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博士論文

Studies on Compatibility and
Innovation in Two-Sided Markets

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Chapter 1

Introduction

With the development of information technology, market channels have changed drastically over the last few decades. For example, buyers and sellers can interact and trade online (e.g., Amazon, eBay, etc). Some content including music, movies, and books are digitized for download through the Internet (e.g., iTunes, Kindle store, etc). Rochet and Tirole (2003) refer to these business models as two-sided markets. Rochet and Tirole (2006) defines two-sided markets as *markets in which one or several platforms enable interactions between end-users and try to get the two (or multiple) sides “on board” by appropriately charging each side*. Examples include credit cards, shopping malls, video games, and digital content, among others. Most important feature of two-sided markets is that there exist indirect network externalities across users on different sides of the platform; i.e., the benefits enjoyed by the users on one side depend on the number of users on the other side. For instance, in video game industry, the larger the number of software developers, the greater benefit consumers can derive. Similarly, the larger the number of consumers, the greater the profit that software developers can

earn. Recently, two-sided markets have attracted much attention from not only academics, but also practitioners.

Since the seminal works of Caillaud and Jullien (2003) and Rochet and Tirole (2003), two-sided market has been investigated by many researchers. Rochet and Tirole (2003) builds a model of platform competition to understand the price allocation between the two sides of the market. Caillaud and Jullien (2003) analyzed a model of competition between intermediation service providers with indirect network externalities. They consider the homogeneous agents on both sides. In the case where all agents can participate only one platform (single-home), all agents participate the same platform. This equilibrium is efficient and always exists. However, inefficient equilibria also exist. They also characterize the divide-and-conquer strategies, where agents on one side are subsidized and profits are made by agents on the other side. In contrast to Caillaud and Jullien (2003), Armstrong (2006) assumes that consumers on both sides are heterogeneous. He shows that, when all agents single-home, the equilibrium prices are determined by the elasticity of participation and the external benefit to the agents on the other side. Therefore, it is optimal to subsidize the agents who benefit the agents on the other side in order to increase the number of transactions on the platforms. On the other hand, he also considers a model of “competitive bottlenecks” where agents on one side can participate in both platforms (multi-home). In this case, the platforms have monopoly market powers for the agents on the multi-homing side because the agents on the multi-homing side must deal with the platforms to interact the agents on the single-homing side. Therefore, platforms compete for the agents on the single-homing side and then make the most profit from the agents on the multi-homing side.

However, these papers does not consider the intra-platform competition. Hagiu (2009) focuses on this aspect. He considers the consumers who prefer variety to model the platform adoption problem by sellers. Weyl (2010) also regards the users' preferences for variety to develop the general non-linear pricing model by the monopoly platform, so-called *insulating tariffs*.

According to Weyl (2010), there are two important features of two-sided markets. First, platforms can set different uniform prices for buyers and sellers. For example, in the video game industry, a platform sets a price of video game consoles for consumers and a participation fee for software developers. Second, there exist indirect network externalities across users on different sides of the platform; i.e., the benefits enjoyed by the users on one side depend on the number of users on the other side. For instance, the larger the number of software developers, the greater benefit consumers can derive. Similarly, the larger the number of consumers, the greater the profit that software developers can earn.

Platforms charge prices for each side taking into account indirect network externalities. As noted above, there has been considerable research on two-sided pricing (Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Hagiu, 2009; Weyl, 2010). However, little has been reported on strategies other than pricing strategies. This dissertation focuses on hardware/software system platforms in particular. That is, a platform sells its hardware device (e.g., video game console, mobile music player, e-book reader, etc) to consumers and operates its software market (e.g., game titles, digital music, e-books, etc). The software developers are charged the license fee by the platform. We analyze the platforms' strategies with regard to *compatibility* and *innovation*, which play a critical role in the hard-

ware/software industries.

Compatibility is one of the most important strategies for digital content markets. Compatibility involves opening the software (application) market to users on rival platforms, which means that a platform provides a converter (also known as translator, emulator, or adapter) that enables users of other hardware devices to use content (application software) that the platform supplies in its marketplace. For example, in the electronic book reader industry, Amazon has adopted compatibility and opened its e-book market (Kindle store) to users who own an Apple iPad. However, Apple does not allow the users of Amazon's Kindle to access iTunes or purchase e-books provided for iPad users. Thus, Apple chooses incompatibility. The advantage of compatibility is that a platform can increase its royalty revenue from software developers because more consumers purchase the software in its software market. On the other hand, the disadvantage of compatibility is that it increases the attractiveness of the rival's hardware device through a wider availability of content. Therefore, compatibility is chosen by the platforms that consider royalty revenue from software sales to be the major profit center. Incompatibility is chosen by the platforms that consider hardware sales to be the major profit center.

Platforms' investment in innovation is also a strategy that depends on the nature of their major profit center. This dissertation focuses on two types of innovation for hardware devices that platforms sell to consumers: process and product innovation. Process innovation refers to increased efficiency of cost-reducing investment. Product innovation refers to improvements in the quality of hardware devices. Platforms that focus on the hardware market have greater incentives to invest in these innovations.

The remainder of this dissertation is organized as follows. In Chapter 2, we show how compatibility decisions change over the product life cycle. In Chapter 3, we investigate the relationship between compatibility decision and process innovation. Chapter 4 focuses on the competition between vertically differentiated platforms and demonstrates the advantages and disadvantages of a high-quality. Chapter 5 summarizes the key findings and concludes the paper by providing directions for further research.

Chapter 2

Compatibility and the Product Life Cycle in Two-Sided Markets

2.1 Introduction

We consider a duopoly model with two competing platforms each of whom chooses whether to make its content compatible with the rival's hardware device. Our main result is that the outcome of these choices depends on the stage of the product life cycle in the industry. Specifically, at the early stage of the product life cycle firms derive most of their profit from the sale of the hardware devices because few customers will have yet purchased them. At this early stage, making content compatible with the rival's hardware device is unprofitable because it increases the attractiveness of the rival's hardware device by wider availability of content. Therefore, incompatibility is the

dominant strategy. But at a mature stage of the product life cycle, many customers already own hardware devices and royalties from the sale of content will have become the major profit center. At the mature stage, making content compatible with the rival's hardware device is the dominant strategy because it supports a wider demand for the content the firm is selling. These results are derived in a stylized model in which the subgame-perfect equilibrium attains one of five possible configurations depending on parameters. Asymmetric equilibria arise when the platform is at an intermediate stage of the product life cycle and there is a large difference of market shares for installed customer base, so that the large firm chooses to make its content incompatible with the other's hardware while the small rival does the opposite.

The sort of example we have in mind is that of an electronic book reader such as the Amazon Kindle or Apple iPad tablet computer and the electronic books that are viewed on these devices. Amazon has chosen to make its electronic "Kindle" books compatible with the iPad tablet computer. While Apple has so far not made its "iBooks" compatible with the Amazon Kindle. Other instances of hardware devices and related content include audio or video players that can but need not be outfitted with a converter, translator, emulator, adapter, or gizmo supplied by a rival that makes the content of the rival compatible with the device.

2.2 Related literature

The seminal research on compatibility with rival’s products focused on network effects.¹ For example the value of a telephone network to any user becomes greater, the larger the number of other users. If subscribers to the telephone service of one supplier are able to call subscribers to rival telephone services, then subscriptions to either service are more valued. These network effects are called “direct network externalities”. Katz and Shapiro (1985) showed that direct network externalities incline the largest incumbent supplier against making its service compatible with the service of rivals. Crémer et al. (2000) adapted the model of Katz and Shapiro (1985) to incorporate locked-in, installed base customers who will not switch to other firms. They showed that a firm with a large installed base prefers to reduce the degree of compatibility with its smaller rivals. Malueg and Schwartz (2006) extended the model of Crémer et al. (2000) to the case in which the largest firm faces any number of small rivals and the incompatibility could produce tipping (all new customers join one network). They showed that a firm with a large installed base is likely to gain from compatibility in growing industries, but not in relatively mature industries. All of this literature studies the issue of compatibility between competitors in settings with single-sided platforms and direct network externalities.

The research on product compatibility most closely related to our model focuses on the strategic aspects of compatibility with complementary products. Matutes and Regibeau (1988, 1992) and Economides (1989) provided

¹See Gilbert (1992), Katz and Shapiro (1994), Economides and White (1994), Economides (1996), Farrell and Klemperer (2007), and Shy (2011) for surveys on network effects and compatibility.

the *mix and match* framework for analysis of compatibility with rival complementary products in a single-sided market without network externalities. The main concern in those papers, and in this one, is not the size of the network that confers consumer benefits when the supplier chooses compatibility, but the wider availability of complementary products. These network effects are called “indirect network externalities”. Our model extends the concept of compatibility with rival complementary products to the case of competing platforms in two-sided markets. Rochet and Tirole (2003) and Armstrong (2006), among others, developed the basic analysis of two-sided markets, defined as any in which a supplier’s sales to one set of demanders also affects its income from a different set of demanders.² So for instance an internet platform provider supplies services to its users, and the number of such users also affects the sale of internet advertising by the same platform provider. In our framework, the competing platforms supply hardware devices (e-book readers) and the number of users of the devices also affects the income from royalties collected from original copyright holders of the content supplied for use with the devices.

There have been a number of papers that address the issue of compatibility in two-sided markets. Doganoglu and Wright (2006) examined the effect of consumers’ multi-homing on compatibility between networks. They showed that platforms have an insufficient incentive to choose compatibility in the presence of multi-homing. Miao (2009) developed the model of two-sided markets and showed that a monopoly platform has an incentive to

²See also the seminal papers by Caillaud and Jullien (2003), Rochet and Tirole (2006), and Armstrong and Wright (2007). For surveys on two-sided markets, see Roson (2005) and Rysman (2009).

foreclose competition in the complementary market by committing to maintain incompatibility. Casadesus-Masanell and Ruiz-Aliseda (2008) examined the case of duopoly. They show that incompatibility gives rise to the situation in which a dominant platform earns more than under compatibility. Vicens (2011) examined platform competition under the assumption that the degree of application compatibility is an exogenous parameter and showed that a small firm will always demand that application compatibility be enforced but a large firm never will.

In sum, contrary to the previous literature that focuses on competition given the structure of compatibility, the unique theoretical contribution of our work lies in showing the equilibrium structure of application compatibility in a two-sided market with indirect network externalities. To the best of our knowledge, this chapter is the first one which shows the interesting point that the equilibrium structure of compatibility changes over the product life cycle, and we will provide the example of the market for electronic book readers such as those of Amazon and Apple which matches this theoretical result.

The remainder of this chapter is organized as follows. In Section 2.3, we develop the simplest possible model of compatibility decisions by platforms in the two-sided markets. Section 2.4 presents the equilibrium outcomes of subgames and comparative static analyses. Section 2.5 derives the subgame-perfect equilibrium. Section 2.6 extends the basic model and Section 2.7 summarizes the key findings and concludes the chapter by providing directions for further research.

2.3 Model

2.3.1 Platforms

Suppose that there are two platforms, $i = 1, 2$. Each platform provides a hardware device i at a price p_i and operates its marketplace i that distributes content for its own hardware device. We consider that there are two kinds of content, $i = 1, 2$, and that content i is exclusively supplied to marketplace i at price ρ_i by the independent content provider and is designed to work only on hardware device i .³ We suppose that each platform chooses whether to make its *content* compatible with the other's hardware device. In this chapter we use the term *application compatibility*, which means that a platform provides a converter (also known as translator, emulator, or adapter) which enables users of other hardware devices to use content (application software) that it supplies in its marketplace.⁴ We assume that each unit of content provides an equal benefit for any consumer, and that the price of a unit of content is the same for any content, $\rho_i = \rho$ ($i = 1, 2$). Each platform charges a royalty rate r ($0 \leq r \leq 1$) for each unit of content sold at its marketplace. Platforms also earn revenue by selling their hardware devices to consumers. We treat the price of content ρ and royalty rate r as exogenous variables in our basic model.

³It is assumed that contents available on the two platforms are mutually exclusive. In reality, however, there are common contents supplied to several platforms (e.g. Amazon and Apple have a large overlap in their selection of e-books). Even when we relax this assumption and allow n kinds of common contents between two platforms in addition to exclusive contents, we can demonstrate the results qualitatively similar to those described in the propositions in Section 2.5. Proof is available upon request.

⁴Farrell and Saloner (1992) study the economics of “converters” in single-sided markets with direct network externalities.

In Section 2.6, we relax this assumption and endogenize the determination of content price ρ , and show that our basic insights continue to hold. The assumption of a common fixed royalty rate is somewhat unusual, but is true for the electronic book (e-book) industry. The owners of digital contents, in other word book publishers, adopt the *agency pricing model*. They set the prices of their e-books, and distributors, like Amazon and Apple, get a fixed percentage fee from the publishers for every book sold (about 30%) Jiang (2012); Knowledge@Wharton (2012).

The profit function of platform i is given by

$$\pi_i = r\rho d_i + p_i D_i, \quad i = 1, 2,$$

where d_i denotes the demand for content supplied at marketplace i and D_i denotes the demand for the hardware device. We assume a constant marginal cost of supplying hardware devices which is normalized to zero.

2.3.2 Consumers

We consider two groups of consumers: existing customers (the installed customer base of each platform), and new customers. The mass of consumers is normalized to 1. Denote by α the fraction of the consumers who are existing customers and by $1 - \alpha$ the remaining fraction who are new customers. Further stipulate that the fraction β of the existing customers have adopted hardware device 1 and the remaining fraction $1 - \beta$ customers have adopted hardware device 2 in the previous period. These customers are the installed bases of the platforms. While the existing customers already own hardware devices and hence only demand content, new customers must buy both hardware devices and content. Platforms compete to add new customers to their

installed bases.

To analyze the consumers' choice of platform, we use a Hotelling model of product differentiation. The hardware devices are differentiated along the unit interval $[0, 1]$, with hardware 1 located at 0, and hardware 2 at 1. Each new customer buys one hardware device only. Ideal points of consumers are distributed uniformly on the unit interval with a unit density, and each consumer incurs a constant proportional disutility t per unit length. We assume that the benefit derived from consumption of the hardware device (that is, the consumer's stand-alone valuation for the hardware device) v is large enough for every new customer to buy one hardware device. Denote by B the utility that any consumer derives from a unit of content, which is assumed to be the same for any content and for any consumer, and satisfies the condition $B > \rho$. Thus, we consider that each consumer buys any usable content.

The utility function of a new customer who is located at x , buys a hardware device i , and uses its available contents is written as

$$u_i = N_i(B - \rho) + v - p_i - t|x - x_i|,$$

where N_i is the amount of available content for hardware device i ($N_i = 1, 2$), and x_i is the location of hardware device i . We will use the notation, $b = B - \rho$.

2.3.3 Game structure

We consider a two-stage game. In stage one, the two platforms independently and simultaneously choose between application compatibility and incompatibility. In stage two, platforms independently and simultaneously set their

hardware prices p_1 and p_2 . Then existing customers purchase content, and new customers purchase both hardware and content.

The model is a one-period and static game, but we focus on the equilibrium of compatibility choices of platforms in different stages of the product life cycle. We use α (the fraction of existing customers) as a measure of the product life cycle in the industry. If α is large, almost all customers have purchased hardware devices, and this industry is in the mature stage of the product life cycle. Otherwise, if α is small, the industry is in the introductory stage of the product life cycle.

2.4 Equilibrium and comparative statics

Given the compatibility decisions in stage one, it follows that there are four possible market structures. The first is one of incompatible platforms in which both platforms choose incompatibility, (IC, IC). The second is one of compatible platforms in which both platforms choose compatibility, (C, C). And the third and fourth are asymmetric market structures in which one platform chooses incompatibility and the other chooses compatibility, (IC, C) and (C, IC). For example, Figure 2.1 illustrates the asymmetric market structure in which firm 1 chooses incompatibility and firm 2 compatibility. In this section, we will derive the equilibrium prices, demands, profits, consumer surplus, and social surplus under the various market structures.

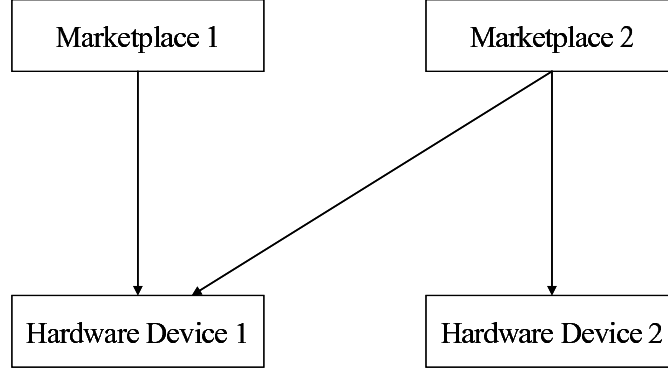


Figure 2.1: Market structure in which firm 2 has chosen compatibility, but firm 1 has chosen incompatibility.

2.4.1 Incompatible platforms

In this case, for each hardware device, only a single content is available . The utility function of a new customer who is located at x can be written as

$$u_i = b + v - p_i - t|x - x_i|, \quad (i = 1, 2).$$

Let $\hat{x}$ be the location of a new customer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{t - p_1 + p_2}{2t}.$$

Hence, the demand for hardware device i is

$$D_i = \frac{(t - p_i + p_j)(1 - \alpha)}{2t} \quad (i = 1, 2, j \neq i).$$

Platform 1 maximizes its profit

$$\pi_1 = r\rho(D_1 + \alpha\beta) + p_1 D_1 = \frac{(p_1 + r\rho)(t - p_1 + p_2)(1 - \alpha)}{2t} + \alpha\beta r\rho$$

with respect to its hardware price p_1 . Similarly, platform 2 maximizes its profit

$$\pi_2 = r\rho(D_2 + \alpha(1 - \beta)) + p_2 D_2 = \frac{(p_2 + r\rho)(t - p_2 + p_1)(1 - \alpha)}{2t} + \alpha(1 - \beta)r\rho$$

with respect to its hardware price p_2 . Taking the first-order conditions and solving for prices, we have the equilibrium prices as follows:

$$p_1(\text{IC}, \text{IC}) = p_2(\text{IC}, \text{IC}) = t - r\rho.$$

We can derive the equilibrium demands, profits, consumer surplus, and social surplus as shown in Table 1.

2.4.2 Compatible platforms

When both platforms choose compatibility, all content is usable on either hardware device. In this case, the utility function of a new customer is

$$u_i = 2b + v - p_i - t|x - x_i|, \quad (i = 1, 2).$$

From this we can derive the demand for hardware as follows:

$$D_i = \frac{(t - p_i + p_j)(1 - \alpha)}{2t} \quad (i = 1, 2, j \neq i).$$

The profit function of platform i is

$$\pi_i = r\rho + p_i D_i = \frac{p_i(t - p_i + p_j)(1 - \alpha)}{2t} + r\rho \quad (i = 1, 2, j \neq i).$$

From the first-order conditions for profit maximization, we have the equilibrium hardware prices, demands, profits, consumer surplus, and social surplus as shown in Table 2.1.

2.4.3 Incompatible-compatible platforms

When platform 1 chooses incompatibility and platform 2 chooses compatibility, all content is usable with hardware device 1, but only content supplied by firm 2 is usable with hardware device 2. In this case, the utility functions of new customers are

$$\begin{cases} u_1 = 2b + v - p_1 - tx \\ u_2 = b + v - p_2 - t(1 - x). \end{cases}$$

From these we can derive the demands for hardware devices as follows:

$$D_1 = \frac{(b + t - p_1 + p_2)(1 - \alpha)}{2t}, \quad D_2 = \frac{(-b + t - p_1 + p_2)(1 - \alpha)}{2t}.$$

The profit functions of the platforms are

$$\begin{cases} \pi_1 = r\rho(D_1 + \alpha\beta) + p_1 D_1 = \frac{(p_1 + r\rho)(b + t - p_1 + p_2)(1 - \alpha)}{2t} + \alpha\beta r\rho, \\ \pi_2 = r\rho + p_2 D_2 = \frac{p_2(-b + t + p_1 - p_2)(1 - \alpha)}{2t} + r\rho. \end{cases}$$

From the first-order conditions for profit maximization, we have the equilibrium prices, demands, profits, consumer surplus, and social surplus as shown in Table 1. The equilibrium in the other asymmetric case is similar.

In this chapter, we exclude cases in which all new customers choose the same hardware device (“tipping”). This requires us to assume that the premise of the following Lemma is true, that the hardware devices of the two firms are inherently differentiated.

Lemma 2.1. *If the hardware of the two firms is sufficiently differentiated that $t > (b + r\rho)/3$, then there exists an equilibrium in which both platforms have positive market shares for new customers.*

Proof. Tipping is the situation in which the market structure is asymmetric and the incompatible platform captures all the new customers. Therefore, in order to exclude tipping it must hold that the market share of the incompatible platform is less than one:

$$\frac{1}{2} + \frac{b + r\rho}{6t} < 1 \quad \Longleftrightarrow \quad \frac{b + r\rho}{6t} < \frac{1}{2} \quad \Longleftrightarrow \quad t > \frac{b + r\rho}{3}$$

□

Table 2.1: Equilibrium price, demands, profits, consumer surplus, and social surplus.

	(IC, IC)	(C, C)	(IC, C)
p_1	$t - r\rho$	t	$t - \frac{2}{3}r\rho + \frac{b}{3}$
p_2	$t - r\rho$	t	$t - \frac{1}{3}r\rho - \frac{b}{3}$
D_1	$\frac{1}{2}(1 - \alpha)$	$\frac{1}{2}(1 - \alpha)$	$\left(\frac{1}{2} + \frac{b+r\rho}{6t}\right)(1 - \alpha)$
D_2	$\frac{1}{2}(1 - \alpha)$	$\frac{1}{2}(1 - \alpha)$	$\left(\frac{1}{2} - \frac{b+r\rho}{6t}\right)(1 - \alpha)$
π_1	$\frac{t}{2}(1 - \alpha) + \alpha\beta r\rho$	$\frac{t}{2}(1 - \alpha) + r\rho$	$\frac{(3t+b+r\rho)^2}{18t}(1 - \alpha) + \alpha\beta r\rho$
π_2	$\frac{t}{2}(1 - \alpha) + \alpha(1 - \beta)r\rho$	$\frac{t}{2}(1 - \alpha) + r\rho$	$\frac{(3t-b-r\rho)^2}{18t}(1 - \alpha) + r\rho$
CS^N	$\left(v + b - \frac{5}{4}t + r\rho\right)(1 - \alpha)$	$\left(v + 2b - \frac{5}{4}t\right)(1 - \alpha)$	$\left(\frac{(b+r\rho)^2}{36t} + v + \frac{3}{2}b - \frac{5}{4}t + \frac{1}{2}r\rho\right)(1 - \alpha)$
CS^E	$\alpha \cdot b$	$\alpha \cdot 2b$	$\alpha(1 + \beta) \cdot b$
SS	$\left(v - \frac{1}{4}t\right)(1 - \alpha) + b + \rho$	$\left(v - \frac{1}{4}t\right)(1 - \alpha) + 2(b + \rho)$	$\left(\frac{5(b+\rho)^2}{36t} + v - \frac{1}{4}t\right)(1 - \alpha) + (b + \rho)\left(\frac{3}{2} + \alpha\left(\beta - \frac{1}{2}\right)\right)$

2.4.4 Comparative statics

We next compare the equilibrium prices, demands, profit, consumer surplus, and social surplus under the various market structures. First, we assert the following proposition regarding prices and demands.

Proposition 2.1. *When the benefit of content is large enough to satisfy the condition $b > 2r\rho$, then the equilibrium prices and demands are ordered as follows:*

$$p_2(\text{IC}, \text{C}) = p_1(\text{C}, \text{IC}) < p_i(\text{IC}, \text{IC}) < p_i(\text{C}, \text{C}) < p_1(\text{IC}, \text{C}) = p_2(\text{C}, \text{IC}),$$

$$D_2(\text{IC}, \text{C}) = D_1(\text{C}, \text{IC}) < D_i(\text{IC}, \text{IC}) = D_i(\text{C}, \text{C}) < D_1(\text{IC}, \text{C}) = D_2(\text{C}, \text{IC}).$$

Proof. These results can be easily shown from the results in Table 2.1. $\square$

This proposition shows the natural result that when consumers get much benefit from content, the relative price and demand for a hardware device increases as it has more usable content compared to competing devices: $p_1(\text{C}, \text{IC}) < p_2(\text{C}, \text{IC})$ and $D_1(\text{C}, \text{IC}) < D_2(\text{C}, \text{IC})$. It is also the case that the prices tend to be higher when both devices are compatible with the others' content compared to the case in which both are incompatible: $p_i(\text{IC}, \text{IC}) < p_i(\text{C}, \text{C})$. That is, compatibility softens price competition. A similar result has been shown in the indirect network externalities literature Schiff (2003); Doganoglu and Wright (2006); Miao (2010). And Matutes and Regibeau (1988) and Economides (1989) showed that even in the absence of network externalities, when firms sell compatible components, cutting the hardware price will increase the demand for *hybrid systems* that use not only the one firm's component but also the rival firm's component (i.e., rival's content). In this way, some benefit from increased demand accrues to the

rival firm, which weakens the one firm's incentive to cut its price. The logic behind our result is subtly different and may be given as follows. Consider the situation of incompatible platforms (IC, IC) and suppose that one platform cuts its hardware price. Then the demand for its hardware increases, which also increases the demand for its content. Because of the increase in revenue from both sides of the two-sided market (the revenue from selling its own hardware and the royalties from selling its content), a firm selling incompatible hardware has a strong incentive to cut its price. However, when the platforms are both compatible (C, C), and one of the firms cuts its hardware price, then the demand for its hardware increases but, in our model, the demand for its content does not increase. This is because of our assumption that any consumer purchases all available content for his hardware device. By this assumption, under compatible platforms, the total demand for either firm's content is constant and equal to 1. Thus, the firms have less incentive to price their hardware aggressively under compatible platforms (C, C) than they would do under incompatible platforms (IC, IC).

We find the same rank ordering for prices and demands under asymmetric market structures. This allows us to derive the relations $p_1(C, IC)D_1(C, IC) < p_1(IC, IC)D_1(IC, IC)$ and $p_1(C, C)D_1(C, C) < p_1(IC, C)D_1(IC, C)$. Thus we obtain the following corollary.

Corollary 2.1. *Consider a single-sided market in which platforms do not charge royalties for content and earn revenue only by selling their hardware devices to consumers. Then, incompatibility is the dominant strategy for both platforms.*

That is, if a platform cannot get profit from selling content in its marketplace, then the only effect of compatibility is to make the rival platform more

attractive to consumers. Thus in this single-sided market, it is the dominant strategy equilibrium for both platforms to choose incompatible platforms.

However, by comparing the profits of platforms summarized in Table 2.1, we have the following proposition.

Proposition 2.2. *It follows that $\pi_i(\text{IC}, \text{IC}) < \pi_i(\text{C}, \text{C})$.*

A similar result has been shown in Schiff (2003) and Doganoglu and Wright (2006). From Corollary 2.1 and Proposition 2.2 we can find that the equilibrium in the single-sided market, that is, (IC, IC), forms a prisoner's dilemma situation and leads to lower profits for both platforms.

Next, when we compare the equilibrium consumer surplus under the various market structures, we have the following result.

Proposition 2.3. *It follows that*

If $b > 7r\rho/5$ and $t > (b + r\rho)/3$, then the consumer surplus is ordered as

(i) $CS^N(\text{IC}, \text{IC}) < CS^N(\text{IC}, \text{C}) = CS^N(\text{C}, \text{IC}) < CS^N(\text{C}, \text{C})$ for new customers, and

(ii) $CS^E(\text{IC}, \text{IC}) < CS^E(\text{IC}, \text{C}) = CS^E(\text{C}, \text{IC}) < CS^E(\text{C}, \text{C})$ for existing customers,

where CS^N denotes consumer surplus for new customers, and CS^E for existing customers.

Proof. See Appendix 1 for the proof. □

While we find in Proposition 2.1 that compatibility leads to higher hardware device prices compared to incompatibility, it also increases the variety

of available content. Thus some consumers are better off with compatibility, while other consumers are worse off. As Proposition 2.3 part (ii) shows, existing customers are always better off when compatibility prevails, because they benefit from wider availability of content without paying higher hardware prices. What about the new customers? Proposition 2.3 part (i) shows that when the benefit of content is large enough, then compatible platforms provide more benefit for new consumers than do incompatible platforms.

Finally, we will compare the social surplus among the different market structures. We assert the following proposition.

Proposition 2.4. *When the degree of differentiation is large enough to satisfy the condition $t > \text{Max}(5(b + \rho)/18, (b + r\rho)/3)$, then the equilibrium social surplus are ordered as follows:*

$$SS(\text{IC}, \text{IC}) < SS(\text{IC}, \text{C}) \leq SS(\text{C}, \text{C}), \quad SS(\text{IC}, \text{IC}) < SS(\text{C}, \text{IC}) \leq SS(\text{C}, \text{C}).$$

Proof. See Appendix 2 for the proof. □

From Propositions 2.3 and 2.4 we can find that the compatible platforms outcome (C, C) is optimal in the light of not only consumer surplus but also social surplus.

2.5 Subgame-perfect equilibrium

In the previous section, we examined the subgame equilibrium in stage two. Here we use the results of the analyses to derive the subgame-perfect equilibrium of the two-stage game using backward induction. To determine the equilibrium, we first compare the equilibrium profits under the various mar-

ket structures in stage two. Using the equilibrium profits shown in Table 2.1, we have the following Lemma 2.2.

Lemma 2.2. *It follows that*

$$\begin{aligned}
\beta > \beta_1(\alpha) &\iff \pi_1(\text{IC}, \text{IC}) > \pi_1(\text{C}, \text{IC}), \\
\beta > \beta_2(\alpha) &\iff \pi_2(\text{IC}, \text{C}) > \pi_2(\text{IC}, \text{IC}), \\
\beta > \beta_3(\alpha) &\iff \pi_1(\text{IC}, \text{C}) > \pi_1(\text{C}, \text{C}), \\
\beta > \beta_4(\alpha) &\iff \pi_2(\text{C}, \text{C}) > \pi_2(\text{C}, \text{IC}),
\end{aligned}$$

where

$$\begin{aligned}
\beta_1(\alpha) &= \frac{6t(b+r\rho) - (b+r\rho)^2}{18tr\rho} - \frac{6t(b-2r\rho) - (b+r\rho)^2}{18tr\rho \cdot \alpha}, \\
\beta_2(\alpha) &= -\frac{6t(b-2r\rho) - (b+r\rho)^2}{18tr\rho} + \frac{6t(b-2r\rho) - (b+r\rho)^2}{18tr\rho \cdot \alpha}, \\
\beta_3(\alpha) &= \frac{6t(b+r\rho) + (b+r\rho)^2}{18tr\rho} - \frac{6t(b-2r\rho) + (b+r\rho)^2}{18tr\rho \cdot \alpha}, \\
\beta_4(\alpha) &= -\frac{6t(b-2r\rho) + (b+r\rho)^2}{18tr\rho} + \frac{6t(b-2r\rho) + (b+r\rho)^2}{18tr\rho \cdot \alpha},
\end{aligned}$$

$$\beta_1 + \beta_2 = 1, \quad \text{and} \quad \beta_3 + \beta_4 = 1.$$

Platforms choose between compatibility and incompatibility in stage one of the game. From Lemma 2.2, depending on parameter values, we can identify four possible subgame-perfect equilibria as follows.

Lemma 2.3. *It follows that*

- (i) *(IC, IC) is an equilibrium when $\beta_1(\alpha) \leq \beta \leq \beta_2(\alpha)$,*
- (ii) *(IC, C) is an equilibrium when $\beta_2(\alpha) \leq \beta$ and $\beta_3(\alpha) \leq \beta$,*

(iii) (C, IC) is an equilibrium when $\beta \leq \beta_1(\alpha)$ and $\beta \leq \beta_4(\alpha)$,

(iv) (C, C) is an equilibrium when $\beta_4(\alpha) \leq \beta \leq \beta_3(\alpha)$.

These results can be immediately shown from Lemma 2.

In Lemma 2.2, we can see that the graph of β_i ($i = 1, 2, 3, 4$) is dependent on the sign of the second term in each equation, that is, the signs of $6t(b - 2r\rho) + (b + r\rho)^2$ and $6t(b - 2r\rho) - (b + r\rho)^2$. Therefore, there are three cases;

$$\begin{aligned}
(\text{Case 1}) \quad & 6t(b - 2r\rho) + (b + r\rho)^2 > 0 \text{ and } 6t(b - 2r\rho) - (b + r\rho)^2 > 0 \\
& \iff b > 2r\rho \text{ and } t > \text{Max}\left(\frac{(b+r\rho)^2}{6|b-2r\rho|}, \frac{b+r\rho}{3}\right), \\
(\text{Case 2}) \quad & 6t(b - 2r\rho) + (b + r\rho)^2 > 0 \text{ and } 6t(b - 2r\rho) - (b + r\rho)^2 < 0 \\
& \iff r\rho < b < 5r\rho \text{ and } \frac{b+r\rho}{3} < t < \frac{(b+r\rho)^2}{6|b-2r\rho|}, \\
(\text{Case 3}) \quad & 6t(b - 2r\rho) + (b + r\rho)^2 < 0 \text{ and } 6t(b - 2r\rho) - (b + r\rho)^2 < 0 \\
& \iff b < 2r\rho \text{ and } t > \text{Max}\left(\frac{(b+r\rho)^2}{6|b-2r\rho|}, \frac{b+r\rho}{3}\right).
\end{aligned}$$

When we use Lemma 2.3 in each case, we can describe the partition of the parameter space (α, β) . The parameter space (α, β) is divided into several regions which have different subgame perfect equilibria. The following proposition gives the partition of the parameter space in *Case 1*.

Proposition 2.5. *Consider Case 1, that is, both the benefit of content and the degree of hardware differentiation are large enough to satisfy the conditions $b > 2r\rho$ and $t > \text{Max}((b + r\rho)^2/(6|b - 2r\rho|), (b + r\rho)/3)$. Then depending on parameter values, the equilibrium market structure is either: (A) incompatible platforms; (B) compatible platforms; (C) incompatible-compatible platforms; (D) compatible-incompatible platforms; or (E) multiple equilibria (IC, C) and (C, IC) . Figure 2.2 illustrates the possibilities in (α, β) space.*

The result can be immediately shown from Lemma 2.3. Suppose that

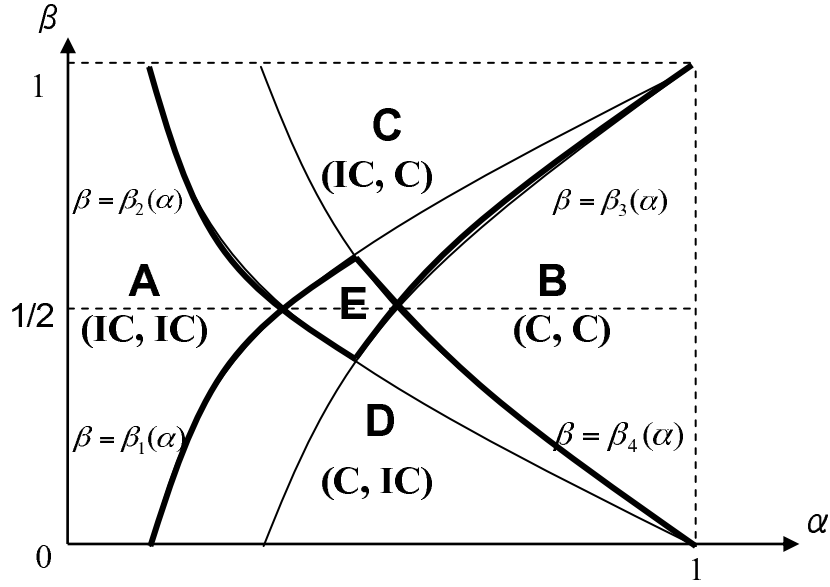


Figure 2.2: Equilibrium Market Structure of *Case 1*.

α is relatively small (as in the introductory stage of the product life cycle) and there are many new customers who need to purchase hardware devices. Then it is of great importance to platforms to sell hardware devices to new customers and add them to their installed bases. In such a situation, compatibility is unprofitable because it increases the attractiveness of the rival's hardware device by providing wider availability of content. Therefore, the unique equilibrium is one in which platforms are incompatible (IC, IC).

In contrast, consider the situation in which α is large (as in the mature stage of the product life cycle) so that almost all customers are in the installed base of some platform. They already have hardware devices and only want to purchase content. Then a platform obtains most of its profit by selling content in its marketplace and obtaining royalties. In such a situation, compatibility is more profitable than incompatibility. Thus, the unique

equilibrium is one in which platforms are compatible (C, C).

Suppose that α is at an intermediate level (i.e., in the growth stage of the product life cycle). First, consider the situation in which there is a large difference between the market shares of two firms. Then for the small platform (with a small installed customer base) the profitable choice is to make its content compatible with the larger rival's hardware device and garner royalties from the sale of content to the installed base of the rival firm. However, for the large platform (with a large installed customer base), the profitable choice is to make its content incompatible with the hardware device of the smaller rival, and maintain its share of the hardware market by preserving the wider availability of content on its hardware devices, as shown in regions C and D of figure 2.2.

Next, consider the situation in which there is little difference between the market shares of the two firms. Then there are multiple equilibria as shown in region E in Figure 2.2. The market has the property of a game of "chicken". That is, either firm will make more profit by making its content incompatible with the rival's hardware device, given that the rival does the opposite.

Figure 2.2 illustrates the possibilities of equilibrium in (α, β) space for given values of the other parameters b and t . Performing comparative statics with respect to the parameters b and t gives the following two corollaries.

Corollary 2.2. *As the parameter b becomes larger, every function β_i shifts to the right. Therefore, the larger benefit from content increases the region A (equilibria with only incompatible platforms) and decreases the region B (equilibria with only compatible platforms).*

Proof. See Appendix 3 for the proof. □

The intuition for this result is the following. When the benefit of content becomes large, new customers are likely to choose the hardware device which has more usable content. Then choosing compatibility is not profitable for any firm because it provides more usable content for the rival's hardware device.

Corollary 2.3. *As the parameter t increases, the functions β_1 and β_3 get closer and the functions β_2 and β_4 also get closer. Therefore, the larger degree of hardware differentiation increases the region A (equilibria with only incompatible platforms) and the region B (equilibria with only compatible platforms).*

Proof. See Appendix 4 for the proof. □

The intuition behind the increase of region B is given as follows. The larger is t , the smaller is the profit loss from hardware device derived from the compatibility decision. So given the profit gain from larger demand of content from the compatibility decision, it is more likely to use the strategy of compatibility. Thus, the region B increases.

Next, we will consider *Case 2*. The following proposition gives the partition of the parameter space in *Case 2*.

Proposition 2.6. *Consider Case 2, that is, both the benefit of content and the degree of hardware differentiation are at an intermediate level that satisfies the conditions $r\rho < b < 5r\rho$ and $(b + r\rho)/3 < t < (b + r\rho)^2/(6|b - 2r\rho|)$. Then, depending on parameter values, the equilibrium market structure is either: (A) there are multiple equilibria (IC, C) and (C, IC); (B) compatible platforms (C, C); (C) incompatible-compatible platforms (IC, C); or (D)*

compatible-incompatible platforms (C, IC). Figure 2.3 illustrates these possibilities in (α, β) space.

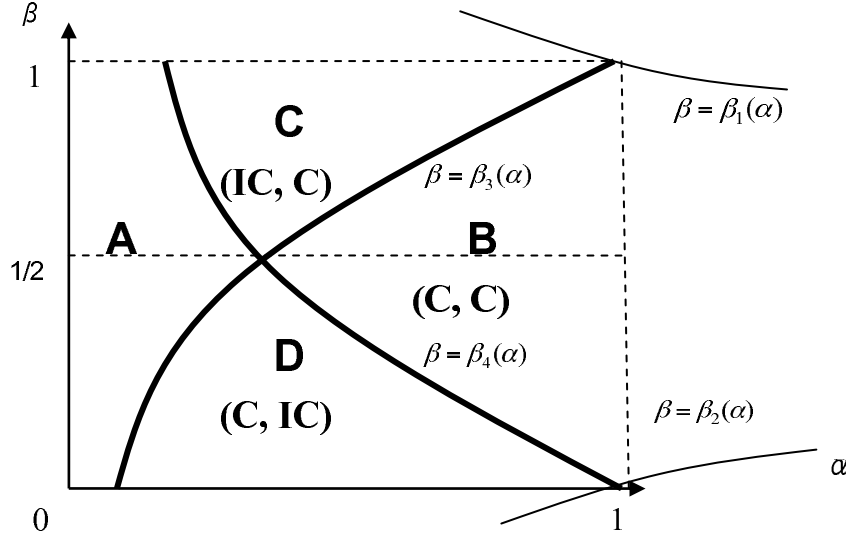


Figure 2.3: Equilibrium Market Structure of *Case 2*.

They can be immediately shown from Lemma 2.3. The difference from Proposition 2.5 is that (IC, IC) is not a possible equilibrium when the degree of hardware differentiation is small. The intuition for this result is the following. When the degree of hardware differentiation is relatively small, there is keen competition in hardware pricing, which reduces the profit from selling hardware. Then platforms are more likely to make their content compatible with the rival's hardware device to garner royalties by selling content to the rival's installed base. Hence, even in the introductory stage of the product life cycle, incompatible platforms (IC, IC) is not an equilibrium market structure.

Finally, consider *Case 3*. The following proposition gives the partition of

the parameter space in *Case 3*.

Proposition 2.7. *Consider Case 3, that is, the benefit of content is small enough to satisfy the condition $b < 2r\rho$ and the degree of hardware differentiation is large enough to satisfy the condition $t > \text{Max}((b + r\rho)^2/(6|b - 2r\rho|), (b + r\rho)/3)$. Then the unique equilibrium market structure is one with compatible platforms (C, C) . Figure 2.4 illustrates the possibilities in (α, β) space.*

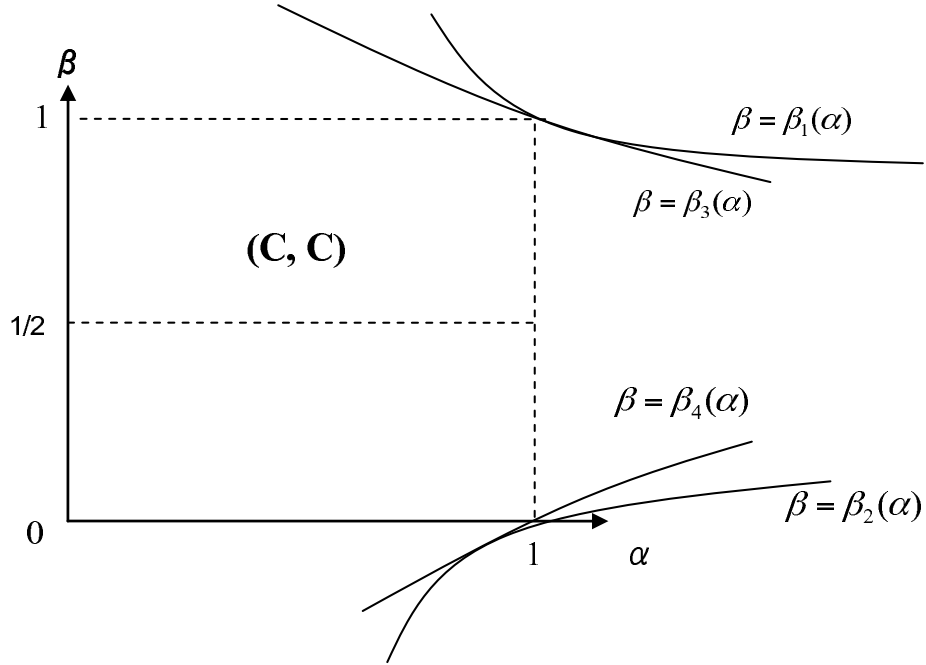


Figure 2.4: Equilibrium Market Structure of *Case 3*.

This can be immediately shown from Lemma 2.3. When the royalty from selling a unit of content $r\rho$ is larger than one half of the marginal benefit from content b , and the degree of hardware differentiation t is relatively

large, making content compatible with the rival's hardware device has little adverse effect on the firm's own sale of hardware devices. In such situation, platforms are likely to make their content compatible with the rival's hardware device, and profit by garnering royalties from the sale of content to the rival's installed base. Thus the unique equilibrium is one with compatible platforms (C, C).

2.6 Model extension

In this section, we relax our assumption that the content price ρ is exogenously fixed to show the robustness of the main propositions obtained in our basic model. We allow the determination of content price by content providers.

We consider a three-stage game. That is, in stage one, the two platforms independently and simultaneously choose between application compatibility and incompatibility. In stage two, platforms independently and simultaneously set their hardware prices. Then the new consumers purchase hardware device. In stage three, content providers independently and simultaneously set their content prices. Then the customers purchase content. Following Church and Gandal (2000), we suppose that the benefit consumers receive from consuming N varieties of content is represented by $w(N)$. We retain our assumption that there are two kinds of content, $N = 1, 2$. Here we assume that $w(1) < w(2)$ and $w(2) - w(1) < w(1)$. Solving the subgame in stage three it can be easily seen that when a single content is available, content price is set at $\rho = w(1)$. When two content are available, content price is set at $\rho = w(2) - w(1)$.

Using these results and solving the game backward, we have the equilibrium profits of platforms under the four different possible market structures in stage two. Comparing those equilibrium profits, we can show that Lemma 2.2 in Section 2.5 is changed as follows.

Lemma 2.2a. *It follows that*

$$\begin{aligned}\beta > \beta_1(\alpha) &\iff \pi_1(\text{IC}, \text{IC}) > \pi_1(\text{C}, \text{IC}), \\ \beta > \beta_2(\alpha) &\iff \pi_2(\text{IC}, \text{C}) > \pi_2(\text{IC}, \text{IC}), \\ \beta > \beta_3(\alpha) &\iff \pi_1(\text{IC}, \text{C}) > \pi_1(\text{C}, \text{C}), \\ \beta > \beta_4(\alpha) &\iff \pi_2(\text{C}, \text{C}) > \pi_2(\text{C}, \text{IC}),\end{aligned}$$

where

$$\begin{aligned}\beta_1(\alpha) &= \frac{6t\{(1-r)W + r\Delta w\} - \{(1-r)W + r\Delta w\}^2}{18tr\Delta w} \\ &\quad - \frac{6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha}, \\ \beta_2(\alpha) &= -\frac{6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2}{18tr\Delta w} \\ &\quad + \frac{6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha}, \\ \beta_3(\alpha) &= \frac{6t\{(1-r)W + r\Delta w\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w} \\ &\quad - \frac{6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha}, \\ \beta_4(\alpha) &= -\frac{6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w} \\ &\quad + \frac{6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha},\end{aligned}$$

$$\beta_1 + \beta_2 = 1, \quad \beta_3 + \beta_4 = 1, \quad W \equiv 2w(1) - w(2), \quad \text{and} \quad \Delta w \equiv w(2) - w(1).$$

Using these four functions, we can separate three cases similarly in Section 2.5, and in each case we have the following propositions which give the

partition of the parameter space.

Proposition 2.5a. *Suppose that both the benefit of content and the degree of hardware differentiation are large enough to satisfy the conditions $0 < r < \{2w(1) - w(2)\}/w(2)$ and $t > \{(1-r)W + r\Delta w\}^2/6\{W - rw(2)\}$. Then, depending on parameter values, the equilibrium market structure is either: (A) incompatible platforms; (B) compatible platforms; (C) incompatible-compatible platforms; (D) compatible-incompatible platforms; or (E) multiple equilibria (IC, C) and (C, IC). Figure 2.2 illustrates the possibilities in (α, β) space.*

Proof. See Appendix 5 for the proof. $\square$

Proposition 2.6a. *Suppose that both the benefit of content and the degree of hardware differentiation are at an intermediate level that satisfies the conditions,*

- (i) $0 < r < \{2w(1) - w(2)\}/w(2)$ and $\{(1-r)W + r\Delta w\}/6 < t < \{(1-r)W + r\Delta w\}^2/6\{W - rw(2)\}$, or
- (ii) $\{2w(1) - w(2)\}/w(2) < r < 2\{2w(1) - w(2)\}/\{3w(1) - w(2)\}$ and $\{(1-r)W + r\Delta w\}/6 < t < \{(1-r)W + r\Delta w\}^2/6\{rw(2) - W\}$.

Then, depending on parameter values, the equilibrium market structure is either: (A) there are multiple equilibria (IC, C) and (C, IC); (B) compatible platforms (C, C); (C) incompatible-compatible platforms (IC, C); or (D) compatible-incompatible platforms (C, IC). Figure 2.3 illustrates these possibilities in (α, β) space.

Proof. See Appendix 5 for the proof. $\square$

Proposition 2.7a. *Suppose that the benefit of content is small enough to satisfy the condition $\{2w(1) - w(2)\}/w(2) < r < 1$ and the degree of hardware differentiation is large enough to satisfy the conditions $t > \text{Max}\left(\{(1 - r)W + r\Delta w\}^2/6\{rw(2) - W\}, \{(1 - r)W + r\Delta w\}/6\right)$. Then the unique equilibrium market structure is one with compatible platforms (C, C) . Figure 2.4 illustrates the possibilities in (α, β) space.*

Proof. See Appendix 5 for the proof. □

By comparing Propositions 2.5 to 2.7 and Propositions 2.5a to 2.7a, we see that even when content price is determined endogenously, the equilibrium outcomes are qualitatively similar.

2.7 Discussion and conclusion

The purpose of this chapter was to understand how the product life cycle affects the compatibility strategy of platforms in two-sided markets. To this end, we proposed a duopoly model of platforms in which the market comprises two segments of consumers: the installed base of customers and the new customers. We have developed a stylized model in which two competing suppliers of platform hardware devices and content each chooses whether to make its content compatible with the other's hardware device. The sort of platform business we have in mind is the market for electronic books. In this market, Amazon has adopted the principle of "application compatibility", meaning that e-books purchased in the Amazon store can be read not only on the "Kindle" hardware devices that it sells but also on the electronic readers sold by its rival Apple, the "iPad", "iPod", and "iPhone". But Apple

has chosen to make the electronic books that it sells incompatible with the Amazon Kindle reader. That is, electronic books purchased at the Apple iTunes store (so-called “iBooks”) can only be read on tablet-like devices also supplied by Apple. They cannot be read on the Amazon Kindle. The main question that our stylized model is intended to address is why these rivals might have chosen opposite strategies with regard to compatibility with one another’s hardware devices of the content each supplies.

This situation can be explained by using our results. Since the e-book market is now in the growth stage of the product life cycle, α is at an intermediate level. In addition, we suppose that the benefit of content (e-book) b is relatively large. In this case, when there is a large difference between the market shares of the two firms, asymmetric equilibria arise, corresponding to regions C and D of Figure 2.2 and Figure 2.3, based on Proposition 2.5 and Proposition 2.6. In fact, Apple has a large installed base of iPad, iPod, and iPhone users, and there is a great difference between the market shares of hardware devices of the two firms, Apple and Amazon. For the small platform (Amazon) the profitable choice is to make its content compatible with the larger rival’s hardware device and gain royalties from expanding the sale of content to the installed base of the rival firm. However, for the large platform (Apple), the profitable choice is to make its content incompatible with the hardware device of the smaller rival, and maintain its share of the hardware market by preserving the wider availability of content on its hardware devices. These roughly correspond to the relative positions of Amazon and Apple in the market for electronic books and the hardware devices for reading them.

In addition, we can show an example in which a switch of the compatibil-

ity choice actually takes place over the product life cycle in the digital music industry. Prior to 2007, online music stores such as Apple iTunes Store and SONY mora have employed DRM (Digital Rights Management) which are the technical systems and technologies to inhibit usage of music purchased and downloaded. Thus, for example, consumers who want to listen to songs downloaded from the iTunes Store had to have an iPod or other Apple device. This structure of compatibility between Apple and SONY corresponded to (IC, IC). However, on May 29, 2007 Apple released version 7.2 of its iTunes software which allows users to purchase DRM-free music (called iTunes Plus music) and play it on rivals' hardware devices such as WALKMAN. Thus the structure changed to (C, IC). Furthermore, on October 1, 2012 SONY mora also began to allow users to purchase DRM-free music. As a result, the structure of compatibility moved to (C, C).

In developing our model, we made a few simplifying assumptions. We assumed that the royalty rate was predetermined and thus unaffected by firms' compatibility choices. In particular, because the royalty rate is exogenous in our model, the platforms cannot use it as a tool to compete for content providers. This is a significant limitation of this chapter as an analysis of two-sided markets. We also assumed that there was no possibility of one platform fully capturing the market (tipping). A more complete analysis of compatibility decisions which includes these issues remains for future work.

2.8 Appendix

Appendix 1: Proof of Proposition 3

From the results in Table 2.1, we have

$$\begin{aligned}
& CS^N(C, C) - CS^N(IC, C) \\
&= \left(v + 2b - \frac{5}{4}t \right) - \left(\frac{(b + r\rho)^2}{36t} + v + \frac{3}{2}b - \frac{5}{4}t + \frac{r\rho}{2} \right) \\
&= \frac{b}{2} - \frac{r\rho}{2} - \frac{(b + r\rho)^2}{36t} \\
&= \frac{18t(b - r\rho) - (b + r\rho)^2}{36t} \\
&> \frac{18 \cdot \frac{b+r\rho}{3} \cdot (b - r\rho) - (b + r\rho)^2}{36t} \\
&= \frac{1}{36t}(b + r\rho)(5b - 7r\rho) > 0 \quad \left(\iff b > \frac{7}{5}r\rho \right), \\
& CS^N(IC, C) - CS^N(IC, IC) \\
&= \left(\frac{(b + r\rho)^2}{36t} + v + \frac{3}{2}b - \frac{5}{4}t + \frac{r\rho}{2} \right) - \left(v + b - \frac{5}{4}t + r\rho \right) \\
&= \frac{b}{2} - \frac{r\rho}{2} + \frac{(b + r\rho)^2}{36t} > 0 \quad \left(\iff b > r\rho \right)
\end{aligned}$$

□

Appendix 2: Proof of Proposition 4

From the results in Table 2.1, we have

$$\begin{aligned}
& SS(C, C) - SS(IC, C) \\
&= \left[\left(v - \frac{1}{4}t \right) (1 - \alpha) + 2(b + \rho) \right] \\
&\quad - \left[\left(\frac{5(b + \rho)^2}{36t} + v - \frac{1}{4}t \right) (1 - \alpha) + (b + \rho) \left(\frac{3}{2} + \alpha \left(\beta - \frac{1}{2} \right) \right) \right] \\
&= -\frac{5(b + \rho)^2}{36t} (1 - \alpha) + (b + \rho) \left(\frac{1}{2} - \alpha \left(\beta - \frac{1}{2} \right) \right) \\
&\quad \text{(This is decreasing function in } \beta \text{.)}
\end{aligned}$$

$$\begin{aligned}
&> -\frac{5(b+\rho)^2}{36t}(1-\alpha) + (b+\rho)\left(\frac{1}{2} - \frac{1}{2}\alpha\right) \quad (\text{for } \beta < 1) \\
&= \left(\frac{1}{2} - \frac{5(b+\rho)}{36t}\right)(b+\rho)(1-\alpha) > 0 \quad \left(\Longleftrightarrow t > \frac{5}{18}(b+\rho) \right) \\
&SS(\text{IC}, \text{C}) - SS(\text{IC}, \text{IC}) \\
&= \left[\left(\frac{5(b+\rho)^2}{36t} + v - \frac{1}{4}t \right)(1-\alpha) + (b+\rho)\left(\frac{3}{2} + \alpha\left(\beta - \frac{1}{2}\right)\right) \right] \\
&\quad - \left[\left(v - \frac{1}{4}t \right)(1-\alpha) + b + \rho \right] \\
&= \frac{5(b+\rho)^2}{36t}(1-\alpha) + (b+\rho)\left(\frac{1}{2} + \alpha\left(\beta - \frac{1}{2}\right)\right) \\
&= \frac{5(b+\rho)^2}{36t}(1-\alpha) + (b+\rho)\left(\frac{1}{2}(1-\alpha) + \alpha\beta\right) > 0 \quad \square
\end{aligned}$$

Appendix 3: Proof of Corollary 2

Here we define $\tilde{\alpha}$ and $\hat{\alpha}$ as

$$\begin{aligned}
\beta_1 = \beta_2 &\Longleftrightarrow \alpha = \frac{(b+r\rho)^2 - 3t(2b-4r\rho)}{(b+r\rho)^2 - 3t(2b-r\rho)} \equiv \tilde{\alpha}, \\
\beta_3 = \beta_4 &\Longleftrightarrow \alpha = \frac{(b+r\rho)^2 + 3t(2b-4r\rho)}{(b+r\rho)^2 + 3t(2b-r\rho)} \equiv \hat{\alpha}.
\end{aligned}$$

The derivatives of $\tilde{\alpha}$ and $\hat{\alpha}$ with respect to b are given by

$$\begin{aligned}
\frac{\partial \tilde{\alpha}}{\partial b} &= \frac{18tr\rho(3t-b-r\rho)}{\{b^2 - 2b(3t-r\rho) + r\rho(3t+r\rho)\}^2} > 0 \quad \text{from Lemma 2.1, and} \\
\frac{\partial \hat{\alpha}}{\partial b} &= \frac{18tr\rho(3t+b+r\rho)}{\{b^2 + 2b(3t+r\rho) - r\rho(3t-r\rho)\}^2} > 0. \quad \square
\end{aligned}$$

Appendix 4: Proof of Corollary 3

The derivatives of $\tilde{\alpha}$ and $\hat{\alpha}$ with respect to t are:

$$\begin{aligned}
\frac{\partial \tilde{\alpha}}{\partial t} &= \frac{9r\rho(b+r\rho)^2}{\{(b+r\rho)^2 - 3t(2b-r\rho)\}^2} > 0, \text{ and} \\
\frac{\partial \hat{\alpha}}{\partial t} &= -\frac{9r\rho(b+r\rho)^2}{\{(b+r\rho)^2 + 3t(2b-r\rho)\}^2} < 0. \quad \square
\end{aligned}$$

Appendix 5: Proof of Lemma 2.2a and Propositions 2.5a, 2.6a, and 2.7a.

In Section 2.6, we relaxed our assumption and allowed the determination of content price by content providers. Then we showed the robustness of the main propositions obtained our basic model. In this section 1 of Appendix, at first we will show the equilibrium prices, demands, and profits under four possible market structures given the compatibility decisions in stage one. Second, we will provide the proofs of Lemma 2.2a, Propositions 2.5a, 2.6a, and 2.7a given in Section 2.6.

Appendix 5.1 Equilibrium and comparative statics.

In this section of Appendix, we will derive the equilibrium prices, demands, and profits under four possible market structures given the compatibility decisions in stage one.

5.1.1 Incompatible platforms: (IC, IC)

In this case, a single content is only available for each hardware device. The utility function of a new customer who is located at x can be written as

$$\begin{cases} u_1 = w(1) + v - \rho_1 - p_1 - tx = v - p_1 - tx \\ u_2 = w(1) + v - \rho_2 - p_2 - t(1 - x) = v - p_2 - t(1 - x). \end{cases}$$

Let $\hat{x}$ be the location of a new customer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{t - p_1 + p_2}{2t}.$$

Platform 1 maximizes its profit

$$\pi_1 = (p_1 + rw(1)) \cdot \frac{t - p_1 + p_2}{2t} \cdot (1 - \alpha) + \alpha\beta rw(1)$$

with respect to its hardware price p_1 . Taking the first-order condition and solving for price, we have the reaction function of platform 1 as follows.

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{t - rw(1) + p_2}{2}.$$

Similarly, we have the reaction function of platform 2 as follows.

$$p_2 = \frac{t - rw(1) + p_1}{2}.$$

Solving two reaction functions for hardware prices, we have the equilibrium prices as follows:

$$\begin{cases} p_1 = t - rw(1) \\ p_2 = t - rw(1). \end{cases}$$

We can derive the equilibrium profits of platforms as follows:

$$\begin{cases} \pi_1 = \frac{t}{2}(1 - \alpha) + \alpha\beta rw(1) \\ \pi_2 = \frac{t}{2}(1 - \alpha) + \alpha(1 - \beta)rw(1). \end{cases}$$

5.1.2 Compatible platforms: (C, C)

In this case, all content is usable on either hardware device. The utility function of a new customer who is located at x can be written as

$$\begin{cases} u_1 = w(2) + v - \rho_1 - \rho_2 - p_1 - tx = 2w(1) - w(2) + v - p_1 - tx \\ u_2 = w(2) + v - \rho_1 - \rho_2 - p_2 - t(1 - x) = 2w(1) - w(2) + v - p_2 - t(1 - x). \end{cases}$$

Let $\hat{x}$ be the location of a new customer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{t - p_1 + p_2}{2t}.$$

Platform 1 maximizes its profit

$$\pi_1 = p_1 \cdot \frac{t - p_1 + p_2}{2t} \cdot (1 - \alpha) + r\{w(2) - w(1)\}$$

with respect to its hardware price p_1 . Taking the first-order condition and solving for price, we have the reaction function of platform 1 as follows.

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{t + p_2}{2}.$$

Similarly, we have the reaction function of platform 2 as follows.

$$p_2 = \frac{t + p_1}{2}.$$

Solving two reaction functions for hardware prices, we have the equilibrium prices as follows:

$$\begin{cases} p_1 = t \\ p_2 = t. \end{cases}$$

We can derive the equilibrium profits of platforms as follows:

$$\begin{cases} \pi_1 = \frac{t}{2}(1 - \alpha) + r\{w(2) - w(1)\} = \frac{t}{2}(1 - \alpha) + r\Delta w \\ \pi_2 = \frac{t}{2}(1 - \alpha) + r\{w(2) - w(1)\} = \frac{t}{2}(1 - \alpha) + r\Delta w. \end{cases}$$

where, $\Delta w \equiv w(2) - w(1)$.

5.1.3 Incompatible-compatible platforms: (IC, C)

When platform 1 chooses incompatibility and platform 2 chooses compatibility, all content is usable with hardware device 1, but only content supplied by firm 2 is usable with hardware device 2. In this case, the utility functions of new customers are

$$\begin{cases} u_1 = w(2) + v - \rho_1 - \rho_2 - p_1 - tx = 2w(1) - w(2) + v - p_1 - tx \\ u_2 = w(1) + v - \rho_2 - p_2 - t(1-x) = v - p_2 - t(1-x). \end{cases}$$

Let $\hat{x}$ be the location of a new customer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{2w(1) - w(2) + t - p_1 + p_2}{2t} = \frac{W + t - p_1 + p_2}{2t}.$$

where, $W \equiv 2w(1) - w(2)$.

Platform 1 maximizes its profit

$$\pi_1 = (p_1 + r\Delta w) \cdot \frac{W + t - p_1 + p_2}{2t} \cdot (1 - \alpha) + \alpha\beta r\Delta w$$

with respect to its hardware price p_1 . Taking the first-order condition and solving for price, we have the reaction function of platform 1 as follows.

$$\frac{\partial \pi_1}{\partial p_1} = 0 \Rightarrow p_1 = \frac{W - r\Delta w + t + p_2}{2}.$$

Platform 2 maximizes its profit

$$\begin{aligned} \pi_2 = & p_2 \cdot \frac{-W + t + p_1 - p_2}{2t} \cdot (1 - \alpha) \\ & + rw(1) \left(\frac{-W + t + p_1 - p_2}{2t} (1 - \alpha) + \alpha(1 - \beta) \right) \end{aligned}$$

$$+r\Delta w \left(\frac{W+t-p_1+p_2}{2t}(1-\alpha) + \alpha\beta \right)$$

with respect to its hardware price p_2 . Taking the first-order condition and solving for price, we have the reaction function of platform 2 as follows.

$$\frac{\partial \pi_2}{\partial p_2} = 0 \Rightarrow p_2 = \frac{t - (1+r)W + p_1}{2}.$$

Solving two reaction functions for hardware prices, we have the equilibrium prices as follows:

$$\begin{cases} p_1 = t + \frac{W}{3} - \frac{rw(2)}{3} \\ p_2 = t - \frac{W}{3} - \frac{r\{3w(1)-2w(2)\}}{3}. \end{cases}$$

We can derive the equilibrium profits of platforms as follows:

$$\begin{cases} \pi_1 = \frac{\{3t+(1-r)W+r\Delta w\}^2}{18t} + \alpha\beta r\Delta w \\ \pi_2 = \frac{9t^2+\{(1-r)W+r\Delta w\}^2}{18t} + \frac{\alpha rw(1)rw(2)-W(1-\alpha-\alpha r+3\alpha\beta r)}{3}. \end{cases}$$

5.1.3 Incompatible-compatible platforms: (IC, C)

Similarly, when platform 1 chooses compatibility and platform 2 chooses incompatibility, we have the hardware prices and profits of platform as follows.

$$\begin{cases} p_1 = t - \frac{W}{3} - \frac{r\{3w(1)-2w(2)\}}{3} \\ p_2 = t + \frac{W}{3} - \frac{rw(2)}{3}. \end{cases}$$

We can derive the equilibrium profits of platforms as follows:

$$\begin{cases} \pi_1 = \frac{9t^2+\{(1-r)W+r\Delta w\}^2}{18t} + \frac{\alpha rw(1)rw(2)-W(1-\alpha-\alpha r+3\alpha(1-\beta)r)}{3} \\ \pi_2 = \frac{\{3t+(1-r)W+r\Delta w\}^2}{18t} + \alpha(1-\beta)r\Delta w. \end{cases}$$

Appendix 5.2 Subgame-perfect equilibrium.

Using the equilibrium profits shown in Appendix 5.1, we have Lemma 2.2a in Section 2.6. The proof is given as follows.

Proof of Lemma 2.2a First we compare the equilibrium profits under (IC, IC) and (C, IC).

$$\begin{aligned}
& \pi_1(\text{IC}, \text{IC}) - \pi_1(\text{C}, \text{IC}) \\
&= \left[\frac{t}{2}(1 - \alpha) + \alpha\beta rw(1) \right] \\
&\quad - \left[\frac{9t^2 + \{(1 - r)W + r\Delta w\}^2}{18t} + \frac{\alpha rw(1)rw(2) - W(1 - \alpha - \alpha r + 3\alpha(1 - \beta)r)}{3} \right] \\
&\geq 0 \\
&\iff \beta \geq \frac{6t\{(1 - 2r)W + rw(1)\} - \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w} \\
&\quad - \frac{6t\{W - rw(2)\} - \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha} \equiv \beta_1(\alpha)
\end{aligned}$$

Second, we compare the equilibrium profits under (IC, IC) and (IC, C).

$$\begin{aligned}
& \pi_2(\text{IC}, \text{IC}) - \pi_2(\text{IC}, \text{C}) \\
&= \left[\frac{t}{2}(1 - \alpha) + \alpha(1 - \beta)rw(1) \right] \\
&\quad - \left[\frac{9t^2 + \{(1 - r)W + r\Delta w\}^2}{18t} + \frac{\alpha rw(1)rw(2) - W(1 - \alpha - \alpha r + 3\alpha\beta r)}{3} \right] \\
&\geq 0 \\
&\iff \beta \leq \frac{6t\{2rw(1) - (1 + r)W\} + \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w} \\
&\quad + \frac{6t\{W - rw(2)\} - \{(1 - r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha} \equiv \beta_2(\alpha)
\end{aligned}$$

Third, we compare the equilibrium profits under (IC, C) and (C, C).

$$\pi_1(\text{IC}, \text{C}) - \pi_1(\text{C}, \text{C})$$

$$\begin{aligned}
&= \left[\frac{9t^2 + \{(1-r)W + r\Delta w\}^2}{18t} + \frac{\alpha rw(1)rw(2) - W(1-\alpha - \alpha r + 3\alpha(1-\beta)r)}{3} \right] \\
&\quad - \left[\frac{t}{2}(1-\alpha) + r\Delta w \right] \\
&\geq 0 \\
&\iff \beta \geq \frac{6t\{(1-r)W + r\Delta w\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w} \\
&\quad - \frac{6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha} \equiv \beta_3(\alpha)
\end{aligned}$$

Finally, we compare the equilibrium profits under (C, IC) and (C, C).

$$\begin{aligned}
&\pi_2(\text{C, IC}) - \pi_2(\text{C, C}) \\
&= \left[\frac{\{3t + (1-r)W + r\Delta w\}^2}{18t} + \alpha(1-\beta)r\Delta w \right] \\
&\quad - \left[\frac{t}{2}(1-\alpha) + r\Delta w \right] \\
&\geq 0 \\
&\iff \beta \leq -\frac{6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w} \\
&\quad + \frac{6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2}{18tr\Delta w \cdot \alpha} \equiv \beta_4(\alpha)
\end{aligned}$$

Thus, Lemma 2.2a holds. $\square$

In Lemma 2.2a, we can see that the graph of β_i ($i = 1, 2, 3, 4$) is dependent on the sign of the second term in each equation, that is, the signs of $6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2$ and $6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2$. Therefore, there are three cases.

Case 1: $6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2 > 0$ and $6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2 > 0$.

Case 2: $6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2 < 0$ and $6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2 > 0$.

Case 3: $6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2 < 0$ and $6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2 < 0$.

Propositions 2.5a, 2.6a, and 2.7a are derived from Cases 1, 2, and 3 respectively.

Proof of Proposition 2.5a We consider Case 1 in which

$$\begin{aligned} & 6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2 > 0 \\ & \text{and } 6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2 > 0 \\ \iff & -6t\{W - rw(2)\} < \{(1-r)W + r\Delta w\}^2 < 6t\{W - rw(2)\} \end{aligned}$$

Then, it must hold that

$$W - rw(2) > 0 \iff r < \frac{2w(1) - w(2)}{w(2)} < 1.$$

In addition, it is necessary to hold that

$$t > \frac{\{(1-r)W + r\Delta w\}^2}{6\{W - rw(2)\}}$$

If these two conditions are satisfied, tipping is excluded. Thus, Case 1 is characterized as the following conditions:

$$\text{Case 1: } 0 < r < \frac{2w(1) - w(2)}{w(2)} \text{ and } t > \frac{\{(1-r)W + r\Delta w\}^2}{6\{W - rw(2)\}}.$$

Thus, we have Proposition 2.5a. □

Proof of Proposition 2.6a We consider Case 2 in which

$$\begin{aligned} & 6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2 < 0 \\ & \text{and } 6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2 > 0 \end{aligned}$$

(i) Suppose that $W - rw(2) > 0 \iff r < \frac{W}{w(2)}$.

Then it is obvious that $6t\{W - rw(2)\} + \{(1 - r)W + r\Delta w\}^2 > 0$. It is easy to see that

$$\begin{aligned} 6t\{W - rw(2)\} - \{(1 - r)W + r\Delta w\}^2 &< 0 \\ \iff t &< \frac{\{(1 - r)W + r\Delta w\}^2}{6\{W - rw(2)\}}. \end{aligned}$$

When we add the condition to exclude tipping solution, Case 2 is characterized as the following conditions:

$$0 < r < \frac{2w(1) - w(2)}{w(2)} \quad \text{and} \quad \frac{(1 - r)W + r\Delta w}{6} < t < \frac{\{(1 - r)W + r\Delta w\}^2}{6\{W - rw(2)\}}.$$

(ii) Suppose that $W - rw(2) < 0 \iff \frac{W}{w(2)} < r < 1$. Then it is obvious that $6t\{W - rw(2)\} - \{(1 - r)W + r\Delta w\}^2 < 0$. We can see that

$$\begin{aligned} 6t\{W - rw(2)\} + \{(1 - r)W + r\Delta w\}^2 &> 0 \\ \iff t &< \frac{\{(1 - r)W + r\Delta w\}^2}{6\{rw(2) - W\}}. \end{aligned}$$

When we add the condition to exclude tipping solution, Case 2 is characterized as the following conditions:

$$\frac{2w(1) - w(2)}{w(2)} < r < 1 \quad \text{and} \quad \frac{(1 - r)W + r\Delta w}{6} < t < \frac{\{(1 - r)W + r\Delta w\}^2}{6\{rw(2) - W\}}.$$

However, in order to satisfy the second conditions it must hold that

$$\frac{\{(1 - r)W + r\Delta w\}^2}{6\{rw(2) - W\}} - \frac{(1 - r)W + r\Delta w}{6} > 0 \iff r < \frac{2W}{3w(1) - 2w(2)}.$$

Hence, we rearrange the conditions for Case 2 as follows:

$$\frac{2w(1) - w(2)}{w(2)} < r < \frac{2W}{3w(1) - 2w(2)}$$

$$\text{and } \frac{(1-r)W + r\Delta w}{6} < t < \frac{\{(1-r)W + r\Delta w\}^2}{6\{rw(2) - W\}}.$$

From the conditions (i) and (ii), Case 2 is characterized the following conditions:

$$\text{Case 2: } \left(\frac{2w(1)-w(2)}{w(2)} < r < 1 \text{ and } \frac{(1-r)W+r\Delta w}{6} < t < \frac{\{(1-r)W+r\Delta w\}^2}{6\{rw(2)-W\}} \right) \text{ or } \\ \left(\frac{2w(1)-w(2)}{w(2)} < r < \frac{2W}{3w(1)-2w(2)} \text{ and } \frac{(1-r)W+r\Delta w}{6} < t < \frac{\{(1-r)W+r\Delta w\}^2}{6\{rw(2)-W\}} \right)$$

Thus, we have Proposition 2.6a. $\square$

Proof of Proposition 2.7a We consider Case 3 in which

$$6t\{W - rw(2)\} - \{(1-r)W + r\Delta w\}^2 < 0 \\ \text{and } 6t\{W - rw(2)\} + \{(1-r)W + r\Delta w\}^2 < 0$$

These conditions are rearranged as

$$6t\{W - rw(2)\} < \{(1-r)W + r\Delta w\}^2 < -6t\{W - rw(2)\} \\ \iff r > \frac{2w(1) - w(2)}{w(2)} \text{ and } t > \frac{\{(1-r)W + r\Delta w\}^2}{6\{rw(2) - W\}}$$

When we add the condition to exclude tipping solution, Case 3 is characterized the following conditions:

$$\text{Case 3: } r > \frac{2w(1)-w(2)}{w(2)} \text{ and } t > \max \left\{ \frac{\{(1-r)W+r\Delta w\}^2}{6\{rw(2)-W\}}, \frac{(1-r)W+r\Delta w}{6} \right\}$$

Thus, we have Proposition 2.7a. $\square$

Chapter 3

Welfare Effects of Process Innovation in Two-Sided Markets: The Role of Compatibility Decisions

3.1 Introduction

We extend the modeling of R&D investment competition into two-sided markets. There are many two-sided markets characterized by hardware-software systems. Examples include video games and video game consoles, digital music and playback devices, and e-books and digital readers. In these two-sided markets, platforms have two sources of revenues: the revenue from selling their hardware to consumers, and the royalties from selling software (content) at their marketplaces. Depending on the situation, platforms must

make strategic choices as to which revenue source to more fully cultivate.

As a matter of fact, compatibility decisions as to whether to make one's software compatible with the other's device are very important for platforms. Compatibility (application compatibility) here means that a platform enables users of others' hardware devices to use content (application software) that it supplies in its marketplace. That is, adopting compatibility means opening up its marketplace for users of other hardware devices. When making its software compatible with the rival's hardware device, on the one hand the platform can support a wider demand for the software the platform is selling, but at the same time it is unprofitable because it increases the attractiveness of the rival's hardware device by wider availability of software. For instance, in the market for e-books, Amazon has adopted the principle of compatibility and made it possible for users of its rival's iPad to purchase e-books from the Amazon Kindle Store. In contrast, Apple has chosen incompatibility and the users of Amazon Kindle cannot purchase e-books at the Apple iTunes Store. Compatibility decisions such as this may depend on whether the platform is more focused on hardware sales or on royalties from the sale of software (content).

Moreover, the compatibility strategy affects the incentive for hardware investment by rival platforms. That is, when one platform selects compatibility with others' content, the rival platform has superior availability of content on its own hardware device. This in turn increases the demand for hardware from rival platforms, and so one may expect that rival platforms will have higher incentives to invest. The aim of this chapter is to provide a framework that accounts for R&D investment competition in two-sided markets that incorporates compatibility decisions by the competing platforms.

We present a model in which there are two platforms, and each platform provides a differentiated hardware device and operates its marketplace which distributes content for its own hardware device. Consumers buy one unit of hardware and, at most, one unit of each content. Platforms earn revenues from the price they charge for hardware, and from the royalties from content that buyers consume with the hardware. We assume that the royalty rate and the content prices are exogenous. We use a Hotelling model of product differentiation to analyze the consumers' choice of platform. We consider a three-stage game. In stage one, each platform chooses between compatibility and incompatibility. In stage two, each platform makes investment in reducing hardware costs. Finally in stage three, each platform sets the price of its hardware device.

In this setting, we argue that process innovation (increased efficiency of cost-reducing investment) of the hardware device can reduce the social surplus. Suppose that the efficiency of investment remains at a lower level, and the hardware devices are not much differentiated. Then we can show that the equilibrium is asymmetric: one platform selects incompatibility (IC) with the others content, while the other platform selects compatibility (C). The intuition behind this result is the following. If the rival platform chooses compatibility, the profitable choice is to make its content incompatible with the rival's hardware device and gain larger profits from hardware sales by preserving the superior availability of content on its own hardware device. Alternatively, if the rival platform chooses incompatibility, then incompatibility is unprofitable because there is stiff competition in hardware pricing when the degree of hardware differentiation is small, which reduces the profit from selling the hardware device. Then the profitable choice is to make its

content compatible with the rival's hardware device to get royalties by selling content to the rival's installed base. Hence, the asymmetric case of (IC, C) or (C, IC) is the equilibrium.

In such a situation, we consider the effect of increased efficiency of investment. We first show that an equilibrium with tipping (corner solution) can arise as one of the equilibria under such an asymmetric case. The reason is that the platform that has chosen incompatibility, say platform 1, can get more demand for its hardware as it has more usable content compared to a competing platform, say platform 2. Then the marginal revenue from investment is larger for platform 1. Hence it is more aggressive on cost-reducing investment, which makes it possible to capture the entire market for hardware devices when the efficiency of investment is increased to higher levels.

In any one of the following three cases, platform 2 does not have an incentive to deviate from compatibility to incompatibility. First, when the royalties from selling content are large enough, platform 2 does not have any incentive to deviate from maintaining compatibility, even if it lost the market of hardware to the platform 1. Second, when the degree of hardware differentiation is very small, or third, when the efficiency of investment is extremely high, it is unprofitable for platform 2 to deviate from compatibility to incompatibility. This is because in addition to the loss of royalties from selling content it is difficult to gain profit from hardware sales due to the stiff price competition through cost-reducing investments.

Alternatively, when the royalties from selling content are small, the degree of hardware differentiation is large, and the efficiency of investment is high enough to lead to an equilibrium with tipping but not extremely high, then

there is an incentive for the platform 2 to deviate from compatibility to incompatibility in order to defend the market for its hardware. Then the case of incompatible platforms (IC, IC) can be an alternative equilibrium.

In sum, in the situation where the asymmetric case of (IC, C) is the unique equilibrium, when there is increased efficiency of cost-reducing R&D investment (process innovation), the process innovation affects the compatibility decision, and another type of (IC, IC) can arise in equilibrium. This yields a socially detrimental result. The social surplus under (IC, IC) becomes lower than that under (IC, C). Because the total demand for hardware is fixed in our Hotelling model, social surplus depends on three things: (i) the extent of available content, (ii) the effect of cost-reducing investment, and (iii) the consumers' trip costs (disutility). First of all, the situation of (IC, IC) is not beneficial for consumers because the variety of available content is less for each hardware device. In addition, it is already explained that the platform 1 that has chosen incompatibility has more incentive for cost-reducing investment under the situation of (IC, C) and sells its hardware to many consumers, thus the total effect of investment is smaller under (IC, IC) than under (IC, C). In the aspect of the consumers' trip costs, it can be easily seen that they are lower under the symmetric situation of (IC, IC) than those under asymmetric case of (IC, C). However, when consumers' trip cost is not so high, it has not much effect. Then the first and second effects dominate, and social surplus becomes lower under (IC, IC) than under (IC, C). The key point of this chapter is that the process innovation in two-sided markets has not only positive direct effects of reducing costs but also negative indirect effects through the change of market structure from (IC, C) to (IC, IC) by the strategic compatibility decisions of competing platforms. Hence we cannot

simply infer that the increased efficiency of cost-reducing R&D investment (process innovation) is socially beneficial in two-sided markets.

For comparison, there can be no such counterintuitive case in a one-sided market, because then each platform has a single source of revenue from hardware sales. When one platform chooses compatibility, it has only a negative effect on its profit as the other platform becomes more attractive. That is, incompatibility is the dominant strategy in a one-sided market. Therefore, it can be said that the main result in this chapter critically depends on the two-sidedness of the market.

3.2 Related literature

This chapter relates to two bodies of research. One is the research of strategic R&D competition. There is much literature dealing with strategic R&D competition in traditional market structures, and the socially undesirable effects of cost-reducing R&D investment have been argued in the technology licensing literature which shows that licensing may reduce welfare if it induces competitors to exit the market (Kabiraj and Marjit, 1992; Lin, 1996), facilitates collusion (Fauli-Oller and Sandonis, 2002), changes R&D organization (Mukherjee, 2005), or induces excessive entry (Mukherjee and Mukherjee, 2008) etc. Furthermore, Chang et al. (2013) considers a three-stage (R&D, technology licensing, and output) oligopoly game in which only one of the firms undertakes cost-reducing R&D and finds that the availability of licensing leads to lower social surplus, if the “efficiency of R&D investment” is high. There is some connection with this finding because it explores the possibility of welfare-reducing innovation. However, the problem we address

here is inherently different. We explore the welfare effect of innovation mediated by compatibility decisions in two-sided markets, while their model is about the welfare effects of licensing innovation in traditional markets.

Focusing on the argument comparing the incentive for R&D investment under different market structures in two-sided markets, there are two strands in the recent literature. One is the literature about open-source and proprietary platforms. Casadesus-Masanell and Llanes (2013) is the seminal article on the incentives to invest in platform quality in proprietary and open-source two-sided markets. They study a two-sided monopoly platform and show conditions under which an open source may lead to higher investment in platform quality than a proprietary platform. Another strand is the literature on network neutrality and investment incentives such as Musacchio et al. (2009), Choi and Kim (2010), and Economides and Hermalin (2012), among others. These papers discuss which regulatory regime (neutral or non-neutral) yields stronger investment incentives in the network infrastructure. In their models of two-sided markets, the market structure (an open-source or a proprietary platform, and net-neutrality or non-neutrality) is assumed to be exogenously given.

This chapter is also related to the literature on compatibility in two-sided markets.¹ The recent papers of Doganoglu and Wright (2006), Miao (2009), Casadesus-Masanell and Ruiz-Aliseda (2008), Viecens (2011), and Maruyama and Zenryo (2013), among others, addressed the issue of compat-

¹Compatibility among complementary components of system goods has been largely studied in the mix-and-match literature (Matutes and Regibeau, 1988, 1992; Economides, 1989; Economides and Salop, 1992). But, little has been reported on compatibility in two-sided markets.

ibility between platforms. Maruyama and Zenryo (2013) contributes to the literature by highlighting the equilibrium structure of compatibility changes over the product life cycle while the other papers focus on competition given the compatibility decisions. Recently, Hermalin and Katz (2013) analyze compatibility decisions endogenously by focusing on the effect of exclusive arrangements between platforms and application providers. These papers, however, do not treat the R&D investment by platforms.

The unique contribution of this chapter lies in examining the welfare effects of process innovation in a model of two-sided markets which endogenizes the determination of compatibility structure among platforms. To the best of our knowledge, this is the first one which explores the possibility of welfare-reducing process innovation in two-sided markets mediated by strategic compatibility decisions of platforms.

The rest of the chapter is organized as follows. The next section develops our basic framework and sets out the model. Section 3.3 presents the sub-game equilibrium. Section 3.4 presents some comparative static analyses. Section 3.5 presents the sub-game perfect equilibrium. Section 3.6 concludes this chapter.

3.3 Model

3.3.1 Platforms

We suppose that there are two platforms, $i = 1, 2$, each selling a differentiated hardware device i at a price p_i ($i = 1, 2$) and operating its marketplace i that distributes content for its own hardware device. Both platforms *ex ante* incur the same marginal cost c per unit of hardware device, and each

platform invests y_i to reduce marginal costs to $c - y_i$. We assume that the cost of the investment is given by ky_i^2 . The parameter k expresses the efficiency of cost-reducing investment where a decrease in k signifies the process innovation of R&D investment. There are two content providers, $i = 1, 2$, and they exclusively supply the content i to marketplace i at its price ρ_i . Each platform charges a content provider for a royalty rate r ($0 \leq r \leq 1$) for each unit of content sold at its marketplace. The unit of content has the same benefit for consumers, and we assume that the price of content is same for all content, $\rho_i = \rho$ ($i = 1, 2$). In this chapter, we treat the royalty rate r and the content price ρ as exogenous parameters.² Thus, the decisions by content providers are not considered in this model. However, we note in advance that the main result we obtain in this chapter is derived from the nature of two-sided markets. Each platform chooses whether to make its content compatible with the rival's hardware device (in what follows we use the term *application compatibility*). Application compatibility enables the users of other hardware devices to use content (application software) that it supplies in its marketplace, which will not only make the rival's hardware more attractive to consumers by wider availability of content, but also make it possible to gain more royalty revenue from a wider demand for the content the platform is selling. We denote by δ_i the following function.

$$\delta_i = \begin{cases} 0 & \text{if platform } i \text{ chooses incompatibility} \\ 1 & \text{if platform } i \text{ chooses compatibility.} \end{cases}$$

²There is a real-life example of a common fixed royalty rate r in the e-book industry (see, for example, Jiang, 2012; Knowledge@Wharton, 2012). Similarly, about the fixed content price, we can suggest that in the iTunes Store almost all music is sold for \$1.29 per track.

The profit function of platform i can be written as

$$\Pi_i = (p_i - c + y_i) \cdot D_i + r\rho \cdot D_i + \delta_i r\rho \cdot D_j - ky_i^2, \quad (i = 1, 2, j \neq i).$$

where D_i denotes the demand for hardware device i .

3.3.2 Consumers

We use the Hotelling model of product differentiation to analyze the consumers' platform choices. Each consumer's preferences for platforms are represented by his location x on a line. Ideal points of consumers are uniformly distributed on the unit interval $[0, 1]$. The hardware device i is located at $i - 1$. Each consumer incurs a constant proportional disutility t per unit length. Denote by v the benefit from hardware device, without loss of generality, which is normalized to zero. We assume that all consumers have the same utility B for any content, which satisfies the condition, $B > \rho$. Therefore, each consumer buys any usable content due to variety-seeking behavior. The utility function of a consumer who is located at x , and buys hardware device i and available content is given by

$$u_i = N_i(B - \rho) - p_i - t|x - (i - 1)|, \quad (i = 1, 2),$$

where N_i is the amount of available content for hardware device i ($N_i = 1, 2$).

We will use the notation, $b = B - \rho$.

In this chapter, we consider the cost-reducing investment that raises the possibility of tipping (corner solution). In the case of sufficiently low value of t , even without the cost-reducing investment, corner solutions are the only equilibria under asymmetric cases. Therefore, in order to exclude such cases, we make the following assumption (See Lemma 2.1):

Assumption 3.1. *The hardware of the two platforms is sufficiently differentiated that $t > \frac{b+r\rho}{3}$.*

3.3.3 Timing of the game

We consider the following non-cooperative three-stage game. In stage one, each platform chooses between application compatibility (C) and incompatibility (IC). In stage two, each platform decides the level of cost-reducing investment, y_i . In stage three, each platform sets the price of hardware device, p_i .

3.4 Sub-game equilibrium

Given the compatibility decisions in stage one, there are four possible cases as follows.

1. (IC, IC); both platforms choose incompatibility.
2. (C, C); both platforms choose compatibility.
3. (IC, C); platform 1 chooses incompatibility and platform 2 chooses compatibility.
4. (C, IC); platform 1 chooses compatibility and platform 2 chooses incompatibility.

In this section, we will derive the equilibrium prices, level of investment, demands, profits, consumer surplus, and social surplus under the various cases.

3.4.1 Incompatible platforms

In this case, only a single content is available for each hardware device. The utility function of a customer who is located at x can be written as

$$u_i = b - p_i - t|x - x_i|, \quad (i = 1, 2).$$

Let $\hat{x}$ be the location of a consumer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{t - p_1 + p_2}{2t}.$$

Hence, the demand for hardware device i is

$$D_i = \frac{t - p_i + p_j}{2t}, \quad (i = 1, 2, j \neq i).$$

Platform i maximizes its profit

$$\begin{aligned} \Pi_i &= (p_i - c + y_i) D_i + r\rho D_i - ky_i^2 \\ &= (p_i - c + y_i + r\rho) \cdot \frac{t - p_i + p_j}{2t} - ky_i^2, \quad (i = 1, 2, j \neq i) \end{aligned}$$

with respect to its hardware price p_i . Taking the first-order conditions and solving for prices, we have the prices as follows:

$$p_i(y_1, y_2) = t + c - r\rho - \frac{2y_i + y_j}{3}, \quad (i = 1, 2, j \neq i).$$

where the second-order conditions are satisfied. Substituting these prices into the profit function of each platform, we have

$$\Pi_i = \frac{(3t + y_i - y_j)^2}{18t} - ky_i^2, \quad (i = 1, 2, j \neq i).$$

Platform i maximizes its profit with respect to its investment level y_i in stage two. Taking the first-order conditions and solving for investment level, we have the equilibrium level of the cost-reducing investment as follows:

$$y_i(\text{IC}, \text{IC}) = \frac{1}{6k}.$$

We assume that the second-order condition is satisfied: $18kt - 1 > 0$. Using this equilibrium value, we can derive the equilibrium hardware price, demand, and profit of platform as follows:

$$\begin{aligned} p_i(\text{IC}, \text{IC}) &= t + c - r\rho - \frac{1}{6k}, \\ D_i(\text{IC}, \text{IC}) &= \frac{1}{2}, \\ \Pi_i(\text{IC}, \text{IC}) &= \frac{t}{2} - \frac{1}{36k}, \quad (i = 1, 2). \end{aligned}$$

The profit of content provider is given by

$$\begin{aligned} \pi_i(\text{IC}, \text{IC}) &= (1 - r)\rho \cdot (D_i(\text{IC}, \text{IC}) + \delta_i D_j(\text{IC}, \text{IC})) \\ &= \frac{(1 - r)\rho}{2}, \quad (i = 1, 2). \end{aligned}$$

Consumer surplus and social surplus are given by

$$\begin{aligned} CS(\text{IC}, \text{IC}) &= \int_0^{D_1(\text{IC}, \text{IC})} u_1(x) dx + \int_{D_1(\text{IC}, \text{IC})}^1 u_2(x) dx \\ &= \frac{1}{6k} + b - \frac{5}{4}t - c + r\rho, \\ SS(\text{IC}, \text{IC}) &= CS(\text{IC}, \text{IC}) + \sum_i \Pi_i(\text{IC}, \text{IC}) + \sum_i \pi_i(\text{IC}, \text{IC}) \\ &= \frac{1}{9k} + b - \frac{t}{4} - c + \rho. \end{aligned}$$

These equilibrium values are summarized in Table 3.1, which appears in the last of this chapter.

3.4.2 Compatible platforms

When both platforms choose compatibility, all content is usable on either hardware device. In this case, the utility function of a customer is

$$u_i = 2b - p_i - t|x - x_i|, \quad (i = 1, 2).$$

Let $\hat{x}$ be the location of a customer who is indifferent between the two hardware devices, and thus it equals the proportion of consumers who buy hardware 1:

$$u_1 = u_2 \Rightarrow \hat{x} = \frac{t - p_1 + p_2}{2t}.$$

Hence, the demand for hardware device i is

$$D_i = \frac{t - p_i + p_j}{2t}, \quad (i = 1, 2, j \neq i).$$

Platform i maximizes its profit

$$\begin{aligned} \Pi_i &= (p_i - c + y_i) D_i + r\rho - ky_i^2 \\ &= (p_i - c + y_i) \cdot \frac{t - p_i + p_j}{2t} + r\rho - ky_i^2, \end{aligned}$$

with respect to its hardware price p_i . Solving the game with backward induction, we can derive the equilibrium investment levels, prices, demands, profits of platform and content provider, consumer surplus, and social surplus, as in Table 3.1. Here, we assume that the second-order condition is satisfied: $18kt - 1 > 0$.

3.4.3 Incompatible-compatible platforms without tipping

When platform 1 chooses incompatibility and platform 2 chooses compatibility, the consumers who have hardware device 1 can use all content, but

the consumers who have hardware device 2 can use only content supplied by platform 2. In this case, we must consider the possibility of tipping (corner solution), where platform 1 that has the advantage of content variety monopolizes the hardware market. In this section, we consider the parameter space in which interior solutions are the equilibrium, and the case of corner solutions is analyzed in the next section. In the case of interior solutions, the utility functions of consumers are

$$\begin{cases} u_1 = 2b - p_1 - tx \\ u_2 = b - p_2 - t(1 - x). \end{cases}$$

From these we can derive the demands for hardware devices as follows:

$$D_1 = \frac{b + t - p_1 + p_2}{2t}, \quad D_2 = \frac{-b + t + p_1 - p_2}{2t}.$$

The profit functions of the platforms are

$$\begin{aligned} \Pi_1 &= (p_1 - c + y_1)D_1 + r\rho D_1 - ky_1^2 \\ &= (p_1 - c + y_1 + r\rho) \cdot \frac{b + t - p_1 + p_2}{2t} - ky_1^2, \\ \Pi_2 &= (p_2 - c + y_2)D_2 + r\rho - ky_2^2 \\ &= (p_2 - c + y_2) \cdot \frac{-b + t + p_1 - p_2}{2t} + r\rho - ky_2^2. \end{aligned}$$

Solving the game with backward induction, we can derive the equilibrium investment levels, prices, demands, profits of platform and content provider, consumer surplus, and social surplus as shown in Table 3.1. We assume that the second-order condition is satisfied: $18kt - 1 > 0$.

Here we can get the following lemma for the existence of interior solution.

Lemma 3.1. *If the efficiency of investment is low enough to satisfy the condition $k > 1/(3(3t-b-r\rho)) \equiv \hat{k}$ and the degree of hardware differentiation is large enough to satisfy the condition $t > (b+r\rho)/3$, then there exist interior solutions under the asymmetric cases.*

3.4.4 Incompatible-compatible platforms with tipping

In the previous section, we considered the case in which there exist interior solutions under asymmetric case. Here we focus on the case of tipping (corner solutions). The demand function of hardware device i can be written as

$$D_1 = \begin{cases} 1 & \text{if } p_1 \leq b - t + p_2 \\ \frac{b+t-p_1+p_2}{2t} & \text{if } b - t + p_2 \leq p_1 \leq b + t + p_2 \\ 0 & \text{if } b + t + p_2 \leq p_1, \end{cases}$$

$$D_2 = \begin{cases} 1 & \text{if } p_2 \leq -b - t + p_1 \\ \frac{-b+t+p_1-p_2}{2t} & \text{if } -b - t + p_1 \leq p_2 \leq -b + t + p_1 \\ 0 & \text{if } -b + t + p_1 \leq p_2. \end{cases}$$

The profit functions of platforms can be written as

$$\Pi_1 = \begin{cases} (p_1 - c + y_1 + r\rho) \cdot 1 - ky_1^2 & \text{if } p_1 \leq b - t + p_2 \\ (p_1 - c + y_1 + r\rho) \cdot \frac{b+t-p_1+p_2}{2t} - ky_1^2 & \text{if } b - t + p_2 \leq p_1 \leq b + t + p_2 \\ -ky_1^2 & \text{if } b + t + p_2 \leq p_1, \end{cases}$$

$$\Pi_2 = \begin{cases} (p_2 - c + y_2) \cdot 1 + r\rho - ky_2^2 & \text{if } p_2 \leq -b - t + p_1 \\ (p_2 - c + y_2) \cdot \frac{-b+t+p_1-p_2}{2t} + r\rho - ky_2^2 & \text{if } -b - t + p_1 \leq p_2 \leq -b + t + p_1 \\ +r\rho - ky_2^2 & \text{if } -b + t + p_1 \leq p_2. \end{cases}$$

From these profit functions, we can derive the following corner solutions.

Lemma 3.2. *If the efficiency of investment and the degree of hardware differentiation are large enough to satisfy the conditions $1/(8t) < k < 1/(2(3t - b - r\rho)) \equiv \bar{k}$ and $t > (b + r\rho)/3$, then there exist the following corner solutions under asymmetric case.*

$$\begin{aligned} y_1(\text{IC}, \text{C})^T &= \frac{1}{2k} \quad , \quad y_2(\text{IC}, \text{C})^T = 0, \\ p_1(\text{IC}, \text{C})^T &= c + b - t \quad , \quad p_2(\text{IC}, \text{C})^T = c, \end{aligned}$$

where suffix T denotes the case of tipping.

3.5 Comparative statics

We compare the equilibrium level of investment, prices, demands, profits, consumer surplus, and social surplus under four market structures.

3.5.1 Non-tipping case

We compare the equilibrium values in the parameter space where there exist the interior solutions under the asymmetric market structure. That is, we perform comparative statics in the parameter space derived in Lemma 3.1, $k > \hat{k}$ and $t > (b + r\rho)/3$. First, we assert the following proposition regarding the investment levels.

Proposition 3.1. *If the efficiency of investment is low enough to satisfy the condition $k > \hat{k}$ and the degree of hardware differentiation is large enough to satisfy the condition $t > (b + r\rho)/3$, then the equilibrium investment levels*

are ordered as follows:

$$y_1(C, IC)^{NT} = y_2(IC, C)^{NT} < y_i(IC, IC) = y_i(C, C) < y_1(IC, C)^{NT} = y_2(C, IC)^{NT}$$

where suffix *NT* denotes the case of non-tipping.

It is easy to see that both platforms choose the same investment level under symmetric market structures. This result depends on the nature of the Hotelling model. Under an asymmetric market structure, the platform having the advantage of software variety has more incentive to invest since its demand function shifts upward by more available content and the marginal revenue of investment is increased. We assume that the marginal cost is large enough to satisfy the condition $c > \max y_i$.³

Next, we have the following proposition by comparing the prices of hardware device summarized in Table 3.1.

Proposition 3.2. *If the efficiency of investment is low enough to satisfy the condition $k > \hat{k}$ and the degree of hardware differentiation and the benefit from a unit of content are large enough to satisfy the conditions $t > (b+r\rho)/3$ and $b > 3r\rho$, then the equilibrium prices are ordered as follows:*

$$p_1(C, IC)^{NT} = p_2(IC, C)^{NT} < p_i(IC, IC) < p_i(C, C) < p_1(IC, C)^{NT} = p_2(C, IC)^{NT}.$$

The order of the equilibrium prices is similar to that of the equilibrium investment levels. The only different thing is that the equilibrium price under compatible platforms (C, C) is higher than that under incompatible platforms (IC, IC). The logic of this result can be explained by the intuition of Proposition 1 in Maruyama and Zenryo (2013).

³Using the following results, it is assumed to satisfy the condition $c > 1/(2k)$.

Next, when we compare the equilibrium consumer surplus under the various market structures, we have the following result.

Proposition 3.3. *If the efficiency of investment is low enough to satisfy the condition $k > \hat{k}$ and the degree of hardware differentiation and the benefit from a unit of content are large enough to satisfy the conditions $t > (b+r\rho)/3$ and $b > 3r\rho$, then the equilibrium consumer surpluses are ordered as follows:*

$$CS(IC, IC) < CS(IC, C)^{NT} = CS(C, IC)^{NT} < CS(C, C).$$

Proposition 3.3 shows that as more platforms choose application compatibility, the higher is consumer surplus. The reason is that when application compatibility prevails, consumers can access more content.

Finally, we will compare the social surplus among the different market structures. We assert the following proposition.

Proposition 3.4. *If the efficiency of investment is low enough to satisfy the condition $k > \hat{k}$ and the degree of hardware differentiation and the benefit from a unit of content are large enough to satisfy the conditions $t > (b+r\rho)/3$ and $b > 3r\rho$, then the equilibrium social surpluses are ordered as follows:*

$$SS(IC, IC) < SS(IC, C)^{NT} = SS(C, IC)^{NT}.$$

In our Hotelling model, social surplus depends on three things: (i) the extent of available content, (ii) the effect of cost-reduction investment, and (iii) the consumers' trip costs (disutility). As for the first thing (i), the extent of available content increases as the number of compatible platform increases.

Thus, this beneficial effect on social surplus is the largest under compatible platforms (C, C) and the lowest under incompatible platforms (IC, IC). As for the second thing (ii), while the level of investment is the same for symmetric cases (IC, IC) and (C, C), it is beyond that level in the asymmetric cases (IC, C) and (C, IC). The platform that has chosen incompatibility has a greater incentive for aggressive investment, enough to offset the negative investment by the compatible platform. Thus the total effect of investment is larger under asymmetric cases than under symmetric cases. Finally, regarding the third thing (iii), consumers' trip costs are lower under the symmetric situations (IC, IC) and (C, C) than those under asymmetric case (IC, C), because the consumers' trip cost is the lowest when the total demand for hardware is evenly divided up between two platforms. Therefore, it is immediately seen that $SS(IC, IC) < SS(C, C)$. Furthermore, from Proposition 4 we find that $SS(IC, IC) < SS(IC, C)^{NT} = SS(C, IC)^{NT}$. However, the comparison of social welfare between the asymmetric case and compatible platforms (C, C) is ambiguous. While the case of compatible platforms (C, C) is the best in terms of social welfare in the absence of R&D investment, we can see that it does not hold when we consider the effect of cost-reducing investment.

3.5.2 Tipping case

Next, we compare the equilibrium values in the parameter space in which there exists tipping (corner solution) under asymmetric market structure, $(IC, C)^T$ or $(C, IC)^T$. That is, we perform comparative statics in the parameter space derived in Lemma 3.2, $k < \bar{k}$ and $t > (b + r\rho)/3$. First, we assert the following proposition regarding the investment levels.

Proposition 3.5. *If the efficiency of investment and the degree of hardware differentiation are large enough to satisfy the conditions $1/(8t) < k < \bar{k}$ and $t > (b + r\rho)/3$, then the equilibrium investment levels are ordered as follows:*

$$y_1(C, IC)^T = y_2(IC, C)^T < y_i(IC, IC) = y_i(C, C) < y_1(IC, C)^T = y_2(C, IC)^T.$$

It is obvious from the results shown in Table 3.1. We can see that when tipping occurs, the monopoly platform in the hardware market (say platform 1 in case of (IC, C)) makes the largest amount of investment and the dominated platform (say platform 2 in such a case) makes the least investment in any case.

Next, when we compare the equilibrium social surplus under the various market structures, we have the following proposition.

Proposition 3.6. *If the efficiency of investment is large enough to satisfy the condition $1/(8t) < k < \bar{k}$ and the degree of hardware differentiation is at an intermediate level that satisfies the condition $(b + r\rho)/3 < t < (b + r\rho)/2$, then the equilibrium social surpluses are ordered as follows:*

$$SS(IC, IC) < SS(IC, C)^T = SS(C, IC)^T.$$

The social surplus under incompatible platforms (IC, IC) is smaller than that under the asymmetric cases, (IC, C)^T and (C, IC)^T. This result is similar to Proposition 3.4 in the case of non-tipping.

3.6 Subgame-perfect equilibrium

In section 3.4, we examined the subgame equilibrium in stage two and stage three. We use these results to solve the three-stage game with backward induction. To determine the equilibrium, we compare the equilibrium profits as shown in Table 3.1.

We analyze the best-response strategy for compatibility decisions in the parameter space in which there exists an interior solution under the asymmetric market structure. We have the following two lemmas.

Lemma 3.3. *If the degree of hardware differentiation is at an intermediate level that satisfies the condition $(b + r\rho)/3 < t < (b + r\rho)/2$ and the benefit from a unit of content is small enough to satisfy the condition $b < 5r\rho$, then it holds that $\Pi_1(C, IC)^{NT} > \Pi_1(IC, IC)$ for all $k > \hat{k}$.*

The intuition for this result is the following. When the degree of hardware differentiation is relatively small, due to the stiff competition in hardware prices, the revenue for platforms from selling hardware devices is also small. Therefore, even if compatibility is chosen, the reduction of the revenue from hardware devices by choosing compatibility is small. In addition, since the royalty revenue is relatively large in this case, it is beneficial for platforms to choose compatibility and gain royalty revenue from consumers who have the rival's hardware device. In such a situation, compatibility is profitable, given that the rival platform chooses incompatibility.

Lemma 3.4. *When both the degree of hardware differentiation and the benefit of content are large enough to satisfy the conditions $t > \frac{b+r\rho}{3}$ and $b > 2r\rho$, then it holds that $\Pi_1(C, C) < \Pi_1(IC, C)^{NT}$ for all $k > \hat{k}$.*

The intuition for this result is the following. Since the royalty revenue is relative small in this case, it is beneficial for the platform to increase the variety of available content by choosing incompatibility.

From Lemma 3.3 and Lemma 3.4, we can derive the following proposition.

Proposition 3.7. *If both the degree of hardware differentiation and the benefit of content are at the intermediate levels that satisfy the conditions $\frac{b+r\rho}{3} < t < \frac{b+r\rho}{2}$ and $2r\rho < b < 5r\rho$, then for all $k > \hat{k}$, the equilibrium market structures are the asymmetric ones without tipping, $(IC, C)^{NT}$ and $(C, IC)^{NT}$.*

It is obvious from Lemma 3.3 and Lemma 3.4.

In Proposition 3.7, we suppose that t is small enough to satisfy the condition $t < (b+r\rho)/2$. When the degree of product differentiation is relatively large, it is possible that symmetric cases (IC, IC) and (C, C) can be equilibria as shown in Chapter 2. In this chapter, we focus on the parameter space in which process innovation has a negative impact on social welfare through the change of compatibility strategies from (IC, C) to (IC, IC) . Therefore, this chapter does not treat the parameter range in which symmetric compatibility decisions can be equilibria even when the process innovation does not occur (k is relatively large).

We can explain why the equilibrium is not symmetric under these conditions even in the symmetric setting. Suppose that the rival chooses incompatibility and the degree of hardware differentiation is not very large. Then, choosing incompatibility leads to a price competition in hardware devices, which reduces the profit from selling hardware devices. Hence, when the royalty from content is large, the platform has an incentive to choose

compatibility and gain royalty revenue from consumers who have the rival's hardware device. On the other hand, if the rival chooses compatibility, then by choosing incompatibility the platform gains the advantage of available content and gets more profit from selling hardware devices. In sum, as long as the efficiency of investment is relatively low, the equilibrium is asymmetric, which is exactly the same as in the absence of R&D investment (See Proposition 2.6).

Next, we analyze the best-response strategy about compatibility decisions in the parameter space in which there exists tipping (corner solution) under asymmetric market structure. We have the following two lemmas.

Lemma 3.5. *If both the degree of hardware differentiation and the benefit of content are large enough to satisfy the conditions $t > (b + r\rho)/3$ and $b > r\rho$, then it holds that $\Pi_1(\text{IC}, \text{C})^T > \Pi_1(\text{C}, \text{C})$ for all $1/(8t) < k < \bar{k}$.*

In the same logic of Lemma 3.4, choosing incompatibility is profitable for the platform when the rival chooses compatibility. If the royalty revenue is relatively small, then the platform has less incentive to choose compatibility in the pursuit of royalty revenue.

Lemma 3.6. *If the degree of hardware differentiation is large enough to satisfy the condition $t > (b + r\rho)/3$, then the following inequality holds.*

$$\begin{cases} \Pi_1(\text{IC}, \text{IC}) > \Pi_1(\text{C}, \text{IC})^T & \text{if } t > 2r\rho \text{ and } \tilde{k} < k < \bar{k}, \\ \Pi_1(\text{IC}, \text{IC}) < \Pi_1(\text{C}, \text{IC})^T & \text{if } (t > 2r\rho \text{ and } 1/(8t) < k < \min\{\bar{k}, \tilde{k}\}) \\ & \text{or } (t < 2r\rho \text{ and } 1/(8t) < k < \bar{k}), \end{cases}$$

where $\tilde{k} \equiv 1/(18(t - 2r\rho))$.

Suppose that the parameter k is relatively large in the range of the parameter space that gives the possibility of tipping (corner solution) as equilibrium under asymmetric market structures. In this situation, given that a rival platform chooses incompatibility, choosing compatibility by a platform makes the rival's hardware device more attractive in the variety of content and the market for hardware devices will be monopolized by the rival platform. Then it is the best-response strategy for a platform to choose incompatibility when a rival platform chooses incompatibility. However, when the parameter k is especially small and both platforms choose incompatibility, they have excessive incentive for cost-reducing investment in hardware devices. Then, through fierce investment competition neither platform gains much revenue from selling hardware devices. Thus it is the best-response strategy for a platform to choose compatibility and support a wider demand for the content the platform is selling.

From Lemmas 3.5 and Lemma 3.6, we can derive the following proposition.

Proposition 3.8. *If both the degree of hardware differentiation and the benefit from content are large enough to satisfy the conditions $t > (b + r\rho)/3$ and $b > r\rho$, then the equilibrium is given by*

$$\left\{ \begin{array}{ll} (\text{IC}, \text{IC}) & \text{if } t > 2r\rho \text{ and } \tilde{k} < k < \bar{k}, \\ (\text{IC}, \text{C})^T \text{ or } (\text{C}, \text{IC})^T & \text{if } (t > 2r\rho \text{ and } 1/(8t) < k < \min\{\bar{k}, \tilde{k}\}) \\ & \text{or } (t < 2r\rho \text{ and } 1/(8t) < k < \bar{k}). \end{array} \right.$$

It is obvious from Lemma 3.5 and Lemma 3.6.

When the parameter t is relatively small (the degree of hardware differentiation is not very large), by using a similar argument with Proposition 3.7,

we can see that the equilibrium is asymmetric. However, if both k and t are not so small, then each platform has an incentive to choose incompatibility in stage one and prevent a rival platform from monopolizing the hardware market.

From Propositions 3.7 and 3.8, the following Corollary 3.1 gives the partition of parameter space when the royalty from a unit of content is relatively large.

Corollary 3.1. *Suppose that $(b + r\rho)/3 < t < (b + r\rho)/2$. When the royalty revenue from a unit of content is large enough to satisfy the condition $2b/7 < r\rho < b/2$, we can derive the partition of equilibrium in the parameter space as shown in Figure 3.1. The equilibrium market structure is either: (A) $(IC, C)^{NT}$ and $(C, IC)^{NT}$; (B) $(IC, C)^{NT}$, $(C, IC)^{NT}$, $(IC, C)^T$, and $(C, IC)^T$; or (C) $(IC, C)^T$ and $(C, IC)^T$.*

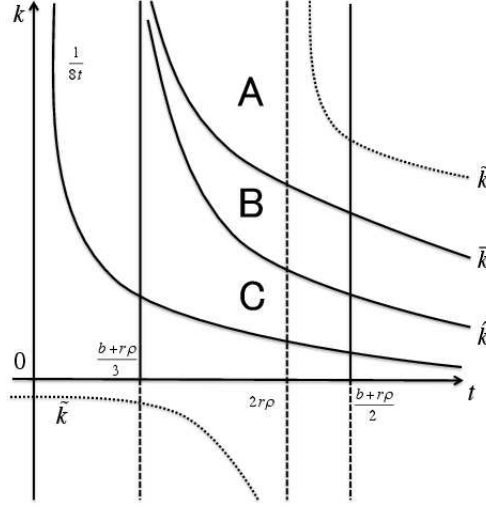


Figure 3.1: $\frac{2}{7}b < r\rho < \frac{b}{2}$

Suppose that the parameter t (the degree of hardware product differentiation) is relatively small. In this situation, if a rival platform chooses compatibility, then a platform has an incentive to choose incompatibility for wider availability of content. If a rival chooses incompatibility, then a platform has an incentive to choose compatibility since choosing incompatibility leads to fierce investment competition. Then, each platform chooses the opposite strategy from the rival's one relating to the compatibility decisions. In addition, when the parameter k is relatively large (i.e., the efficiency of R&D investment is relatively low), the equilibrium is the asymmetric one with non-tipping, $(IC, C)^{NT}$ and $(C, IC)^{NT}$. Here, under the asymmetric market structure, the platform that chooses incompatibility has a relatively larger demand for its hardware. Then, it has more incentive to invest in its hardware device than does the rival platform.

As the parameter k decreases (i.e., the efficiency of R&D investment increases), it makes easier for the platform that chooses incompatibility to monopolize the hardware market. When the royalty revenue from a unit of content is relatively large, the platform that chooses compatibility and has no hardware demand can get the relatively large royalty revenue from all consumers. Therefore, then the equilibrium is the asymmetric one with tipping, $(IC, C)^T$ and $(C, IC)^T$.

Corollary 3.1 also shows that, in the parameter range $\hat{k} < k < \bar{k}$, there exist multiple equilibria, $(IC, C)^{NT}$ and $(IC, C)^T$. In this range, the equilibrium path depends on what belief each platform holds.⁴ If each platform believes that the rival plays strategies $(y_i(IC, C)^{NT}, p_i(IC, C)^{NT})$ after platform 1

⁴We assume that two platforms have symmetric belief about rival's strategies each other.

chooses incompatibility and platform 2 chooses compatibility in stage one, then the equilibrium outcome becomes $(IC, C)^{NT}$. Similarly, if each platform believes that the rival plays strategies $(y_i(IC, C)^T, p_i(IC, C)^T)$, then $(IC, C)^T$ becomes an equilibrium.

Finally, when the royalty from a unit of content is relatively small, we can derive the following corollary, which gives the partition of the equilibrium.

Corollary 3.2. *Suppose that $(b + r\rho)/3 < t < (b + r\rho)/2$. When the royalty revenue from a unit of content is small enough to satisfy the condition $b/5 < r\rho < 2b/7$, we can derive the partition of equilibrium in the parameter space as shown in Figures 3.2 and 3.3. The equilibrium market structure is either: (A) $(IC, C)^{NT}$ and $(C, IC)^{NT}$; (B) $(IC, C)^{NT}$, $(C, IC)^{NT}$, $(IC, C)^T$, and $(C, IC)^T$; (C) $(IC, C)^T$ and $(C, IC)^T$; (D) (IC, IC) ; or (E) $(IC, C)^T$, $(C, IC)^T$, and (IC, IC) .*

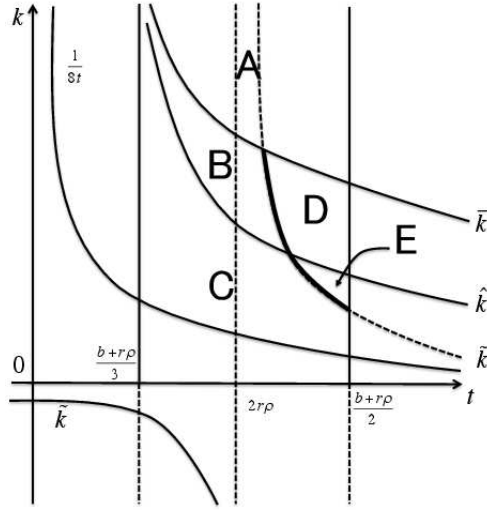


Figure 3.2: $\frac{b}{5} < r\rho < \frac{5}{19}b$

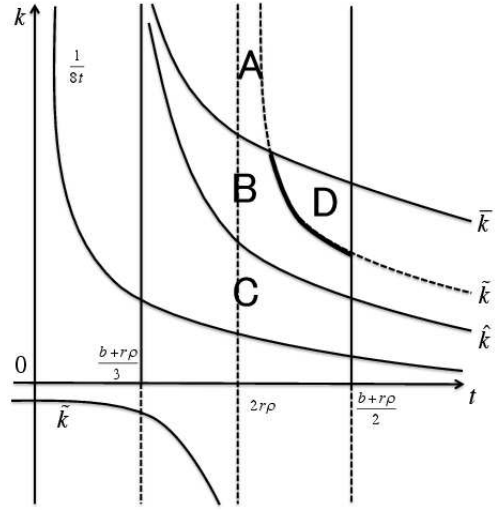


Figure 3.3: $\frac{5}{19}b < r\rho < \frac{2}{7}b$

When royalty revenue from a unit of content is smaller, the merit of

choosing compatibility and supporting a wider demand for its content is not so large. Hence, it is more likely to deviate from compatibility to incompatibility. In this situation, as shown in Figures 3.2 and 3.3, there exists a parameter range (D and E) in which (IC, IC) is the equilibrium compatibility decisions.

We can see that both (IC, IC) and (IC, C)^{NT} become equilibrium outcomes in some parameter spaces (e.g., the range (D) in Figure 3.2 and 3.3). But this multiple equilibria does not indicate that platform 2 is indifferent between choosing incompatibility and compatibility in stage one. Suppose that platform 1 only chooses incompatibility in stage one. When each platform believes that the rival plays the strategies $(y_i(\text{IC}, \text{C})^{\text{NT}}, p_i(\text{IC}, \text{C})^{\text{NT}})$, then the equilibrium outcome becomes (IC, C)^{NT}. On the other hand, when each platform believes that the rival plays the strategies $(y_i(\text{IC}, \text{C})^{\text{T}}, p_i(\text{IC}, \text{C})^{\text{T}})$, platform 2 has an incentive to deviate from compatibility to incompatibility in stage one to avoid being dominated in the hardware device market. Therefore, the equilibrium outcome becomes (IC, IC).

3.7 Discussion and conclusion

From Corollary 3.2 and Proposition 3.4, we can indicate the following paradoxical result. The increased efficiency of cost-reducing investment (process innovation) affects the compatibility decisions by platform. Hence, it leads to the change of market structure, which has the negative impact on social welfare.

In sum, the process innovation has a positive direct effect on social welfare. But it may also lead to an equilibrium with inefficient market structure,

(IC, IC) by affecting the compatibility decisions. Suppose that the efficiency of investment is increased through process innovation, which is shown as the reduction of parameter k_1 to k_2 , and that each platform believes that rival plays the strategies $(y_i(\text{IC}, C)^T, p_i(\text{IC}, C)^T)$. Then social welfare increases from W_A to W_B by a positive direct effect under the market structure $(\text{IC}, C)^{NT}$. However, social welfare decreases, from W_B to W_C , by a negative indirect effect through the change of market structure to (IC, IC) . If the positive direct effect exceeds the negative indirect one as shown in Figure 3.4, the process innovation increases social welfare. However, if this negative indirect effect exceeds the positive direct one as shown in Figure 3.5, the process innovation reduces social welfare.

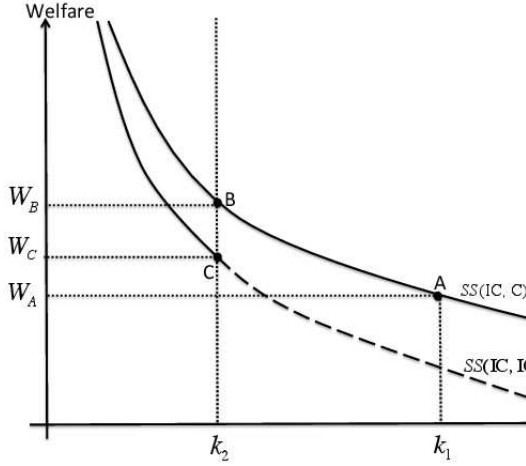


Figure 3.4:

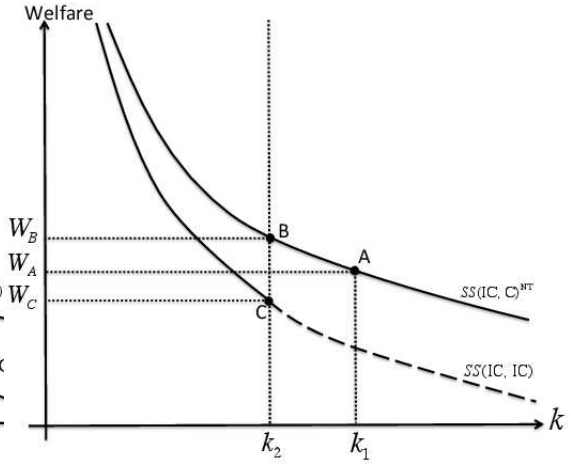


Figure 3.5:

In this chapter, we considered the effect of process innovation (the increased efficiency of cost-reducing investment) on social welfare in a two-sided market model which endogenizes the determination of compatibility structure between platforms. While process innovation directly confers socially

benefits, we have shown that it might nevertheless reduce social welfare by inducing change of market structure. Therefore, subsidies for cost-reducing investment may reduce social welfare. Indeed, attaining a first-best might actually require taxing investment, to prevent the platforms from choosing inefficient market structures.

There is a limitation in assuming that the content price is exogenously given. But even if we apply the model of Church and Gandal (2000) and allow the determination of content price ρ_{ij} by the content provider, our main insights from the baseline model also hold. The intuition behind this is as follows. Suppose that the market structure is asymmetric in which one platform selects compatibility (C) with the other's content, while the other platform selects incompatibility (IC). It can be easily seen that the price of content supplied for the rival's hardware is decreased by price competition between content providers. As a result, since royalties from the sale of content will be decreased, the platform which selected compatibility has a heightened incentive to deviate to select incompatibility. Therefore, when we relax the assumption that content price is exogenously given, process innovation has a greater potential of leading to an inefficient market structure (IC, IC) and having a negative impact on social welfare. However, our model ignores some of the main features in two-sided markets, such as the coordination problem between the two sides of the market (Caillaud and Jullien, 2003), and the platforms' contracting possibilities (Armstrong, 2006; Weyl, 2010), which may affect the results. The more complete analysis of the determination of content prices and royalties remains for future work.

3.8 Appendix

1. *Proof of Lemma 3.1.*

$$\begin{aligned}
 & 0 < D_i < 1 \quad (i = 1, 2) \\
 \Leftrightarrow & \frac{3k(b + r\rho)}{2(9kt - 1)} < \frac{1}{2} \\
 \Leftrightarrow & k > \frac{1}{3(3t - b - r\rho)} \equiv \hat{k} > 0
 \end{aligned}$$

□

2. *Proof of Lemma 3.2.* The platform 1 chooses the price that leads to the tipping by its own ($D_1 = 1$) when the profit function can be drawn as shown in Figure 3.6.

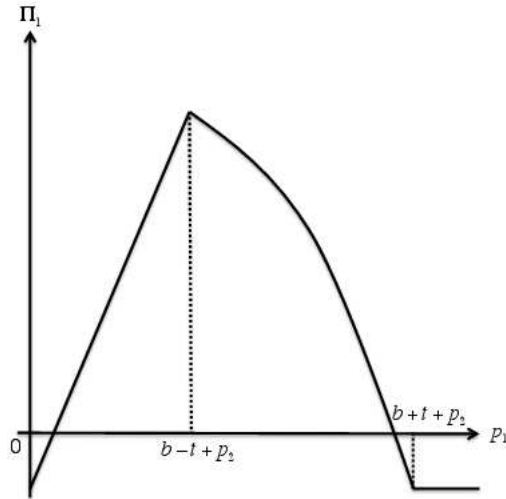


Figure 3.6: Platform 1's profit function

The condition that tipping is a best-response strategy for platform 1 is

given by:

$$\begin{aligned} \lim_{p_1 \rightarrow (b-t+p_2)+0} \frac{\partial \Pi_1}{\partial p_1} &= 1 - \frac{b-t+p_2-c+y_1+r\rho}{2t} \leq 0, \\ \iff y_1 &\geq 3t-b-r\rho+c-p_2. \end{aligned} \quad (3.1)$$

Then the best response function of platform1 can be written as

$$BR_1(p_2) = b - t + p_2.$$

Similarly, the platform 2 accepts the price that leads to tipping by rival platform ($D_1 = 1$) when the profit function of platform 2 is shown in Figure 3.7.

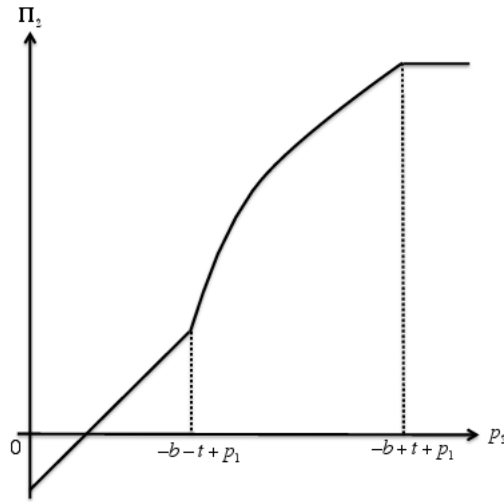


Figure 3.7: Platform 2's profit function

The condition that being tipped is a best-response strategy for platform 2 is given by:

$$\lim_{p_2 \rightarrow (-b+t+p_1)-0} \frac{\partial \Pi_2}{\partial p_2} = -\frac{-b+t+p_1-c+y_2}{2t} \geq 0,$$

$$\Longleftrightarrow y_2 \leq b - t + c - p_1. \quad (3.2)$$

Then the best response function of platform 2 can be written as

$$BR_2(p_1) = \{p_2 | p_2 \geq -b + t + p_1\}.$$

Here, when there exists an equilibrium with tipping, the profit of platform 2 can be written as $\Pi_2 = r\rho - ky_2^2$. Therefore, we can immediately get the following investment level of platform 2:

$$y_2(\text{IC}, C)^T = 0.$$

From this investment level, we can rewrite the condition (3.2) as follows:

$$p_1 \leq b - t + c. \quad (3.3)$$

Therefore we have the line $p_2 = -b + t + p_1$ ($p_1 \leq b - t + c$) as the set of common point of both platforms' best response functions. So we cannot derive the unique equilibrium.

However, using the *trembling hand perfect equilibrium* as equilibrium concept, we have the following unique equilibrium:

$$p_1 = b - t + c, \quad p_2 = c.$$

Because, if platform 2 set the price below marginal cost, $p_2 < c$, and platform 1 accidentally set the slightly higher price than the best response one, then consumer who locates on $x \in [1 - \epsilon, 1]$ buys hardware device 2 and platform 2 incurs some losses from the revenue of hardware device.

The profit of platform 1 can be written as $\Pi_1 = b - t + y_1 + r\rho - ky_1^2$.

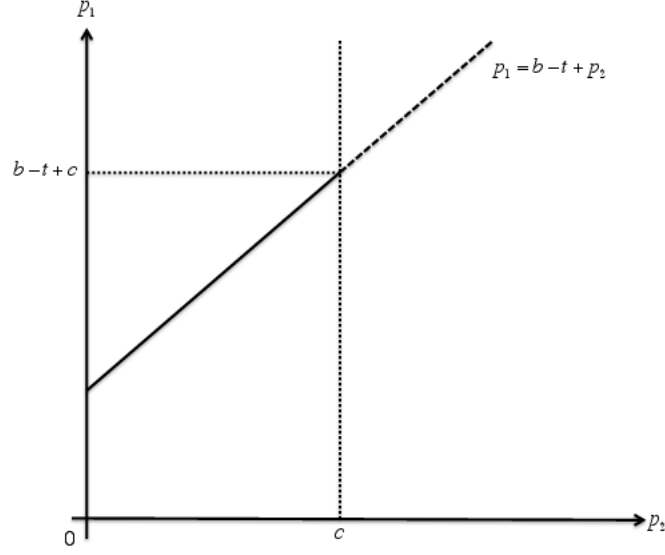


Figure 3.8: Firms' best response functions

Taking the first-order conditions and solve for the investment level, we can derive the following investment level:

$$\frac{\partial \Pi}{\partial y_1} = 1 - 2by_1 = 0 \iff y_1(\text{IC}, C)^T = \frac{1}{2k}.$$

Using this, we can get the equilibrium prices, demands, profits of platform and content provider, consumer surplus, and social surplus as shown in Table 3.1.

There exists a corner solution when $p_1(\text{IC}, C)^T$, $p_2(\text{IC}, C)^T$, $y_1(\text{IC}, C)^T$, and $y_2(\text{IC}, C)^T$ satisfy the equation (3.1) and (3.3). We can check that the equation (3.3) is obviously satisfied. So we can derive the condition for the existence of this corner solution by substituting the equilibrium value $(y_1(\text{IC}, C)^T = 1/(2k)$ and $p_2(\text{IC}, C)^T = c$) into equation (3.1):

$y_1(\text{IC}, \text{C})^T > 3t - b - r\rho + c - p_2(\text{IC}, \text{C})^T$. This implies that

$$\frac{1}{2k} > 3t - b - r\rho \iff k < \frac{1}{2(3t - b - r\rho)} \equiv \bar{k}.$$

Finally, given that platform 1 chooses $y_1(\text{IC}, \text{C})^T = \frac{1}{2k}$ and $p_1(\text{IC}, \text{C})^T = c - t + b$, we need to confirm that platform 2 has no incentive to deviate from the equilibrium strategies, $y_2(\text{IC}, \text{C})^T = 0$ and $p_2(\text{IC}, \text{C})^T = c$. Suppose that platform 2 chooses different investment level, $\hat{y}_2 > 0$. Equation (3.2) does not hold. That is, platform 2 can get a strictly positive hardware demand. We can rewrite platform 2's profit function as follows.

$$\begin{aligned} \Pi_2 &= (p_2 - c + \hat{y}_2) \cdot \frac{-b + t + p_1(\text{IC}, \text{C})^T - p_2}{2t} + r\rho - k\hat{y}_2^2 \\ &= (p_2 - c + \hat{y}_2) \cdot \frac{c - p_2}{2t} + r\rho - k\hat{y}_2^2 \end{aligned}$$

Taking the first-order condition for profit maximization with respect to price and solving for price yield

$$\frac{\partial \Pi_2}{\partial p_2} = 0 \iff p_2 = c - \frac{\hat{y}_2}{2}.$$

Using this price, we can derive platform 2's profit, $\frac{\hat{y}_2^2}{8t} + r\rho - k\hat{y}_2^2$. Differentiating this profit with respect to $\hat{y}_2$ yields

$$\begin{aligned} \frac{\partial \Pi_2}{\partial \hat{y}_2} &= \frac{\hat{y}_2}{4t} - 2k\hat{y}_2, \\ &= \left(\frac{1}{4t} - 2k \right) \hat{y}_2. \end{aligned}$$

When it holds that $k > \frac{1}{8t}$, it is easy to show that $\frac{\partial \Pi_2}{\partial \hat{y}_2} < 0$ and thus platform 2 has no incentive to deviate from the equilibrium strategies.

□

3. Proof of Proposition 3.1

First,

$$\begin{aligned}
y_1(\text{IC}, \text{C})^{\text{NT}} - y_i(\text{C}, \text{C}) &= \frac{3k(3t + b + r\rho) - 1}{6k(9kt - 1)} - \frac{1}{6k} \\
&= \frac{b + r\rho}{2(9kt - 1)} > 0, \\
\iff y_1(\text{IC}, \text{C})^{\text{NT}} &> y_i(\text{C}, \text{C}).
\end{aligned}$$

Next,

$$\begin{aligned}
y_i(\text{IC}, \text{IC}) - y_2(\text{IC}, \text{C})^{\text{NT}} &= \frac{1}{6k} - \frac{3k(3t - b - r\rho) - 1}{6k(9kt - 1)} \\
&= \frac{3k(b + r\rho)}{2(9kt - 1)} > 0, \\
\iff y_i(\text{C}, \text{C}) &> y_2(\text{IC}, \text{C})^{\text{NT}}.
\end{aligned}$$

□

4. Proof of Proposition 3.2

First, it is obvious that $p_i(\text{IC}, \text{IC}) < p_i(\text{C}, \text{C})$. Next,

$$\begin{aligned}
p_1(\text{IC}, \text{C})^{\text{NT}} - p_i(\text{C}, \text{C}) &= p_i(\text{IC}, \text{IC}) - p_2(\text{IC}, \text{C})^{\text{NT}} \\
&= \frac{b}{3} - \frac{2}{3}r\rho - \frac{b + r\rho}{6(9kt - 1)} \\
&= \frac{6kt(b - 2r\rho) - (b - r\rho)}{2(9kt - 1)} \\
&> \frac{6 \cdot \frac{1}{3t} \cdot t(b - 2r\rho) - (b - r\rho)}{2(9kt - 1)} \quad (\text{if } k > \hat{k} > \frac{1}{3t}) \\
&= \frac{b - 3r\rho}{2(9kt - 1)} > 0 \quad (\text{if } b > 3r\rho), \\
\iff p_1(\text{IC}, \text{C})^{\text{NT}} &> p_i(\text{C}, \text{C}) \quad , \quad p_i(\text{IC}, \text{IC}) > p_2(\text{IC}, \text{C})^{\text{NT}}.
\end{aligned}$$

□

5. Proof of Proposition 3.3

We can express consumer surplus as follows:

$$\begin{aligned}
 CS(IC, C)^{NT} &= \int_0^{D_1(IC, C)^{NT}} (2b - p_1(IC, C)^{NT} - tx) dx \\
 &\quad + \int_{D_1(IC, C)^{NT}}^1 (b - p_2(IC, C)^{NT} - t(1 - x)) dx, \\
 CS(IC, IC) &= \int_0^{\frac{1}{2}} (b - p_1(IC, IC) - tx) dx + \int_{\frac{1}{2}}^1 (b - p_2(IC, IC) - t(1 - x)) dx.
 \end{aligned}$$

From Proposition 3.2, we can see that $b - p_2(IC, C)^{NT} - t(1 - x) > b - p_2(IC, IC) - t(1 - x)$. So it is easy to derive that $CS(IC, C)^{NT} > CS(IC, IC)$ as shown in Figure 3.9.

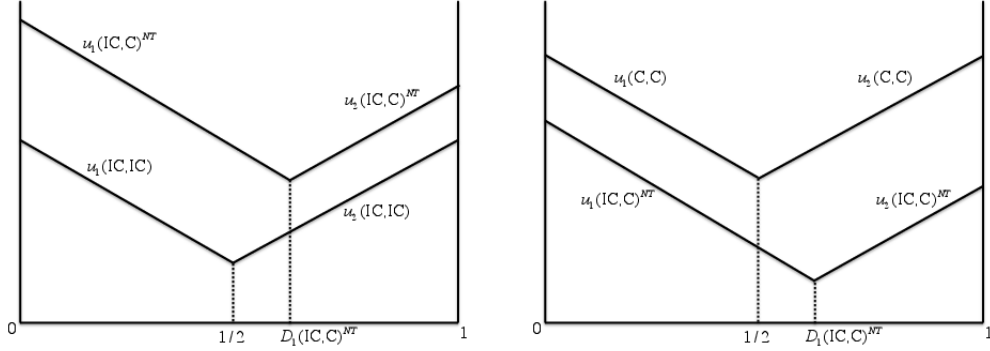


Figure 3.9: $CS(IC, C)^{NT}$ and Figure 3.10: $CS(IC, C)^{NT}$ and $CS(C, C)$

Next,

$$\begin{aligned}
 CS(C, C) &= \int_0^{\frac{1}{2}} (2b - p_1(C, C) - tx) dx + \int_{\frac{1}{2}}^1 (2b - p_2(C, C) - t(1 - x)) dx, \\
 CS(IC, C)^{NT} &= \int_0^{D_i(IC, C)^{NT}} (2b - p_1(IC, C)^{NT} - tx) dx \\
 &\quad + \int_{D_i(IC, C)^{NT}}^1 (b - p_2(IC, C)^{NT} - t(1 - x)) dx.
 \end{aligned}$$

Here we can confirm the following two inequalities:

$$\begin{aligned} \left(2b - p_2(C, C) - t(1 - x)\right) &- \left(b - p_2(IC, C)^{NT} - t(1 - x)\right) \\ &= \frac{2(2b - r\rho)}{3} + \frac{b + r\rho}{6(9kt - 1)} > 0, \end{aligned}$$

and

$$\begin{aligned} \left(2b - p_1(C, C) - tx\right) &- \left(2b - p_1(IC, C)^{NT} - tx\right) \\ &= p_1(IC, C)^{NT} - p_1(C, C) > 0, \quad (\text{Proposition 3.2}). \end{aligned}$$

Therefore, it is easy to see that $CS(C, C) > CS(IC, C)^{NT}$ as shown in Figure 3.10. Then, we can derive that $CS(IC, IC) < CS(IC, C)^{NT} = CS(C, IC)^{NT} < CS(C, C)$. $\square$

6. Proof of Proposition 3.4

First of all, it is obvious that $CS(IC, IC) < CS(IC, C)^{NT} = CS(C, IC)^{NT}$ from Proposition 3.3. Next,

$$\begin{aligned} &\sum_i \Pi_i(IC, C)^{NT} - \sum_i \Pi_i(IC, IC) \\ &= \left(\frac{t}{2} - \frac{1}{36k}\right) \left[\left\{1 + \frac{3k(b + r\rho)}{9kt - 1}\right\}^2 + \left\{1 - \frac{3k(b + r\rho)}{9kt - 1}\right\}^2 - 2 \right] + r\rho \\ &= 2\left(\frac{t}{2} - \frac{1}{36k}\right) \left\{ \frac{3k(b + r\rho)}{9kt - 1} \right\}^2 + r\rho > 0, \\ &\iff \sum_i \Pi_i(IC, C)^{NT} > \sum_i \Pi_i(IC, IC), \end{aligned}$$

and

$$\begin{aligned} &\sum_i \pi_i(IC, C)^{NT} - \sum_i \pi_i(IC, IC) \\ &= \left[(1 - r)\rho \cdot D_1(IC, C)^{NT} + (1 - r)\rho \right] - \left[\frac{(1 - r)\rho}{2} + \frac{(1 - r)\rho}{2} \right] \end{aligned}$$

$$\begin{aligned}
&= (1-r)\rho \cdot \left[\frac{3k(b+r\rho)}{2(9kt-1)} + \frac{1}{2} \right] > 0, \\
&\iff \sum_i \pi_i(\text{IC}, \text{C})^{\text{NT}} > \sum_i \pi_i(\text{IC}, \text{IC}).
\end{aligned}$$

Therefore, social surplus is expressed as $SS = CS + \sum_i \Pi_i + \sum_i \pi_i$, we can show that $SS(\text{IC}, \text{IC}) < SS(\text{IC}, \text{C})^{\text{NT}} = SS(\text{C}, \text{IC})^{\text{NT}}$. $\square$

7. Proof of Proposition 3.6

From Table 3.1,

$$\begin{aligned}
SS(\text{IC}, \text{C})^T - SS(\text{IC}, \text{IC}) &= \left(2(b+\rho) + \frac{1}{4k} - \frac{t}{2} - c \right) - \left(b + \rho + \frac{1}{9k} - \frac{t}{4} - c \right) \\
&= b + \rho + \frac{1}{4k} - \frac{1}{9k} - \frac{t}{2} + \frac{t}{4} \\
&= b + \rho + \frac{5}{36k} - \frac{t}{4} \\
&> b + \rho + \frac{5}{36k} - \frac{1}{4} \cdot \frac{b+r\rho}{2} \quad (t < \frac{b+r\rho}{2}) \\
&> b + \rho + \frac{5}{36k} - \frac{1}{4} \cdot \frac{b+\rho}{2} \quad (r < 1) \\
&= \frac{7}{8}(b+\rho) + \frac{5}{36k} > 0.
\end{aligned}$$

$\square$

8. Proof of Lemma 3.3

$$\begin{aligned}
&\Pi_1(\text{C}, \text{IC})^{\text{NT}} - \Pi_1(\text{IC}, \text{IC}) \\
&= \left(\frac{t}{2} - \frac{1}{36k} \right) \left(\frac{3k(3t-b-r\rho)-1}{9kt-1} \right)^2 + r\rho - \left(\frac{t}{2} - \frac{1}{36k} \right) \\
&= r\rho - \frac{18kt-1}{12} \cdot \frac{(b+r\rho)\{2(9kt-1)-3k(b+r\rho)\}}{(9kt-1)^2}.
\end{aligned}$$

Here, we define Δ_- as

$$\Delta_- \equiv \frac{18kt-1}{12} \cdot \frac{(b+r\rho)\{2(9kt-1)-3k(b+r\rho)\}}{(9kt-1)^2}.$$

Therefore,

$$\begin{aligned}
\frac{\partial \Delta_-}{\partial k} &= -\frac{b+r\rho}{4(9kt-1)^3} \{27kt(2t-b-r\rho) - (6t-b-r\rho)\} > 0 \quad (\text{if } t < \frac{b+r\rho}{2}), \\
\lim_{k \rightarrow \infty} \Delta_- &= \lim_{k \rightarrow \infty} \frac{(b+r\rho)(18t - \frac{1}{k}) \{2(9t - \frac{1}{k}) - 3(b+r\rho)\}}{12(9t - \frac{1}{k})^2} \\
&= \frac{(b+r\rho)(6t-b-r\rho)}{18t} \equiv \max_k \Delta_-.
\end{aligned}$$

In addition,

$$\begin{aligned}
r\rho - \max_k \Delta_- &= \frac{18tr\rho - (b+r\rho)(6t-b-r\rho)}{18t} \\
&= \frac{1}{18t} \left((b+r\rho)^2 - 6t(b-3r\rho) \right) \\
&> \frac{1}{18t} \left((b+r\rho)^2 - 6 \cdot \frac{b+r\rho}{2} \cdot (b-3r\rho) \right) \quad (\text{if } t < \frac{b+r\rho}{2}) \\
&= \frac{b+r\rho}{18t} \left((b+r\rho) - 3(b-3r\rho) \right) \\
&= \frac{b+r\rho}{18t} (-2b+10r\rho) > 0 \quad (\text{if } b < 5r\rho).
\end{aligned}$$

Then, for all $k > \hat{k}$, it holds that $r\rho > \Delta_-$. Hence $\Pi_1(\text{C}, \text{IC})^{\text{NT}} > \Pi_1(\text{IC}, \text{IC})$ holds. $\square$

9. Proof of Lemma 3.4

$$\begin{aligned}
&\Pi_1(\text{C}, \text{C}) - \Pi_1(\text{IC}, \text{C})^{\text{NT}} \\
&= \left(\frac{t}{2} + r\rho - \frac{1}{36k} \right) - \left(\frac{t}{2} - \frac{1}{36k} \right) \left(\frac{3k(3t+b+r\rho)-1}{9kt-1} \right)^2 \\
&= r\rho - \frac{18kt-1}{12} \cdot \frac{(b+r\rho)\{2(9kt-1)+3k(b+r\rho)\}}{(9kt-1)^2}.
\end{aligned}$$

Here, we define Δ_+ as

$$\Delta_+ \equiv \frac{18kt-1}{12} \cdot \frac{(b+r\rho)\{2(9kt-1)+3k(b+r\rho)\}}{(9kt-1)^2}.$$

Therefore,

$$\begin{aligned}
\frac{\partial \Delta_+}{\partial k} &= -\frac{b+r\rho}{4(9kt-1)^3} \{6t(9kt-1) + (b+r\rho)(27kt-1)\} < 0 \quad (\text{if } k > \hat{k}), \\
\lim_{k \rightarrow \infty} \Delta_+ &= \lim_{k \rightarrow \infty} \frac{(b+r\rho)(18t - \frac{1}{k}) \{2(9t - \frac{1}{k}) + 3(b+r\rho)\}}{12(9t - \frac{1}{k})^2} \\
&= \frac{(b+r\rho)(6t+b+r\rho)}{18t} \equiv \min_k \Delta_+.
\end{aligned}$$

In addition,

$$\begin{aligned}
r\rho - \min_k \Delta_+ &= \frac{18tr\rho - (b+r\rho)(6t+b+r\rho)}{18t} \\
&= -\frac{(b+r\rho)^2 + 6t(b-2r\rho)}{18t} < 0 \quad (\text{if } b > 2r\rho).
\end{aligned}$$

Then, for all $k > \hat{k}$, it holds that $r\rho < \Delta_+$. Hence $\Pi_1(\text{C}, \text{C}) < \Pi_1(\text{IC}, \text{C})^{\text{NT}}$ holds. $\square$

10. Proof of Lemma 3.5

$$\begin{aligned}
\Pi_1(\text{IC}, \text{C})^{\text{T}} - \Pi_1(\text{C}, \text{C}) &= \left(\frac{1}{4k} - t + b + r\rho \right) - \left(\frac{t}{2} + r\rho - \frac{1}{36k} \right) \\
&= \frac{5}{18k} - \frac{3}{2}t + b \\
&> \frac{5}{18 \cdot \frac{1}{2(3t-b-r\rho)}} - \frac{3}{2}t + b \quad (\text{if } k < \bar{k}) \\
&= \frac{t}{6} + \frac{4}{9}b - \frac{5}{9}r\rho \\
&> \frac{1}{6} \cdot \frac{b+r\rho}{3} + \frac{4}{9}b - \frac{5}{9}r\rho \quad (\text{if } t > \frac{b+r\rho}{3}) \\
&= \frac{b-r\rho}{2} > 0 \quad (\text{if } b > r\rho).
\end{aligned}$$

$\square$

11. Proof of Lemma 3.6

First, from $\Pi_1(\text{IC}, \text{IC}) = \frac{t}{2} - \frac{1}{36k}$, $\Pi_1(\text{C}, \text{IC})^{\text{T}} = r\rho$, it doesn't hold

that $\Pi_1(\text{IC}, \text{IC}) > \Pi_1(\text{C}, \text{IC})^T$ unless the $t > 2r\rho$ holds. When it holds that $t > 2r\rho$, we can derive the following condition of the parameter k for $\Pi_1(\text{IC}, \text{IC}) > \Pi_1(\text{C}, \text{IC})^T$.

$$\begin{aligned}\Pi_1(\text{IC}, \text{IC}) - \Pi_1(\text{C}, \text{IC})^T &= \left(\frac{t}{2} - \frac{1}{36k}\right) - r\rho > 0 \\ \iff k &> \frac{1}{18(t - 2r\rho)} \equiv \tilde{k}.\end{aligned}$$

□

Table 3.1: Equilibrium investment, price, demands, profits of platform, profit of content provider, consumer surplus, and social surplus.

	(IC, IC)	(C, C)	(IC, C) ^{NT}	(IC, C) ^T
y_1	$\frac{1}{6k}$	$\frac{1}{6k}$	$\frac{3k(3t+b+r\rho)-1}{6k(9kt-1)}$	$\frac{1}{2k}$
y_2	$\frac{1}{6k}$	$\frac{1}{6k}$	$\frac{3k(3t-b-r\rho)-1}{6k(9kt-1)}$	0
p_1	$t + c - r\rho - \frac{1}{6k}$	$t + c - \frac{1}{6k}$	$t + c + \frac{b}{3} - \frac{2}{3}r\rho - \frac{1}{6k} - \frac{b+r\rho}{6k(9kt-1)}$	$c - t + b$
p_2	$t + c - r\rho - \frac{1}{6k}$	$t + c - \frac{1}{6k}$	$t + c - \frac{b}{3} - \frac{1}{3}r\rho - \frac{1}{6k} + \frac{b+r\rho}{6k(9kt-1)}$	c
D_1	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} + \frac{3k(b+r\rho)}{2(9kt-1)}$	1
D_2	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2} - \frac{3k(b+r\rho)}{2(9kt-1)}$	0
Π_1	$\frac{t}{2} - \frac{1}{36k}$	$\frac{t}{2} + r\rho - \frac{1}{36k}$	$\left(\frac{t}{2} - \frac{1}{36k}\right)\left(\frac{3k(3t+b+r\rho)-1}{9kt-1}\right)^2$	$\frac{1}{4k} - t + b + r\rho$
Π_2	$\frac{t}{2} - \frac{1}{36k}$	$\frac{t}{2} + r\rho - \frac{1}{36k}$	$\left(\frac{t}{2} - \frac{1}{36k}\right)\left(\frac{3k(3t-b-r\rho)-1}{9kt-1}\right)^2 + r\rho$	$r\rho$
π_1	$\frac{(1-r)\rho}{2}$	$(1-r)\rho$	$(1-r)\rho \cdot D_1(\text{IC}, \text{C})^{\text{NT}}$	$(1-r)\rho$
π_2	$\frac{(1-r)\rho}{2}$	$(1-r)\rho$	$(1-r)\rho$	$(1-r)\rho$
CS	$\frac{1}{6k} + b - \frac{5}{4}t - c + r\rho$	$\frac{1}{6k} + 2b - \frac{5}{4}t - c$	$\int_0^{D_1(\text{IC}, \text{C})^{\text{NT}}} u_1(x) dx + \int_{D_1(\text{IC}, \text{C})^{\text{NT}}}^1 u_2(x) dx$	$b + \frac{t}{2} - c$
SS	$\frac{1}{9k} + b - \frac{t}{4} - c + \rho$	$\frac{1}{9k} + 2b - \frac{t}{4} - c + 2\rho$	$CS(\text{IC}, \text{C})^{\text{NT}} + \sum_i \Pi_i(\text{IC}, \text{C})^{\text{NT}} + \sum_i \pi_i(\text{IC}, \text{C})^{\text{NT}}$	$\frac{1}{4k} - \frac{t}{2} + 2(b + \rho) - c$

Chapter 4

High Profitability of Low-Quality Firms in Two-Sided Markets

4.1 Introduction

In the hardware/software systems industry, in which various platforms produce hardware devices and independent programmers develop software for these devices, we can easily discern a quality gap between platforms (e.g., Sony's PlayStation 3 vs. Nintendo's Wii; Apple's iPad vs. Amazon's Kindle). Conventional wisdom argues that firms choose these different qualities to avoid fierce price competition (Mussa and Rosen, 1978; Gabszewicz and Thisse, 1979). It is well known that higher-quality firms have an advantage in competition, unless higher-quality products require greater costs to produce. For example, Shaked and Sutton (1982) show that a higher-quality

firm may be more profitable than its rival when the level of quality does not depend on production costs. Conversely, Wang (2003) suggests that if the variable cost function increases with quality, the higher-quality firm does not necessarily enjoy greater profit than its lower-quality rival. The failure of higher-quality firms occurs frequently in two-sided hardware/software systems markets. The purpose of this chapter is to better understand the relationship between platform quality and profitability in such markets.

In two-sided markets literature, most studies assume symmetric equilibrium pricing (Rochet and Tirole, 2003; Armstrong, 2006). Recently, several papers employ an asymmetric approach, including the vertical differentiation. Gabszewicz and Wauthy (2014) assume that consumers differ in their valuation of the indirect network externalities, and show that platform competition induces a vertical differentiation structure, despite the fact that the *ex ante* settings of their model are symmetric. Belleflamme and Peitz (2010) consider that the sellers' investment raises their net gains from trade. These two papers regard platform quality as an extension of indirect network externalities.

As an alternative, Lin et al. (2011) focus on the quality level of products and investigate how this affects the optimal pricing strategies. This chapter differs in the following two respects from their model. First, we consider a situation where platforms choose the qualities of their devices while focusing on the quality choices by sellers. Second, unlike this analysis, the utility of consumers does not depend on the number of sellers. We assume that consumers benefit from the large variety of software developers (indirect network externalities). Njoroge et al. (2013) is relevant to this chapter because they argue that platforms choose the qualities of their devices. They show

that platform investment in quality increases the profit of software developers. However, they did not assume that the development cost of software increased with hardware quality. In hardware/software system industries, high-quality hardware devices tend to increase the development cost for software (Corts and Lederman, 2009). Thus, we consider the platforms' quality choice in the situation where the development cost of software increases with hardware quality.

This chapter also relates to the literature on two-sided pricing theory, in which many existing studies examine the pricing mechanism (Armstrong, 2006; Weyl, 2010). In the hardware/software system industry, royalty fees are more common than lump-sum fees as the prevailing fee structure for software developers. Most empirical (Corts and Lederman, 2009; Dubé et al., 2010) and theoretical (Hagiu, 2006, 2009) studies assume this type of fee structure. In addition, we consider a sequential-move game in which software developers decide upon their chosen platform before users as in Hagiu (2006). That is, platforms decide their royalty rates (for software developers) before setting their hardware prices (for consumers).

In this chapter, after analyzing the model that includes these real-world examples, we derive the following two results. First, we show the potential higher profitability of the lower-quality platform under the parameter range in which the extent of the indirect network externalities is relatively large compared with the quality of the hardware device. The intuition is as follows. The high-quality hardware device tends to decrease the range of software developed for the device. However, given indirect network externalities, consumers prefer a platform with a large variety of software. Therefore, when the advantage of software variety dominates the disadvantage of low

hardware quality, the lower-quality platform has the opportunity to enjoy greater profit than does its higher-quality rival.

Second, we demonstrate that, in a sequential-move game, platforms select the same royalty rate in equilibrium, even with quality asymmetries across the hardware devices. The intuition is as follows. If a given platform selects a higher royalty rate than its rival, it may reduce the variety of software developed, which will then decrease its advantage in the final stage (where it determines the price for the device). Therefore, both platforms have no incentive to set a higher royalty rate and therefore both will choose the same royalty rate.

The remainder of the chapter is organized as follows. Section 4.2 introduces the model. Section 4.3 characterizes the equilibrium of this model. Section 4.4 provides a profit comparison between higher- and lower-quality platforms, where we show that the lower-quality platform can enjoy greater profit than its higher-quality rival in at least some parameter space. Section 4.5 concludes the chapter by providing directions for further research.

4.2 Model

There are two platforms, $i = 1, 2$. Each platform provides a hardware device i at a price p_i and enables the interaction between consumers and software developers “on board”. Denote by b the utility any consumer derives from a unit of software, which is assumed to be the same for all consumers. Similarly, denote by π the revenue that any software developer earns from a unit mass of consumers, which is also assumed to be the same for all software developers. For simplicity, we assume that the extents of indirect network externalities,

b and π , are exogenous parameters.¹ Consumers need to purchase a device to use the software. Platform i decides the quality q_i and price p_i of its devices. Without loss of generality, we suppose that platform 1 produces lower-quality hardware device q_L while platform 2 produces higher-quality one q_H . In addition, platform i sets a royalty rate r_i ($0 \leq r_i \leq 1$) for software developers. Each of the software developers then needs to pay a fraction r_i of its software revenue to the platform. Therefore, the profit function of platform i is given by

$$\Pi_i = p_i D_i + r_i \pi D_i \cdot n_i = (p_i + r_i \pi n_i) D_i,$$

where D_i denotes the demand for device i and n_i denotes the number of software products developed for device i .

Suppose now that software developers for the higher-quality platform incur higher development costs in producing their software. We assume that software developers are heterogeneous in terms of cost efficiency, which is denoted by c_j and uniformly distributed between zero and infinity with a unit density.² A software developer with a smaller c_j can develop software at a lower fixed cost. In this analysis, we assume that the fixed cost of developers is given by $c_j q_i$. Thus, the profit that a software developer j gains from developing a software product for device i is given by

¹Following Caillaud and Jullien (2003), we can also interpret this as meaning that the total gross gain from trade between a consumer and a software developer is $b + \pi$, where the allocation is negotiated as a function of their bargaining power. We assume that because of bargaining, the consumer receives b and the software developer earns π .

²Following Katz and Shapiro (1985), we assume that the parameter c_j has no finite upper limit to avoid the need to consider corner solutions.

$$\pi_{ij} = (1 - r_i)\pi D_i - c_j q_i$$

Each developer can develop software for either platform, and independently decides whether to develop for each platform. Therefore, the profit of software developer j is given by $\pi_j = \sum_i \pi_{ij}$.

Consumers are also heterogeneous in taste regarding their preferred hardware quality level, which is denoted by θ and distributed on the interval $[0, 1]$ with a unit density. We suppose that the utility to a consumer with a quality preference θ from a device i is given by θq_i . In addition, consumers benefit (in terms of utility) not only from the device they own but also from the variety of software available for the device. Thus, the utility function of a consumer with preference θ who purchases device i and its available software is written as

$$u_i(\theta) = b n_i + \theta q_i - p_i.$$

Here, we assume the following decision rule for consumers. Each consumer purchases only one device. If the consumer obtains the same utility from both devices, the consumer purchases the device with the larger range of software. All else being equal, the consumer chooses device i with probability $\alpha_i > 0$ ($\sum_i \alpha_i = 1$). We consider a two-stage game with the following timing.

1. Each platform i sets the royalty rate (r_i). Software developers then independently decide whether to develop software for platform i .
2. Each platform i sets a hardware price (p_i). Consumers then purchase one device and its available software.

4.3 Equilibrium

4.3.1 Stage 2: Optimal price for devices

Platform 2 produces the higher-quality device, $(q_1, q_2) = (q_L, q_H)$. The type of consumer that is indifferent between device 1 and 2 can be characterized by

$$u_1 = u_2 \iff \theta = \frac{b(n_1 - n_2) - p_1 + p_2}{q_H - q_L} \equiv \hat{\theta}.$$

We can now derive the demand functions for device i ,

$$\begin{cases} D_1 = \hat{\theta} = \frac{b(n_1 - n_2) - p_1 + p_2}{q_H - q_L}, \\ D_2 = 1 - \hat{\theta} = 1 - \frac{b(n_1 - n_2) - p_1 + p_2}{q_H - q_L}, \end{cases}$$

and the profit functions,

$$\begin{aligned} \Pi_1 &= (p_1 + r_1\pi n_1) \frac{b(n_1 - n_2) - p_1 + p_2}{q_H - q_L}, \\ \Pi_2 &= (p_2 + r_2\pi n_2) \frac{q_H - q_L - b(n_1 - n_2) + p_1 - p_2}{q_H - q_L}. \end{aligned}$$

Taking the first-order conditions and solving for prices, we have the following prices.

$$\begin{aligned} \tilde{p}_1 &= \frac{q_H - q_L + b(n_1 - n_2) - 2r_1\pi n_1 - r_2\pi n_2}{3} \\ \tilde{p}_2 &= \frac{2(q_H - q_L) + b(n_2 - n_1) - r_1\pi n_1 - 2r_2\pi n_2}{3} \end{aligned}$$

We rewrite these demands and profits as follows.

$$\begin{aligned} \tilde{D}_1 &= \frac{q_H - q_L + b(n_1 - n_2) + r_1\pi n_1 - r_2\pi n_2}{3(q_H - q_L)} \\ \tilde{D}_2 &= \frac{2(q_H - q_L) + b(n_2 - n_1) - r_1\pi n_1 + r_2\pi n_2}{3(q_H - q_L)} \end{aligned}$$

$$\begin{aligned}\tilde{\Pi}_1 &= \frac{\{q_H - q_L + b(n_1 - n_2) + r_1\pi n_1 - r_2\pi n_2\}^2}{9(q_H - q_L)} \\ \tilde{\Pi}_2 &= \frac{\{2(q_H - q_L) + b(n_2 - n_1) - r_1\pi n_1 + r_2\pi n_2\}^2}{9(q_H - q_L)}\end{aligned}$$

4.3.2 Stage 1: Optimal royalty rate for software developers

In Stage 1, content providers rationally forecast consumer demand $D_i = \tilde{D}_i$ for Stage 2 and make their decisions about whether to develop software for each device. We use this to calculate the marginal type of software developer that is indifferent between developing software for device i and not.

$$\begin{aligned}\pi_{ij} &= (1 - r_i)\pi\tilde{D}_i - c_j q_i \geq 0 \\ \iff c_j &< \frac{(1 - r_i)\pi\tilde{D}_i}{q_i} = n_i \quad (i = 1, 2)\end{aligned}\tag{4.1}$$

Solving these equations ($i = 1, 2$), we derive the range of software developed for device i as follows.

$$\begin{cases} \hat{n}_1 = \frac{(1-r_1)\pi\{q_H(q_H-q_L)-(1-r_2)\pi(b+r_2\pi)\}}{3q_Lq_H(q_H-q_L)-(1-r_2)\pi(b+r_2\pi)q_L-(1-r_1)\pi(b+r_1\pi)q_H} \\ \hat{n}_2 = \frac{(1-r_2)\pi\{2q_L(q_H-q_L)-(1-r_1)\pi(b+r_1\pi)\}}{3q_Lq_H(q_H-q_L)-(1-r_2)\pi(b+r_2\pi)q_L-(1-r_1)\pi(b+r_1\pi)q_H} \end{cases}\tag{4.2}$$

Substituting this range of software ($\hat{n}_1, \hat{n}_2$) in $(\tilde{\Pi}_1, \tilde{\Pi}_2)$, we have

$$\begin{aligned}\hat{\Pi}_1 &= \tilde{\Pi}_1(\hat{n}_1, \hat{n}_2) \\ &= \frac{\{q_H - q_L + b(\hat{n}_1 - \hat{n}_2) + r_1\pi\hat{n}_1 - r_2\pi\hat{n}_2\}^2}{9(q_H - q_L)}, \\ \hat{\Pi}_2 &= \tilde{\Pi}_2(\hat{n}_1, \hat{n}_2) \\ &= \frac{\{2(q_H - q_L) + b(\hat{n}_2 - \hat{n}_1) - r_1\pi\hat{n}_1 + r_2\pi\hat{n}_2\}^2}{9(q_H - q_L)}.\end{aligned}$$

We rewrite the first-order condition for platform 1's profit maximization as follows.

$$\begin{aligned} \frac{\partial \hat{\Pi}_1}{\partial r_1} &= \frac{2\{q_H - q_L + b(\hat{n}_1 - \hat{n}_2) + r_1\pi\hat{n}_1 - r_2\pi\hat{n}_2\}}{9(q_H - q_L)} \\ &\times \frac{\partial[(b + r_1\pi)\hat{n}_1 - (b + r_2\pi)\hat{n}_2]}{\partial r_1} = 0 \end{aligned}$$

It holds that $q_H - q_L + b(\hat{n}_1 - \hat{n}_2) + r_1\pi\hat{n}_1 - r_2\pi\hat{n}_2 > 0$, given that we analyze the interior solution, $\tilde{D}_i \in (0, 1)$. Given the need to satisfy the first-order condition such that $\{\partial[(b + r_1\pi)\hat{n}_1 - (b + r_2\pi)\hat{n}_2]\} / \partial r_1 = 0$, we can similarly analyze platform 2. Accordingly, we rewrite the platforms' profit maximization problems as

$$\frac{\partial \hat{\Pi}_i}{\partial r_i} = 0 \iff \frac{\partial[(b + r_i\pi)\hat{n}_i - (b + r_j\pi)\hat{n}_j]}{\partial r_i} = 0. \quad (4.3)$$

That is, platform i maximizes $b(\hat{n}_i - \hat{n}_j) + r_i\pi\hat{n}_i - r_j\pi\hat{n}_j$ with respect to r_i for profit maximization. This expresses the advantage the platform can derive if it marginally increases the number of content providers for its device. The first term, $b(n_i - n_j)$, indicates the difference in indirect network externalities. In two-sided markets, the profit-maximizing price is adjusted downward by the external benefit to the other group (Armstrong, 2006). The second and third terms, $r_i\pi\hat{n}_i - r_j\pi\hat{n}_j$, indicates the difference in such reduced prices between the two platforms. Next, by rewriting equation (4.1) with $i = 1$, we derive the following equation.

$$\begin{aligned} (1 - r_1)\pi\tilde{D}_1(\hat{n}_1, \hat{n}_2) &= \hat{n}_1q_L \\ \iff (1 - r_1)\pi \cdot \frac{q_H - q_L + b(\hat{n}_1 - \hat{n}_2) + r_1\pi\hat{n}_1 - r_2\pi\hat{n}_2}{3(q_H - q_L)} &= \hat{n}_1q_L \\ \iff (b + r_1\pi)\hat{n}_1 - (b + r_2\pi)\hat{n}_2 &= \frac{3q_L(q_H - q_L)\hat{n}_1}{\pi(1 - r_1)} - (q_H - q_L) \end{aligned}$$

Taking the partial derivative of both sides with respect to r_1 , we derive the following equation.

$$\frac{\partial[(b + r_1\pi)\hat{n}_1 - (b + r_2\pi)\hat{n}_2]}{\partial r_1} = \frac{3q_L(q_H - q_L)}{\pi} \cdot \frac{\partial}{\partial r_1} \left[\frac{\hat{n}_1}{1 - r_1} \right]$$

We can similarly calculate this for platform 2, and summarize the result as follows.

$$\frac{\partial[(b + r_i\pi)\hat{n}_i - (b + r_j\pi)\hat{n}_j]}{\partial r_i} = 0 \iff \frac{\partial}{\partial r_i} \left[\frac{\hat{n}_i}{1 - r_i} \right] = 0 \quad (4.4)$$

Here, by using equation (4.2), we can derive the following equation.

$$\frac{\hat{n}_1}{1 - r_1} = \frac{\pi\{q_H(q_H - q_L) - (1 - r_2)\pi(b + r_2\pi)\}}{3q_Lq_H(q_H - q_L) - (1 - r_2)\pi(b + r_2\pi)q_L - (1 - r_1)\pi(b + r_1\pi)q_H}$$

The maximization of $\hat{n}_1/(1 - r_1)$ is replaced by the maximization of $(1 - r_1)(b + r_1\pi)$. This means that platform 1's optimal decision about its royalty rate is unaffected by any of the strategies of its rival. This result also holds for platform 2. Thus, we rearrange the first-order conditions as follows.

$$\frac{\partial}{\partial r_i} \left[\frac{\hat{n}_i}{1 - r_i} \right] = 0 \iff \frac{\partial[(1 - r_i)(b + r_i\pi)]}{\partial r_i} = 0 \quad (4.5)$$

From equations (4.3), (4.4), and (4.5), we summarize the platforms' first-order conditions for profit maximization and obtain the equilibrium royalty rates r_i^* as follows.

$$\frac{\partial \hat{\Pi}_i}{\partial r_i} = 0 \iff \frac{\partial[(1 - r_i)(b + r_i\pi)]}{\partial r_i} = 0 \iff r_i^* = \frac{\pi - b}{2\pi}$$

This result has two interesting aspects. First, equilibrium royalty rates do not depend on the quality of the device, as both platforms make their decisions to maximize the indirect network externalities. Therefore, the equilibrium royalty rates depend only on two parameters reflecting the degree of the indirect network externalities. This equilibrium results from the same mechanism in the case where both platforms produce devices of the same quality.

Second, the equilibrium royalty rate chosen by the higher-quality platform equals that chosen by its lower-quality rival, despite the fact that platform 2 produces a higher-quality device than platform 1. The intuition behind this result is as follows. Platform 1, which produces a lower-quality device, has no incentive to set a higher royalty rate than platform 2. On the other hand, if platform 2, which produces a higher-quality device, sets a higher royalty rate than its rival, it cannot use the higher quality to its best advantage, as setting the higher royalty rate decreases the number of software developers for its device, which prevents it from effectively internalizing the indirect network externalities. Therefore, the platforms set the same royalty rate.³

These equilibrium royalties satisfy the second-order condition (see the Appendix for the proof). Using this result, we derive other equilibrium values as follows.

³In the video game industry, software developers are charged about \$7–8 as a royalty fee by Nintendo, Sony or Microsoft, even though there is little difference in the price of the game titles across these video game console manufacturers.

Proposition 4.1. *Suppose that platform 1 produces a lower-quality hardware device and platform 2 produces a higher-quality device. If the quality is sufficiently high to satisfy the condition*

$$q_H > \max \left\{ \frac{q_L + \sqrt{q_L^2 + (b + \pi)^2}}{2} \equiv f_L(q_L), \quad q_L + \frac{(b + \pi)^2}{8q_L} \equiv f_H(q_L) \right\}, \quad (4.6)$$

we derive the interior equilibrium royalty rates, device prices, number of software developers, consumer demand for each device, and platform profits as follows.

$$\begin{aligned} r_1^* &= \frac{\pi - b}{2\pi} \\ r_2^* &= \frac{\pi - b}{2\pi} \\ p_1^* &= \frac{\{4q_L(q_H - q_L) + b^2 - \pi^2\}\{4q_H(q_H - q_L) - (b + \pi)^2\}}{4\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}} \\ p_2^* &= \frac{\{4q_H(q_H - q_L) + b^2 - \pi^2\}\{8q_L(q_H - q_L) - (b + \pi)^2\}}{4\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}} \\ n_1^* &= \frac{(b + \pi)\{4q_H(q_H - q_L) - (b + \pi)^2\}}{2\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}} \\ n_2^* &= \frac{(b + \pi)\{8q_L(q_H - q_L) - (b + \pi)^2\}}{2\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}} \\ D_1^* &= \frac{q_L\{4q_H(q_H - q_L) - (b + \pi)^2\}}{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)} \\ D_2^* &= \frac{q_H\{8q_L(q_H - q_L) - (b + \pi)^2\}}{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)} \\ \Pi_1^* &= \frac{q_L^2(q_H - q_L)\{4q_H(q_H - q_L) - (b + \pi)^2\}^2}{\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}^2} \\ \Pi_2^* &= \frac{q_H^2(q_H - q_L)\{8q_L(q_H - q_L) - (b + \pi)^2\}^2}{\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}^2} \end{aligned}$$

Proof. See the Appendix.

4.4 Profit comparison

We now compare profits between the higher- and lower-quality platforms. From this comparison, we derive the following proposition.

Proposition 4.2. *If inequality (4.6) and $q_H < (b + \pi)^2/(4q_L) \equiv f(q_L)$ hold, a lower-quality platform can earn more profit than a higher-quality platform. The parameter space that satisfies these conditions is shown in Figure 4.1.*

Proof. See the Appendix.

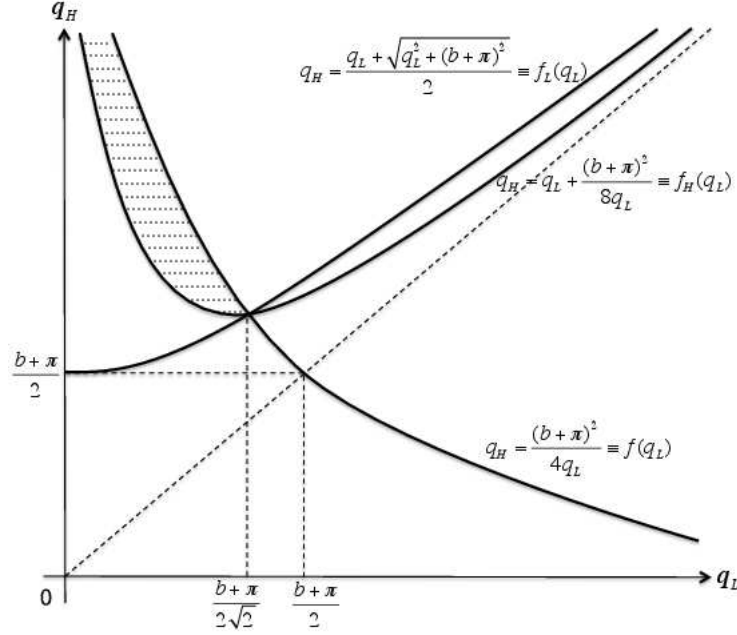


Figure 4.1: Lower-quality platform enjoys greater profit.

Proposition 4.2 shows that even with some difference in hardware quality, the lower-quality device has a larger range of software, and this software superiority may dominate its disadvantage in hardware quality. However, if the

quality difference is larger than the extent of the indirect network externalities, only the quality of the device affects consumers' decisions. Therefore, a lower-quality platform can derive more profit than its higher-quality rival only when the product of qualities $q_L \cdot q_H$ is not sufficiently large compared with the degree of indirect network externalities, $4q_L \cdot q_H < (b + \pi)^2$.

4.5 Conclusion

This chapter investigated the competition between vertically differentiated platforms in two-sided markets. We derived a result that is counterintuitive to conventional wisdom. In two-sided markets, the lower-quality platform may be able to gain more profit than its higher-quality rival. The intuition behind this result is as follows. The higher quality of the device implies that the software developers incur higher development costs, which in turn decreases the number of software developers. This reduction in software variety decreases the platform's utility to consumers through the indirect network externalities. Therefore, when the advantage of software variety dominates the disadvantage of low hardware quality, the lower-quality platform has the opportunity to enjoy greater profit than its rival.

This chapter also contributes to the two-sided pricing literature by suggesting an interesting mechanism by which asymmetric platforms may still choose symmetric strategies. When platforms decide their royalty rates before setting their hardware prices, they choose the same royalty rate in equilibrium even if there are asymmetries in the quality of the devices they produce. This result depends on the platform fee structure we adopt. Under a royalty fee structure, the increase (decrease) in the number of software devel-

operators increases (decreases) the number of consumers. However, it also raises (lowers) the payments from software developers to the platform. Therefore, when compared with a lump-sum fee, the platform royalty decision strongly affects, via the indirect network externalities, the decision made in the subsequent stage. Hence, platforms have no incentive to employ a royalty rate that differs from the rival. Further analysis of the robustness of these symmetric strategies remains a task for future work.

In our model, we made a few simplifying assumptions. One of these is that we regard the benefits consumers gain from software as independent of the hardware quality. We also assume that platforms choose their hardware quality level from only two alternatives, high and low. Changing these assumptions is a potentially fruitful direction for future research.

4.6 Appendix

1. Proof of the second-order condition.

$$\begin{aligned}\frac{\partial^2 \hat{\Pi}_1}{\partial r_1^2} &= -\frac{16\pi^2 q_L^2 q_H (q_H - q_L) \{4q_H (q_H - q_L) - (b + \pi)^2\}^2}{\{12q_L q_H (q_H - q_L) - (b + \pi)^2 (q_L + q_H)^2\}^3} < 0 \\ \frac{\partial^2 \hat{\Pi}_2}{\partial r_2^2} &= -\frac{16\pi^2 q_L q_H^2 (q_H - q_L) \{8q_L (q_H - q_L) - (b + \pi)^2\}^2}{\{12q_L q_H (q_H - q_L) - (b + \pi)^2 (q_L + q_H)^2\}^3} < 0\end{aligned}$$

□

2. Proof of Proposition 4.1.

If $4q_H(q_H - q_L) - (b + \pi)^2 > 0$ and $8q_L(q_H - q_L) - (b + \pi)^2 > 0$ hold, both platforms can attract a positive number of software developers and consumers and earn a positive profit ($n_i^* > 0$, $0 < D_i^* < 1$, $\Pi_i^* > 0$)

for $i = 1, 2$). Solving these inequalities for $q_H(> 0)$ yields

$$\begin{aligned}
& 4q_H(q_H - q_L) - (b + \pi)^2 > 0 \\
\Longleftrightarrow \quad & q_H > \frac{q_L + \sqrt{q_L^2 + (b + \pi)^2}}{2} \equiv f_L(q_L), \\
& 8q_L(q_H - q_L) - (b + \pi)^2 > 0 \\
\Longleftrightarrow \quad & q_H > q_L + \frac{(b + \pi)^2}{8q_L} \equiv f_H(q_L).
\end{aligned}$$

Thus, a necessary and sufficient condition for the existence of an inner solution is given by $q_H > \max \{f_L(q_L), f_H(q_L)\}$. $\square$

3. Proof of Proposition 4.2.

$$\begin{aligned}
& \Pi_1^* - \Pi_2^* \\
= & \frac{q_H - q_L}{\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}^2} \\
& \cdot [q_L^2\{4q_H(q_H - q_L) - (b + \pi)^2\}^2 - q_H^2\{8q_L(q_H - q_L) - (b + \pi)^2\}^2] \\
= & \frac{q_H - q_L}{\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}^2} \\
& \cdot [q_L\{4q_H(q_H - q_L) - (b + \pi)^2\} + q_H\{8q_L(q_H - q_L) - (b + \pi)^2\}] \\
& \cdot [q_L\{4q_H(q_H - q_L) - (b + \pi)^2\} - q_H\{8q_L(q_H - q_L) - (b + \pi)^2\}] \\
= & \frac{(q_H - q_L)^2\{(b + \pi)^2 - 4q_Lq_H\}}{\{12q_Lq_H(q_H - q_L) - (b + \pi)^2(q_L + q_H)\}^2} > 0 \\
\Longleftrightarrow \quad & q_L < \frac{(b + \pi)^2}{4q_H}
\end{aligned}$$

$\square$

Chapter 5

Conclusion

This dissertation focused on compatibility and innovation in two-sided markets. In Chapter 2, we showed how the product life cycle affects the compatibility strategy of platforms. At the introductory stage, it is of great importance to platforms to sell hardware devices to new customers. In such a situation, compatibility is unprofitable because providing wider availability of content across devices increases the attractiveness of rival hardware devices. Thus, incompatibility is a dominant strategy for platforms. At the mature stage, almost all customers have purchased an installed base of some platform, and only want to purchase content for the platform. In this case, a platform obtains most of its profit by selling content in its marketplace and obtaining royalties. In such a situation, compatibility is a dominant strategy. At the intermediate stage, the share of the hardware device market plays a critical role. With sufficiently large differences in the market share between competing platforms, the smaller platform gives up increasing its market share and chooses compatibility to earn more royalty revenue. On the other hand, the large platform has an incentive to select incompatibil-

ity for a bigger market share. Without sufficiently large differences in the market share, there are multiple asymmetric equilibria. The market has the characteristics of a game of “chicken.” That is, either firm will make more profit by making its content incompatible with the rival’s hardware device, given that the rival does the opposite.

In Chapter 3, we showed how process innovation (increased efficiency of cost-reducing investment for hardware devices) affects the platforms’ compatibility decisions. Process innovation changes platforms’ incentives to invest in their hardware devices. The change in investment incentives affects compatibility decisions. For example, suppose that one platform chooses incompatibility while the other chooses compatibility. In this case, the platform that chooses incompatibility has a larger market share for the hardware device from its advantage in terms of software variety. On the other hand, the platform that chooses compatibility can earn more royalty revenue instead of increasing its market share. If process innovation occurs, both platforms can easily invest in cost reductions in their hardware device. The platform that chooses incompatibility and obtains a greater market share has an incentive to monopolize the hardware device market by investing aggressively. Therefore, the platform that chooses compatibility, changes its compatibility decisions to prevent it being monopolized by a larger rival. As a result, both platforms choose incompatibility in equilibrium. These changes in competitive structure have the following three effects. First, the profit of the large platform that chose incompatibility before process innovation decreases. Second, the profit of the small platform that chose compatibility before process innovation increases conversely. Finally, consumer surplus and social welfare may decrease.

In Chapter 4, we focused on the competition between vertically differentiated platforms. Higher-quality increases the utility of consumers directly. On the other hand, it also increases the fixed development cost of software developers. This leads to a reduction in the software variety, which reduces the attractiveness to consumers of high-quality hardware devices. Therefore, the higher-quality has a positive direct effect and a negative indirect effect for competition. When the negative indirect effect dominates the positive direct one, the lower-quality platform may be able to gain greater profit than the higher-quality rival may. We also show that the vertically differentiated platforms charge *the same royalty rate* to software developers in equilibrium.

In this dissertation, we focused on investment and innovation only for hardware devices (consumer side). However, in reality, consumers benefit not only from the quality of hardware devices, but also from the quality of software products. The existence of indirect network externalities is one of the most important features of two-sided markets. However, the previous literature assumes that users on each side benefit only from the number of users on the other side. However, users' benefits depend on the quality of the users on the other side. Therefore, further analysis of an extension of our focus in this direction remains a task for future work.

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