_n^{(\pm)}\right) \quad , \quad (3.27c)$$

with a normalization factor

$$N_n^{(\pm)} \equiv \left| 1 - \frac{\tan \varphi_n^{(\pm)} \sin^2 \alpha_n^{(\pm)}}{\varphi_n^{(\pm)}} \right| \quad , \quad (3.28)$$

which reduces to 1 for $v = 0$. $f_o^{(\pm,n)}(y)$ is a odd function of y , while $f_{Le}^{(\pm,n)}(y)$, $f_{Re}^{(\pm,n)}(y)$ are even functions for left- and right-handed quarks.

4 Anomalous Higgs interactions

In this section we focus on the Higgs interaction with quarks.

In any gauge theory with spontaneous gauge symmetry breaking, the mass term of fermion ψ can be written in a form

$$m(v) \bar{\psi} \psi, \quad (4.1)$$

where “mass function” $m(v)$ is a function of the Higgs VEV v . In our case the mass term for generic KK mode n is written as

$$m_n(v) \bar{\psi}_n \psi_n \quad (4.2)$$

where $m_n(v)$ is just $m_n^{(\pm)}$ given by the relation (3.22). Since (3.22) cannot be solved for $m_n^{(\pm)}$ analytically, the mass function is non-linear function of v in general.

Since the physical Higgs h denotes the deviation of Higgs field from its VEV, it is a reasonable guess to expect that interactions of the Higgs h with quarks are obtained by a replacement

$$v \rightarrow v + h. \quad (4.3)$$

Namely, the Higgs interaction is expected to be described by

$$m_n(v + h)\bar{\psi}_n\psi_n. \quad (4.4)$$

Because of the non-linearity of the mass function, such obtained Higgs interaction should be also non-linear in the Higgs field h , including e.g. $h^2\bar{\psi}_n\psi_n$, which clearly does not exist in the SM at least at the classical level. Already in this sense, Higgs interactions in the GHU scenario is anomalous.

4.1 Yukawa coupling

In particular, the Yukawa interaction of the Higgs with the KK modes of quark is expected to be described as

$$f_n h \bar{\psi}_n \psi_n, \quad (4.5)$$

with

$$f_n = \frac{dm_n(v)}{dv}, \quad (4.6)$$

which is not a constant in general and again some non-linear function of v .

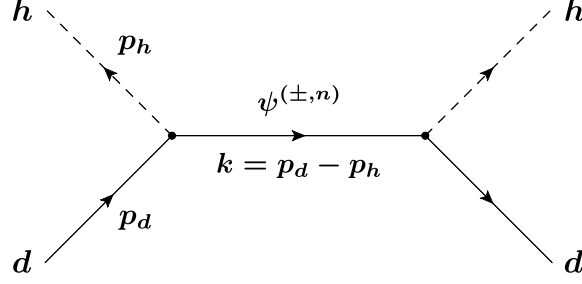
By the way, in the lagrangian (3.10) the interaction of the Higgs field h ($A_y^{6(0)}$) with quarks is through covariant derivative along the extra dimension and therefore is inevitably linear in the Higgs field. Then a question is why the non-linear Higgs interaction with quarks is possible starting from such linear interaction.

Actually, if we write the Yukawa coupling, which is (of course) linear in h , in a $\infty \times \infty$ matrix in the base of KK modes of the quark, this matrix turns out to be non-diagonal, in general. For instance we have Yukawa couplings which connect KK zero mode with KK non-zero modes of quark

$$h\bar{\psi}^{(n)}\psi^{(0)}. \quad (4.7)$$

We can confirm that such off-diagonal elements exist by explicit calculation of overlap integral of mode functions, namely by inserting the KK mode expansion given in (3.24) and by replacing v by h in the relevant lagrangian (3.14) (after $\bar{\psi}$ is transformed back to ψ) and by performing y integral. The calculated (n, m) matrix element of the ‘‘Yukawa coupling matrix’’ M_Y is given as

$$(M_Y)_{nm} = -i\frac{g_4}{2} \int_{-\pi R}^{\pi R} dy \left(A_n^* B_m f_{Le}^{(n)*} f_{Re}^{(m)} - B_n^* A_m f_o^{(n)*} f_o^{(m)} \right). \quad (4.8)$$


 Figure 2: Quadratic Higgs interaction with d quark

with the results of y integrals,

$$\int_{-\pi R}^{\pi R} dy f_o^{(n)*} f_o^{(m)} = -\frac{1}{\sqrt{N_n N_m}} \left\{ \frac{\sin(\varphi_n + \varphi_m)}{\varphi_n + \varphi_m} - \frac{\sin(\varphi_n - \varphi_m)}{\varphi_n - \varphi_m} \right\}. \quad (4.9)$$

$$\int_{-\pi R}^{\pi R} dy f_{Le}^{(n)*} f_{Re}^{(m)} = -\frac{1}{\sqrt{N_n N_m}} \left\{ \frac{\sin(\varphi_n + \varphi_m + \alpha_n - \alpha_m) - \sin(\alpha_n - \alpha_m)}{\varphi_n + \varphi_m} + \frac{\sin(\varphi_n - \varphi_m + \alpha_n + \alpha_m) - \sin(\alpha_n + \alpha_m)}{\varphi_n - \varphi_m} \right\}. \quad (4.10)$$

The mixing between KK zero mode and non-zero modes leads to the non-linear Higgs interactions. For instance $h^2 \psi^{(0)} \bar{\psi}^{(0)}$ stems from the Feynman diagram shown in Fig.2.

From this point of view, the Yukawa coupling obtained by the replacement $m_n(v) \rightarrow m_n(v + h)$, (4.6), just coincide to the diagonal element $(M_Y)_{n,n}$ of Yukawa coupling matrix. This expectation has been confirmed in [17] by comparing $(M_Y)_{n,n}$ obtained from (4.4) with $\frac{dm_n(v)}{dv}$ obtained by taking derivative of the equation (3.22) with respect to v . The results obtained by these two different methods have been shown to coincide with each another:

$$\frac{dm_n}{dv} = \frac{g_4}{2} \frac{\varphi_n \cos \alpha_n}{\varphi_n \cot \varphi_n - \sin^2 \alpha_n} \cot \left(\frac{g_4}{2} v \pi R \right). \quad (4.11)$$

4.2 Anomalous Yukawa coupling

We now focus on the Yukawa coupling of KK zero mode of quark:

$$f = \frac{dm(v)}{dv}, \quad (4.12)$$

where $m(v)$ denotes the mass eigenvalue of zero mode quark.

As already mentioned above, the zero mode of light quarks have 4D mass eigenvalues which behaves as a trigonometric function. Let us now discuss how such function is obtained approximately. Though the Yukawa coupling is also obtained from (4.11), here we consider the case $m_0 \ll M$, which is justified for light quarks, in the relation (3.21). As we easily find in this case m_0 is approximately given by

$$m(v) = m_0 \simeq \frac{M}{\sinh(\pi RM)} \sin\left(\frac{g_4}{2} v \pi R\right). \quad (4.13)$$

Thus,

$$f = \frac{dm(v)}{dv} \simeq \frac{M}{\sinh(\pi RM)} \frac{g_4}{2} \pi R \cos\left(\frac{g_4}{2} v \pi R\right). \quad (4.14)$$

Thus obtained Yukawa coupling is not a constant and varies as the function of dimensionless parameter

$$x \equiv \frac{g_4}{2} v \pi R. \quad (4.15)$$

Let us note that the Yukawa coupling even vanishes for specific value of x , i.e. $x = \frac{\pi}{2}$ as was pointed out in [10, 11, 12, 13]. Though this drastic possibility has been ruled out by the recent discovery of the Higgs boson at LHC, this clearly indicates that the Yukawa coupling in our model deviates from the prediction of the SM.

Such anomalous Yukawa coupling generally arises in this scenario for arbitrary non-vanishing x . We now give a formula for the extent of the deviation from the SM prediction for arbitrary x . In the SM the Yukawa coupling of the quark is given by $f_{SM} = \frac{m_0}{v}$. By use of (4.13) this can be written as

$$f_{SM} \simeq \frac{M}{v \sinh(\pi RM)} \sin\left(\frac{g_4}{2} v \pi R\right) \quad (4.16)$$

Thus, the ratio of the prediction of GHU to that of the SM is given as

$$\frac{f}{f_{SM}} \simeq \frac{g_4}{2} v \pi R \cot\left(\frac{g_4}{2} v \pi R\right) = x \cot x. \quad (4.17)$$

In Figure 3 we have compared the results of exact numerical calculation by use of the formula (3.21) for the case of $m_0 = m_b$ (taking bottom quark as light quark) and of the approximated formula (4.17). The dots stand for the numerical results and the curvy line stands for the function $x \cot x$. We see that the exact result is very well approximated by an approximated formula $x \cot x$. As is seen in the Figure 3, in the scenario of GHU, predicted Yukawa coupling is always smaller than that of the SM and even vanishes at $x = \frac{\pi}{2}$ as we have already mentioned. The

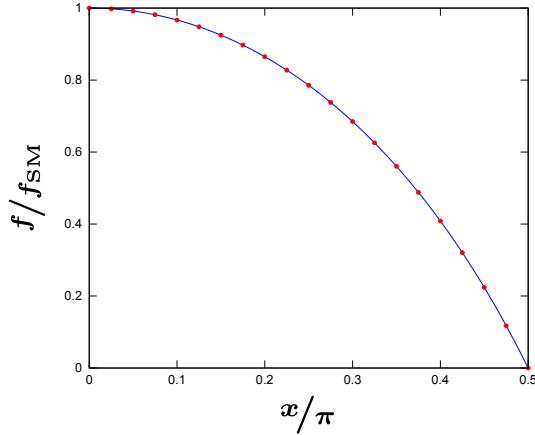


Figure 3: The ratio of Yukawa coupling to its standard model prediction for bottom quark

ratio, however, approaches to 1 for $x \ll 1$, namely when the weak scale is much smaller than the compactification scale $M_c = \frac{1}{R}$: $M_W \ll M_c$. This is reasonable since in this limit all non-zero KK modes are expected to decouple from the low energy effective theory (“decoupling limit”) and the theory is expected to reduce to the SM.

5 The essential causes for the anomalous Higgs interactions

From the argument extended so far it seems that the essential cause of the anomalous Higgs interaction is the periodicity of physical observables in the Higgs field. Surely this periodicity should be an essential cause for the anomalous Higgs interactions. We, however, can present an example where the Yukawa coupling is (almost) normal, i.e. the same as the SM prediction (for the broad range of the parameter x), while the periodicity of the quark mass spectrum is maintained.

In fact, in the case of heavy quark such as top quark, the Z_2 -odd bulk mass M is switched off to avoid the exponential suppression factor e^{-MR} , and the mass spectrum of KK modes is just linear in the VEV of the Higgs v ((3.23)) just as in the SM, see Fig. 4. Thus the Yukawa coupling constant is just a constant as in the case of the SM:

$$f = \frac{dm(v)}{dv} = \frac{d}{dv}(g_4 v) = g_4 \quad (\text{for } 0 \leq x \leq \frac{\pi}{2}). \quad (5.1)$$

Namely, in this case there is no anomaly in the Yukawa coupling.

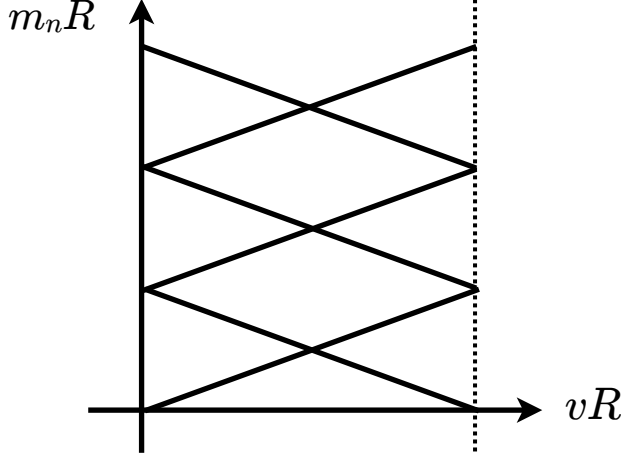


Figure 4: The mass spectrum for a heavy quark as the function of v

Let us make a comment on a specific case of $x = \frac{\pi}{2}$, for vanishing bulk mass $M = 0$. In this case from (3.23) we realize $m_n^{(+)} = m_{n+1}^{(-)}$. In particular, there appears a “level crossing” between KK zero mode m_0 and the first KK mode $m_1^{(-)}$. However, in the absence of the bulk mass M , the translational invariance along the extra space is maintained and the extra space component of the momentum p_y is preserved (strictly speaking, the presence of the fixed points the direction of the momentum p_y is not preserved but its absolute value is still conserved) and therefore there does not appear mixing between two degenerate state even at the point of level crossing.

In the presence of Z_2 -odd bulk mass M , however, these levels can mix with each other, because of the violation of translational invariance in the direction of extra space, and therefore the mass degeneracy is lifted by an amount $\mathcal{O}(M)$ (see Figure 5), treating the bulk mass term as a perturbation.

In this way the linearity of the quark mass spectrum is spoilt and finally for sufficiently large M , as is required for light quarks such as d, s, b quarks, the mass function of KK zero mode becomes sin-function as in (4.13), and anomalous Higgs interaction arises. Thus here we have learned an important lesson. Namely, another essential cause of the anomalous Higgs interaction is the violation of translational invariance along the direction of the extra space.

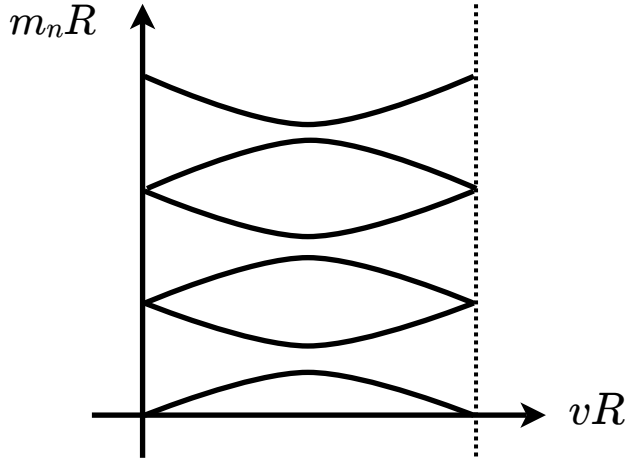


Figure 5: The mass spectrum in the case of non-vanishing Z_2 -odd bulk mass M

6 On the reality of the model

For the scenario of the GHU to be viable as a physics beyond the standard model (BSM), in addition to the theoretical aspects of the model, the reality of the model also should be addressed. In this section, we pick up some typical and important issues from such a point of view.

First we point out that our model, based on a simple gauge group $SU(3)$ on a flat space-time, has a few serious phenomenological problems, which and whose possible solutions have been discussed in the literature, see e.g. [15]. First it predicts too large Weinberg angle, $\sin^2 \theta_W = \frac{3}{4}$. It, however, may be avoided by different choices of the unified gauge group, e.g. G_2 [18], which can predict $\sin^2 \theta_W = \frac{1}{4}$, $SU(6)$ in the framework of GUT predicting $\sin^2 \theta_W = \frac{3}{8}$ [19]. Or it may be avoided also by introducing brane-localized kinetic terms for standard model gauge bosons [15].

Realizing the observed top quark mass is also nontrivial since we need an enhancement factor of roughly 2, $m_t \simeq 2m_W$, which cannot be achieved even if we switch off the bulk mass M . In the flat space GHU scenario, this enhancement factor can be obtained from the group theoretical factor of large irreducible representation adopted for the top quark [20]. In the GHU on a warped space, it

has been known that the enhancement factor comes from the product of curvature scale and compactification radius [21].

Secondly, the unitarity of the scattering amplitudes of the longitudinal W and Z bosons is of special interest, in view of the fact that for the specific choice of the VEV, $x = \frac{94}{2}\pi Rv = \frac{\pi}{2}$, the Higgs coupling with these gauge bosons should vanish. As was pointed out in the case of “Higgsless” model [22], for the initialization to be achieved, the compactification scale should be around 1 TeV. Fortunately, in our model the problem of unitarity should be absent except for the specific choice of $x = \frac{\pi}{2}$, whose possibility now seems to be ruled out by the recent result from LHC experiment, claiming the discovery of the “Higgs-like” particle. The reason is that the Higgs interactions with massive gauge bosons, W and Z , are just as in the standard model and is “almost normal” just as in the case of Yukawa coupling of heavy quark. The essential reason is that because of the presence of translational invariance along the extra space in the sector of free lagrangian for the gauge bosons, there is no mixing between different KK modes, and therefore the mass spectrum is linear in the VEV v , just as in the standard model. The situation should be different for the case of GHU on the warped space, where the warp factor violates the translational invariance and causes the mixing and therefore anomalous Higgs interaction with massive gauge bosons [10].

We also would like to comment on the implication of our model to the precision test of electro-weak parameters S and T of Peskin-Takeuchi. In the Higgsless model on the warped space [23] or in the model of light composite Higgs [24], which have some similarity to our model, it has been discussed that the S parameter is too large and positive already at the tree level. On the other hand, S and T parameters were discussed and concretely calculated some time ago [25] in the same model as what we adopt here and S and T have been shown to vanish at the tree level and appear only at the quantum level, just as in the case of the standard model. Therefore in our model S and T parameters are not problematic.

We would like to see why such difference arises concerning the precision test. Let us recall that in the standard model S and T parameters vanish at the tree level, since the gauge invariant operators including Higgs field responsible for the S and T parameters are of higher mass dimension ($d > 4$). For instance the operator relevant for the T parameter is $(\phi^\dagger D_\mu \phi)(\phi^\dagger D^\mu \phi) - \frac{1}{3}(\phi^\dagger D_\mu D^\mu \phi)$, where ϕ is the Higgs doublet and D_μ denotes a covariant derivative. This operator has mass dimension 6 and therefore does not exist at the tree level and induced only at the quantum level. In our model, being one of higher dimensional ($D = 5$) gauge theories, the lagrangian is described by the operators all with mass dimension 4 (from 4-dimensional point of view). This is the reason why in our model S and T get contributions only at the quantum level. We, however, should note that even if the original lagrangian does not have higher mass dimensional operators, which still

may be induced at the tree level, through the mixing between zero-mode and non-zero KK modes. The typical example is nothing but the process shown in Figure 2, which is induced by the off-diagonal Yukawa coupling of fermion and yields $d = 5$ operator $\bar{d}dh^2$ with respect to the zero mode quark and Higgs fields, which has some similarity to the $d = 5$ operator responsible for the see-saw mechanism of neutrino masses. Thus if there is off-diagonal Higgs interaction, or in other words if there is anomalous Higgs interaction, higher dimensional ($d > 4$) operators may be induced even at the tree level. Thus S and T parameters potentially get contribution already at the tree level. Thus, the reason why S and T do not get contribution at the tree level in our model is that the gauge boson mass spectrum is linear in the VEV v in our model, since there is no source of the violation of translational invariance along the extra dimension. On the other hand, on the R-S type space-time the translational invariance is “universally” violated by the warp-factor in every sectors, not only in the quark sector but also in the gauge boson sector. That may be the reason why in [23] the authors got tree level contribution to the S parameter. For instance, the interaction vertex coming from $\text{Tr}([A_\mu, A_y][A^\mu, A_y])$ may cause off-diagonal Higgs interaction, $A_\mu^{(n)}A^{(0)\mu}h^2$, which through the exchange of non-zero KK modes $A_\mu^{(n)}$ ($n = 1, 2, \dots$) may yield an $d = 6$ operator, such as $A_\mu^{(0)}A^{(0)\mu}h^4$.

Part II

Anomalous Higgs Interaction in dimensional deconstruction

7 Introduction for Part II

The purpose of this second part of the thesis is to discuss anomalous Higgs interaction in the scenario of dimensional deconstruction (DD) [8].

As was already discussed in the introduction for the thesis as the whole, dimensional construction is the scenario where Higgs is originally a (pseudo-) Nambu-Goldstone (NG) boson composed by a pair of fermion and anti-fermion by strong interactions, just as the pions are composed by pairs of quarks and anti-quarks in QCD. The specific feature of the model is that after the confinement by the strong interactions the remaining (weak) gauge symmetries are of the form of direct products of the same type of gauge group: $SU_1(m) \times SU_2(m) \times \dots \times SU_N(m)$, as is schematically shown by the “moose diagram”.

From a different point of view, the DD scenario can be interpreted as a sort of

gauge-Higgs unification (GHU) in 5D space-time [3], [4], [5], [6], where the extra space is “latticed” and the number N denotes the number of the lattice sites. In fact, the non-linear realization of the Higgs field ϕ

$$U = e^{i\sqrt{N}\frac{\phi}{f}} \quad (7.1)$$

just corresponds to the Wilson-loop in the scenario of GHU. Here f is the “decay constant” of ϕ .

Such close relation of DD with GHU suggests that similar anomalous Higgs interaction to that in GHU is expected in DD scenario. We show in this part of the thesis that it is really the case. We, however, will also see that the anomalous Higgs interaction obtained in DD also has a different characteristic feature from that in GHU, in these sense that the anomalous interaction arises even if the bulk mass term for the fermion is absent.

In the first part of the thesis, discussing the anomalous Higgs interactions in GHU, we have seen that the anomalous interactions arises as the result of the following two properties of the theory:

Periodicity of physical observable in the Higgs field

The violation of translational invariance along the extra space

We easily understand that such conditions are met also in DD scenario, we are now interested in.

First, the periodicity also exists in this theory, since the Higgs field is non-linearly realized as a sort of phase factor as is seen in (7.1). Secondly, it is clear that the translational invariance is violated by the fact that the extra space is latticed, once DD is understood as a latticed 5D GHU. Thus it is promising that we get anomalous Higgs interactions in the scenario of DD.

We, however, also should note that there is a qualitatively distinct difference in the anomalous interaction present in DD scenario from the one in the scenario of GHU. Namely, it is because of the violation of the translational invariance by latticing the extra space, not because of the “ Z_2 -odd” bulk mass for fermion in the case of GHU. As far as the violation is due to the property of the space-time on which the theory is constructed, the anomalous interactions should arise not only in the sector of matter fermion but basically in every sector of the theory. The situation, in such a sense, may be similar to the case of GHU formulated on the Randall-Sundrum type 5D space-time, where the translational invariance is violated by the presence of the “warp-factor” $e^{-\kappa|y|}$ (κ is a constance to denote brane-localized tension and y is the extra-space coordinate) and anomalous interactions appear not only in the fermion sector but also the gauge boson sector as well [10], [11], [12], [13]. Thus we expect that the anomaly goes away in the limit $a \rightarrow 0$ (a : lattice spacing), unless there is no other source of violation of translational invariance.

In this discussion, we take an attitude that the DD is equivalent to a latticed GHU and study the anomalous Higgs interaction in the GHU with latticed extra-space. What we adopt as a model is 5D scalar QED (quantum electro-dynamics) with latticed extra space as its 5th dimension [26].

8 5D gauge theory with latticed extra-space

Before discussing 5D QED, here we consider a general 5-dimensional (5D) gauge theory where the extra dimension is compactified S^1 of radius R and circumference $L = 2\pi R$, which is latticed to N lattice sites with extra space coordinates y_i ($i = 1 - N$).

For a generic matter field $\psi(x^\mu, y_i)$, a local gauge transformation is given as

$$\psi(x^\mu, y_i) \rightarrow \psi'(x^\mu, y_j) = g(x^\mu, y_j)\psi(x^\mu, y_j), \quad (8.1)$$

where $g(x^\mu, y_j)$ is a member of the gauge group $\mathcal{G}$. In the limit of $N \rightarrow \infty$ this transformation reduces to a 5D local gauge transformation. On the other hand, we may regard it as the 4D local gauge transformation whose gauge group is the direct product

$$G_1 \times G_2 \times \dots \times G_N \quad (8.2)$$

where each of G_i , with the group element $g(x^\mu, y_j)$, belongs to the same group $\mathcal{G}$. (8.2) is just equivalent to the gauge symmetry shown by the “moose diagram” in the original DD scenario [8], where G_i are “weak” gauge symmetries remaining after the confinement due to the strong forces. In this way, we can confirm that the scenario of DD is equivalent to the GHU where the extra-space is latticed.

Hereafter, we change the notation of the field as

$$\psi(x^\mu, y_j) \rightarrow \psi_i(x^\mu). \quad (8.3)$$

The fields $\psi_i(x^\mu)$ ($i = 1, 2, \dots, N$) may also be regarded as N pieces of 4D fields. Because of the S^1 compactification, there is a periodic boundary conditions as follows,

$$\psi_{N+i}(x^\mu) = \psi_i(x^\mu). \quad (8.4)$$

The “derivative” along the extra-space is given by a difference,

$$\partial_y \psi_i(x^\mu) \equiv \frac{\psi_{i+1}(x^\mu) - \psi_i(x^\mu)}{a}, \quad (8.5)$$

where $a(> 0)$ is a distance between neighboring sites: lattice spacing, satisfying

$$L = 2\pi R = Na. \quad (8.6)$$

The covariant derivative along 4D space-time is just an ordinary one

$$D_\mu \psi_i = \partial_\mu \psi_i - ig A_{i\mu} \psi_i. \quad (8.7)$$

where $A_{i\mu}$ is the gauge field of G_i and g is the gauge coupling constant.

Covariant derivative along extra dimension is given by

$$D_y \psi_i = \frac{\psi_{i+1} - U_i \psi_i}{a}, \quad (8.8)$$

where

$$U_i(x^\mu) = e^{iga A_{iy}(x^\mu)} \quad (8.9)$$

is a "link variable" (Wilson-line) to be introduced for gauge covariance, which connects the i and $(i+1)$ -th sites. $A_{iy}(x^\mu)$ corresponds to the extra-space component of gauge field $A_y(x^\mu, y)$ in GHU. In order to guarantee the gauge covariance, the link variable should transform under the local gauge transformation as follows:

$$U_i \rightarrow U'_i = g_{i+1} U_i g_i^\dagger, \quad (8.10)$$

where g_i, g_{i+1} are group elements of G_i, G_{i+1} , respectively. Namely, U_i behaves as bi-fundamental repr. of (G_i, G_{i+1}) .

We also need covariant derivative for a link variable U_i in order to get the kinetic term for A_{iy} . Since U_i behaves as the bi-fundamental repr. of (G_i, G_{i+1}) , its covariant derivative (along 4D space-time) is given as

$$D_\mu U_i = \partial_\mu U_i - ig A_{i+1,\mu} U_i + ig U_i A_{i\mu}. \quad (8.11)$$

9 5D Scalar QED

As the simplest example to see the anomalous Higgs interaction, we take the model of 5D scalar QED model on the latticed extra -space, according to the formalism discussed in the previous section.

The model is composed of a 5D scalar electron $\phi_i(x^\mu)$ with electric charge $-e$ and the photon

$(A_{i\mu}(x^\mu), A_{iy}(x^\mu))$. The lightest 4D field, corresponding to the Kaluza-Klein (KK) zero-mode in GHU, of $A_{iy}(x^\mu)$ is identified with the Higgs field and is supposed to have a VEV. Thus scalar electron has masses due to the VEV, though the gauge symmetry is not broken in this U(1) Abelian gauge theory. We expect in a realistic model with non-Abelian gauge symmetry to incorporate the standard model, gauge symmetry is broken through Hosotani-mechanism [5].

The 4D lagrangian, which corresponds the one obtained by the integral over the extra-space coordinate y of 5D lagrangian in GHU, is given by

$$\mathcal{L} = a \sum_{i=1}^N \left\{ -\frac{1}{4} F_i^{\mu\nu} F_{i\mu\nu} + \frac{1}{(ae)^2} (D^\mu U_i)^* D_\mu U_i + (D^\mu \phi_i)^* D_\mu \phi_i - (D_y \phi_i)^* D_y \phi_i - m^2 \phi_i^* \phi_i \right\} \quad (9.1)$$

where

$$F_i^{\mu\nu} = \partial^\mu A_i^\nu - \partial^\nu A_i^\mu, \quad (9.2)$$

$$U_i = e^{iaeA_{iy}}, \quad (9.3)$$

$$\begin{aligned} D^\mu U_i &= \partial^\mu U_i - ieA_{i+1}^\mu U_i + ieU_i A_i^\mu \\ &= i(ea) \left(\partial^\mu A_{iy} - \frac{A_{i+1}^\mu - A_i^\mu}{a} \right) U_i = i(ea) (\partial^\mu A_{iy} - \partial_y A_i^\mu) U_i, \end{aligned} \quad (9.4)$$

$$D^\mu \phi_i = \partial^\mu \phi_i + ieA_i^\mu \phi_i, \quad (9.5)$$

$$D_y \phi_i = \frac{\phi_{i+1} - U_i^* \phi_i}{a}. \quad (9.6)$$

So far the charge e and all fields are regarded to be 5D coupling and fields, respectively. We later introduce 4D electric charge e_4 .

10 Kaluza-Klein modes and mass eigenvalues

10.1 Kaluza-Klein mode expansion - discretized Fourier transformation -

We now perform “discretized Fourier transform” for each 5D field in order to get 4D mass eigenstates.

First, let us note that although the translational invariance along the extra-space is violated by latticization, there still remain a symmetry in the theory under the following discrete transformation

$$D : y_i \rightarrow y_{i+1}, \text{ i.e. } \phi_i \rightarrow \phi_{i+1}, \text{ etc.} \quad (10.1)$$

On the other hand, repeating D N times should be identical transformation. Thus, the eigenvalues of D should be

$$(\omega_N)^n \quad (\omega_N \equiv e^{i\frac{2\pi}{N}}, \quad n = 0, 1, 2, \dots, N-1). \quad (10.2)$$

Thus, the “KK” mode functions (vectors with N elements) can be easily found

without solving eigenvalue equations for 4D mass eigenvalues:

$$\begin{pmatrix} (\omega_N)^n \\ (\omega_N)^{2n} \\ \vdots \\ (\omega_N)^{(N-1)n} \\ 1 \end{pmatrix}. \quad (10.3)$$

By use of these eigenvectors we easily get (discretized) Fourier series expansions of each field as follows,

$$A_{i\mu}(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} A_\mu^{(n)}(x^\mu) (\omega_N)^{in}, \quad (10.4)$$

$$A_{iy}(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} A_y^{(n)}(x^\mu) (\omega_N)^{in} \quad (10.5)$$

$$\phi_i(x^\mu) = \frac{1}{\sqrt{L}} \sum_{n=0}^{N-1} \phi^{(n)}(x^\mu) (\omega_N)^{in}, \quad (10.6)$$

where $L = Na = 2\pi R$ and $\phi^{(n)}(x^\mu)$ etc. are 4D fields with proper canonical mass dimension ($d = 1$) of the KK mode n . The KK zero-mode of A_{yi} , $A_y^{(0)}$, is identified with the Higgs field. The reality of the gauge fields $A_{i\mu}$, A_{iy} means

$$A_\mu^{(N-n)} = (A_\mu^{(n)})^*, \quad A_y^{(N-n)} = (A_y^{(n)})^*. \quad (10.7)$$

Let us note that the KK zero-mode of $A_{i\mu}$, i.e. $A_\mu^{(0)}$ appears at each lattice site as $\frac{1}{\sqrt{L}} A_\mu^{(0)}(x^\mu)$ (see (10.4)). Thus, the 4D electric charge e_4 , which is nothing but the coupling constant of $A_\mu^{(0)}(x^\mu)$ with the scalar electron is given by

$$e_4 = \frac{e}{\sqrt{L}} = \frac{e}{\sqrt{2\pi R}}, \quad (10.8)$$

just as in the case of GHU.

10.2 4D mass eigenvalues

Getting mass eigenstates, we now calculate 4D mass eigenvalues of each KK mode.

10.2.1 4D masses of gauge-Higgs sector

First we discuss the gauge-Higgs sector, i.e. the sector of $A_\mu^{(n)}$ and $A_y^{(n)}$. Note that this sector does not acquire the masses due to the VEV v of the Higgs field $A_y^{(0)}$.

We should also note that except for the Higgs field $A_y^{(0)}$, all non-zero KK modes of A_y are absorbed by a sort of Higgs mechanism to the corresponding massive KK modes of A_μ .

Substituting the mode expansion (10.4) and (10.5) in the relevant part of the lagrangian (9.1),

$$a \sum_i \frac{1}{(ae)^2} (D^\mu U_i)^* D_\mu U_i, \quad (10.9)$$

and performing the sum over i we get

$$\sum_{n=0}^{N-1} (\partial^\mu A_y^{(n)*} - \frac{(\omega_N)^{-n} - 1}{a} A_\mu^{(n)*}) (\partial_\mu A_y^{(n)} - \frac{(\omega_N)^n - 1}{a} A_\mu^{(n)}). \quad (10.10)$$

Here we have used the ortho-normal condition

$$\sum_{i=1}^N (\omega_N)^n (\omega_N)^m = N \delta_{n+m}, \quad (10.11)$$

where $n + m$ is in mod N . By the re-phasing of the fields $A_\mu^{(n)} \rightarrow -i(\omega_N)^{-\frac{n}{2}} A_\mu^{(n)}$, the coefficients $\frac{(\omega_N)^n - 1}{a}$ is made real:

$$\frac{(\omega_N)^n - 1}{a} \rightarrow -i \frac{(\omega_N)^{\frac{n}{2}} - (\omega_N)^{-\frac{n}{2}}}{a} = \frac{2 \sin(\frac{n}{N} \pi)}{a}. \quad (10.12)$$

In this way it is clear that for the sector of non-zero KK modes Higgs-like mechanism is operative and the 4D mass eigenvalues of massive gauge bosons are given as

$$m_n = \frac{2 \sin(\frac{n}{N} \pi)}{a}. \quad (10.13)$$

Note that in the limit $N \rightarrow \infty$ ($a \rightarrow 0$) keeping L , the it reduces to

$$m_n \rightarrow 2 \frac{n}{Na} \pi = \frac{2n\pi}{L} = \frac{n}{R} \quad (L = Na = 2\pi R), \quad (10.14)$$

which is nothing but the well-known KK masses in higher dimensional theories. These eigenvalues and the eigenvectors given in (10.3) are just the same as those in the system of coupled harmonic oscillators: the system of springs and balls,

10.2.2 4D masses of matter field

Next, we discuss the mass eigenvalues of the scalar electron ϕ . Again by substituting the mode expansion (10.6) in the part relevant for the mass-squared term and replacing A_i by its VEV, we get

$$a \sum_i \{ (D_y \phi_i)^* D_y \phi_i + m^2 \phi_i^* \phi_i \} = \sum_{n=0}^{N-1} m_n^2 |\phi^{(n)}|^2, \quad (10.15)$$

where

$$m_n^2 = \frac{1}{a^2} |(\omega_N)^n - e^{-iae_4v}|^2 + m^2 = \left\{ \frac{2}{a} \sin\left(\frac{n\pi}{N} + \frac{ae_4v}{2}\right) \right\}^2 + m^2. \quad (10.16)$$

Again, at the limit $N \rightarrow \infty$ ($a \rightarrow 0$) keeping L , the m_n reduces to

$$m_n^2 \rightarrow \left(\frac{n}{R} + e_4v\right)^2 + m^2, \quad (10.17)$$

recovering the result in 5D QED with S^1 compactification [6].

11 The coupling constants of Higgs interaction

Our main purpose is to investigate whether the Higgs couplings with matter fields show some anomalous behaviour. In this section, we thus focus on the Higgs couplings with scalar electron, which is regarded as the counterpart of the Yukawa coupling in the realistic model including the standard model (SM).

We have obtained the mass-squared term for n -th KK mode of scalar electron ϕ (see (10.15), (10.16)):

$$\left\{ \frac{2}{a} \sin\left(\frac{n\pi}{N} + \frac{ae_4v}{2}\right) \right\}^2 + m^2 \phi^{(n)}(x^\mu)^* \phi^{(n)}(x^\mu). \quad (11.1)$$

Since the physical Higgs field is nothing but the deviation of the Higgs field from its VEV, the Higgs interactions with ϕ are obtained by the following replacement in (11.1):

$$v \rightarrow v + h(x^\mu) \quad (11.2)$$

where h denotes the physical Higgs. In the case of GHU non-linearity of the mass eigenvalues came from the violation of translational symmetry along the extra-space due to the presence of the Z_2 -odd bulk mass term for the fermion, and the Yukawa coupling was found to have “off-diagonal” couplings between different KK modes [17] and the Higgs interactions obtained by the prescription in (11.2) were argued to represent the “diagonal” couplings of the same KK mode. It is interesting to note that in our model based on the scenario of DD, the non-linearity comes from the fact that in the covariant derivative the Higgs is non-linearly realized from the beginning. Thus Higgs interactions are expected to be diagonal in the basis of KK modes, in contrast to the case of GHU.

For instance, the coupling constant of 3-point coupling between Higgs and n -th KK mode of matter scalar $h\phi^{(n)*}\phi^{(n)}$ is given by the first derivative of the mass-squared m_n^2 in (11.1), given by (10.16), with respect to the VEV v ,

$$\frac{dm_n^2}{dv} = \frac{2e_4}{a} \sin\left(\frac{2n\pi}{N} + ae_4v\right). \quad (11.3)$$

Similarly, the coupling constant of 4-point coupling $h^2\phi^{(n)*}\phi^{(n)}$ is calculated to be

$$\frac{1}{2} \frac{d^2 m_n^2}{dv^2} = e_4^2 \cos\left(\frac{2n\pi}{N} + ae_4 v\right). \quad (11.4)$$

Epecially the coupling constants with KK zero-mode of matter scalar are given as

$$\text{3-point coupling : } \frac{2e_4}{a} \sin(ae_4 v), \quad (11.5)$$

$$\text{4-point coupling : } e_4^2 \cos(ae_4 v). \quad (11.6)$$

We realize that, for instance, the 4-point coupling (11.6) shows quite different behaviour from that of ordinary 4D theory such as SM, and even vanishes for a specific value of the VEV,

$$v = \frac{\pi}{2ae_4}. \quad (11.7)$$

12 Anomalous Higgs interaction

In this section we finally discuss anomalous Higgs interaction, namely we argue how our predictions on the Higgs couplings deviate from the corresponding predictions in the SM.

First we discuss what interaction in this model should be compared with the Yukawa coupling in the SM and then we compare our prediction on that observable with that of the SM.

12.1 What is the counterpart of the Yukawa coupling in the SM ?

Our model, latticed 5D scalar QED, is just a toy model and is not realistic especially in the following sense:

- (a) Gauge group is Abelian U(1) and does not contain the gauge symmetry $SU(2) \times U(1)$ of the SM (even if we realistic ourself to the electro-weak sector).
- (b) The matter field is a scalar, not fermions to describe quarks and leptons.

One reason not to have introduced fermions is to avoid the cumbersome problem which arises when we formulate the theory of fermions on a lattice. Although we believe that the anomalous Higgs interaction pointed out in this work has its origin in the fact that the extra dimension is latticed and therefore is a general feature of the latticed higher dimensional gauge theories, when we wish to compare our prediction with that of SM it is not trivial what interaction in our model should be counterpart of the interaction in the SM we are familiar with, such as the Yukawa coupling.

From such a point of view, it may be useful to think of the situation in the supersymmetric theory, where the relation between the coupling constants of matter fermion and matter scalar becomes clear, though we do not intend to introduce supersymmetry to our theory. In MSSM, for instance, a term in the superpotential

$$W = fh\bar{t}t + \dots \quad (12.1)$$

provides not only the Yukawa coupling constant f for the top quark, but also 4-point interaction between the Higgs and stop $\tilde{t}$, the super-partner of t , $h^2|\tilde{t}|^2$ with a coupling constant f^2 :

$$fh\bar{t}t \xleftrightarrow{\text{SUSY}} f^2 h\tilde{t}^*\tilde{t}. \quad (12.2)$$

This argument suggests that the counterpart of the Yukawa coupling in our model is the coupling constant of 4-point function given in (11.6), which we write as f_{DD}^2 from now on, rather than the coupling constant of 3-point function (11.5):

$$f_{\text{DD}}^2 \equiv e_4^2 \cos(ae_4 v). \quad (12.3)$$

12.2 Quantitative analysis of the anomaly

We now compare the prediction of our model f_{DD}^2 given in (12.3) with the squared Yukawa coupling in the SM, f_{SM}^2 . In this analysis we set $m = 0$. This seems to be reasonable, since in a realistic theory, ordinary matter fields are expected to have masses only through spontaneous gauge symmetry breaking.

In the standard model, the Yukawa coupling is simply given by the ratio of the fermion mass, given by $m^{(0)}$ in our model, to the VEV of the Higgs. Hence we identify the Yukawa coupling of the SM as

$$f_{\text{SM}} = \frac{m^{(0)}}{v_{\text{SM}}}. \quad (12.4)$$

One non-trivial thing here is whether the v_{SM} is identical with v in our model. Though the VEV is determined so that it provides correct weak scale M_W , M_W may not be linear in v in our model in contrast to the case of the SM where M_W is linear in v_{SM} , $M_W = \frac{g}{2}v_{\text{SM}}$. We have to wait until a realistic model incorporating the SM is constructed in order to make a definite claim on this issue. So now we assume two typical possible cases and discuss the anomalous Higgs interaction in these cases separately.

12.2.1 Case 1: $v_{\text{SM}} = v$

We first consider the case where M_W is linear function of the VEV in the DD scenario: $M_W = \frac{g}{2}v$, i.e.

$$v_{\text{SM}} = v. \quad (12.5)$$

Then from (12.4) $f_{\text{SM}} = \frac{m^{(0)}}{v}$ and therefore we get from (12.3)

$$\frac{f_{\text{DD}}^2}{f_{\text{SM}}^2} = \frac{e_4^2 v^2 \cos(ae_4 v)}{\left\{\frac{2}{a} \sin\left(\frac{ae_4 v}{2}\right)\right\}^2} = x^2 (\cot^2 x - 1) \quad (12.6)$$

where the dimensionless parameter x is defined as

$$x \equiv \frac{ae_4 v}{2}. \quad (12.7)$$

For $x > 0$ the ratio in (12.6) deviates from unity and the Yukawa coupling (its counterpart) becomes anomalous.

12.2.2 Case 2: $v_{\text{SM}} \neq v$

Since the violation of the translational invariance is due to the property of the extra space, we may naturally expect that in a realistic model the gauge boson mass M_W is also given by trigonometric function of v like that in (10.16) with $n = 0$ and $m = 0$:

$$M_W = \frac{2}{a} \sin\left(\frac{agv}{4}\right), \quad (12.8)$$

which reduces to $\frac{g}{2}v$ in the limit $a \rightarrow 0$. Thus we find

$$v_{\text{SM}} = \frac{4}{ag} \sin\left(\frac{agv}{4}\right). \quad (12.9)$$

Since in our toy model e_4 should be identified with $\frac{g}{2}$ in a realistic model, we replace $\frac{g}{2}$ in (12.9) by e_4 to get

$$v_{\text{SM}} = \frac{2}{ae_4} \sin\left(\frac{ae_4 v}{2}\right). \quad (12.10)$$

Thus we get simple results

$$f_{\text{SM}} = \frac{m^{(0)}}{v_{\text{SM}}} = e_4. \quad (12.11)$$

and therefore

$$\frac{f_{\text{DD}}^2}{f_{\text{SM}}^2} = \cos(ae_4 v) = \cos(2x). \quad (12.12)$$

Again, for $x > 0$ the ratio in (12.12) deviates from unity and the Yukawa coupling (its counterpart) becomes anomalous.

12.2.3 Continuum limit and decoupling limit

We finally consider two limits

(i) Continuum limit: $N \rightarrow \infty$, $a \rightarrow 0$ keeping $L = Na = 2\pi R$ invariant.

(ii) “Decoupling limit”: $\frac{M_W}{M_c} \rightarrow 0$ keeping N as a finite integer ($M_c \equiv \frac{1}{R}$).

Clearly (i) is the limit where original 5D GHU with S^1 compactification is recovered. Since there is no other source of the violation of the translational invariance, we expect that the SM prediction is recovered in this continuum limit. (ii) is a limit where the masses of all non-zero KK modes of the order of M_c (compactification mass scale) are much greater than the weak scale and these massive KK particles are expected to decouple from the low energy effective theory, thus recovering the SM. In our toy model M_W should be regarded as $\sim e_4 v$ and the decoupling limit is equivalent to $e_4 v R \sim e_4 v L = e_4 v Na \sim e_4 v a \rightarrow 0$.

It is now easy to know that in both limits $x \rightarrow 0$. Interestingly, in both (12.6) and (12.12), at the limit of $x \rightarrow 0$, the ratio $\frac{f_{\text{DD}}}{f_{\text{SM}}} \rightarrow 1$ and the anomaly just goes away, as we expected.

13 Summary

In this thesis we discussed properties of the Higgs interaction in two candidates of physics beyond the standard model aiming to solve the hierarchy problem in the Higgs sector, i.e. gauge-Higgs unification and dimensional deconstruction. We pointed out that in these theories there exist anomalous Higgs interactions, namely those which deviate from the prediction of the standard model.

As the characteristic feature of these scenarios, the mass spectrum of Kaluza-Klein (KK) modes of matter fields is not linear in the VEV of the Higgs v but some non-linear function of v , in clear contrast to the case of the standard model. Accordingly, Higgs interactions with the matter fields, expected to be obtained by a replacement $v \rightarrow v + h$ (recall that physical Higgs is understood as a shift from the VEV of the Higgs field), are also non-linear in the Higgs field h : we obtain e.g. $h^n \bar{\psi}\psi$ ($n = 2, 3, \dots$) interactions with fermion already at the classical level (tree level).

Among possible anomalous Higgs interactions we mainly focused on the Yukawa coupling with quark or corresponding coupling with matter scalar. The coupling “constants” of these interactions are no longer constant, as in the case of the standard model, but turn out to depend on the VEV v . We derived general formulae to show the deviation of these couplings from the corresponding standard model predictions as the function of the v . Although in the limit where the weak scale is much smaller than the scale of new physics the coupling is known to recover the standard model prediction, we found that we generally have anomalous couplings.

In the end of analysis we argued what are essential causes of such anomalous Higgs interactions and identified two key ingredients. Namely, we argued that the interplay between the periodicity of physical observables in the Higgs field and the violation of the translational invariance along the extra space leads to the anomalous Higgs interactions.

In the scenario of gauge-Higgs unification, the origin of Higgs is KK zero mode of extra space component of higher dimensional gauge field and Higgs field appears only through a Wilson-loop (or Aharonov-Bohm) phase for a non-simply-connected extra space like S^1 . As far as Higgs appears only through the phase factor, the periodicity of physical observables in the Higgs field is an inevitable consequence. Also, in this scenario in order to obtain chiral theory in four dimension we adopt an orbi-fold as the extra space. In this thesis we took S^1/Z_2 orbi-folding. To guarantee the Z_2 invariance of the orbi-fold, we can introduce “ Z_2 -odd” bulk mass term which clearly violates translational invariance along the extra space. This enables us to realize naturally small hierarchical quark masses, because Z_2 -odd bulk mass term causes the localization of mode functions of chiral fermions at different fixed points depending on their chiralities and the overlap integral of their mode functions is

exponentially suppressed. We thus have found that for light quarks such as d, s, b quarks the bulk mass is considerably large and remarkable anomalous Yukawa coupling arises as the consequence.

On the other hand, in the scenario of dimensional deconstruction the origin of the Higgs is pseudo Nambu-Goldstone boson ϕ which is non-linearly realized,

$$U = e^{i\frac{\phi}{f}}. \quad (13.1)$$

Thus we naturally expect the periodicity of physical observables in the Higgs field in this case too.

In this scenario the gauge symmetries are of the form of direct product of the same type of gauge group $SU_1(m) \times SU_2(m) \times \cdots \times SU_N(m)$, as is schematically displayed by the moose diagram. From a different point of view, this scenario can be interpreted as five dimensional gauge-Higgs unification where the extra space S^1 is latticed to N lattice sites. So the violation of translational invariance along the extra space is obviously realized by latticizing the extra dimension.

In this way, in both scenarios, which are in close relationship as mentioned above, two essential causes for the anomalous Higgs interactions exist and similar anomalous Higgs interactions are concluded. We, however, should note that there exists some qualitative difference between the anomalous Higgs interactions in two scenarios. Namely, in the scenario of dimensional deconstruction anomalous Higgs interaction arises even though we do not introduce “ Z_2 -odd” bulk mass as in the case of gauge-Higgs unification. This is because in the dimensional deconstruction the property of the extra dimension, i.e. latticized extra dimension, itself violates the translational invariance and anomalous Higgs interaction appears universally irrespectively of what kind of particle is interacting with the Higgs. This fact in turn means that the anomalous Higgs interactions all disappear in the continuum limit, i.e. the limit where lattice spacing $a \rightarrow 0$, as we have explicitly shown by concrete calculation, unless some other sources to violate translational invariance is introduced.

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