Asset Bubbles and Macroeconomic Analysis

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資産価格バブルとマクロ経済分析

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Contents

1. Introduction

A bubble on asset is defined as a deviation of an asset's market value from its fundamental value. Economic history has repeatedly witnessed severe financial crises accompanied by the bursting of bubbles. Aliber and Kindleberger (2015) report that prices of various assets, including stocks, land, and real estate, often deviate upward from their fundamental values, and possibly affect various economic activities, such as consumption, investment, economic growth and unemployment. They state that during bubble periods, in some cases the bubble would arise on a single type of asset, while in other cases it would arise on various types of assets, with different effects on the economy. Additionally, bubbles are frequently observed when the economy is booming and the economic growth rate is high (Martin and Ventura, 2012; Famer and Schelnast, 2013, ch. 6). Indeed, empirical studies have shown that there is a negative relationship between stock market wealth and the unemployment rate. A correlation between stock market and unemployment is found to hold for European countries (Fitoussi et al., 2000). Moreover, the US stock market boom of the 1990s was accompanied by a reduction in the unemployment rate (Phelps, 1999).

As a result, the purpose of this paper is as follows. First, we construct an equilibrium model, in which two types of asset bubbles exist simultaneously and analyze their effects on the economy. Second, we develop an overlapping-generations model with labor market frictions to examine the relationship among bubbles, unemployment and economic growth.

In his seminal study, Tirole (1985) using an overlapping-generations model, shows that pure bubbles on intrinsically useless assets, such as fiat money, can exist in the equilibrium.[1] If the equilibrium without bubbles is dynamically inefficient, that is, too much capital is being accumulated in the equilibrium, bubbles can arise in the equilibrium and affect real economic activity, such as consumption, capital accumulation, and production. When bubbles arise, they crowd out savings from capital accumulation, thus leading to a change in the intergenerational resource allocation. This property is called the crowd-out effect of bubbles. Saint-Paul (1992), Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) extend Tirole's (1985) model to an endogenous growth framework. Their studies re-examine

[1] Kamihigashi (2001, 2003, 2005), using a transversality condition, shows that asset bubbles are impossible in infinitely-lived agent models with continuous and discrete time. Kamihigashi (2015) assumes a sequential budget constraint and establishes a simple non-bubble theorem that can be used to reject asset bubbles in a wide range of infinitely-lived agent models.

the necessary condition for the existence of bubbles and show that even if the equilibrium without bubbles is not dynamically inefficient, bubbles can exist in the equilibrium. In the endogenous growth framework, when bubbles arise in the economy, they divert savings from capital accumulation, which leads to lower economic growth. Kunieda and Shibata (2016) call the above mentioned literature first-generation models, because only crowd-out effects on capital accumulation arise in these models.

On the other hand, using the research and development (R&D)-based endogenous growth model developed by Romer (1990), Olivier (2000) considers a bubble on a stock asset rather than an intrinsically useless asset, and shows that bubbles can enhance economic growth. When bubbles arise on stocks in new firms created by R&D activities, which lead to the higher return of R&D activities, households allocate more labor inputs to the R&D sector, and realize a higher growth rate. This property of stimulating R&D activities is called the growth enhancing effect. However, Olivier's model does not consider the crowd-out effect of bubbles stated by Grossman and Yanagawa (1993). Tanaka (2011) uses an endogenous growth model different from Romer's (1990), to re-examine the robustness of Olivier's (2000) results. He constructs a model where bubbles on new firm's stocks created by R&D activities have not only a growth enhancing effect, but also a crowd-out effect; moreover, this shows that a bubble on stock assets can enhance economic growth through a different mechanism compared to Olivier's model. However, all of the above studies consider only one type of bubbly asset, that is, a bubble on an intrinsically useless asset having a crowd-out effect or a bubble on a stock having growth enhancing effect.[2]

In chapter 2, we construct a model economy in which two types of asset bubbles can exist at the same time and investigate conditions for them to arise. One of them is a bubble on an intrinsically useless asset, that is, nonproductive savings, and the other is a bubble on the stocks of firms newly created by R&D activities. We call the first type pure bubble and the second type stock bubble, and derive a condition for the two types of asset bubbles to simultaneously arise in a steady state equilibrium. If the supply of pure bubbles grows at a constant rate, a steady state equilibrium with two types of asset bubbles can exist in the economy. Additionally, we show that pure and stock bubbles enhance economic growth in the steady state equilibrium.

[2] Clain-Chamosset-Yvrard and Kamihigashi (2016) considers two types of asset bubbles in the two-country model. However, they only consider the crow-out effect and do not analyze the relationship between bubbles and economic growth.

The drawback of the first-generations models is that their theoretical results are inconsistent with real world data that bubbles seem to enhance economic growth. To overcome this, the literature on asset bubbles and economic growth has been recently focusing on the presence of asset bubbles promoting capital accumulation and economic growth. In this literature, financial market imperfections and productivity differences across agents are key factors in producing a situation such that asset bubbles enhance capital accumulation. Farhi and Tirole (2012), Martin and Ventura (2012), Carvalho et al. (2012), and Kunieda (2014) create such situations using the overlapping generations framework of Samuelson (1958), Tirole (1985), or Blanchard (1985). To produce the same situation, Aoki and Nikolov (2015) and, Hirano and Yanagawa (2016) use dynamic general equilibrium models, in which asset bubbles arise in equilibrium despite the assumption that infinitely-lived agents, and the presence of bubbles promotes economic growth through a mechanism similar to that founded by Mitsui and Watanabe (1989). These models all use an asset bubble on intrinsically useless assets. Using utility depends on wealth, Kamihigashi (2008) shows how stock market bubbles cause output fluctuations and affect output positively if the production function exhibits increasing returns to scale. However, these second- and first-generations models do not analyze how unemployment is affected by the presence of asset bubbles. As such, in chapters 3, 4, and 5, we introduce a search and matching model, where unemployment arises in the equilibrium into an overlapping-generations model, and investigate the relationship among bubbles, unemployment and capital accumulation. In chapters 3 and 4, we use a first-generation model according to literature on bubble, and in chapter 5, we use a second-generation model.

In chapter 3, we construct a continuous-time overlapping-generations model with labour market frictions to examine the relationships among unemployment, asset bubbles, and economic growth. We show that the existence of asset bubbles is contingent upon the unemployment rate: a bubble (non-bubble) regime arises in equilibrium when unemployment is relatively low (high). Our framework focuses on the boom and bust of asset bubbles caused by changes in fundamental variables, not a stochastic probability. Then, as labor market frictions generate a negative relationship between the unemployment rate, the interest rate and economic growth, we find that the bubble regime exhibits a higher growth rate than the non-bubble one. Furthermore, we show that policy or parameter changes that have a positive influence on the labor market shift the economy from a non-bubble to a bubble regime.

In chapter 4, using an overlapping-generations model of R&D-based growth with labor market frictions, we examines how employment changes induced by labor market frictions influence asset bubbles and economic growth. We show that the existence of bubbles is contingent upon the equilibrium employment rate. Asset bubbles can (not) exist when employment rate is high (low), which leads to higher (lower) economic growth through labor market efficiency. This result is similar to one chapter 3. We explore the steady state and transitional dynamics of bubbles, economic growth, and employment. Furthermore, we show that policy or parameter changes that have a negative influence on the labor market leads to a bubble burst.

In chapter 5, a tractable overlapping-generations model with asset bubbles is presented to demonstrate that a financial crisis caused by bubbles bursting increase unemployment rates. Without asset bubbles, all agents engage in capital production regardless of their idiosyncratic productivity shocks. A bubbly asset has a positive market value, because selling the asset is a fund-raising method for agents who draw sufficiently high productivity to initiate an investment project, whereas purchasing the bubbly asset is the sole saving method for agents who draw very low productivity. The presence of asset bubbles corrects allocative inefficiency, reallocating investment resources from low productive agents to highly productive one. Additionally, under mild parameter conditions, the presence of asset bubbles promotes capital accumulation and reduces the unemployment rate. However, a self-fulfilling financial crisis caused by extrinsic uncertainty would result in high unemployment rates.

2. An equilibrium model with two types of asset bubbles

2.1. Introduction

Aliber and Kindleberger (2015) highlight that bubbles have occurred throughout history, often with major impacts on local economies. Some examples include the current recession in the United States and other countries, the Japanese experience in the late 1980s and 1990s, and 1929 crash. They state that during bubble periods, in some cases the bubble would arise on a single type of asset, while in other cases it would arise on various types of assets, with different effects on the economy. Additionally, assets markets worldwide are very volatile and the prices of various assets such as stocks, land, and real estate rise rapidly (Miao 2015). The purpose of this chapter is to construct model economy in which various asset bubbles can exist at same time, and investigate relationship between them analytically. More specifically, we focus on two types of asset bubbles that are bubbles on the intrinsically useless asset and on stock of firm and analyze the effect of two types of bubbles on the economy.

In his seminal study, Tirole (1985) examines the condition for the existence of bubbles on intrinsically useless assets in an overlapping generations model. Bubbles on intrinsically useless asset can be positive in the overlapping generations model if the steady state equilibrium without bubbles is dynamically inefficient; that is, equilibria with too capital accumulation. Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) extend Tirole's model to an endogenous growth framework. They re-examine the conditions necessary for bubbles to exist and show the relationships between bubbles and economic growth. In their models, there is too little rather than too much capital. In addition, a bubble arising in the economy diverts savings from capital accumulation and retards economic growth. This property is called the crowding out effect.

For an alternative perspective on bubbles, using an R&D-based model of endogenous growth, Olivier (2000) considers bubbles not as useless assets, but as assets tied to capital goods. As such, he shows that when bubbles arise in R&D firms, bubbles can increase economic growth. This is growth enhancing effect. Bubbles on stocks of firms positively affect the grow rate by encouraging the creation of new firms. However, Olivier (2000) does not consider the crowding out effect of bubbles, as Grossman and Yanagawa (1993) emphasize. Tanaka(2011), using an alternative endogenous growth model, re-examines Olivier's (2000) properties of stock bubbles and shows that stock bubbles have both a growth enhancing effect and a crowding out effect. He derives the different results from Olivier(2000). However, all of the above studies consider only a single type of a bubbly asset, that is, bubbles on intrinsically useless asset or on stocks.

Based on Tanaka (2011), we construct a model economy in which bubbles on an intrinsically useless asset and on stocks can exist simultaneously and analyze the relationship between them. We call the first type a pure bubble and the second type a stock bubble. Pure and stock bubbles have deferent properties in an economy. The former crowds out productive savings away from capital accumulation through the crowding out effect, which lowers economic growth. the later type has both a crowding out effect and a growth enhancing effect which, enhances growth by stimulating R&D activities.

We derive a condition for pure and stock bubbles to exist in the equilibrium. When the supply of pure bubbles is constant, the condition for pure bubbles to exist in a steady state equilibrium is when the economic growth rate equals the market interest rate. On the other hand, stock bubbles can exist only if the growth rate is greater than the market interest rate. Thus, pure and stock bubbles can-not coexist in the equilibrium. However, if the supply of pure bubbles grows at a constant rate, the steady state equilibrium with pure and stock bubbles can exist in the economy because the condition for the existence of pure bubbles becomes when economic growth rate is greater than the market interest rate.

We show that pure and stock bubbles increase the economic growth rate. The change in the initial arising in the stock price of a new firm through R&D activity has three effects on the growth rate. First, an increase in the initial bubble increases the quantity of stock bubbles, which strengthens crowding out effect of stock bubbles. Second, the initial bubble decreases pure bubbles and weakens their crowding out effect. Third, an increase in the initial bubble increase the return in the R&D sector, and hence has a positive effect on a growth rate. The positive effect dominates the negative crowding out effect in a steady state equilibrium with pure and stock bubbles. The rate of supply of a pure bubble positively affects the growth rate of an economy with two types of bubbles due to an increase in asset holdings. When governments distribute a new pure bubble asset to households as a transfer payment as a lump-sum, households believe they are wealthier and want to save more. This leads to the higher the growth rate in an economy with two types of bubbles.

The remainder of this paper proceeds as follows. Section 2 outlines the features of the model. Section 3 derives the condition for bubbles to exist in a steady state equilibrium and investigate the effects parameter changes on the growth rate. The final section summarizes our findings and concludes the chapter.

2.2 The Model

This section develops a two-period overlapping generations model with two types of asset bubbles. The economy begins in period 0, and the cohort born in period t is generation t. Each generation has a constant labor size (L) , which is supplied inelastically. The economy has two types of asset bubbles, defined as the difference between the fundamental and market values of an asset. First, following Tirole (1985), we consider a bubble on an intrinsically useless asset; that is the fundamental value is zero, which we call a pure bubble. The second type is a bubble on a stocks of new firms created via R&D activities, which we call a stock bubble. On the production side, the economy consists of three sectors: a final goods sector, an intermediate goods sector, and an R&D sector. The labor market is open only in the final goods sector. In accordance with Rivera-Batiz and Romer (1991) and Barro and Sala-i-Martin (2004, Chap. 6), we regard final goods as the production factor in both the intermediate goods and R&D sectors. R&D firms invent blueprints of intermediate goods and launch these goods into the market. Each intermediate good is produced by a single monopoly firm. Each final good is produced by competitive firms using labor and a variety of imperfectly substitutable intermediate goods as input.

2.2.1. The final goods sector

In the final goods sector, the many homogenous firms produce final goods with the same production technology. We normalize the number of firms to one without loss of generality. A firm needs workers and intermediate goods to produce final goods. The production function of the firm is given by

$$Y_t = AL_t^{1-\alpha}\int_0^{N_t}(x_t(j))^{\alpha}\,dj\,, \tag{1}$$

where A, N_t, L_t and $x_t(j)$ are the productivity of the technology, the number of varieties available at period t, the labor input, and the input of intermediate goods for product variety j, respectively. The final good is set as the model numeraire, so the firm's profit is

$$\pi_t^y = AL_t^{1-\alpha}\int_0^{N_t}(x_t(j))^{\alpha}\,dj - \int_0^{N_t} p_t^x(j)x_t(j)dj - w_tL_t\,, \tag{2}$$

where $p_t^x(j)$ and $x_t(j)$ are the price and the input of intermediate goods for product variety j, respectively. Because the factor market is competitive, we can get first order conditions from maximization problem, as follows:

$$(1-\alpha)AL_t^{-\alpha}\int_0^{N_t}(x_t(j))^{\alpha}dj = w_t, \tag{3}$$

$$\alpha AL_t^{1-\alpha}(x_t(j))^{\alpha-1} = p_t^x(j). \tag{4}$$

Using (4), the demand function for an intermediate good for variety *j* is given by

$$x_t(j) = \left(\frac{\alpha A}{p_t^x(j)}\right)^{\frac{1}{1-\alpha}} L_t. \tag{5}$$

2.2.2 The intermediate goods sector

There are N_t types of intermediate goods at the beginning of period *t* and each intermediate good *j* is produced by monopolistically competitive firms that hold a blueprint for intermediate good *j*. We assume that one unit of final goods is required to produce one unit of an intermediate good, and the operating profit of each intermediate goods producer $\pi_t(j)$ is $\pi_t(j) = \left(p_t^x(j)-1\right)x_t(j)$, where $x_t(j)$ is the supply of intermediate good *j*. Under monopolistic competition, each firm maximizes its profits given the demand function (5) by establishing a price that is equal to a constant markup over unit cost:

$$p_t^x(j) = p^x = \frac{1}{\alpha}. \tag{6}$$

Thus, we can drop the firm-specific index in the intermediate goods sector and express profits as follows:

$$\pi = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L, \tag{7}$$

in which we use the labor market equilibrium condition $L_t = L$. We define the fundamental value of firm *j*'s stock price at period *t* as

$$D_t = \frac{\pi}{1+r_{t+1}} + \frac{\pi}{(1+r_{t+1})(1+r_{t+2})} + \cdots, \tag{8}$$

where r_t represents the market interest rate at period *t*.

2.2.3. The R&D sector

R&D technology development requires final goods as inputs. We assume that the cost of inventing new blueprints is η units of the final goods between periods *t* and *t* + 1. Competitive

R&D firms can invent one unit of $N_{t+1} - N_t$ new blueprints; thus, we express the output of R&D firms as follows:

$$I_t^R = \eta(N_{t+1} - N_t). \tag{9}$$

where I_t^R denotes final goods devoted to the R&D sector. R&D firms can sell these blueprints to intermediate goods firms at their market values of D_t. Hence, the benefit from R&D activity in period t is D_t. In this study, however, we consider an economy in which the market value of stock is greater than the fundamental value of the firm. More specifically, following Tanaka(2011), bubbles arise on the stocks of new firms holding a new blueprint. The market value of the stock at the beginning of period t is $D_t + B(t,t)$; thus, the value of R&D activity is $D_t + B(t,t)$ in period t. Here we define $B(s,t)$ as a bubble occuring in period t on the firm created by R&D activity in period s (this new firm can produce intermediate goods in period s+1). $B(t,t)$ is a bubble that arise in period t on the firm just created by R&D activity in the same period t (this firm can produce intermediate goods in period t+1), and we assume that $B(t,t) = B$ is constant over time. We call this B the initial bubble. Assuming free entry in the R&D sector, the following condition holds in an equilibrium with a finite size of R&D activity:

$$D_t + B = \eta. \tag{10}$$

We next consider no-arbitrage conditions. We represent the market value of a firm's stock created by R&D activity at period s at the beginning of period t as

$$V(s,t) = D_t + B(s,t) \qquad (s \in [-1,t]). \tag{11}$$

The market value of intermediate goods firms $V(s,t)$ is related to the risk-free interest rate r_{t+1}. Shareholders of intermediate goods firms who purchased these shares during period t at price $V(s,t)$ obtain dividends of π_{t+1} during period t + 1 and can sell these shares to the subsequent generation at a value of $V(s,t+1)$. In the financial market, the rate of return on holding this stock must be equal to the risk-free interest rate 1+ r_{t+1}, which implies the following no-arbitrage condition: for all t, the return on one unit of the stock must be equal to the interest rate:

$$\frac{\pi_{t+1} + V(s,t+1)}{V(s,t)} = 1 + r_{t+1}. \tag{12}$$

By substituting (10) and (11) into (12), we can obtain the interest rate

$$r = \frac{\pi}{\eta - B}. \tag{13}$$

Thus, the interest rate is constant over time. Using (8), (11) and (12), the initial bubble grows at the interest rate.

$$B(s,t+1) = (1+r)B(s,t) \qquad (s \in [-1,t]). \tag{14}$$

Above equation (14) implies that the bubble on a stock must grow at the market interest rate to satisfy the no-arbitrage condition.

2.2.4. Households

We refer to the first and second periods of household' lifetimes as young and old, respectively. The cohort born in period *t* is called generation *t*. We normalize the number households to one, so the total number of household is two and constant over time during any period in the economy. A household derives utility from consumption during young C_t^y and consumption during old C_{t+1}^o. The lifetime utility of generation *t* is

$$U_t = \log C_t^y + \beta \log C_{t+1}^o. \tag{15}$$

where β is the discount factor. During the first period, the young household is endowed with *L* units of labor and supply it inelastically. Households allocate wage income and transfers from the government to consumption and savings to maximize their lifetime utility. The young household allocates its savings to interest-bearing and pure bubble assets. The young will buy pure bubble assets only if they can resell them at a positive price to the unborn young of the next generation. In the second period, the household spends their savings on old-age consumption. Therefore, the budget constraints for generation *t* are expressed as follows:

$$C_t^y + S_t + P_t m_t = w_t L + \tau_t. \tag{16}$$

$$C_{t+1}^o = (1+r)S_t + P_{t+1} m_t. \tag{17}$$

where S_t is the interest-bearing asset, m_t is the demand for pure bubble assets, P_t is the price of pure bubble assets at time *t* in real terms of final goods and τ_t is government transfer at time *t*. $P_t m_t$ is the real value of pure bubble assets at time *t*. To hold pure bubble assets in equilibrium, the price of pure bubble assets must satisfy the following arbitrage condition:

$$\frac{P_{t+1}}{P_t} = 1 + r, \tag{18}$$

Equation (18) implies that the rate of return on one unit of a pure bubble asset equals the rate of return on one unit of interest-bearing assets. Solving the households' maximization problem, we obtain the following optimal plan for savings:

$$S_t + P_t m_t = \frac{\beta}{1+\beta}(w_t L + \tau_t). \tag{19}$$

2.2.5. Government

Following Futagami and Shibata (2000), we consider the case where the government supplies an intrinsically useless asset to households. The government gives pure bubble assets to the old household of first generation (-1) at time 0 and keeps the expansion rate of pure bubble assets constant at rate μ, and distributes it to each household. That is, a new pure bubble asset is distributed to households as a lump-sum transfer payment. Then, the supply of pure bubble assets is

$$M_t = (1+\mu) M_{t-1}, \tag{20}$$

where M_t is the total nominal supply of pure bubbly assets. The government's flow budget constraint at period *t* in real terms of final goods is

$$\tau_t = P_t \mu M_{t-1}. \tag{21}$$

2.2.6. Aggregate stock bubbles

We define the market value of total stock assets at the beginning of period t as

$$W_t = \sum_{s=-1}^{t-1} V(s,t)(N_{s+1} - N_s) = D_t N_t + B_t^A \tag{22}$$

where $B_t^A \equiv \sum_{s=-1}^{t-1} B(s,t)(N_{s+1} - N_s)$. B_t^A represents the aggregate stock bubble.

In this study, new bubbles appear in the economy when the growth rate is strictly positive. Using the definition of B_t^A and (14), we obtain the following dynamics of the aggregate stock bubble

$$B_{t+1}^A = (1+r)\left(B_t^A + B(N_{t+1} - N_t)\right). \tag{23}$$

2.2.7. Equilibrium

Consider the equilibrium dynamics of the economy. First, we derive the equilibrium dynamics of pure bubbles. The pure bubble equilibrium condition is:

$$m_t = M_t. \tag{24}$$

Using the arbitrage condition (18), we have the dynamics of pure bubbles:

$$P_{t+1}M_{t+1} = (1+r)(1+\mu)P_tM_t. \tag{25}$$

Using $b_t^m \equiv P_tM_t / N_t$, we obtain the dynamics of the normalized pure bubbles as follows:

$$b_{t+1}^m = \frac{(1+r)(1+\mu)}{1+g_t} b_t^m, \tag{26}$$

where $g_t \equiv (N_{t+1} - N_t)/N_t$ is the growth rate of the variety. Next, we consider the dynamics of aggregate stock bubbles. Dividing (25) by N_t, we obtain the dynamics of the normalized aggregate stock bubbles as follows:

$$b_{t+1}^s = \frac{(1+r)}{(1+g_t)}\left(b_t^s + Bg_t\right), \tag{27}$$

where $b_t^s \equiv B_t^A/N_t$ is the normalized aggregate stock bubble. For simply, we call b^s stock bubbles in the following.

Next, we derive the equilibrium growth rate of the production variety in this economy. The final goods market equilibrium condition is

$$Y_t = C_t^y + C_t^o + x_tN_t + I_t^R, \tag{28}$$

Using equation (23), we can obtain the following asset market equilibrium condition (the derivation is provided in Appendix A):[3]

$$S_t = DN_t + B_t^A + \eta(N_{t+1} - N_t). \tag{29}$$

Equation (29) implies that the interest-bearing assets consist of the existing stocks held by the old household at the beginning period t, $DN_t + B_t^A$, and the investing in the R&D sector. On

[3] Tanaka and Iwaisako (2011) and Tanaka (2011) show an analogous asset market equilibrium condition.

the other hand, using (19) and (21), the holdings of the interest-bearing assets S_t can be given by:

$$S_t = \frac{\beta}{1+\beta}(w_t L + \tau) - P_t M_t. \tag{30}$$

By dividing equation (29) by N_t, and substituting (10) and (30) into (29), we can obtain the growth rate of the economy[4]:

$$g_t = \frac{\Gamma + B - \eta - ab_t^m - b_t^s}{\eta}, \tag{31}$$

where $\Gamma \equiv \frac{\beta}{1+\beta}(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L$, $a \equiv \frac{1+\beta+\mu}{(1+\beta)(1+\mu)}$. From the (31), the growth rate of the equilibrium in which pure and stock bubbles do not arise is given by:

$$\mathrm{g}(b_m, b_s = 0) = \frac{\Gamma - \pi/r}{\eta}, \tag{32}$$

where $\mathrm{g}(b^m, b^s = 0)$ is the growth rate of an economy in the bubble-less equilibrium in which pure and stock bubbles do not arise. We assume, like Oliver (2000) and Tanaka (2011), the following condition:

$$g(b^n, b^s = 0) > r^{B=0} \Leftrightarrow \Gamma - \frac{\pi}{r} - \pi > 0, \tag{33}$$

where $r^{B=0}$ denotes the market interest rate in the bubble-less equilibrium. As we see the later, this condition guarantees a positively valued stock bubble in equilibrium. From (31), we find a negative relationship between the growth rate and pure and stock bubbles (b_t^m, b_t^s). This property is the crowding out effect of bubbles, as in Grossman and Yanagawa (1993). On the other hand, there is a positive relationship between the growth rate g_t and the initial bubble *B*. This relationship represents the growth enhancing effect, as in Olivier (2000) and Tanaka (2011).

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[4] where we use $w_t L = (1-\alpha)Y_t = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} A^{\frac{1}{1-\alpha}} L N_t$

2.3. Pure and stock Bubbles

2.3.1. The condition for pure and stock bubbles

In this section, we derive the conditions under which pure and stock bubbles coexist in an equilibrium. From the dynamics of the pure bubbles described by equation (26), we obtain two types of steady states for pure bubbles: a positive pure bubble equilibrium ($b^m > 0$) and a pure bubble-less equilibrium ($b^m = 0$).

First, we consider the case of b^m=0, which means that only stock bubbles exist and pure bubbles do not arise in the steady state equilibrium, as in Tanaka (2011). We present the results in this scenario for later reference, though do not claim originality here. Using (31) and b^m=0, the growth rate in the equilibrium in which only stock bubbles arise is

$$g_t(b^m = 0) = \frac{\Gamma + B - \eta - b_t^s}{\eta}, \tag{34}$$

By substituting (13) and (34) into (27), we then have the dynamics of stock bubbles:

$$b_{t+1}^s = \frac{\eta}{\Gamma + B - b_t^s}\left(1 + \frac{\pi}{\eta - B}\right)\left[B\left(\frac{\Gamma + B - \eta - b_t^s}{\eta}\right) + b_t^s\right]. \tag{35}$$

The dynamics described by equation (35) has two steady state equilibria with only stock bubbles. Let E_1 and E_2 be the steady states with stock bubbles, respectively.[5] The effects of a change in the initial bubble (B) are quite different. In the steady state equilibrium, the following hold:

$$\frac{\partial g_1}{\partial B} < 0, (E_1) \tag{36}$$

$$\frac{\partial g_2}{\partial B} > 0, (E_2) \tag{37}$$

Hence, in the steady state equilibrium E_2, the initial bubble (B) has growth enhancing effect.

Next, we investigate the condition for pure and stock bubbles to coexist in the equilibrium. First, suppose an expansion rate of pure bubbles of zero ($\mu = 0$); then, the dynamics of a pure bubble is

[5] See Appendix B for the mathematical derivation of the steady state equilibrium and its properties.

$$b_{t+1}^{m} = \frac{1+r}{1+g_t} b_t^{m},\tag{38}$$

The dynamics of pure bubbles (38) imply that, when the market interest rate equals the economy's growth rate ($g = r$), pure bubbles can exist in a steady state equilibrium at a positive value (b^m>0). Using equation (29), the steady state of stock bubbles is

$$b^{s} = \frac{(1+r)Bg}{g-r}.\tag{39}$$

Equation (39) implies that the steady state of stock bubbles can exist only if the economy's growth rate is higher than the market interest rate (g $> r$). Hence, if the expansion rate of pure bubbles is zero ($\mu = 0$), pure and stock bubbles do not coexist in the steady state equilibrium.

Next, we suppose a positive expansion rate of pure bubbles ($\mu > 0$). Then, from (26), we obtain the following condition for pure bubbles to arise in the steady state equilibrium:

$$g(b^{m}, b^{s} > 0) = r + \mu + r\mu,\tag{40}$$

where $g(b^m, b^s > 0)$ is the growth rate in the steady state equilibrium with pure and stock bubbles. The growth rate depends on the market interest rate and the expansion rate of pure bubbles. Equation (40) implies that the growth rate is higher than the market interest rate; thus, the steady state of an aggregate stock bubble is defined by positive value because the aggregate stock bubbles are positive when g $> r$ (See equation (39)). We can obtain the equilibrium stock bubble value by substituting (13) and (40) into (39) as follows:

$$b^{s} = \frac{B}{\mu}\left(\frac{\pi}{\eta - B}(1+\mu) + \mu\right).\tag{41}$$

By substituting (13), (40) and (41) into (31) and then rearranging the terms, we can obtain the value of equilibrium pure bubbles:

$$b^{m} = \frac{1}{a}\left(\Gamma - \eta(1+\mu)\left(1 + \frac{\pi}{\eta - B}\right) - \frac{B(1+\mu)}{\mu}\frac{\pi}{\eta - B}\right)\tag{42}$$

From equation (42), a positive pure bubble equilibrium requires

$$\Gamma > \eta(1+\mu)\left(1 + \frac{\pi}{\eta - B}\right) + \frac{B(1+\mu)}{\mu}\frac{\pi}{\eta - B}.\tag{43}$$

Therefore, we have the following proposition:

Proposition 1: *If the expansion rate of pure bubbles is zero ($\mu = 0$) , pure and stock bubbles can not coexist in the steady state equilibrium. If the expansion rate of pure bubbles is positive* $\mu > 0$ *and equation (43) is satisfied, then pure and stock bubbles can coexist in an economy.*

2.3.2. The property of steady states

This subsection analyzes the properties of the steady state equilibrium with pure and stock bubbles. In the steady state equilibrium with two types of bubbles, we can use equations (13) and (40), to obtain the following expression:

$$g(b^m, b^s > 0) = \frac{\pi}{\eta - B}(1+\mu) + \mu. \tag{44}$$

In the equilibrium with pure and stock bubbles, the growth rate positively depends on the initial bubble (*B*) and the expansion rate of pure bubbles (μ). Here, we consider the effect of permanent change in the initial bubble (*B*) and the expansion rate of pure bubbles on the growth rate. Using equations (13), (41), (42), and (44), we obtain the followings:

$$\frac{\partial r}{\partial B} > 0, \frac{\partial b^s}{\partial B} > 0, \frac{\partial b^m}{\partial B} < 0, \frac{\partial g}{\partial B} > 0, \frac{\partial g}{\partial \mu} > 0. \tag{45}$$

Thus, in the steady state equilibrium with pure and stock bubbles, the initial bubble has a growth enhancing effect. A change in the initial bubble (*B*) has three effects on the growth rate. First, an increase the initial bubble increase the stock bubbles (b^s), which strengthens the quantity of the crowding out effect of stock bubbles. Second, the initial bubble lowers the pure bubble, which weakens the crowding out effect of pure bubble on the growth rate. Finally, an increase in the initial bubble increase the return on investing in the R&D sector, and has a positive effect on the growth of variety; that is a growth enhancing effect. In the steady state equilibrium with pure and stock bubbles, the positive effects of the change in the initial bubble dominate the negative crowding out effect; hence, an increase the initial bubble enhances the growth rate. The increase in the pure bubble expansion rate (μ) increases the growth rate in the steady state equilibrium with both bubbles. Futagami and Shibata (2000) also find this positive effect on the growth rate in the steady state. When the government distributes a new pure bubble asset to households as a lump-sum transfer payment, households believe they are wealthier and want to save more. This leads to a higher economic growth rate with two types of bubbles. Therefore, we obtain the following proposition;

Proposition 2: *In the steady state equilibrium with pure and stock bubbles, the initial bubble B and the expansion rate of pure bubbles μ have a growth enhancing effect.*

In addition, we can consider the effects of the interaction between two types of assets bubbles. From (44), we obtain the following property:

$$\frac{\partial^2 g}{\partial \mu \partial B} = \frac{\pi}{(\eta - B)^2} > 0. \tag{46}$$

In the steady state with pure and stock bubbles, the growth enhancing effect of the initial bubble (*B*) is increased by the expansion rate of pure bubbles (μ).

2.3.3. Dynamics of the two types of bubbles

This subsection analyzes the dynamics of the equilibrium path of pure and stock bubbles described by (26) and (27). We refer to the locus on the plane (b_t^s, b_t^m) representing $b_{t+1}^m = b_t^m$ as the b^m locus and $b_{t+1}^s = b_t^s$ as the b^s locus. Substituting (13) and (31) into (26) gives the dynamics of pure bubbles:

$$b_{t+1}^m = b^m(b_t^m, b_t^s) \Leftrightarrow b_{t+1}^m = \frac{\eta(1+\mu)}{\Gamma + B - ab_t^m - b_t^s}\left(1 + \frac{\pi}{\eta - B}\right)b_t^m. \tag{47}$$

We transform the dynamics above as follows:

$$b_{t+1}^m \geq b_t^m \Leftrightarrow \begin{cases} b_t^m \geq \Omega(b_t^s) \equiv -\frac{1}{a}b_t^s + \frac{1}{a}\left[\Gamma + B - \eta(1+r)(1+\mu)\right] \\ b_t^m = 0 \end{cases}. \tag{48}$$

The b^m locus has two lines. Figure 1 shows the dynamics of pure bubbles (48). Using (13), (27), and (31), we can derive the dynamics of stock bubbles as follows;

$$b_{t+1}^s = b^s(b_t^m, b_t^s) \Leftrightarrow b_{t+1}^s = \frac{\eta}{\Gamma + B - ab_t^m - b_t^s}\left(1 + \frac{\pi}{\eta - B}\right)\left[B\left(\frac{\Gamma + B - \eta - ab_t^m - b_t^s}{\eta}\right) + b_t^s\right]. \tag{49}$$

Furthermore, using (49), we can get the following expression:

$$b_{t+1}^s \geq b_t^s \Leftrightarrow b_t^m \geq \Phi(b_t^s) \equiv \frac{-{b_t^s}^2 + \left(\Gamma + B - (\eta - B)(1+r)\right)b_t^s - (\Gamma + B - \eta)(1+r)B}{a\left(b_t^s - B(1+r)\right)}, \tag{50}$$

Figure 2 shows the dynamics of stock bubbles (49). The steady state equilibrium with two types of asset bubbles is depicted at point *E* in the Figure 3. E_1 and E_2 denote the steady state equilibrium in which only stock bubbles only. We can show that the equilibrium path to point E, E_1, and E_2 are the saddle point, sink, and source in the neighborhood of steady state equilibrium, respectively.

Finally, we compare growth rates in steady states. Using (10), (31), and (32), we obtain the following relationship:

$$\mathrm{g}(b^m, b^s > 0) = \mathrm{g}(b^m, b^s = 0) - \frac{b^s + b^m}{\eta}. \tag{51}$$

Because the second term of (51) is negative, we obtain the following relationship:

$$g(b^m, b^s = 0) > g(b^m, b^s = 0). \tag{52}$$

Thus, the growth rate in the steady state equilibrium with two types of bubbles is lower than that of a bubble-less steady state equilibrium.

2.4. Conclusion

In this chapter, we developed an overlapping-generation model with pure and stock bubbles and examined the conditions for the existence of both types. In contrast to previous studies that only consider one type of bubbles, we introduce two types into an endogenous growth model. We show both types can coexist in a steady state equilibrium. When the total supply of pure bubbles is constant, the existence of pure bubbles requires that the growth rate of the economy equal the market interest rate. On the other hand, stock bubbles can exist only if the growth rate is greater than the market interest rate. Thus, pure and stock bubbles can-not coexist in the equilibrium. However, if the total supply of pure bubbles grows at a constant rate, both can coexist in the economy. In addition, a steady state equilibrium with two types of bubbles has a growth enhancing effect. A permanent change in the initial bubble and an expansion of the pure bubble positively affects the growth rate. In addition, pure and stock bubbles interact. In a steady state equilibrium with pure and stock bubbles, the degree of the growth enhancing effect of the initial bubble depends on the expansion rate of the pure bubble.

Appendix

Appendix A: Asset market equilibrium condition (29) derivation

Using (1), (2), (6), and fact that $\pi_t^y = 0$ hold s in the equilibrium, the output of final goods is

$$Y_t = w_t L + p^x x N_t. \tag{A1}$$

Thus, we express the final goods market equilibrium condition as follows:

$$w_t L + p^x x N_t = C_t^y + C_t^o + x_t N_t + I_t^R, \tag{A2}$$

Using $\pi = (p^x - 1)x$, (9), (10), (16),(17),(20),(21) and (24),

$$S_t + P_t M_t + (p^x - 1)xN_t = (1+r)S_{t-1} + P_t(1+\mu)M_{t-1} + \eta(N_{t+1} - N_t)$$

$$\Leftrightarrow S_t + \pi N_t = (1+r)S_{t-1} + \eta(N_{t+1} - N_t)$$

$$\Leftrightarrow S_t + \pi N_t = (1+r)S_{t-1} + (D_t + B)(N_{t+1} - N_t)$$

$$\Leftrightarrow S_t - D_t N_{t+1} - B(N_{t+1} - N_t) = (1+r)(S_{t-1} - D_{t-1}N_t) \tag{A3}$$

Adding the term $-B_t^A$, and using (14) and (23), we can express (A3) as follows:

$$S_t - D_t N_{t+1} - B_t^A - B(N_{t+1} - N_t) = (1+r)[S_{t-1} - D_{t-1}N_t - B_{t-1}^A - B(N_t - N_{t-1})]$$

Because initial assets are given by $S_{-1} = (D_{-1} + B)N_0$, we can have (29) for any period *t*.

Appendix B: Mathematical derivation of the steady state equilibrium with stock bubbles.

we can transform equation (35) as follows:

$$f(b^s) = b^{s2} - \left(\Gamma - \frac{\pi}{r} - \pi + B\right)b^s + \left(\Gamma - \frac{\pi}{r}\right)(1+r)B. \tag{B1}$$

Equation (B1) has the following solutions;

$$b^s = \frac{(\Gamma - \pi/r - \pi + B) \pm \sqrt{(\Gamma - \pi/r - \pi + B)^2 - 4(1+r)(\Gamma - \pi/r)B}}{2}. \tag{B2}$$

Equation (B2) has dual stationary stock bubbles equilibria if the following three conditions are satisfied:

$$\begin{aligned} &f(0) = \Gamma - \pi/r > 0 \\ &f'(0) = -(\Gamma - \pi/r - \pi + B) < 0 \\ &F = (\Gamma - \pi/r - \pi + B)^2 - 4(1+r)(\Gamma - \pi/r)B > 0 \end{aligned} \tag{B3}$$

where *F* is the determinant of (B2). We assume the following condition, as in Oliver (2000) and Tanaka (2011):

$$g(b^n, b^s = 0) > r^{B=0} \Leftrightarrow \Gamma - \frac{\pi}{r} - \pi > 0, \tag{B4}$$

where $g(b^m, b^s = 0)$ and $r^{B=0}$ denote the growth rate and the market interest rate in the bubble-less equilibrium, respectively. This condition guarantees a positively valued stock bubble in equilibrium. Using (B4), we can verify that $f(0) > 0$ and $f'(0) < 0$. The sign of the determinant F depends on the size of initial bubble (B). We can show the following:

$$F'(B) < 0 \text{ and } F(0) \tag{B5}$$

This implies that the sign of the determinant is positive for a relatively small value of the initial bubble.

Appendix C: The dynamics of aggregate stock bubbles (47)

Differentiating (51) with respect to b^s yields:

$$\Phi'(b^s) = -\frac{1}{a} + \frac{(1+r)\eta Br}{a\left(b^s - B(1+r)\right)^2}. \tag{C1}$$

From equation (C1), we obtain the following:

$$\Phi'(b^s) = 0 \Leftrightarrow \begin{aligned} b_1^s &= B(1+r) + \sqrt{\eta Br(1+r)} \\ b_2^s &= B(1+r) - \sqrt{\eta Br(1+r)} \end{aligned} \tag{C2}$$

Additionally, from (C2), the following condition are satisfied:

$$\begin{aligned} \eta r > B(1+r) \Leftrightarrow b_2^s < 0 \\ \eta r < B(1+r) \Leftrightarrow b_2^s > 0 \end{aligned} \tag{C3}$$

Figure 4 shows illustrates (C1).

Differentiating (C2) with respect to b^s, yields

$$\Phi''(b^s) = -\frac{2\eta Br(1+r)}{a\left(b^s - B(1+r)\right)^3}. \tag{C4}$$

The sign of (C4) is:

$$\begin{aligned} b^s < B(1+r) \Leftrightarrow \Phi''(b^s) > 0 \\ b^s = B(1+r) \Leftrightarrow \Phi''(b^s) = \infty \\ b^s > B(1+r) \Leftrightarrow \Phi''(b^s) < 0 \end{aligned} \tag{C5}$$

Using the definition of the normalized aggregate stock bubble (b^s), $b^s > (1+r)B$ holds in this economy. The b^s locus represents the inverse -U shape in the region $b^s > (1+r)B$.

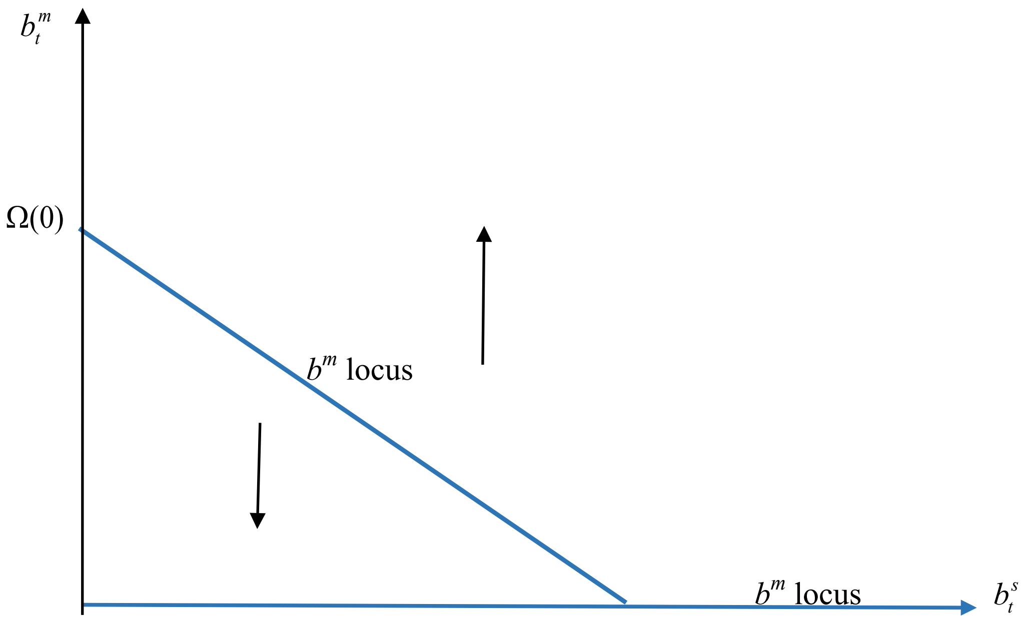

Figure 1. The dynamics of pure bubbles

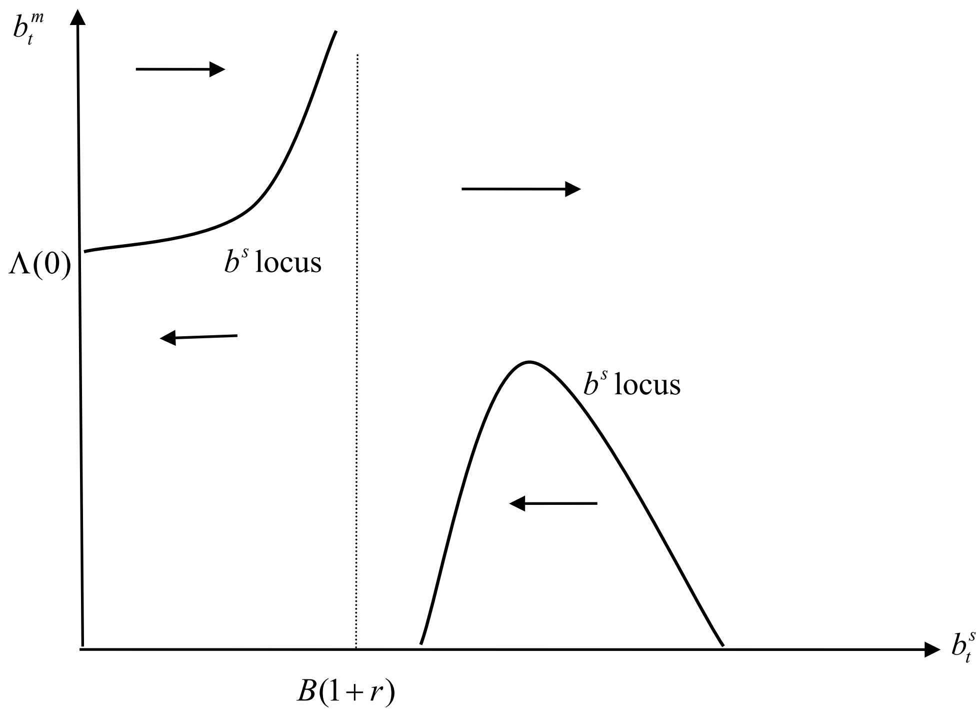

Figure 2. The dynamics of aggregate stock bubbles

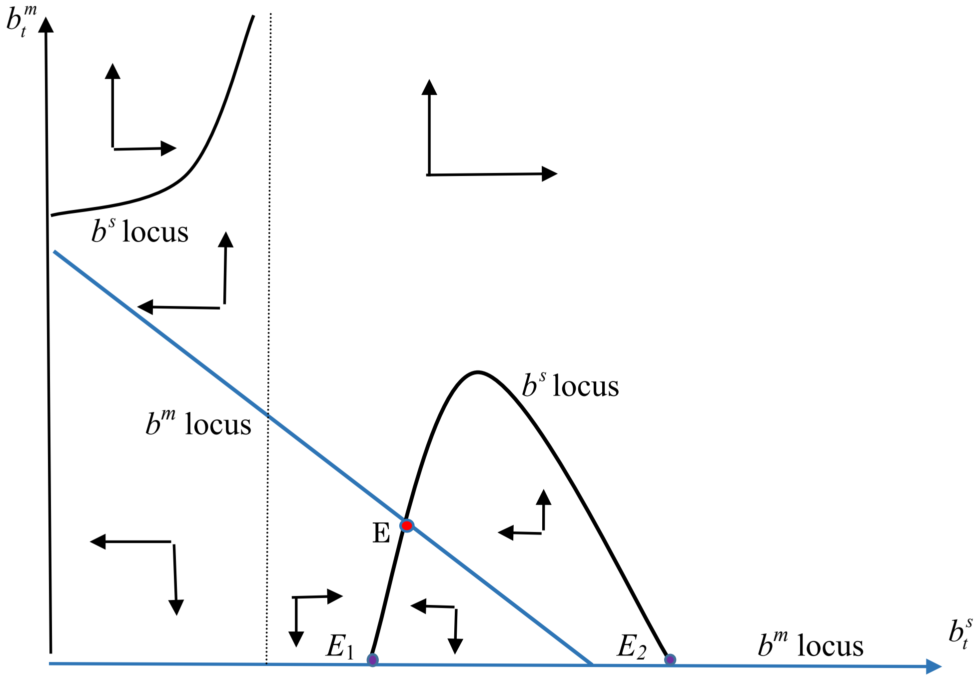

Figure 3. Dynamics of pure and aggregate stock bubbles

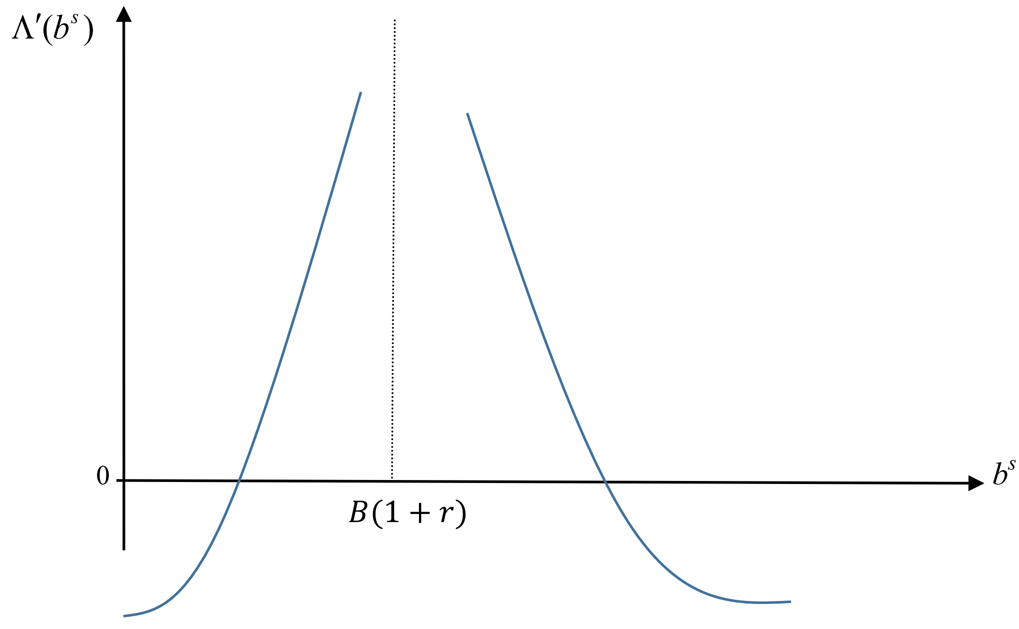

Figure 4.1. The function (C1), ($\eta r > B(1+r)$)

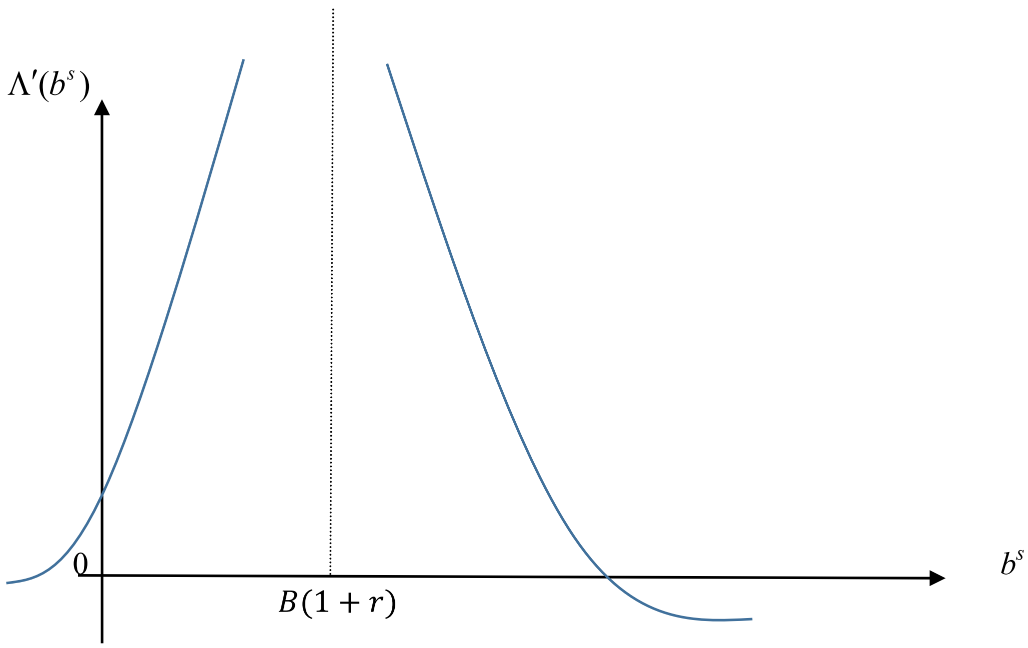

Figure 4.2. The function (C1), ($\eta r < B(1+r)$)

3. Bubbles and unemployment in an endogenous growth model[†]

3.1. Introduction

Economic bubbles have occurred throughout history, often with major impacts on the economies of the countries concerned. These bubbles are frequently observed when economic activities are booming and GDP growth rates are high (Martin and Ventura, 2012; Farmer and Schelnast, 2013, ch. 6). Indeed, empirical studies have shown that there is a negative relationship between stock market wealth and unemployment. For example, the US stock market boom of the 1990s was accompanied by a reduction in the unemployment rate (Phelps, 1999). In addition, a similar correlation between the stock market and unemployment has been found to hold for numerous European countries (Fitoussi et al., 2000). Kunieda and Shibata (2016) provide historical observations of the negative correlation between asset bubbles and economic recessions for Japan and the US. Despite this empirical evidence, however, no study has addressed the theoretical relationships among unemployment, asset bubbles, and economic growth. The purpose of this study is to examine these relationships analytically.

In his seminal study, Tirole (1985) examines the effects of bubbles on intrinsically useless assets in an overlapping generations model. Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) extend Tirole's (1985) model to an endogenous growth framework. These scholars re-examine the necessary conditions for the existence of bubbles and investigate the relationship between bubbles and economic growth. These studies find that when asset bubbles occur, they divert savings from capital accumulation and thereby retard economic growth. Taking an alternative approach, Olivier (2000) considers bubbles that are tied to capital goods, rather than useless assets, and shows that bubbles can have a positive effect on economic growth. A key feature of the above studies is that they do not consider the influences of unemployment.

There is now a broad literature which argues that unemployment is created by frictions in the labour market. Diamond (1982), Mortensen (1982), and Pissarides (1985) develop search and matching models of unemployment, which scholars have since applied in a wide variety of fields.[6] Eriksson (1997) introduces labour market frictions into the standard dynamic

[†] This chapter is based on Hashimoto and Im (2016): "Bubbles and unemployment in an endogenous growth model", *Oxford Economic Papers*, 68 (4), pp. 1084-106.

[6] See Pissarides (2000) for an introduction to search friction models.

optimizing (Ramsey) model with capital stock externalities generated by learning by doing that ensure long-run economic growth. He then examines the effects of various policies (e.g., capital taxes and unemployment benefits) on economic growth and unemployment.[7]

Unlike the overlapping-generations model, rational bubbles cannot be generated in an infinitely-lived, representative-agent (Ramsey-type) model (Tirole, 1985; Santos and Woodford, 1997).[8] Other strands of the literature on asset bubbles with rational agents take a microeconomic approach. Allen et al. (1993), Conlon (2004), and Doblas-Madrid (2012) examine bubbles based on market-timing games under asymmetric information. Allen and Gorton (1993), Allen and Gale (2000), and Barlevy (2014) develop models of credit-driven bubbles by focusing on agency problems.[9] These studies, however, do not treat the determinants of unemployment and economic growth endogenously.

To the best of our knowledge, no study has introduced an endogenous growth model to analyse the relationship between asset bubbles and unemployment. To fill this void, this paper presents a theoretical framework to examine the necessary conditions for the existence of bubbles in an economy with endogenous unemployment, and to investigate the relationships between unemployment, bubbles, and economic growth. To this end, we merge the endogenous growth and labour market friction approach of Eriksson (1997) with the continuous-time overlapping-generations model of Weil (1989).

In our framework, unemployment arises as result of labour market frictions, and labour market efficiency is reflected in the interest rate, as the marginal productivity of capital influences the interest rate. Because asset returns are linked with the interest rate, the existence of asset bubbles depends on labour market conditions. Specifically, we find that unemployment is a key factor in the existence of bubbles; when the unemployment rate is relatively low and the interest rate is high, asset bubbles may exist in equilibrium. We define two equilibrium regimes: a “bubble” regime that features multiple equilibria which may or may not exhibit asset

[7] See Bean and Pissarides (1993), Aghion and Howitt (1994), Caballero and Hammour (1996), and Haruyama and Leith (2010) for alternative models addressing the role of labour market frictions in the relationship between growth and unemployment.

[8] As Santos and Woodford (1997) point out, it is difficult to generate rational bubbles in an infinitely-lived agent model without market frictions. For an approach that focuses on financial market frictions, see Hirano and Yanagawa (2010) and Martin and Ventura (2012). These studies examine the existence of bubbles and find that bubbles can be growth-enhancing or growth-impairing, depending on the restrictiveness of the collateral constraint.

[9] See Brunnermeier and Oehmke (2015) for a survey of the theoretical literature on bubbles.

bubbles and a "non-bubble" regime in which bubbles never occur. We show that bubbles divert savings away from physical capital accumulation and lower the output growth rate, a result which supports a common finding in the literature. Comparing the bubble regime with the non-bubble regime, however, we find that the rate of output growth is always higher under the former than under the latter.

Finally, we use the framework to study the effects of changes in labour market policy and model parameters on unemployment, asset bubbles, and economic growth. For example, we find that, because unemployment benefits raise the value of unemployment, they negatively influence employment, a standard conclusion of models with search frictions. Thus, reducing unemployment benefits increases the employment rate, making the labour market more efficient, raising the interest rate, and consequently shifting the economy from the non-bubble regime to the bubble regime. In this case, there is a negative relationship between unemployment and economic growth.[10]

The remainder of this paper is organized as follows. Section 2 outlines the features of the model. Section 3 describes the steady-state equilibrium with and without bubbles. In Section 4, the effects of policy and parameter changes under the two regimes on bubbles, economic growth, and unemployment are examined. Section 5 briefly discusses the implications of the results. Section 6 concludes.

3.2. The Model

Consider an economy with a number of infinitely-lived dynastic households. At each moment in time, new and identical dynastic households appear at a rate n. Thus, normalizing the size of each household to unity and setting the total initial population N_0, the total population of households at time t is $N_t = N_0 e^{nt}$. For the remainder of the paper, we suppress time notation when it is not required for the exposition.

3.2.1. Matching

In the labour market, unemployed workers and firms with vacant positions strive to find each other. In our framework, unemployment is generated by matching frictions. Denoting the

[10] Many empirical studies find a negative relationship between unemployment and economic growth (Ball and Moffitt, 2001; Muscatelli and Tirelli, 2001; Staiger et al., 2001; Tripier, 2006; Pissarides and Vallanti, 2007).

number of successful matches between unemployed workers and firms as f, the matching process is described by the following matching function:

$$f(uN, \upsilon N),$$

where u and υ represent the unemployment rate and the vacancy rate. As such, uN is the number of unemployed workers and υN is the number of vacant jobs in the economy. Following the standard assumptions of the search literature,[11] the matching function is assumed to be concave, homogeneous of degree one, and increasing in both of its arguments. Defining the tightness of the labour market as

$$\theta \equiv \frac{\upsilon N}{uN} = \frac{\upsilon}{u},$$

the probability that a firm with vacancies is matched with an unemployed worker has the following property:

$$\frac{f(uN, \upsilon N)}{\upsilon N} = f\left(\frac{1}{\theta}, 1\right) \equiv q(\theta), \text{ where } \frac{\partial q(\theta)}{\partial \theta} < 0.$$

3.2.2. Firms

We assume that the number of firms equals the number of households N. The production function of firm j is described by

$$y_j = Ak_j^{\alpha}(\bar{k}l_j)^{1-\alpha}, \; 0 < \alpha < 1,$$

where A, k_j, and l_j represent the productivity, capital stock, and number of workers employed by firm j. Labour productivity is captured by $\bar{k}$, and is assumed to rise over time as a result of spillovers that emanate from the firm's accumulated investment per worker, similar to the spillovers proposed by Romer (1986). To ensure the existence of a long-run growth path, we assume that $\bar{k}$ takes the form $\bar{k} = \left(\int_0^N k_j dj\right) \Big/ N$, which represents the average capital stock.

To create matches, firm j must advertise its job vacancies. If firm j has v_j vacancies, then $q(\theta)v_j$ workers are hired by firm j at each moment of time.[12] Furthermore, firm j fires or loses

[11] See Petrongolo and Pissarides (2001) for a discussion on matching functions.

[12] As θ is a given for all firms, the probability $q(\theta)$ is the same for all firms.

workers at a rate of δl_j, where δ represents the exogenous separation rate. Summing these two flows, the labour force size changes according to the following equation:

$$\dot{l}_j = q(\theta)v_j - \delta l_j, \tag{1}$$

where a dot over a variable denotes differentiation with respect to time. In accordance with Eriksson (1997), Mortensen and Pissarides (1999), and Pissarides and Vallanti (2007), the cost of recruiting a worker for a vacancy is assumed to be proportional to the wage rate, γw, where w is the wage rate and γ is a cost parameter.[13] To keep the model simple, we do not consider an adjustment cost for investment. Then, firm j's profit maximization problem can be written as

$$\max \int_t^{\infty} \left(y_j - rk_j - wl_j - \gamma w v_j\right) e^{-\int_t^i r_\iota d\iota} di, \text{ subject to (1)}, \tag{2}$$

where r represents the interest rate.

3.2.3. Households

We model households following the approach of Weil (1989). Although the first generation has an initial set of assets, new households enter the economy with no asset wealth. The first generation of households is distinguished from those born at time 0 (denoted 0^+) by denoting them as 0^-. The lifetime utility of a representative household in generation s is given by $\int_t^{\infty} \ln c(s,i) e^{-\rho(i-t)} di$, where $c(s, t)$ represents the consumption of generation s at time t, and ρ is the subjective discount rate.

To eliminate any uncertainty regarding employment, the model assumes that each household has a large number of members normalized to unity, with $(1-u)$ employed members and u unemployed members. Then, each household receives an expected wage income from production of $w(1-u)$ and unemployment benefits of λwu, which are proportional to the wage rate, $\lambda \in (0, 1)$. Each household allocates its total assets (z) between physical capital (k) and the bubble asset (m), where the bubble asset is an intrinsically useless paper asset, specifically money. The price of the bubble asset in terms of goods ($1/p$) must satisfy the arbitrage condition $(\dot{1/p})/(1/p) = -\dot{p}/p = r$ or, rather, the return on one unit of the bubble asset must equal r. This

[13] This assumption is required to ensure a balanced growth path.

relationship leads to the following flow/stock budget constraint expressed in terms of real goods:[14]

$$\frac{\mathrm{d}z(s,t)}{\mathrm{d}t} = rz(s, t) + w(1 - u) + \lambda wu + v^I - \tau - c(s, t), \tag{3}$$

$$z(s, t) = k(s, t) + m(s, t),$$

where $z(s, t)$ represents the total asset holdings of generation s at time t, τ is a lump-sum tax, and v^I is the income from vacancies defined as $v^I = \left(\int_0^N \gamma w v_j dj \right) \Big/ N$. Thus, the optimization problem of a representative household in generation s is given by

$$\max \int_t^\infty \ln c(s,i) e^{-\rho(i-t)} di \text{, subject to (3).} \tag{4}$$

3.2.4. Government

The government supplies a useless paper asset, B, which is priced in terms of goods at $1/p$. We define the real value of the supply of this asset as $M \equiv B/p$. The government gives the asset to the first generation (0^-) at time 0 and continues to supply it to each household at a constant rate of μ ($= \dot{B}/B$). Since the government's real revenue is the sum of the lump-sum tax (τN) plus the bubble expansion (μM), and its expenditure is the unemployment benefit (λwuN), the government budget constraint is

$$\tau N + \mu M = \lambda wuN. \tag{5}$$

3.2.5. Market clearing conditions

The aggregate variable X_t is defined as follows (Weil, 1989):

$$X_t \equiv x(0^{-1}, t) N_0 + \int_0^t x(s,t) n e^{ns} N_0 ds.$$

Using the above definition, we formulate the economy-wide dynamics for total assets and bubble assets as follows:

$$\dot{Z} = rZ + \omega N - C, \tag{6}$$

$$\dot{M} = (\mu + r) M, \tag{7}$$

[14] See Appendix 1 for a detailed explanation of the derivation of the flow/stock budget constraint.

where $Z = K + M$ and $\omega \equiv w(1 - u) + \gamma wv + \mu M/N$. See Appendix 2 for the derivations of the above equations. Total output is defined as $Y \equiv \int_0^N y_j dj$. Therefore, the goods market equilibrium condition is[15]

$$Y = C + \dot{K}, \tag{8}$$

where the left side is total output (Y) and the right side comprises aggregate consumption (C) and capital accumulation ($\dot{K}$).

3.2.6. Definition of equilibrium on a balanced growth path

In this section, we provide a definition for the balanced growth path of the economy. An equilibrium consists of prices (r, p, w) and allocations for firms (k_j, l_j, v_j) and for households ($c(s, t)$, $z(s, t)$). These variables must satisfy the following conditions: (i) firms solve the optimization problem in (2), (ii) households solve the optimization problem in (4), (iii) the governmental budget constraint (5) holds, (iv) the goods market (8) clears, and (v) the wage is determined by negotiation between workers and firms (as given in (16) and discussed in Subsection 2.7).

A balanced growth path in a steady state is an equilibrium in which the interest rate (r), unemployment rate (u), and labour market tightness ($\theta \equiv \upsilon/u$) are constant over time, whereas total output, aggregate consumption, and physical capital grow at the same rate, while aggregate supply (Y) and aggregate demand (C and $\dot{K}$) grow proportionally.

3.2.7. Solving the model

In this subsection, we solve the optimization problems for households and firms. First, consider the optimization problem (4) of a representative household in generation s. The solution to the inter-temporal optimization problem is the Euler equation:

$$\frac{\mathrm{d}c(s,t)}{\mathrm{d}t} = (r - \rho)c(s,t), \tag{9}$$

with the transversality condition given by $\lim_{t\to\infty} z(s,t)e^{-\int_s^t r_i di} = 0$. Using (3) and (9), we obtain the following consumption function:

[15] See Appendix 3 for the derivation of (8).

$$c(s, t) = \rho[z(s, t) + h_t], \quad (10)$$

where $h_t \equiv \int_t^{\infty} \left(w_i(1-u_i) + \lambda w_i u_i + v_i^I - \tau_i \right) e^{-\int_s^i r_\iota d\iota} di$.

Then, aggregating the Euler equation yields the following economy-wide dynamic equation:[16]

$$\dot{C} = (r - \rho + n)C - n\rho Z. \quad (11)$$

Next, we consider firm j's profit maximization problem (2). This inter-temporal maximization problem can be solved using the current-value Hamiltonian function $H = (y_j - rk_j - wl_j - \gamma w v_j) + \chi(q(\theta)v_j - \delta l_j)$, where χ is the current shadow value of labour. Assuming that the market shares are small enough that each firm takes average capital ($\bar{k}$) and market tightness (θ) as constants, the first-order conditions are $\partial H/\partial k_j = 0$, $\partial H/\partial v_j = 0$, and $\partial H/\partial l_j = r\chi - \dot{\chi}$. Combining these conditions yields

$$\left(\frac{\partial y_j}{\partial k_j} = \right) \; \alpha A k_j^{\alpha-1} (\bar{k} l_j)^{1-\alpha} = r, \quad (12)$$

$$\left(\frac{\partial y_j}{\partial l_j} = \right) \; (1-\alpha) A k_j^{\alpha} \bar{k}^{1-\alpha} l_j^{-\alpha} = w + \frac{\gamma w}{q(\theta)} \left(r + \delta - \frac{\dot{w}}{w} + \frac{q'(\theta)\dot{\theta}}{q(\theta)} \right). \quad (13)$$

All firms are considered identical because of the symmetry in production technology and, as such, $\bar{k} = k_j = k$ in equilibrium. Moreover, the vacancy rate is equal to the number of firm j's vacancies, $\upsilon \equiv \left(\int_0^N v_j dj \right) \Big/ N = v_j = v$. Thus, the tightness of the labour market is determined by the ratio of the number of vacancies per firm to the unemployment rate, θ ($\equiv \upsilon N/uN$) $= v/u$. Furthermore, from the definition of the employment rate, we have

$$1 - u \equiv \left(\int_0^N l_j dj \right) \Big/ N = l. \quad (14)$$

As a result, per capita output can be written as $y = Ak(1-u)^{1-\alpha}$, and the total output Y ($\equiv \int_0^N y_j dj$ $= yN$) becomes

$$Y = AK(1-u)^{1-\alpha}. \quad (15)$$

[16] See Appendix 2 for the derivation of (11).

Following previous studies that address search frictions (Pissarides, 2000), we assume that a given worker and a given firm negotiate wages after they meet. When a match is made, the firm employs the worker in production and saves on the vacancy cost. Hence, the upper boundary of the wage is the marginal benefit of labour, which is determined by the marginal product and the marginal value of the saved vacancy cost ($\partial y/\partial l + \theta\gamma w$).[17] The lower boundary, on the other hand, is a worker's opportunity income; in other words, the unemployment benefit, λw. It is assumed that negotiation between a firm and a worker results in a wage somewhere between these two extremes (Eriksson, 1997), such that $w = (1 - \beta)\lambda w + \beta(\partial y/\partial l + \theta\gamma w)$, where $\beta \in (0, 1)$ denotes the worker's bargaining power.[18] Consequently, the wage rate can be expressed as follows:

$$w = \frac{\beta}{1-(1-\beta)\lambda - \beta\gamma\theta} \cdot \frac{\partial y_j}{\partial l_j}. \tag{16}$$

3.2.8. The steady state conditions

Using (1) and (14) in conjunction with the definition $v = \theta u$ yields $\dot{u} = -\theta q(\theta)u + \delta(1 - u)$. Consequently, the following labour market equilibrium condition is satisfied in the steady state:

$$u = \frac{\delta}{\delta + \theta q(\theta)}, \tag{17}$$

where $\partial u/\partial\delta > 0$ and $\partial u/\partial\theta < 0$.[19] This expression represents the Beveridge curve, which implies that the unemployment rate rises when the separation rate increases or the labour market becomes less tight.

Also, based on (12) and using (14) and the fact that $\bar{k} = k_j = k$, the interest rate can be written as

[17] The upper boundary on the wage can be derived as follows: $\partial(y - \gamma wv)/\partial l$ with $v = \theta u = \theta(1 - l)$. This is consistent with the marginal benefit of labour with an internalized unemployment rate.

[18] This formulation of the wage equation is broadly used in the literature on unemployment that includes search frictions (Aghion and Howitt, 1994; Caballero and Hammour, 1996; Eriksson, 1997). See Pissarides (2000) and Hall (2005) for discussions on the determination of the wage equation.

[19] Based on the matching function property provided in Section 2.1, we have $\partial(\theta q(\theta))/\partial\theta > 0$, which implies $\partial u/\partial\theta < 0$.

$$r = \alpha A(1-u)^{1-\alpha}. \tag{18}$$

Therefore, a negative relationship between the unemployment rate and the interest rate is easily confirmed.

The growth rate of total output is defined as $g \equiv \dot{Y}/Y$. Then, based on (14)-(16) and the fact that $\partial y/\partial l = (1-\alpha)y/l$, in a steady state where u and θ are constant, the wage growth rate equals the per capita output growth rate ($\dot{y}/y = \dot{Y}/Y - n$), such that

$$\dot{w}/w = g - n.$$

As such, substituting the above equation and (16) into (13) yields

$$\left[\frac{(1-\beta)(1-\lambda)}{\beta\gamma} - \theta\right] q(\theta) = r + \delta - g + n. \tag{19}$$

The variables normalized by aggregate output are defined as $c \equiv C/Y$ and $m \equiv M/Y$. Using the definitions (c, m), the fact that $Z = K + M$, as well as (7), (11), and (15), and recalling that the steady-state levels of c and m are constant when $\dot{C}/C = \dot{K}/K = \dot{Y}/Y = g$ are on a balanced growth path, we derive the following equilibrium conditions:

$$(r-\rho+n-g)c - n\rho\left(\frac{1}{A(1-u)^{1-\alpha}} + m\right) = 0, \tag{20}$$

$$(\mu + r - g)m = 0. \tag{21}$$

Then, dividing (8) by Y and using (15) and (18) yields the consumption-output ratio as

$$c = 1 - g\frac{\alpha}{r}. \tag{22}$$

From this expression, it is clear that $g < r/\alpha$ is required for a positive consumption-output ratio ($c > 0$). Together (17)-(22) give the equilibrium values of c, m, g, r, u, and θ.

Considering the case in which the population growth rate is zero ($n = 0$), as in a representative-agent (Ramsey) model with one dynasty (Eriksson, 1997), we can easily confirm that $g = r - \rho$ for $c > 0$, based on (20). Consequently, based on (21), the value of the asset bubble must become zero ($m = 0$). As a result, asset bubbles cannot exist in equilibrium in a Ramsey-type representative-agent model.

3.3. The conditions for and consequences of bubbles

3.3.1. The conditions for bubbles

This section derives the conditions required for asset bubbles to exist in equilibrium. Under a bubble regime, there are two possible types of steady states: a positive bubble equilibrium ($m > 0$), and a bubble-less equilibrium ($m = 0$). Using (21), we obtain the aggregate output growth rate (g) associated with a positive bubble as

$$g = r + \mu. \tag{23}$$

In a positive bubble equilibrium, the output growth rate depends positively on the interest rate and the bubble expansion rate.

The equilibrium bubble value can be found by substituting (18), (22), and (23) into (20), and reorganizing the result:

$$m = \frac{\alpha}{n\rho r}\left[\left(\frac{r}{\alpha} - g\right)(n-\rho-\mu) - n\rho\right] = \frac{(n-\rho-\mu)(1-\alpha)}{n\rho r}\left[r - \frac{\alpha}{1-\alpha}\left(\mu + \frac{n\rho}{n-\rho-\mu}\right)\right]. \tag{24}$$

Given the unemployment rate, u (which determines the interest rate (18)), both the aggregate output growth rate and the bubble satisfy (23) and (24). Based on the first equality in (24), a positive value of m requires $n - \rho - \mu > 0$ under the condition $r/\alpha > g$ necessary for $c > 0$ from (22). From the second equality, the positive bubble equilibrium requires

$$r > \bar{r} \equiv \frac{\alpha}{1-\alpha}\left(\mu + \frac{n\rho}{n-\rho-\mu}\right). \tag{25}$$

Then, using $r = \alpha A(1-u)^{1-\alpha}$ from (18), we can derive the following condition on the unemployment rate for a positive bubble equilibrium:

$$u < \bar{u} \equiv 1 - \left[\frac{1}{(1-\alpha)A}\left(\mu + \frac{n\rho}{n-\rho-\mu}\right)\right]^{\frac{1}{1-\alpha}}. \tag{26}$$

Thus, from (26) we have the following proposition.

Proposition 1: *If (26) is satisfied in equilibrium, then bubbles can exist in the economy; if not, then bubbles cannot exist.*

If the equilibrium unemployment rate is below the threshold $\bar{u}$, and the interest rate is greater than $\bar{r}$, bubble assets exist in equilibrium.

Under full employment ($u = 0$), because the interest rate is determined solely by production parameters, $r = \alpha A$, the existence of bubbles depends fully on the condition $\alpha A > \bar{r}$.[20] With labour market frictions, however, the growth rate and the interest rate are contingent on the unemployment rate. Thus, the unemployment rate plays a crucial role in determining whether or not bubbles occur. When the equilibrium unemployment is less (higher) than the threshold level $\bar{u}$, bubbles can (cannot) arise. The following subsection examines the equilibrium unemployment rate in detail.

3.3.2. Equilibrium in the non-bubble regime

This subsection considers the steady-state equilibrium of the non-bubble regime. Using (18), (20), and (22), the growth rate in the non-bubble regime is denoted as g^{NB}, and defined as

$$g^{NB} = \Gamma(r) \quad \Leftrightarrow \quad (g - r - n + \rho)\left(g - \frac{r}{\alpha}\right) - n\rho = 0, \tag{27}$$

where $\Gamma(0) < 0$.[21] In equilibrium, $g < r + n - \rho$ must be satisfied because $g < r/\alpha$. In addition, taking the total derivative of (27), we have the following property for $\Gamma(r)$:

$$\Gamma'(r) = \frac{\partial g}{\partial r} = \frac{(r/\alpha - g) + (r + n - \rho - g)/\alpha}{(r/\alpha - g) + (r + n - \rho - g)} > 1.$$

Furthermore, using (23) and (27), it can be confirmed that, at the threshold level $\bar{r}$ given in (25), the growth rate in the non-bubble regime is equivalent to the growth rate in the bubble regime; that is, $g = \bar{r} + \mu = \Gamma(\bar{r})$, as shown in Figure 1.

Next, the equilibrium unemployment rate determines the interest rate $r = r(u;\, A) \equiv \alpha A(1-u)^{1-\alpha}$. It is clear from (17) that $\theta = \theta(u;\, \delta)$, with $\partial\theta/\partial u < 0$ and $\partial\theta/\partial\delta > 0$. Then, substituting (27) and $r = r(u;\, A)$ into (19), the equilibrium unemployment rate in the non-bubble regime (u^{NB}) is determined by

$$\Phi_L(u^{NB};\beta,\lambda,\gamma,\delta) = \Phi_R(u^{NB};\delta,A,n), \tag{28}$$

[20] See King and Ferguson (1993) and Futagami and Shibata (2000) for discussions on the conditions required for the existence of bubbles in an endogenous growth model with full employment (and no labour market frictions).

[21] See Appendix 4 for the explicit expression of $\Gamma(r)$.

where $\Phi_L(u^{NB};\beta,\lambda,\gamma,\delta) \equiv \left[\dfrac{(1-\beta)(1-\lambda)}{\beta\gamma} - \theta(u^{NB};\delta)\right] q(\theta(u^{NB};\delta))$,

$$\Phi_R(u^{NB};A,\delta,n) \equiv r(u^{NB};A) + \delta - \Gamma(r(u^{NB};A)) + n,$$

$$\Phi'_L(u) > 0, \text{ and } \Phi'_R(u) > 0.$$

Defining $\Psi(u) \equiv \Phi_L(u) - \Phi_R(u)$, we have $\Psi(0) < 0$ and $\Psi(1) > 0$. Furthermore, assuming that the equilibrium is unique, we find that $\Psi'(u) > 0$ ($\Phi'_L(u) > \Phi'_R(u)$) must be satisfied. Figure 2 (a) shows the equilibrium unemployment rate for the non-bubble regime at Point E^{NB}. Furthermore, the intersection of $\Phi_R(u)$ and $\delta + n - \mu$ gives the threshold level of $\bar{u}$ in (26), since $g^{NB} = g = \bar{r} + \mu$ at the threshold unemployment rate $\bar{u}$. From (18) and (27), we derive the equilibrium interest rate and growth rate for the non-bubble regime: $r^{NB} = \alpha A(1- u^{NB})^{1-\alpha}$ and $g^{NB} = \Gamma(r^{NB})$. These relationships are depicted at Point E^{NB} in Figure 3.

3.3.3. Equilibrium in the bubble regime

This subsection analyses the properties of the bubble regime, under which there are two equilibria: the positive bubble equilibrium (B) and the bubble-less equilibrium (N). The variables in this regime are denoted as u_ι^B, r_ι^B, and g_ι^B, where $\iota = (B, N)$. The variables associated with the bubble-less equilibrium are given by u_N^B, r_N^B, and g_N^B, and are equivalent to those of the non-bubble regime described in Subsection 3.2. The bubble-less equilibrium unemployment rate (u_N^B) is given by $\Phi_L(u) = \Phi_R(u)$ and shown at Point E_N^B in Figure 2 (b). Then, the equilibrium interest rate and growth rate, $r_N^B = \alpha A(1-u_N^B)^{1-\alpha}$ and $g_N^B = \Gamma(u_N^B)$, are obtained from (18) and (27). This relationship is depicted at Point E_N^B in Figure 3.

Next, we consider the unemployment rate for the positive bubble equilibrium. When there is a positive bubble, the growth rate is $g_B^B = r + \mu$. Substituting this expression into (20), the equilibrium condition for the unemployment rate (u_B^B) is obtained as

$$\Phi_L(u_B^B;\beta,\lambda,\gamma,\delta) = \delta + n - \mu. \tag{29}$$

As shown at Point E_B^B in Figure 2 (b), the unemployment rate associated with the positive bubble equilibrium is larger than that of the bubble-less equilibrium; that is, $u_B^B > u_N^B$, which implies that $r_B^B < r_N^B$. The interest rate and growth rate of the positive bubble equilibrium are $r_B^B = \alpha A(1-u_B^B)^{1-\alpha}$ and $g_B^B = r_B^B + \mu$. This relationship is depicted at Point E_B^B in Figure 3.

3.4. Policy implications

This section analyses how changes in labour market policy, productivity in production, and the supply of bubbles in the asset market affect the unemployment rate and the growth rate.

3.4.1. Comparative statics

The properties of the equilibrium level of u, depending on various parameters, including β, λ, γ, δ, A, and n can be examined with (28) and (29). Using Φ_R and Φ_L, as defined in (28), we obtain the following conditions:

$$\frac{\partial \Phi_L}{\partial \beta} < 0, \frac{\partial \Phi_L}{\partial \lambda} < 0, \frac{\partial \Phi_L}{\partial \gamma} < 0, \frac{\partial \Phi_L}{\partial \delta} < 0, \frac{\partial \Phi_R}{\partial A} < 0, \frac{\partial \Phi_R}{\partial \delta} > 0, \frac{\partial \Phi_R}{\partial n} > 0.$$

See Appendix 5 for explicit expressions of the above conditions.

First, we consider the non-bubble regime by calculating the following partial derivatives of u^{NB}:

$$\frac{\partial u^{NB}}{\partial \beta} = -\frac{\partial \Phi_L}{\partial \beta}\frac{1}{\Psi'} > 0,\ \frac{\partial u^{NB}}{\partial \lambda} = -\frac{\partial \Phi_L}{\partial \lambda}\frac{1}{\Psi'} > 0,\ \frac{\partial u^{NB}}{\partial \gamma} = -\frac{\partial \Phi_L}{\partial \gamma}\frac{1}{\Psi'} > 0, \tag{30}$$

$$\frac{\partial u^{NB}}{\partial \delta} = \left[-\frac{\partial \Phi_L}{\partial \delta} + \frac{\partial \Phi_R}{\partial \delta}\right]\frac{1}{\Psi'} > 0,\ \frac{\partial u^{NB}}{\partial A} = \frac{1}{\Psi'}\frac{\partial \Phi_R}{\partial A} < 0,\ \frac{\partial u^{NB}}{\partial n} = \frac{1}{\Psi'}\frac{\partial \Phi_R}{\partial n} > 0.$$

The result, using (30) in conjunction with (18) and (27), is

$$\text{sign}\frac{\partial g^{NB}}{\partial i} = \text{sign}\frac{\partial r^{NB}}{\partial i} = \text{sign}\ -\frac{\partial u^{NB}}{\partial i}, \quad (\text{for } i = \beta, \lambda, \gamma, \delta, A, n). \tag{31}$$

Thus, we can derive the following proposition from (30) and (31).

Proposition 2: *In the non-bubble regime, the bargaining power of labour (β), unemployment benefits (λ), the vacancy cost (γ), the separation rate (δ), and the population growth rate (n) all serve to increase the unemployment rate, so that the equilibrium interest rate and output growth rate fall. Productivity (A), on the other hand, decreases the unemployment rate, so that the equilibrium interest and output growth rates rise.*

Intuitively, an increase in the bargaining power of workers (β) leads to higher wages, implying reduced incentives for job creation and a smaller number of vacancies offered by firms. The result is a decrease in the tightness of the labour market ($\theta = v/u$) and an increase in the steady-state unemployment rate. Increases in the vacancy cost (γ) and unemployment benefits (λ) have similar effects to those described above. In addition, a rise in the separation rate (δ) shifts the Beveridge curve down, raising unemployment. These results conform to those of the standard search friction model (Pissarides, 2000).

From Section 2.8, because the wage growth rate is negatively related with the population growth rate, $\dot{w}/w = g - n$, an increase in the population growth rate dampens wage growth, lowering future vacancy costs. As a result, firms postpone the creation of vacancies, and the labour market contracts, causing a rise in the unemployment rate. On the other hand, an improvement in productivity (A) directly increases the growth rate, and thereby raises future vacancy costs. As such, firms increase the current number of vacancies, and the unemployment rate falls.

Changes in the parameters $(\beta, \lambda, \gamma, \delta, n)$ that lead to an increase the unemployment rate will also have a negative effect on the interest rate and the growth rate (see (18) and (27)). A rise in productivity (A) has direct and indirect effects that both lead to an increase in the interest rate and accelerate output growth.

Next, consider the case of an economy with bubbles. Using (29), the partial derivatives of u_B^B are obtained as follows:

$$\frac{\partial u_B^B}{\partial \beta} = -\frac{\partial \Phi_L}{\partial \beta}\frac{1}{\Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial \lambda} = -\frac{\partial \Phi_L}{\partial \lambda}\frac{1}{\Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial \gamma} = -\frac{\partial \Phi_L}{\partial \gamma}\frac{1}{\Phi'_L} > 0, \tag{32}$$

$$\frac{\partial u_B^B}{\partial \delta} = \left[1 - \frac{\partial \Phi_L}{\partial \delta}\right]\frac{1}{\Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial n} = \frac{1}{\Phi'_L} > 0, \quad \frac{\partial u_B^B}{\partial \mu} = \frac{-1}{\Phi'_L} < 0.$$

Then, using (32) in conjunction with (18) and (23), we have

$$\text{sign}\frac{\partial g_B^B}{\partial i} = \text{sign}\frac{\partial r_B^B}{\partial i} = \text{sign}\ -\frac{\partial u_B^B}{\partial i} < 0, \text{(for } i = \beta, \lambda, \gamma, \delta, n\text{)}, \tag{33}$$

$$\text{sign}\frac{\partial r_B^B}{\partial \mu} = \text{sign}\ -\frac{\partial u_B^B}{\partial \mu} > 0, \quad \frac{\partial g_B^B}{\partial \mu} = \frac{\partial r_B^B}{\partial \mu} + 1 > 0, \ \text{sign}\frac{\partial g_B^B}{\partial A} = \text{sign}\frac{\partial r_B^B}{\partial A} > 0.$$

Consequently, we can derive the following proposition using (32) and (33).

Proposition 3: *In the positive bubble equilibrium, the bargaining power of labour (β), unemployment benefits (λ), the vacancy cost (γ), the separation rate (δ), and the population growth rate (n) increase the unemployment rate so that the equilibrium interest rate and output* *growth rate fall. Productivity (A) has a positive effect on the equilibrium interest rate and output growth rate, but the unemployment rate is not affected. The bubble expansion rate (μ), on the other hand, decreases the unemployment rate and increases the equilibrium interest rate and output growth rate.*

The effects of β, λ, γ, δ, and n on unemployment, the interest rate, and the output growth rate are all analogous to those for the non-bubble regime. However, because the relationship between the interest rate and the growth rate is given by $g = r + \mu$ in an economy with bubbles (23), the job creation curve in (19) or (29) is free of the effect of productivity (A). Thus, productivity has no effect on the unemployment rate. Consequently, the interest rate is affected by the direct positive effect of productivity so that it increases the growth rate of output.

The bubble expansion rate (μ) has two positive effects on the output growth rate. The first occurs directly, by means of an increase in asset holdings (asset effect) (Futagami and Shibata, 2000), which shifts the $g_B^B = r_B^B + \mu$ curve upward. Furthermore, this increase in economic growth increases growth in wages, which, in turn, increases the number of operating vacancies so that the unemployment rate falls. The second effect stems from the improvement in employment, which induces economic growth through an increase in the interest rate. This effect moves the positive bubble economy along the g_B^B curve at interest rate r_B^B. Figure 4 depicts how the presence of bubbles makes E_B^B (the bubble economy) approach E_N^B (the bubble-less economy).

In addition, the effects of an interaction between the two types of policies (labour market policy and bubble supply policy) can be considered. Based on the above results, a decrease in the bubble supply, implemented with the aim of moderating asset bubbles, lowers the employment rate and economic growth. On the other hand, a decrease in unemployment benefits could improve the employment rate and economic growth. Therefore, labour market policies could weaken the negative effects of the bubble supply on the employment rate and economic growth. Thus, when devising policies, policy makers should consider the extent to which labour market policies' effects on the employment rate and economic growth depend on the effects of bubble supply policies and vice versa in a macroeconomy.

3.4.2. Comparison of the two regimes

The economy will be in a non-bubble or bubble regime when the equilibrium unemployment rate is higher or lower, respectively, than the threshold level. Thus, if changes in policies or parameters cause a decrease in the unemployment rate (e.g., a reduction in unemployment benefits), the economy will shift from a non-bubble regime to a bubble regime. As shown in Figure 3, the output growth rate is always higher under the bubble regime than under the non-bubble regime, even when bubbles occur.

Additionally, under a bubble regime, steady-state equilibrium can be achieved in either the presence or absence of bubbles (Tirole, 1985). As for equilibrium unemployment, the relationship $u_N^B < u_B^B < \bar{u}$ holds under the bubble regime. Thus, the conditions $r_N^B > r_B^B > \bar{r}$ and $g_N^B > g_B^B$ are satisfied. Regarding the growth rate, bubbles create a crowd-out effect by reducing capital accumulation (Grossman and Yanagawa, 1993; Futagami and Shibata 2000).

The above properties can be formally restated as follows:

Proposition 4: *The growth rate (g^{NB}) in a non-bubble regime ($u > \bar{u}$) is lower than the growth rate (g_ι^B) in a bubble regime ($u < \bar{u}$); that is, $g^{NB} < g_\iota^B$. Under a bubble regime, the growth rate is lower when there are bubbles (g_B^B) than when there are not (g_N^B); that is, $g_B^B < g_N^B$.*

3.5. Discussion

3.5.1. Related literature on bubbles with labour market frictions

This paper responds to the literature on asset pricing models with labour market frictions. Regarding empirical and theoretical studies on asset pricing and unemployment, Kuehn et al. (2012) develop a model with a stock market and a labour market search mechanism in a dynamic stochastic general equilibrium framework, and investigate the correlation between the equity premium (stock market volatility) and labour market tightness. Farmer (2012) also provides an empirical and theoretical study of the relationship between the stock market crash of 2008 and the Great Recession in the US, and finds a strong correlation between unemployment and the price of capital. Miao et al. (2016) observe a historical relationship between monthly price-earnings ratio data and the unemployment rate during recessions in the

US, and investigate the relationship between unemployment and stock market bubbles in an economy with labour and financial market frictions.

Direct comparisons of our results with the findings of the empirical literature are difficult, as empirical studies have generally focused on how macroeconomic shocks affect the volatility of variables measuring asset prices and unemployment. We can consider our framework, however, with respect to the empirical evidence of a negative correlation between asset bubbles and unemployment. A crucial point of our model is that, in contrast to the existing literature, we endogenously consider economic growth. In particular, our framework introduces a capital externality that enables an investigation of the role of labour market frictions in the determination of economic growth and asset prices. Changes in asset prices are captured by a shift in the economy from a non-bubble to a bubble equilibrium. Thus, our framework demonstrates a positive relationship between asset bubbles and economic growth that is consistent with empirical evidence (Martin and Ventura, 2012; Farmer and Schelnast, 2013).

3.5.2. A boom and bust pattern of a bubble

As pointed out by Aliber and Kindleberger (2015), the term "bubble" itself foreshadows the end of an economic bubble. Following the discussion of the boom and bust properties of bubbles in Farhi and Tirole (2012), there are two types of causes leading to the end of an asset bubble. The first type is associated with changes in fundamental variables.[22] The second type is the realization of a sunspot (extrinsic uncertainty).

Because incorporating the second type of cause (the stochastic probability of a bubble bursting) into a continuous time model is difficult, it is not treated. In our framework, we show that labour market frictions can lead to a bubbly steady state (i.e., a bubble regime). Therefore, our framework focuses on the burst of asset bubbles caused by changes in fundamental variables, such as labour market conditions, productivity, and government policy. We examine the boom and bust of bubbles through regime shifts resulting from changes in parameter conditions. In our model, policy and parameter changes (Propositions 2 and 3) may shift the economy from a bubble regime to a non-bubble regime (Proposition 4), leading to a bubble burst that is accompanied by an increase in the unemployment rate and decreases in the interest rate and economic growth rate. Figure 5 summarizes these correlations.

[22] Brunnermeier and Oehmke (2015) point out that an initial boom in asset prices (potentially an asset price bubble) is often triggered by fundamentals.

3.6. Conclusion

In this paper, we present a continuous-time overlapping-generation model with labour market frictions and consider the conditions required for the existence of asset bubbles. We demonstrate the theoretical relationships between unemployment, bubbles, and economic growth. In contrast to previous studies that only account for the production technology in calculations of the interest rate, our framework introduces labour market frictions into an endogenous growth model, thereby linking the interest rate with labour market conditions. This link, in turn, determines the efficiency of labour in production. Allowing for unemployment, economic fluctuations induced by labour market shocks determine whether the economy exhibits bubbles or not. In particular, we find that bubbles are more likely to arise when the unemployment rate is relatively low and the interest rate is relatively high. Furthermore, we show that economic growth is deeply contingent upon the employment situation, as labour market efficiency affects capital accumulation through the marginal product of capital.

Based on our findings that bubbles may (not) occur when the unemployment rate is low (high) and the interest rate and economic growth rate are high (low), it is reasonable to conclude that policies that positively influence the labour market (e.g., a reduction in unemployment benefits) could improve employment and while creating an asset bubble, raising the interest rate and accelerating economic growth.

Given these preliminary results, an interesting extension of our framework might be an analysis of stochastic bubbles which have an exogenous probability of collapsing. For example, Tanaka (2007) introduces the confidence of asset bubble in a two-period overlapping-generation model and investigates the relationship between the confidence and economic growth. A simplified version of the model presented in this paper might allow a stochastic probability of the bubble burst in an endogenous growth model with labour market frictions. We leave this point as a topic for future work.

Appendix

A1. Derivation of the flow budget constraint (3)

Bubble asset holdings are defined in nominal terms as $b(s, t)$. Then, the flow budget constraint of generation s at time t is

$$[\mathrm{d}b(s, t)/\mathrm{d}t]/p + \mathrm{d}k(s, t)/\mathrm{d}t = rk(s, t) + w(1 - u) + \lambda wu + v^I - \tau - c(s, t).$$

Because the real value of a bubble asset is given by $m(s, t) = b(s, t)/p$, $\mathrm{d}m(s, t)/\mathrm{d}t = [\mathrm{d}b(s, t)/\mathrm{d}t]/p + rm(s, t)$ is obtained using the arbitrage condition $-(\dot{p}/p) = r$. Substituting this into the above equation yields

$$\mathrm{d}m(s, t)/\mathrm{d}t - rm(s, t) + \mathrm{d}k(s, t)/\mathrm{d}t = rk(s, t) + w(1 - u) + \lambda wu + v^I - \tau - c(s, t).$$

Therefore, using $z(s, t) = k(s, t) + m(s, t)$ gives the flow budget constraint (3).

A2. Derivations of (6), (7), and (11)

Aggregating the flow budget constraint gives

$$\dot{Z} = rZ + w(1 - u)N + \lambda wuN + v^IN - \tau N - C.$$

Given firm symmetry, we combine the budget constraint of government (5) and the income from vacancies, $v^I = \gamma wv$, to obtain

$$\dot{Z} = rZ + [w(1 - u) + \gamma wv + \mu M/N]N - C,$$

which is equivalent to (6).

The real value of useless assets is given by $M = B/p$. Using the government policy condition $\dot{B}/B = \mu$ and the arbitrage condition $-(\dot{p}/p) = r$, we obtain the following dynamic equation for M:

$$\dot{M}/M = \dot{B}/B - \dot{p}/p = \mu + r,$$

which is equivalent to (7).

Aggregating the consumption function (10) yields

$$C = \rho[Z + H]. \tag{A.1}$$

Then, using (9), (A.1), and $z(0, 0) = 0$, and differentiating the aggregate consumption with respect to time, we have

$$\dot{C} = (r-\rho)C + \rho n H = (r-\rho+n)C - n\rho Z,$$

which is equivalent to (11).

A3. Derivation of the equilibrium condition for the goods market (8)

The following equation is obtained from (6), (7), $Z = K + M$ and $v^I = \gamma w v$:

$$\dot{K} = rK + [w(1-u) + \gamma w v]N - C. \quad \text{(A.2)}$$

With perfect competition in the market for production factors, firms earn zero profits. Using this information in conjunction with (14) yields

$$Y = rK + [w(1-u) + \gamma w v]N, \quad \text{(A.3)}$$

which implies that total output is distributed as capital income, wage income, and vacancy income. Substituting (A.2) into (A.3) yields the equilibrium condition for the goods market (8).

A4. The explicit expression of $\Gamma(r)$

Using (27), the quadratic equation for the growth rate (g) can be solved for the two following solutions: $g = \left[(r+n-\rho+r/\alpha) \pm \sqrt{(r+n-\rho+r/\alpha)^2 - 4((r+n-\rho)r/\alpha - n\rho)}\right]/2$. Because $g < r/\alpha$ is required, however, in order for c to take a positive value from (22), only one solution may be used:

$$g = \Gamma(r) \equiv \frac{1}{2}\left[(r-\rho+n+r/\alpha) - \sqrt{(r-\rho+n+r/\alpha)^2 - 4((r-\rho+n)r/\alpha - n\rho)}\right],$$

where $\Gamma(0) \equiv \left[(n-\rho) - \sqrt{(n-\rho)^2 + 4n\rho}\right]/2 < 0$.

A5. Derivation of the partial derivatives of Φ_L and Φ_R

Differentiating Φ_L and Φ_R with respect to β, λ, γ, δ, A, and n yields

$$\frac{\partial \Phi_L}{\partial \beta} = -\frac{(1-\lambda)q}{\beta^2 \gamma} < 0,$$

$$\frac{\partial \Phi_L}{\partial \lambda} = -\frac{(1-\beta)q}{\beta\gamma} < 0\,,$$

$$\frac{\partial \Phi_L}{\partial \gamma} = -\frac{(1-\beta)(1-\lambda)q}{\beta\gamma^2} < 0,$$

$$\frac{\partial \Phi_L}{\partial \delta} = -\left[1 + \left\{q - \frac{\partial q}{\partial \theta}\left[\frac{(1-\beta)(1-\lambda)}{\beta\gamma} - \theta\right]\right\}\frac{\partial \theta}{\partial \delta}\right] < 0,$$

$$\frac{\partial \Phi_R}{\partial A} = -\left[\Gamma'(r) - 1\right]\frac{\partial r}{\partial A} < 0\,,$$

$$\frac{\partial \Phi_R}{\partial \delta} = \frac{\partial \Phi_R}{\partial n} = 1 > 0\,.$$

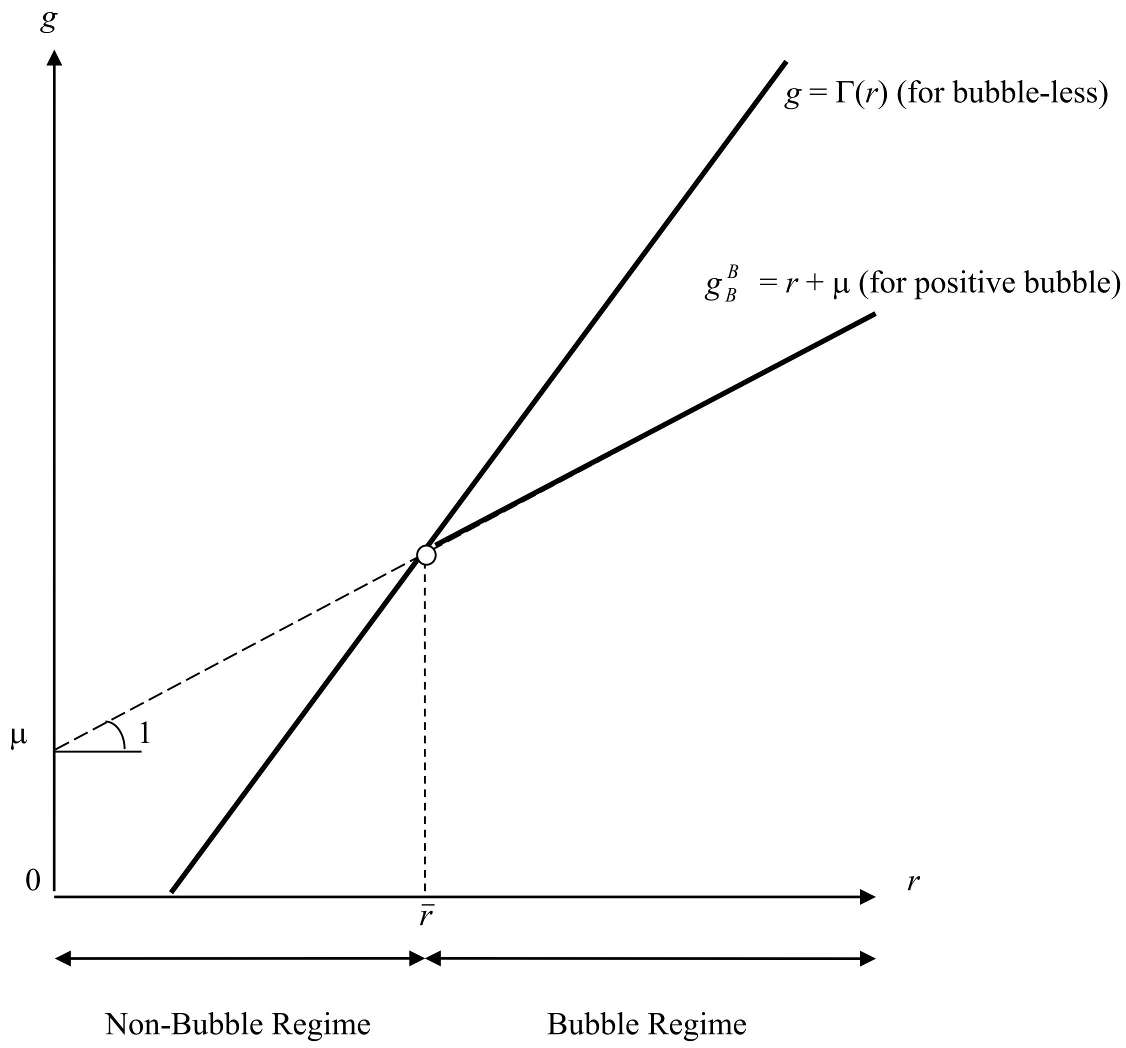

Figure 1. The threshold between a non-bubble regime and a bubble regime

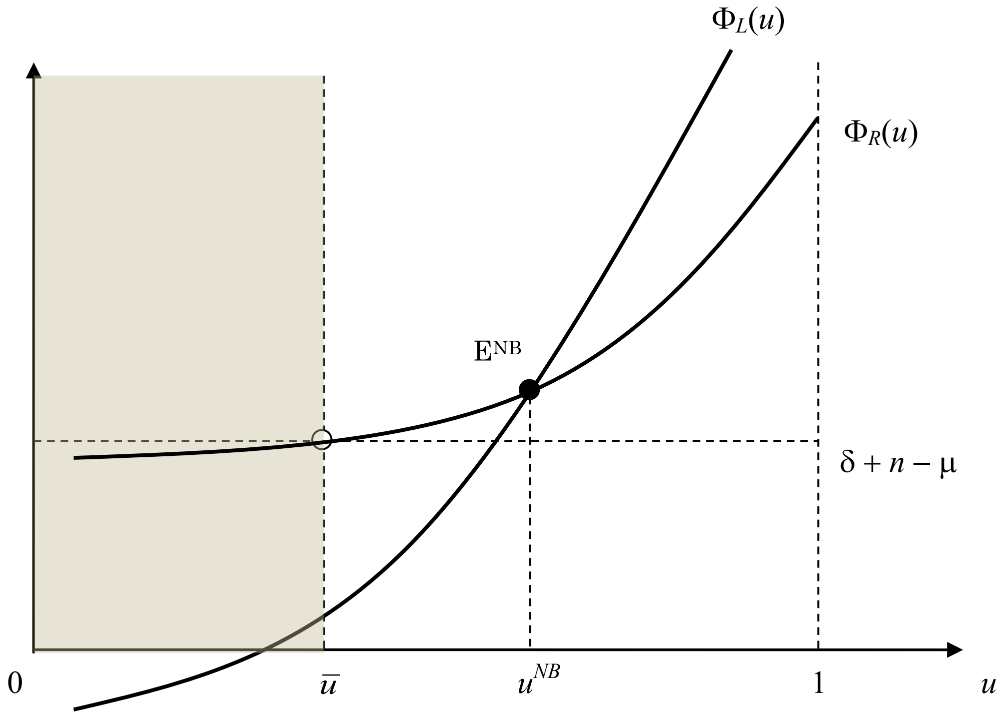

Figure 2 (a). Steady-state unemployment under a non-bubble regime ($u > \bar{u}$)

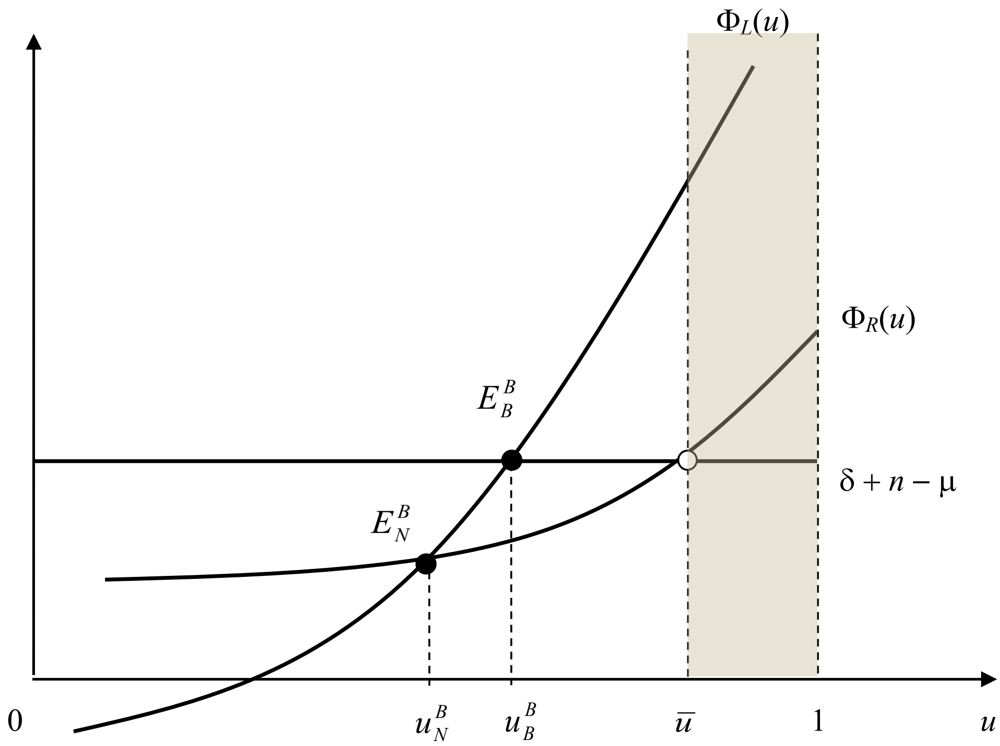

Figure 2 (b). Steady-state unemployment under a bubble regime ($u < \bar{u}$)

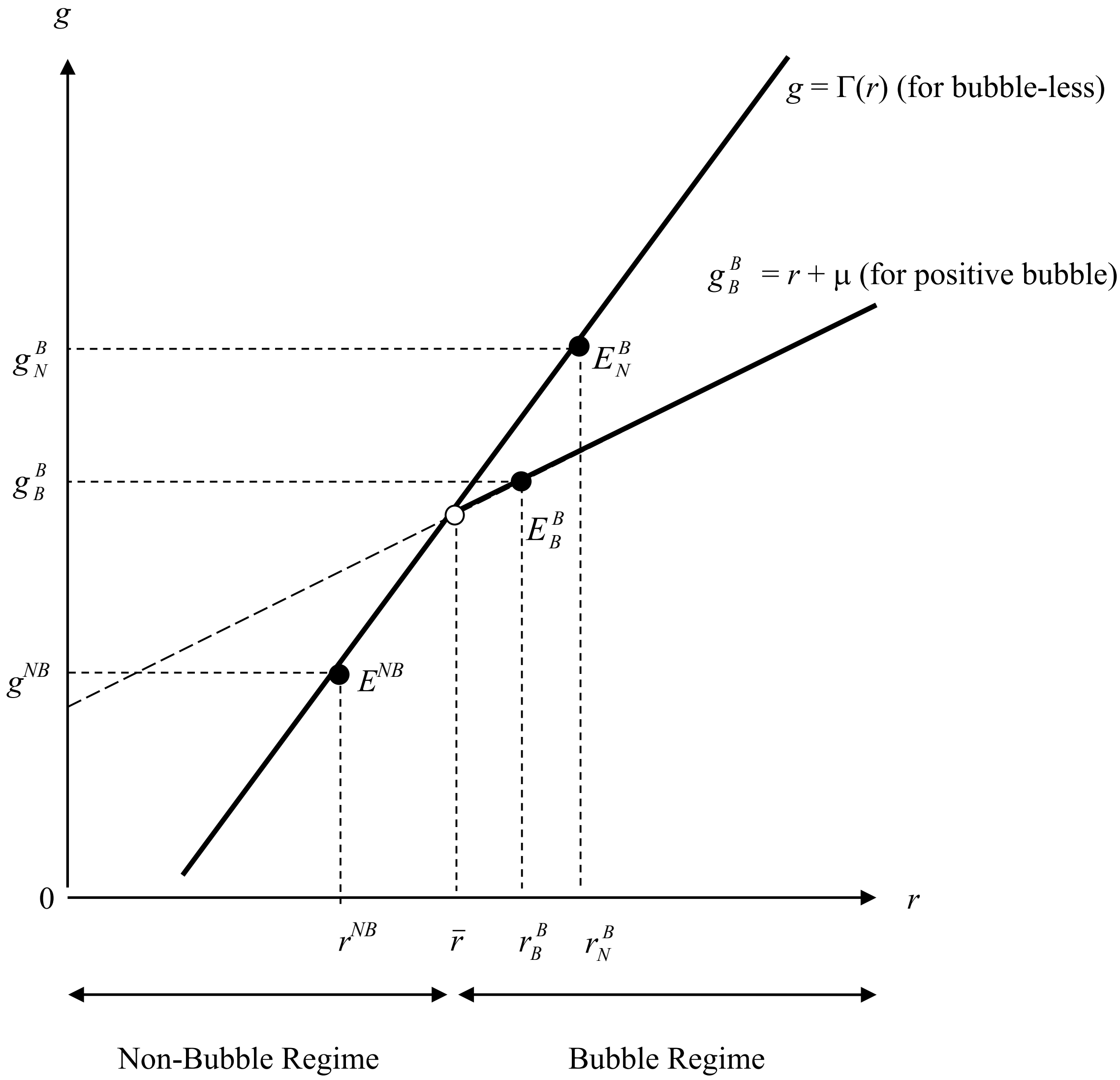

Figure 3. Comparison between regimes

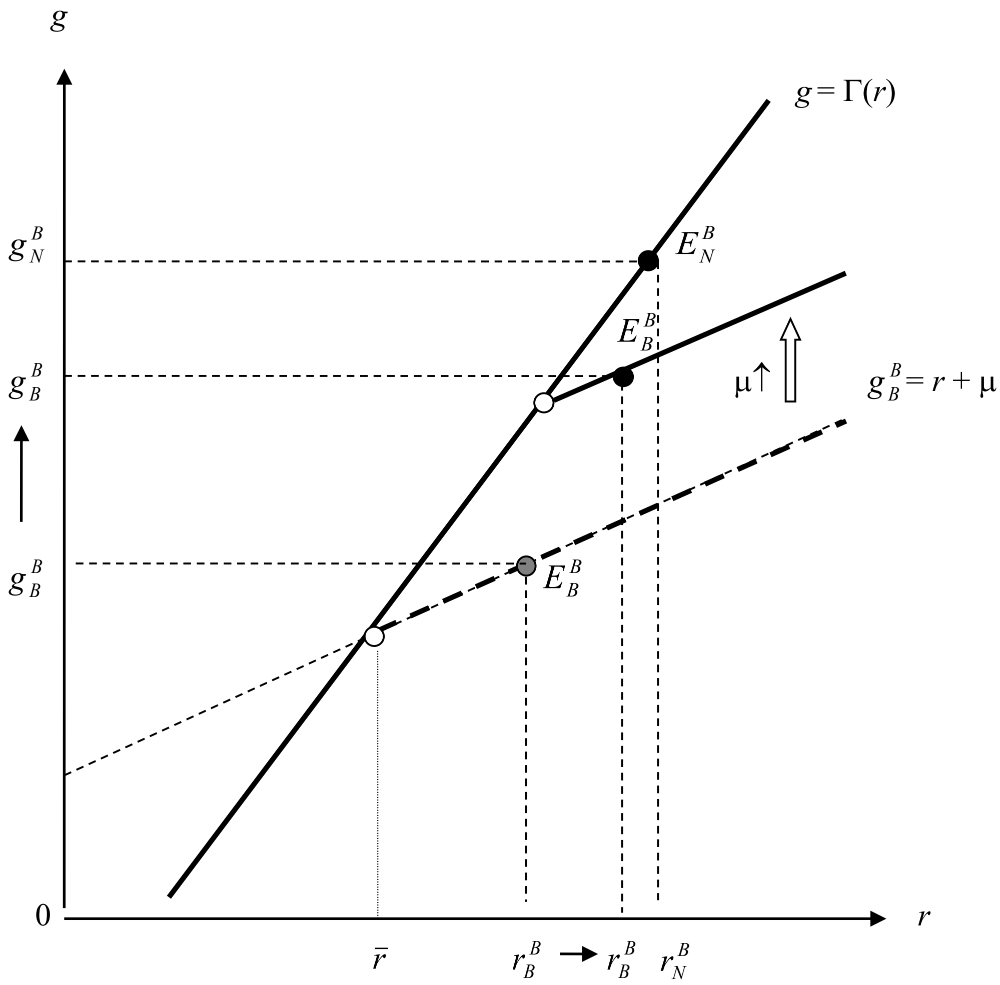

Figure 4. The effect of bubble growth (μ)

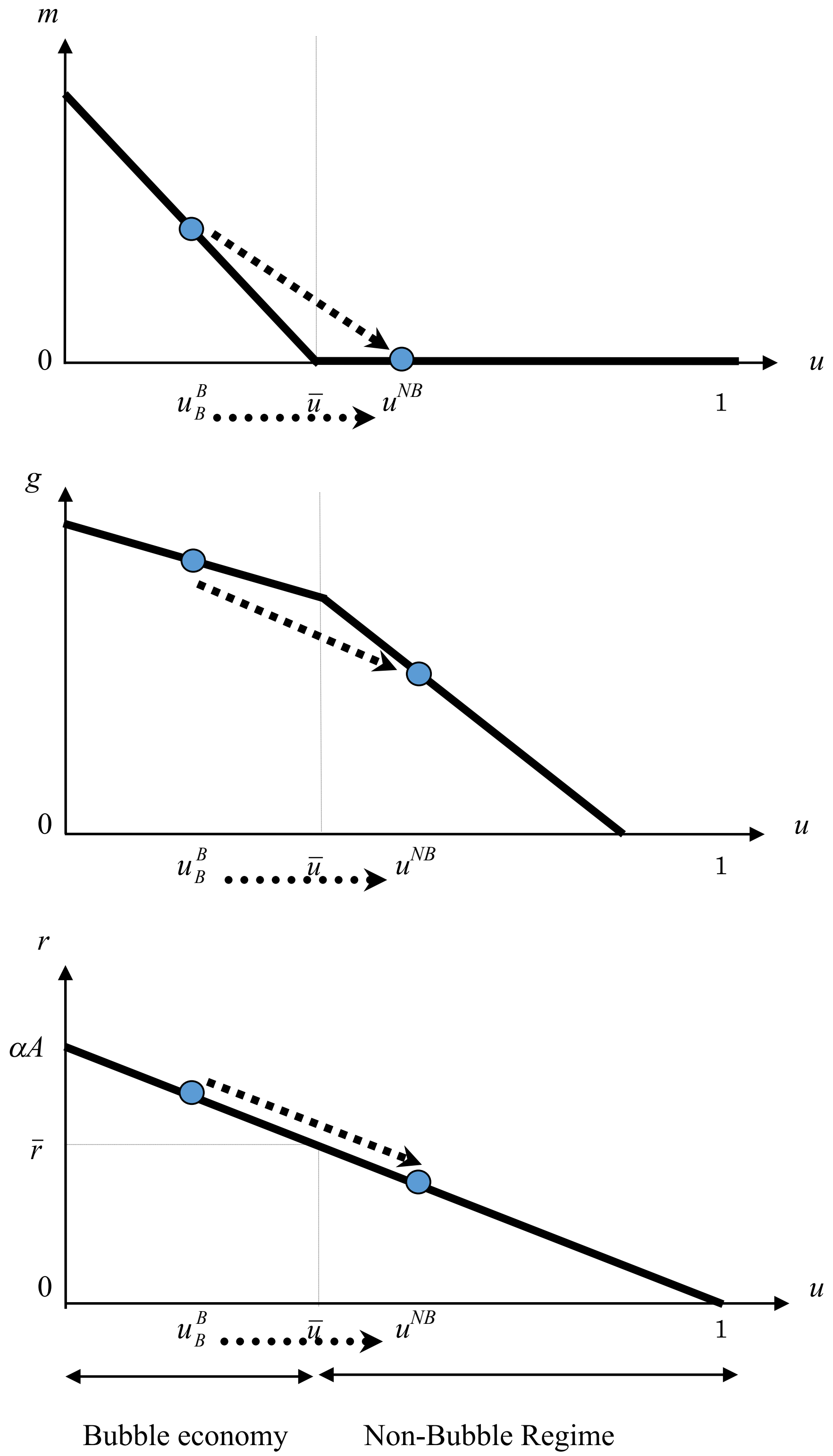

Figure 5. The correlation of unemployment rate (u):
asset bubbles (m), the growth rate (g), the interest rate (r)

4. Asset bubbles, labor market frictions, and R&D-based growth

4.1. Introduction

As mentioned by Aliber and Kindleberger (2015), economic bubbles have occurred throughout history, often with major impacts on the economies of the countries concerned. Examples include the current recession in the United States and other countries, the Japanese experience in the late 1980s and 1990s and the 1929 crash. These bubbles featured spectacular booms that lasted for a few years followed by dramatic crashes. In 1990 stock prices collapsed, and Japan's deepest and longest depression began and the average growth rate for the decade was 1.7 percent, and the year 1998 recorded a minus growth. Unemployment rose from 2.1% in 1990 to 4.7 % in 1999 (Kaihara, 2008). In this way, bubbles have been frequently observed when economic activity is booming and the growth rate of GDP is high (Martin and Ventura, 2012; Farmer and Schelnast, 2013). Also, empirical studies show that asset bubbles are accompanied by a reduction in the unemployment rate (Phelps, 1999; Fitoussi et al., 2000).

In this paper, we analyze the interaction between bubbles, unemployment and the long-run growth rate of the economy. Because technological progress via R&D innovation has been identified as the primary driving force of modern economic growth (e.g., Romer 1990), we are particularly interested in the effects of these interactions and transitional dynamics on R&D-based innovations.

In the literature of asset bubbles and economic growth, Grossman and Yanagawa (1993), King and Ferguson (1993), and Futagami and Shibata (2000) examine the conditions that are necessary for bubbles to exist in an overlapping generations economy. In their studies, when a bubble arises in the economy, it diverts savings from capital accumulation and retards economic growth. For an alternative approach that focuses on the financial market imperfections, Hirano and Yanagawa (2010), Martin and Ventura (2012) and Kunieda and Shibata (2016) show that asset bubbles can be growth enhancing or growth impairing depending on the restrictiveness of the collateral constraint.[23] However, these do not consider the possibility of unemployment.

[23] Most of these studies are based on a model that considers the accumulation of physical capital, technological progress via learning by doing, or knowledge spillovers that occur during production as fundamental drivers of growth. Consequently, these studies are unable to analyze the effects of bubbles on R&D-based innovations, which play a crucial role in modern technological development.

There are theories that state that equilibrium unemployment occurs as a result of friction in the labor market. In the economy with labor market frictions, the wage rate is endogenously determined by agents' negotiation. Bean and Pissarides (1993) introduces labor market search frictions in a standard overlapping generation model, where the wage is negotiated by vacant firm and worker,[24] and analyze the relationship between economic growth and unemployment. Corneo and Marquardt (2000) consider a monopolistic trade union in an endogenous growth model, where wages and employment rate are set by the unions.[25] To the best of our knowledge, however, no studies have used an endogenous growth model to analyze the connection between asset bubbles and unemployment.

In order to fill this void, we develop an endogenous growth framework with which to examine the conditions for bubbles to exist in the economy with labor market frictions. A study close to ours is Hashimoto and Im (2016), who use a continuous-time overlapping generations model (Weil, 1989) and consider the relationship between bubbles and unemployment in an endogenous growth framework (AK model) through a learning-by-doing technological capital externality. However, they give the ad-hoc setting in the determination of wage rates and the analysis focuses on steady state only. In contrast to it, this paper follows the standard labor market frictions where wage rate is endogenously determined by Nash-bargaining negotiation between a vacant firm and a worker. In this framework we construct a simple overlapping-generations model of R&D-based growth with labor market frictions and explore the steady state and transitional dynamics of bubbles, economic growth, and employment.[26]

In our model, where unemployment stems from labor market friction, labor market efficiency is reflected in the interest rate. Then, because asset returns are related to the interest rate, the existence of bubbles depends on conditions in the labor market. As such, we find that the equilibrium employment rate is a key factor in the existence of bubbles; when it is over a certain level and interest rate is high, bubbles asset can exist. When the conditions are satisfied

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[24] See Pissarides (2000) for an introduction to search friction models.

[25] See Aghion and Howitt (1994), Eriksson (1997), Caballero and Hammour (1996), and Haruyama and Leith (2010) for other models of the relationship between growth and unemployment that address labor market frictions.

[26] Miao et al. (2016) and Kocherlakota (2011) present studies that are similar to our own. Miao et al. (2016) investigate the relationship between unemployment and stock market bubbles in an economy with labor market friction and financial market friction. Kocherlakota (2011) assumes that output is determined by household demand, and as such, he does not consider the firm's behavior and capital stock accumulation in an economy with matching frictions and bubbles. However, these studies do not consider economic growth endogenously.

for bubbles to exist, we say that the economy is in a “bubble regime”; conversely, when it is not possible for them to exist, we say that the economy in a “non-bubble regime.” In a bubble regime, there are multiple equilibria, such that a steady state can exist either with bubbles or without. We show that bubbles divert savings away from the resource of R&D sector and lower the output growth rate, which is a common finding in the literature. On the other hand, when we compare bubble regimes to non-bubble regimes, we find that the output growth rate is always higher under the former than the latter.

With our model we can examine the effects of labor market policy or parameter changes on bubbles, economic growth, and employment. For example, we find that, because a rise in search cost decreases the number of firms with vacant position, it has a negative impact on employment (which is a standard conclusion among models with search friction). Thus, if search costs are increased in the economy with bubbles, then the employment rate should decrease and the labor market should become more inefficient, which would lower the interest rate and consequently shift the economy from a bubble regime to a non-bubble regime. As a result, employment rate and economic growth fall down associated with bubble bursting. In this case, there would be a positive relationship between employment and economic growth.[27]

The remainder of this paper is organized as follows: Section 2 outlines the features of the model. Section 3 discusses the labor market structure. Section 4 describes the traditional dynamics and the steady-state equilibrium with and without bubbles, and compares the effects of policy and parameter changes under the two regimes on bubbles, economic growth, and employment. The final section summarizes our findings and concludes the paper.

4.2. The model

This section develops an overlapping generations model with labor market frictions. A new generation is born in each period $t = 0, 1, \cdots$ and lives for three periods: young, adult, and old age. Each generation has constant population size (L). The economy consists of three sectors: a final goods sector, an intermediate goods sector, and an R&D sector. Labor market is opened in a final goods sector only. In accordance with Rivera-Batiz and Romer (1991) and Barro and Sala-i-Martin (2004, Chapter 6), we regard final goods as the production factor in both an intermediate goods sector and an R&D sector. R&D firms invent blueprints of intermediate

[27] In fact, many empirical studies show a positive relationship between employment rate and economic growth (Ball and Moffitt, 2001; Muscatelli and Tirelli, 2001; Staiger et al., 2001; Tripier, 2006; Pissarides and Vallanti, 2007).

goods and conduct the market launches of these goods. Each intermediate good is produced by a single monopoly firm by contrast. Each final good is produced by competitive firms which are successful in matching with labor, with a variety of imperfectly substitutable intermediate goods as inputs.

4.2.1. Final goods sector

In the final goods sector, many identical firms produce unique final goods with the same production technology. A firm needs one worker and intermediate goods to produce final goods. In the labor market, there are young households and firms with vacant position to find each other. When firm is success in matching with one worker, then it operates final good production with inputs of intermediate goods.

Consider the behavior of the operating production firm. Firm *i* produces final goods $y_{i,t}$ at time *t* with a following production technology:

$$y_{i,t} = \int_0^{N_t} \left(x_{i,t}(j)\right)^{\alpha} dj, \quad 0 < \alpha < 1, \tag{1}$$

where $x_{i,t}(j)$ and N_t the input of intermediate goods for product variety *j* and the number of varieties available at period *t*, respectively. Then, the operating profits, which is the remainder of output to be allotted between firm *i* and its worker, is given by

$$\pi_{i,t}^{Y} \equiv y_{i,t} - \int_0^{N_t} p_t(j) x_{i,t}(j) dj, \tag{2}$$

where $p_t(j)$ represents the price of intermediate good *j*. Because the factors market is competitive, from the profit maximization problem, we can get the firm *i*'s demand function for intermediate goods as:

$$x_{i,t}(j) = x_t(j) = \left(\frac{\alpha}{p_t(j)}\right)^{\frac{1}{1-\alpha}}. \tag{3}$$

The firm-specific index in the final goods sector can be dropped because of the symmetricity in production technology; $y_{i,t} = y_t$ and $\pi_{i,t}^{Y} = \pi_t^{Y}$.

4.2.2. Intermediate goods sector

Each intermediate good *j* is produced by monopolistically competitive firms that hold a blueprint for the intermediate good *j*. One unit of final goods is required to produce one unit of an intermediate good, and the operating profit of each intermediate goods producer $\pi_t^X(j)$ is expressed as follows: $\pi_t^X(j) = (p_t(j) - 1)X_t(j)$ where $X_t(j)$ represents the supply of intermediate good *j*. Under monopolistic competition, each firm maximizes its profits given a demand curve for its brand. Because final good firms need one worker to produce, the number of active firms producing final goods in time *t* equals to the total number of workers $\sigma_t L$, where σ_t represents employment rate. Then, the aggregate demand for product variety *j* is defined as $X_t(j) \equiv \int_0^{\sigma_t L} x_{i,t}(j)di = x_t(j)\sigma_t L$. By using this definition and equation (3), we can obtain the demand curve for intermediate good *j*:

$$X_t(j) = \left(\frac{\alpha}{p_t(j)}\right)^{\frac{1}{1-\alpha}} \sigma_t L. \tag{4}$$

Then, the optimization problem for intermediate good firm *j* establishes a price that is equal to a constant markup over unit cost:

$$p_t(j) = p_t = \frac{1}{\alpha}. \tag{5}$$

Thus, the firm-specific index in the intermediate goods sector can be dropped, and profits may therefore be expressed as follows:

$$\pi_t^X = (1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}} \sigma_t L. \tag{6}$$

4.2.3. R&D sector

The development of R&D technology requires final goods as its input. Denoting η units of the final goods between periods *t* and *t* + 1, competitive R&D firms can invent one unit of $N_{t+1} - N_t$ new blueprints, and sell these blueprints to intermediate goods firms at their market values of D_t. Thus, output is expressed as follows:

$$N_{t+1} - N_t = \frac{1}{\eta} I_t^R, \tag{7}$$

where I_t^R represents R&D inputs. Under the assumption of free entry in the R&D sector, the expected gain of $D_t I_t^R / \eta$ from R&D must not exceed the cost of I_t^R for a finite size of R&D activities at equilibrium. We assume that the R&D cost is given by $\eta = \overline{\eta} L$, which expresses the dilution effect that removes scale effect as in Laincz and Peretto (2006) and Peretto and Connolly (2007). Thus, we have the following conditions:

$$D_t = \overline{\eta} L. \tag{8}$$

We next consider no-arbitrage conditions. The market value of intermediate goods firms D_t (i.e., the market value of blueprints) is related to the risk-free interest rate r_t. Shareholders of intermediate goods firms that purchased these shares during period t obtain dividends of π^X_{t+1} during period $t + 1$ and can sell these shares to the subsequent generation at a value of D_{t+1}. In the financial market, the total returns from holding the stock of a particular intermediate firm must be equal to the returns on the risk-free asset $(1+r_{t+1})D_t$, which implies the following no-arbitrage condition: for all t, the return on one unit of the stock must be equal to the interest rate:

$$1 + r_{t+1} = \frac{\pi^X_{t+1} + D_{t+1}}{D_t}. \tag{9}$$

Then, substituting (6) and (8) into (9) gives the interest rate as follows:

$$r_{t+1} = r(\sigma_{t+1}); \quad r(\sigma_{t+1}) \equiv \frac{\pi^X_{t+1}}{D_t} = \frac{1}{\overline{\eta}}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\sigma_{t+1}. \tag{10}$$

4.2.4. Agents

The first, second and third periods of agents' lifetime are referred to as young, adult, and old, respectively. The cohort born in period $t-1$ become active workers in period t. Thus, we call this cohort generation t. Note that the superscript ι denotes an agent's employment status: $\iota = e$ if employed and $\iota = u$ if unemployed, which is an outcome of job search. An individual derives utility from consumption in old age c^ι_{t+1}, then the life time utility of individuals in generation t is expressed as $U^\iota_t = c^\iota_{t+1}$.

During the first period, individuals are endowed with one unit of labor. If match with a firm is successful in the first period (young), the agent can work and receives wage income w_t in the second period of their lives (adult). Otherwise the adult receives the unemployment benefit from the government z_t. Individuals transfer lump-sum tax τ_t to the government and save the after-tax income. The allocation of saving is devoted to the interest-bearing asset and bubbly assets. Following Tirole (1985), we consider bubbly assets. Bubbles are intrinsically useless, that is, the fundamental value of the bubbles is zero. The adult will buy bubble only if he will be able to resell it at a positive price to the next generation. In the third period, they are retired and spend their savings on old-age consumption. Thus, the budget constraint for generation *t* is expressed as follows:

$$s_t^{\iota} + p_t^B m_t^{\iota} = \omega_t^{\iota} - \tau_t, \quad \omega_t^e = w_t \text{ for employed, } \omega_t^u = z_t \text{ for unemployed,}$$

$$c_{t+1}^{\iota} = (1 + r_{t+1}) s_t^{\iota} + p_{t+1}^B m_t^{\iota},$$

where s_t^i is the interest-bearing asset holdings, m_t^i is the demand for bubble assets, p_t^B is the price of bubble asset at time *t* in real terms of goods. In order to hold bubbles in equilibrium, the price of bubbles must satisfy the arbitrage condition $p_{t+1}^B / p_t^B = 1 + r_{t+1}$, that is, return of bubbles equals the interest rate. Then, an agent's lifetime utility is given by

$$U_t^{\iota} = (1 + r_{t+1})(\omega_t^{\iota} - \tau_t). \tag{11}$$

4.2.5. Government

Government finances the unemployment benefit using lump-sum tax on households. From the condition of a balanced government budget, we have

$$\tau_t L = z_t (1 - \sigma_t) L.$$

The left-hand side denotes aggregate tax revenue and the right-hand side represents the payment for unemployment benefit. The unemployment benefits are paid to unemployed workers following such a policy that $z_t = \bar{z} w_t$, where $\bar{z} \in [0,1)$. In other words, the benefit payment to each unemployment worker is proportional to but less than the wage rate in the current period.

4.3 Labor market

4.3.1. Matching mechanism

As discussed in the previous section, young agents and employers search for each other in the labor market. The matching mechanism follows from the standard model of unemployment. Because young agents and firms face matching frictions in the current economy, unemployment occurs in equilibrium although each agent is born endowed with one unit of labor supplied inelastically.

Now, consider matching mechanism in this economy. By denoting the number of successful matches as F, this process can be given by matching function $F(L, \upsilon_{t-1})$, where L and υ_{t-1} represent the number of young agents and the number of firms with vacancy, respectively. Following the standard assumptions, the matching function is to be concave, homogeneous of degree one, increasing in both of its arguments, and $0 \le F(L, \upsilon_{t-1}) \le \min[L, \upsilon_{t-1}]$. The tightness of the labor market is expressed by $\theta_{t-1} \equiv \upsilon_{t-1}/L$, then the probability that a firm with vacancy matches with a young agent is given by $F(L,\upsilon_{t-1})/\upsilon_{t-1} = F(1/\theta_{t-1},1) \equiv q(\theta_{t-1})$. Note that the probability $q(\theta)$ holds the following properties; $q(\theta)\in[0, 1]$, $q'(\theta) < 0$, $\lim_{\theta\to 0} q(\theta) = 1$ and $\lim_{\theta\to\infty} q(\theta) = 0$.[28]

If the search is successful in time $t-1$, employment is realized in the next period (time t). Using the definition of employment rate σ_t, because the realized number of employment is equal to the number of successful of matches, it follows that $F(L,\upsilon_{t-1}) = \sigma_t L$, which is rewritten as:

$$\sigma_t = \theta_{t-1} q(\theta_{t-1}), \tag{12}$$

It shows that the relationship between the employment rate and the tightness of the labor market, from which we obtain $d\sigma_t / d\theta_{t-1} > 0$ because $\partial[\theta_{t-1} q(\theta_{t-1})]/\partial\theta_{t-1} = \partial F(1,\theta_{t-1})/\partial\theta_{t-1} > 0$.[29] Therefore, (12) provides a positive relationship between the employment rate and the tightness of the labor market, which is so-called Beveridge curve. Thus, when the labor market tightness θ approaches to zero (infinity), employment rate σ becomes zero (unity).

If match is made successfully, the firm can produce the final goods and earn operating profits. The probability that a firm will be matched with a worker in periods t is given by $q(\theta_{t-1})$. Thus, $1 - q(\theta_{t-1})$ is the probability that a firm with vacancy cannot match to a worker. Let V_t and J_t be the value of a vacant job and an occupied job in period t, respectively. Then, the value of a vacant job is as follows:

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[28] See den Haan et al. (2000) and Petrongolo and Pissarides (2001) for a discussion on matching functions.

[29] Using $\theta_{t-1} \equiv \upsilon_{t-1}/L$ and (12), it finds that the number of final goods production firms in time t are successful in matching in time $t-1$; $\upsilon_{t-1} q(\theta_{t-1}) = \sigma_t L$.

$$V_{t-1} = -k_{t-1} + \frac{1}{1+r_t}\left[q(\theta_{t-1})J_t + (1-q(\theta_{t-1}))V_t\right], \tag{13}$$

where k_{t-1} denotes the search cost.[30] The second term of the right hand side represents the expected current value of a successful and an unsuccessful match. The value of an occupied job is given by,

$$J_t = \pi_t^Y - w_t. \tag{14}$$

Since the period of employment is one period (adult age), the value of an occupied job is one period profit (implying the full separation rate in one period).

We assume that the final product firms enter the market freely. Then, from free entry condition, the value of a vacant job is $V_t = 0$ for all t. Consequently, from (13), the value of an occupied job becomes $J_t = (1+r_t)k_{t-1}/q(\theta_{t-1})$, and substituting it into (14) yields:

$$\pi_t^Y - w_t = \frac{(1+r_t)k_{t-1}}{q(\theta_{t-1})}. \tag{15}$$

4.3.2. Nash bargaining

The remainder of output after payments to intermediate goods is allotted to a firm and tis worker. We assume that the wage rate is negotiated and determined by Nash bargaining. The household surplus and the firm surplus are given by $U_t^e - U_t^e$ and $J_t - V_t$, respectively. Using (11) and (14) with free entry condition $V_t = 0$, the shares to each are determined by maximizing the following Nash product with respect to the wage:

$$w_t = \arg\max\left(U_t^e - U_t^u\right)^{\beta}\left(J_t - V_t\right)^{1-\beta} = \arg\max\left((1+r_{t+1})(w_t - z_t)\right)^{\beta}\left(\pi_t^Y - w_t\right)^{1-\beta},$$

where $\beta \in (0,1)$ denotes the worker's bargaining power. Then, the wage rate is given by;

$$w_t = (1-\beta)z_t + \beta\pi_t^Y. \tag{16}$$

Using (1), (2), (3) and (5), the output and operating profit of final goods are given by

$$y_t = \alpha^{\frac{2\alpha}{1-\alpha}}N_t, \quad \pi_t^Y = (1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}}N_t. \tag{17}$$

[30] The search cost would cover the recruitment activities such as job interviews and the evaluation of reference letters, which are done by using the firm's operating resources.

Then, using (17) and the unemployment benefit policy $z_t = \bar{z}w_t$, the wage rate is given by the following equation:

$$w_t = \Omega(\bar{z},\beta)(1-\alpha)\alpha^{\frac{2\alpha}{1-\alpha}} N_t, \tag{18}$$

where $\Omega(\bar{z},\beta) \equiv \beta/(1-(1-\beta)\bar{z}) \in (0, 1)$ represents the worker's output share of π_t^Y. Then the following conditions hold; $\partial\Omega/\partial\bar{z} > 0$ and $\partial\Omega/\partial\beta > 0$. It means that the larger outside option that the worker faces leads to the greater share of π_t^Y, and the larger Nash bargaining power enables the worker to obtain the greater share of π_t^Y.

Furthermore, in accordance with Mortensen and Pissarides (1999), Pissarides and Vallanti (2007), and Miyamoto and Takahashi (2011), we assume the form of search cost is as follows: $k_t = \bar{k}y_t$, that is, the search cost is proportionate to the scale of production, $\bar{k} \in (0,1)$.[31] Then, substituting $k_t = \bar{k}y_t$, (17), and (18) into (15) yields:

$$(1-\alpha)(1-\Omega(\bar{z},\beta)) = \frac{(1+r_t)\bar{k}}{(1+g_{t-1})q(\theta_{t-1})}, \tag{19}$$

where $g_{t-1} \equiv (N_t - N_{t-1})/N_{t-1}$ is growth rate of the variety. Eq. (19) is referred as the job creation condition (Pissarides, 2000). It shows that at higher $\Omega(\bar{z},\beta)$ or $\bar{k}$, θ_{t-1} is lower. Also, the growth rate on the firm's effective rate of discount, $(1+g_{t-1})/(1+r_t)$ has positive effect on a job creation (higher θ_{t-1}), which is so-called "capitalization effect" (Aghion and Howitt, 1994). The following section examines the equilibrium growth rate and employment rate.

4.4. Equilibrium

4.4.1. The equilibrium dynamics

Consider the equilibrium dynamics of the economy. First, we derive growth rate of the production variety. The final goods market equilibrium condition is given by

$$Y_t = C_t + N_t X_t + I_t^R + \upsilon_t k_t, \tag{20}$$

[31] This assumption is required to ensure a balanced growth path in which the search cost follows the pace of economic growth.

where Y_t and C_t are the aggregate final goods, and the aggregate consumption, respectively; $Y_t \equiv y_t \sigma_t L$ and $C_t \equiv c_t^e \sigma_{t-1} L + c_t^u (1-\sigma_{t-1}) L$. We can obtain the following asset market equilibrium condition (the derivation is provided in Appendix A):

$$S_t = D_t N_{t+1} + \upsilon_t k_t. \tag{21}$$

It finds that the interesting-bearing assets consist of the patent of varieties $D_t N_{t+1}$ and the total search cost of the matching process for the final goods production $\upsilon_t k_t$.[32] On the other hand, from the budget constraint of households, the aggregate holdings of the interesting-bearing assets $S_t \left(\equiv s_t^e \sigma_t L + s_t^u (1-\sigma_t) L \right)$ can be given by:

$$S_t = w_t \sigma_t L - B_t, \tag{22}$$

where $B_t \equiv p_t^B \left[m_t^e \sigma_t L + m_t^u (1-\sigma_t) L \right]$ represents aggregate demand of asset bubbles. Then, denoting the growth rate of varieties as $g_t \equiv (N_{t+1} - N_t)/N_t$, we obtain the following equation (the derivation is provided in Appendix B):

$$1 + g_t = \frac{\dfrac{1}{\eta}\left[\dfrac{\Omega}{\alpha} \eta r(\sigma_t) - b_t \right]}{\left[1 + \dfrac{1-\Omega}{\alpha} \dfrac{r(\sigma_{t+1})}{1 + r(\sigma_{t+1})} \right]} \tag{23}$$

where $b_t \equiv B_t / L N_t$ is defined as the normalized bubbles. It finds that the growth rate depends on variables of employment rate and bubbles; $g_t = g(\sigma_{t+1}, \sigma_t, b_t)$.

Then, using (10), (19) and (23) with $\theta_t = \theta(\sigma_{t+1})$ from (12), we obtain the dynamics of the employment rate:

$$\sigma_{t+1} = \sigma(\sigma_t, b_t) \;\Leftrightarrow\; \frac{1 + r(\sigma_{t+1})}{1 + g(\sigma_{t+1}, \sigma_t, b_t)} = \frac{1}{\Delta} q(\theta(\sigma_{t+1})), \tag{24}$$

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[32] Using (9), the holding of the patent of varieties can be rewritten as the current value of the return: $D_t N_{t+1} = \dfrac{\pi_{t+1}^X + D_{t+1}}{1 + r_{t+1}} N_{t+1}$. Also, using (12) and (15) with $\theta_t \equiv \upsilon_t / L$, the total search cost can be rewritten as the current value of the return: $\upsilon_t k_t = \dfrac{\pi_{t+1}^Y - w_{t+1}}{1 + r_{t+1}} \sigma_{t+1} L$. Therefore, it finds that the interest-bearing asset is devoted to the investments to the expected profits of final goods sector and intermediate goods sector.

where $\Delta \equiv \bar{k}/(1-\alpha)(1-\Omega(\beta,\bar{z}))$ can be interpreted as the cost parameter of firm's entry because the parameter Δ increases in both search cost $\bar{k}$ and worker's profit share $\Omega(\beta,\bar{z})$. Using (12), the probability q is described as a function of σ; $q'(\sigma) < 0$, $q(0) = 1$ and $q(1) = 0$. Then, we obtain the following properties; $\partial\sigma_{t+1}/\partial\sigma_t > 0$ and $\partial\sigma_{t+1}/\partial b_t < 0$ (the derivation is provided in Appendix C).

Let $B_t = p_t^B M$ be the real value of the bubble at time t, where M is total nominal supply of bubbles; then the equilibrium condition is given by $M = m_t^e\sigma_t L + m_t^u(1-\sigma_t)L$. By the arbitrage condition, we have the dynamics of bubbles: $B_{t+1} = (1+r_{t+1})B_t$. Using $b_t \equiv p_t^B M/LN_t$, then the dynamics of the normalized bubbles can be obtained as follows:

$$b_{t+1} = b(\sigma_t, b_t) \iff b_{t+1} = \frac{1+r(\sigma_{t+1})}{1+g(\sigma_{t+1},\sigma_t,b_t)} b_t. \tag{25}$$

using (24). The equilibrium of this economy is completely described by these equations; (24) and (25) in σ_t and b_t.

The phase diagram can be drawn on the (σ_t, b_t) plane. We refer to the locus on the plane (σ_t, b_t) representing $\sigma_{t+1} = \sigma_t$ as the σ locus and that representing $b_{t+1} = b_t$ as the b locus. Using (23), from (24) and (25), the σ and b loci are represented as equal parts of (26) and (27), respectively;

$$\sigma_{t+1} \geq \sigma_t: \; b_t \leq \Gamma(\sigma_t) \equiv \eta\left[\frac{\Omega}{\alpha} r(\sigma_t) - \frac{\Delta}{q(\sigma_t)}\left(1 + r(\sigma_t) + \frac{1-\Omega}{\alpha} r(\sigma_t)\right)\right], \tag{26}$$

$$b_{t+1} \geq b_t: \begin{cases} b_t \geq \Phi(\sigma_t) \Leftrightarrow b_t \geq \eta\left[\frac{\Omega}{\alpha} r(\sigma_t) - \left(1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha} r(\sigma_{t+1})\right)\right], \\ b_t = 0. \end{cases} \tag{27}$$

where $\sigma_{t+1} = \sigma(\sigma_t, b_t)$ from (24). The phase diagram is as shown in Figure 1. It shows that the slope of the σ locus represents the inversed-U shape, in which $\Gamma(0) < 0$, $\Gamma(1) < 0$ and $\Gamma''(\sigma_t) < 0$ are satisfied. The b locus has two lines; one is represented by horizontal line in $b = 0$, and the another is represented by an increase curve $\Phi'(\sigma_t) > 0$ in $b > 0$.[33] Then, we can show that there are two phases; "non-bubble regime" and "bubble regime" as shown in Figure 1 (i) and (ii),

[33] See Appendix D for the slope of σ locus and b locus.

respectively. In "non-bubble regime", there is a unique saddle path to the point E^{NB}, since the path to point E is source. In this regime, bubbles can not occur. In "bubble regime", there are two equilibrium; E^{N} and E^{B}, since the path to point E is source. The equilibrium path to E^{N} is sink, while the equilibrium path to E^{B} is saddle-point stable.[34] In this regime, bubble equilibrium can occur at the point E^{B}.

4.4.2. The condition for bubbles

In this section, we derive the condition under which bubbles exist in a steady state. In the bubble equilibrium, by using equation (25) with (10), the growth rate with a positive bubble g^{B} can be expressed by:

$$g^{B} = r(\sigma) \equiv \frac{1}{\overline{\eta}}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\sigma, \tag{28}$$

then the growth rate depends only on interest rate. By substituting (28), (12) and (19) into (23) and then rearranging them, we can get the value of equilibrium bubble

$$b = \overline{\eta}\left[r(\sigma)\left(\frac{\Omega}{\alpha} - \frac{1-\Omega}{\alpha} - 1\right) - 1\right]. \tag{29}$$

We assume the parameter condition $\Omega > (1+\alpha)/2$ for the possibility of bubble equilibrium.[35] From (29) with (10), we obtain the condition of the employment rate for a bubble regime,

$$\sigma > \hat{\sigma} \equiv \frac{\overline{\eta}}{(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}\left(\frac{\Omega}{\alpha} - \frac{1-\Omega}{\alpha} - 1\right)}. \tag{30}$$

Then, bubble regime where positive bubble can exist ($b > 0$) holds for $\sigma > \hat{\sigma}$, while non-bubble regime where bubble cannot exist value ($b = 0$) holds for $\sigma \leq \hat{\sigma}$. Thus, we obtain the following proposition.

Proposition 1: *If the equilibrium employment rate is over a threshold level $\hat{\sigma}$ in (27), then bubbles can exist in the equilibrium; if not, then bubbles cannot exist.*

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[34] See Appendix E for the local stability analysis of these equilibrium paths in each regime.

[35] This assumption is imposed to allow bubble equilibrium to occur; otherwise, the possibility of bubbles is intrinsically avoided. Note that this assumption is the possibility of a non-bubble equilibrium is not eliminated. In fact, the following proposition obtains the non-bubble equilibrium under it.

It implies that the equilibrium employment rate plays an important role in the existence of bubbles. The following subsection examines the steady state equilibrium of employment rate in greater detail.

4.4.3. Equilibrium employment and growth

On the non-bubble economy, from (23) with $b = 0$, the growth rate without bubble g^N can be rewritten as:

$$1+g^N(\sigma)=\frac{\Omega}{\frac{\alpha}{r(\sigma)}+\frac{1-\Omega}{1+r(\sigma)}}. \tag{31}$$

The relationship between the growth rate without bubble and the growth rate with bubble is described in Figure 2. The economy will be in a bubble regime (non-bubble regime) when the equilibrium employment rate is higher (lower) than the threshold level.

Now we derive the equilibrium employment. In the steady state, (24) gives the level of the employment rate with (31) in bubble-less equilibrium, and with (28) in bubble equilibrium.

$$\frac{1+r(\sigma)}{1+g^N(\sigma)}=\frac{1}{\Delta}q(\sigma) \text{ for bubble-less equilibrium,} \tag{32}$$

$$1=\frac{1}{\Delta}q(\sigma) \text{ for bubble equilibrium,} \tag{33}$$

where $\Delta \equiv \bar{k}/(1-\alpha)(1-\Omega(\beta,\bar{z}))$. The relationship between the employment rate with and without bubble is described in Figure 3 (a) and (b). These equations (32) and (33) determine the equilibrium of employment rate. We summarize the determinants of employment rate in the following lemma.

Lemma 1: *An increase in the search cost ($\bar{k}$) decreases employment rate. An Increase in the R&D cost (η) has negative effect on the employment rate in the bubble-less equilibrium, while it has no effect on the employment rate in the bubble equilibrium. Increases in unemployment benefit rate ($\bar{z}$) and the bargaining power of worker (β) decrease the employment rate in the bubble equilibrium, while under bubble-less economy they increase employment rate for* $\Omega(\beta,\bar{z})<\Omega^*$ *and decrease it for* $\Omega(\beta,\bar{z})>\Omega^*$.

Proof. See Appendix F.

An increase in $\bar{k}$ is captured by down shift of right hand side of (32) and (33), which leads to a negative effect on employment rate. This is because an increase in search cost decrease the entry of firms with vacant job, which decreases the labor market tightness and employment rate falls. An increase in $\bar{\eta}$ shifts the left hand side of (32) downward, then the employment rate decreases in bubble-less equilibrium. The effect of R&D cost increases the relative interest rate to growth rate, which decreases the expected current value of profit, therefore the entry of firms with vacant job decreases and employment rate falls. Since under bubble regime the growth rate always equal to the interest rate, the R&D cost has no impact on the determinant of employment. Analogous to the case of $\bar{k}$, an increase in $\Omega(\beta, \bar{z})$, which is increases by β or $\bar{z}$, shifts the right hand side of (32) and (33) downward, then employment rate decrease. In addition to the effect, an increase in $\Omega(\beta, \bar{z})$ has positive effect on the growth rate through an increase in household income, which shifts the left hand side of (32) downward, therefore it increases employment rate under bubble-less economy. As shown in Appendix F, in the low (high) range of $\Omega(\beta, \bar{z})$, unemployment benefit rate ($\bar{z}$) and the bargaining power of worker (β) has a positive (negative) effect on employment rate, as the positive effect (income effect) dominates the negative effect (entry cost effect).

4.4.4. The dynamics of boom and bust of bubbles

The economy will be in a non-bubble or bubble regime when the equilibrium employment rate is lower or higher, respectively, than the threshold level. As pointed out by Aliber and Kindleberger (2015), the term "bubble" itself foreshadows the end of an economic bubble. If the cause of bubble bursting stems from the realization of a sunspot, then the bubble equilibrium shifts to the non-bubble equilibrium under bubble regime, which leads to higher economic growth. Thus, the bubble burst caused by a sunspot results in bringing a crowd-out effect, a negative relationship between bubble and growth (Grossman and Yanagawa, 1993). In our framework, we follow the approach of the bubbles boom-bust by Brunnermeier and Oehmke (2015), who point out that an initial boom in asset bubble is often triggered by fundamentals.[36] In our model, it shows that labor market frictions can lead to a bubbly steady

[36] See Farhi and Tirole (2012) for the discussion of the boom and bust properties of bubbles; two types of causes leading the asset bubbles.

state (i.e., a bubble regime). Therefore, our framework focuses on the boom-bust of asset bubbles caused by changes in fundamental variables, such as labor market conditions or R&D production technologies.

We can examine the boom and bust of bubbles through regime shifts resulting from changes in parameter conditions. If changes in policies or parameters cause a decrease in the employment rate (Lemma 1; e.g., a rise in search cost or a fall in R&D technology), the economy will shift from a bubble regime to a non-bubble regime. It finds that the equilibrium employment E^B in Figure 3(b) changes to E^{NB} in Figure 3(a). A bubble burst is accompanied by a decrease in employment rate and economic growth rate. Then, as shown in Figure 2, the output growth rate is always higher under the bubble regime than under the non-bubble regime, even when bubbles occur.

Furthermore, using comparative dynamics in response to changes in parameters, we can analyze the dynamic properties of the boom and bust of a bubble in the phase diagram. Consider a rise in search cost ($\bar{k}$), which leads to decrease employment rate in steady state. From (26) and (27) with (24), σ locus shifts downward and b locus shifts upward.[37] Therefore, if changes in search cost occur under bubble equilibrium, bubbles can suddenly burst and both employment rate and growth rate converge to a lower steady state. Conversely, a decrease in search cost can lead to a bubble boom and both employment rate and economic growth rate converge to a high steady state. The dynamic behavior of a bubble burst is shown in Figure 4 (a) and bubble boom in Figure 4 (b).

On the effect of production technologies, a fall in R&D productivity (an increase in $\overline{\eta}$) has no effect on the employment rate under bubble steady state equilibrium (lemma 1), while it decreases the region of bubble regime (an increase in the threshold level $\hat{\sigma}$ from Proposition 1). Then, from (26) and (27) with (24), both σ locus and b locus shift downward.[38] Therefore, if a negative shock of R&D productivity occur and the threshold level exceeds the employment rate under bubble equilibrium, bubbles can suddenly burst and employment rate and growth rate converges to a lower steady state under non-bubble equilibrium.

These results are formally stated in the following proposition.

[37] See Appendix G for the mathematical derivation of these shifts of σ locus and b locus; $\partial\Gamma/\partial\bar{k} < 0$ and $\partial\Phi/\partial\bar{k} > 0$.

[38] See Appendix G for the mathematical derivation of these shifts of σ locus and b locus; $\partial\Gamma/\partial\overline{\eta} < 0$ and $\partial\Phi/\partial\overline{\eta} < 0$.

Proposition 2: *If policies or parameters change to cause a decrease (an increase) in the employment rate under bubble economy (under non bubble economy), asset bubbles can burst (boom) immediately and employment rate converges to a lower (higher) equilibrium which leads to a lower (higher) growth rate.*

Figure 5 summarizes the dynamic paths of bubbles, employment rate and economic growth after bubble bust.

4.5. Conclusion

In this paper, we developed an overlapping-generation model with labor market friction and examined the conditions for bubbles. We showed theoretical relationships between bubbles, economic growth, and employment. In contrast to previous studies, we introduce labor market friction into an endogenous growth model, so that the interest rate depends on labor market conditions. Allowing for unemployment, fluctuations induced by the labor market determine the type of regime that the economy will be under bubble-less or bubble equilibrium. Based on our finding that bubbles can (not) occur when the equilibrium employment rate is high (low), and the interest and economic growth rates are high (low), we conclude that policies that have a positive impact on the labor market (e.g., a decrease in the search cost) can improve employment and place the economy under a bubble regime. This, in turn, will raise both the interest rate and the economic growth rate.

Appendices

Appendix A: The derivation of the equation (21)

From (15) and (1), the output of a firm can be expressed by

$$y_t = w_t + \frac{(1+r_t)k_{t-1}}{q(\theta_{t-1})} + p_t x_t N_t . \tag{A1}$$

Based on equation (A1), and using (12), $\theta_{t-1} \equiv \upsilon_{t-1} / L$, $X_t \equiv x_t \sigma_t L$ and the fact that the number of firms in the final goods sector are equal to the number of successful of matches $\sigma_t L$, we can obtain the aggregate the output $Y_t \equiv y_t \sigma_t L$ as follows:

$$Y_t \equiv y_t \sigma_t L = w_t \sigma_t L + (1+r_t)k_{t-1}\upsilon_{t-1} + p_t X_t N_t . \tag{A2}$$

Therefore, the market-clearing condition (20) for final goods is expressed in the following manner:

$$w_t\sigma_t L + (1+r_t)\upsilon_{t-1}k_{t-1} + p_t X_t N_t = C_t + N_t X_t + I_t^R + \upsilon_t k_t. \qquad \text{(A3)}$$

Using $\pi_t^X = (p_t - 1)X_t, \eta = \overline{\eta}L$, (7), (8) and (9), we obtain the following expression:

$$S_t + p_t^B M + (1+r_t)\upsilon_{t-1}k_{t-1} + (p_t - 1)X_t N_t = (1+r_t)S_{t-1} + p_t^B M + D_t(N_{t+1} - N_t) + \upsilon_t k_t$$

$$\Leftrightarrow \quad S_t - D_t N_{t+1} - \upsilon_t k_t = (1+r_t)[S_{t-1} - D_{t-1}N_t - \upsilon_{t-1}k_{t-1}].$$

Because initial assets are given by $S_{-1} = D_{-1}N_0 + \upsilon_{-1}k_{-1}$, we obtain (18) for any period $t > 0$.

Appendix B: The derivation of (23)

By dividing equation (21) by LN_t and substituting equations (8), (18), (22) and $k_t = \bar{k}y_t$ into (21) yields the growth rate of varieties $g_t \equiv (N_{t+1} - N_t)/N_t$ as follows:

$$1 + g_t = \frac{1}{\overline{\eta}}\left[\Omega\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)\sigma_t - \theta_t\bar{k}\alpha^{\frac{2\alpha}{1-\alpha}} - b_t\right]. \qquad \text{(B1)}$$

From (19) with (12), the following condition can be obtained;

$$(1-\alpha)(1-\Omega)\sigma_{t+1}\frac{1+g_t}{1+r_{t+1}} = \theta_t\bar{k}. \qquad \text{(B2)}$$

After using (B1) and (B2) with (10) to eliminate $\theta_t\bar{k}$, we obtain (23) as follows:

$$1 + g_t = \frac{1}{\overline{\eta}}\left[\Omega\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)\sigma_t - (1-\Omega)\alpha^{\frac{2\alpha}{1-\alpha}}(1-\alpha)\sigma_{t+1}\frac{1+g_t}{1+r_{t+1}} - b_t\right]$$

$$\Leftrightarrow \quad (1+g_t)\left[1 + \frac{1-\Omega}{\alpha}\frac{r(\sigma_{t+1})}{1+r(\sigma_{t+1})}\right] = \frac{1}{\overline{\eta}}\left[\frac{\Omega}{\alpha}\overline{\eta}r(\sigma_t) - b_t\right]$$

$$\Leftrightarrow \quad 1 + g_t = \frac{\frac{1}{\overline{\eta}}\left[\frac{\Omega}{\alpha}\overline{\eta}r(\sigma_t) - b_t\right]}{\left[1 + \frac{1-\Omega}{\alpha}\frac{r(\sigma_{t+1})}{1+r(\sigma_{t+1})}\right]}.$$

Appendix C: The property of σ(σ$_t$, *b*$_t$)

Using (23), Eq. (24) can be rewritten as:

$$\frac{1+r(\sigma_{t+1})}{1+g(\sigma_{t+1},\sigma_t,b_t)}=\frac{1}{\Delta}q(\sigma_{t+1})\Leftrightarrow$$

$$\overline{\eta}\left\{1+r(\sigma_{t+1})+\frac{1-\Omega}{\alpha}r(\sigma_{t+1})\right\}\frac{1}{q(\sigma_{t+1})}=\frac{1}{\Delta}\left[\frac{\Omega}{\alpha}\overline{\eta}r(\sigma_t)-b_t\right]. \tag{C1}$$

Totally differentiating (C1) leads to:

$$[A]d\sigma_{t+1}=\left[\frac{1}{\Delta}\frac{\Omega}{\alpha}\overline{\eta}r'\right]d\sigma_t+\left[\frac{-1}{\Delta}\right]db_t,$$

where $$A\equiv\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}}-\frac{q'(\sigma_{t+1})}{\Delta}$$

$$=\overline{\eta}r'\left(1+\frac{1-\Omega}{\alpha}\right)\frac{1}{q(\sigma_{t+1})}+\overline{\eta}\left\{1+r(\sigma_{t+1})+\frac{1-\Omega}{\alpha}r(\sigma_{t+1})\right\}\frac{-q'(\sigma_{t+1})}{q(\sigma_{t+1})^2}>0,$$

$r'=\frac{1}{\overline{\eta}}(1-\alpha)\alpha^{\frac{1+\alpha}{1-\alpha}}$ is positive and constant.

Therefore, we have

$$\frac{\partial\sigma_{t+1}}{\partial\sigma_t}=\frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right]=\frac{1}{A}\left[\frac{1}{\Delta}\frac{\Omega}{\alpha}\overline{\eta}r'\right]>0, \tag{C2}$$

$$\frac{\partial\sigma_{t+1}}{\partial b_t}=\frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right]=\frac{1}{A}\left[\frac{-1}{\Delta}\right]<0. \tag{C3}$$

Appendix D: The slope of σ locus and *b* locus

First consider the slope of σ locus. From (26), we have the slope of σ locus as follows:

$$\frac{db_t}{d\sigma_t}=\Gamma'(\sigma_t)=\overline{\eta}\left[\frac{\Omega}{\alpha}r'-\frac{\Delta}{q(\sigma_t)}\left(r'+\frac{1-\Omega}{\alpha}r'\right)+\frac{q'(\sigma_t)\Delta}{q(\sigma_t)^2}\left(1+r(\sigma_t)+\frac{1-\Omega}{\alpha}r(\sigma_t)\right)\right].$$

Furthermore, we obtain the second derivatives as follows:

$$\frac{d^2b_t}{d\sigma_t^{\;2}}=\Gamma''(\sigma_t)=\frac{\overline{\eta}\Delta}{q^3}\left[2q'q\left(r'+\frac{1-\Omega}{\alpha}r'\right)+\left[-2(q')^2+q''q\right]\left(1+r+\frac{1-\Omega}{\alpha}r\right)\right].$$

Then, the condition $\Gamma''(\sigma_t) < 0$ holds as long as the probability $q(\cdot)$ is not too convex. We assume the functional form $q(\cdot)$ to satisfy $\Gamma''(\sigma_t) < 0$. In addition to the above properties, using σ locus, $\sigma_{t+1}(\sigma_t, b_t) = \sigma_t$, we have

$$\frac{\partial\sigma_{t+1}}{\partial b_t} db_t = \left[1 - \frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right] d\sigma_t .$$

Thus, the slope of σ locus is positive when $\partial\sigma_{t+1}/\partial\sigma_t > 1$, while the slope is negative when $\partial\sigma_{t+1}/\partial\sigma_t \in (0,1)$.

Next consider the slope of b locus. Totally differentiating (27) gives;

$$\left[1 + \left(1 + \frac{1-\Omega}{\alpha}\right)\overline{\eta} r' \frac{\partial\sigma_{t+1}}{\partial b_t}\right] db_t = \left[\frac{\Omega}{\alpha} - \left(1 + \frac{1-\Omega}{\alpha}\right)\frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right]\overline{\eta} r' d\sigma_t . \tag{D1}$$

Using (C2), (C3) and the condition $q(\sigma_{t+1})/\Delta = 1$ in positive bubble equilibrium ($b_t > 0$), each coefficient of db_t and $d\sigma_t$ is given by

$$\left[1 + \left(1 + \frac{1-\Omega}{\alpha}\right)\overline{\eta} r' \frac{\partial\sigma_{t+1}}{\partial b_t}\right] = \frac{1}{A}\overline{\eta}\left\{1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha} r(\sigma_{t+1})\right\}\frac{-q'(\sigma_{t+1})}{q(\sigma_{t+1})^2} > 0, \tag{D2}$$

$$\left[\frac{\Omega}{\alpha} - \left(1 + \frac{1-\Omega}{\alpha}\right)\frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right]\overline{\eta} r' = \frac{1}{A}\overline{\eta} r' \overline{\eta}\frac{\Omega}{\alpha}\overline{\eta}\left\{1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha} r(\sigma_{t+1})\right\}\frac{-q'(\sigma_{t+1})}{q(\sigma_{t+1})^2} > 0. \tag{D3}$$

Therefore, the slope of b locus, $b_t = \Phi(\sigma_t)$, is positive; $db_t / d\sigma_t = \Phi'(\sigma_t) > 0$ in $b_t > 0$.

Appendix E: Dynamic stability

Totally differentiating (24) and (25) leads to:

$$d\sigma_{t+1} = \frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right] d\sigma_t + \frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right] db_t , \tag{E1}$$

$$db_{t+1} = b\left(\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}}\frac{\partial\sigma_{t+1}}{\partial\sigma_t} + \frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right) d\sigma_t + \left[\frac{1+r_{t+1}}{1+g_t} + b\left(\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}}\frac{\partial\sigma_{t+1}}{\partial b_t} + \frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right)\right] db_t . \tag{E2}$$

Then, the Jacobian matrices of this system are as follows:

$$\begin{bmatrix} \frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right] & \frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right] \\ b\left(\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}}\frac{\partial\sigma_{t+1}}{\partial\sigma_t}+\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right) & \frac{1+r_{t+1}}{1+g_t}+b\left(\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}}\frac{\partial\sigma_{t+1}}{\partial b_t}+\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right) \end{bmatrix}.$$

First, consider the stability of the equilibrium under non bubble regime ($b = 0$). The following conditions are satisfied around the steady state (E^{NB}):

$$\frac{\partial\sigma_{t+1}}{\partial\sigma_t}=\frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right]>0,\ \frac{\partial\sigma_{t+1}}{\partial b_t}=\frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right]<0,\frac{\partial b_{t+1}}{\partial\sigma_t}=0,\frac{\partial b_{t+1}}{\partial b_t}=\frac{1+r}{1+g^N}>1.$$

Furthermore, around the steady state at the point E^{NB} in Figure 3 (a), we have

$$\frac{\partial\sigma_{t+1}}{\partial\sigma_t}<1 \quad\Leftrightarrow\quad \frac{-q'(\sigma)}{\Delta}>\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}}+\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}. \tag{E3}$$

Denote the trace and determinant of the Jacobian matrices as T and D respectively. In addition, the eigenvalues are denoted as λ_j (j = 1, 2), the characteristic polynomial is expressed as $\xi(\lambda)\equiv\lambda^2-T\lambda+D$. Under non-bubble regime $\xi(0)>0$ and $\xi(1)<0$ are obtained. As is well known (Azariadis, 1993; Chapter 6), the steady state is a saddle if the relations $\xi(0)>0$ and $\xi(1)<0$ hold simultaneously. Therefore, the steady state (E^{NB}) under non bubble regime is stable and a saddle.

Next, consider the equilibrium under bubble regime; bubble-less equilibrium (E^N) and bubble equilibrium (E^B). Around the steady state under the bubble regime with bubble-less equilibrium (E^N) the following conditions are satisfied:

$$\frac{\partial\sigma_{t+1}}{\partial\sigma_t}=\frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t}\right]<1,\ \frac{\partial\sigma_{t+1}}{\partial b_t}=\frac{1}{A}\left[-\frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial b_t}\right]<0,\frac{\partial b_{t+1}}{\partial\sigma_t}=0,\frac{\partial b_{t+1}}{\partial b_t}=\frac{1+r}{1+g^N}<1,$$

using (E3). Then, $\xi(0) > 0$, $\xi(1) > 0$ and $T < 2$ are obtained. Therefore, the steady state (E^N) under bubble regime is a sink.

Around the steady state under the bubble regime with bubble equilibrium (E^B) the following conditions are satisfied:

$$\frac{\partial\sigma_{t+1}}{\partial\sigma_t} = \frac{1}{\hat{A}}\frac{\partial g_t}{\partial\sigma_t} > 0, \quad \frac{\partial\sigma_{t+1}}{\partial b_t} = \frac{1}{\hat{A}}\frac{\partial g_t}{\partial b_t} < 0,$$

$$\frac{\partial b_{t+1}}{\partial\sigma_t} = \frac{1}{1+g} b\left[\left(\frac{\partial r_{t+1}}{\partial\sigma_{t+1}} - \frac{\partial g_t}{\partial\sigma_{t+1}}\right)\frac{\partial\sigma_{t+1}}{\partial\sigma_t} - \frac{\partial g_t}{\partial\sigma_t}\right],$$

$$\frac{\partial b_{t+1}}{\partial b_t} = 1 + \frac{1}{1+g} b\left[\left(\frac{\partial r_{t+1}}{\partial\sigma_{t+1}} - \frac{\partial g_t}{\partial\sigma_{t+1}}\right)\frac{\partial\sigma_{t+1}}{\partial b_t} - \frac{\partial g_t}{\partial b_t}\right],$$

where $\hat{A} \equiv -\frac{q'(\sigma)}{q'(\sigma)}(1+g) + \frac{\partial r_{t+1}}{\partial\sigma_{t+1}} - \frac{\partial g_t}{\partial\sigma_{t+1}} > 0$. Then, we obtain $\xi(1) \equiv 1 - T + D$ and $\xi(-1) \equiv 1 + T + D$ as:

$$\xi(-1) = -\frac{b}{1+g}\left[\mathrm{H}\frac{\partial\sigma_{t+1}}{\partial b_t} - \left(1 - \frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right)\frac{\partial g_t}{\partial b_t}\right] > 0,$$

$$\xi(1) = 2 + 2\frac{\partial\sigma_{t+1}}{\partial\sigma_t} + + \frac{b}{1+g}\left[\mathrm{H}\frac{\partial\sigma_{t+1}}{\partial b_t} - \left(1 - \frac{\partial\sigma_{t+1}}{\partial\sigma_t}\right)\frac{\partial g_t}{\partial b_t}\right] < 0,$$

where $\mathrm{H} \equiv \frac{\partial r_{t+1}}{\partial\sigma_{t+1}} - \frac{\partial g_t}{\partial\sigma_{t+1}} - \frac{\partial g_t}{\partial\sigma_t} = \frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_{t+1}} + \frac{\partial\left(\frac{1+r_{t+1}}{1+g_t}\right)}{\partial\sigma_t} < 0$.

Therefore, the steady state with bubbles (E^B) under bubble regime is stable and a saddle, since $\xi(-1) > 0$ and $\xi(1) < 0$ hold simultaneously.

Appendix F: The proof of lemma 1

Under bubble regime, totally differentiating (33) gives:

$$-q'(\sigma)d\sigma = -\frac{\partial\Delta}{\partial\bar{k}}d\bar{k} - \frac{\partial\Delta}{\partial\Omega}\frac{\partial\Omega}{\partial\beta}d\beta - \frac{\partial\Delta}{\partial\Omega}\frac{\partial\Omega}{\partial\bar{z}}d\bar{z},$$

where $\Delta \equiv \bar{k}/(1-\alpha)(1-\Omega)$ and $\Omega \equiv \beta/(1-(1-\beta)\bar{z})$. Thus, we obtain

$$\bar{k}\uparrow, \beta\uparrow, \bar{z}\uparrow \Rightarrow \sigma^B \downarrow.$$

Under bubble-less economy, substituting (31) into (32) leads to

$$\left(\frac{1+r}{1+g^N}=\right)\ \frac{\alpha}{\Omega}\frac{1+r(\sigma)}{r(\sigma)}+\frac{1-\Omega}{\Omega}=\frac{1}{\Delta}q(\sigma). \qquad \text{(F1)}$$

Totally differentiating (F1) gives:

$$\bar{A}\,d\sigma=\frac{-1}{\Delta^2}\frac{\partial\Delta}{\partial\bar{k}}q(\sigma)d\bar{k}+[\Xi]\left(\frac{\partial\Omega}{\partial\beta}d\beta+\frac{\partial\Omega}{\partial\bar{z}}d\bar{z}\right)-\frac{\partial\left(\frac{1+r}{1+g^N}\right)}{\partial\bar{\eta}}d\bar{\eta},$$

$$\text{where } \bar{A}\equiv\left[\frac{-q'(\sigma)}{\Delta}+\frac{\partial\left(\frac{1+r}{1+g^N}\right)}{\partial\sigma}\right]>0,\quad \frac{\partial\left(\frac{1+r}{1+g^N}\right)}{\partial\bar{\eta}}>0,$$

$$\Xi\equiv\left[-\frac{\partial\left(\frac{1+r}{1+g^N}\right)}{\partial\Omega}+\frac{-1}{\Delta^2}\frac{\partial\Delta}{\partial\Omega}q(\sigma)\right]=\frac{1}{\Omega^2}\left(\alpha\frac{1+r}{r}+1\right)-\frac{1}{1-\Omega}\left(\frac{\alpha}{\Omega}\frac{1+r}{r}+\frac{1-\Omega}{\Omega}\right)$$

$$=\frac{1}{\Omega^2}\left[\left(\alpha\frac{1+r}{r}+1\right)-\frac{\Omega}{1-\Omega}\left(\alpha\frac{1+r}{r}+1-\Omega\right)\right].$$

We can easily confirm that $\Xi>0$ when Ω approaches to 0 and $\Xi<0$ when Ω approaches to 1. Then, there is the threshold value of $\Omega=\Omega^*$ to satisfy $\Xi=0$,

$$\Omega^*=\psi-\sqrt{\psi^2-\psi},\qquad \text{where } \psi\equiv\alpha\frac{1+r}{r}+1.$$

Thus, we obtain

$$\bar{k}\uparrow,\bar{\eta}\uparrow\Rightarrow\sigma^N\downarrow,$$

$$\beta\uparrow,\bar{z}\uparrow\Rightarrow\sigma^N\uparrow(\downarrow)\ \text{ if }\ \Omega(\beta,\bar{z})<(>)\Omega^*.$$

Appendix G: Comparative dynamics: $\bar{k}$ and $\bar{\eta}$

From (26), we obtain the following conditions:

$$\frac{\partial\Gamma(\sigma_t)}{\partial\bar{k}}=-\frac{\bar{\eta}}{q(\sigma_t)}\left(1+r(\sigma_t)+\frac{1-\Omega}{\alpha}r(\sigma_t)\right)\frac{\partial\Delta}{\partial\bar{k}}<0.$$

$$\frac{\partial \Gamma(\sigma_t)}{\partial \overline{\eta}} = -\frac{\Delta}{q(\sigma_t)} < 0.$$

Using (D1), totally differentiating (27) gives;

$$\frac{\partial \Phi(\sigma_t)}{\partial \bar{k}} = \frac{-1}{\Theta} \overline{\eta} r' \left(1 + \frac{1-\Omega}{\alpha}\right) \frac{\partial \sigma_{t+1}}{\partial \bar{k}}, \tag{G1}$$

$$\frac{\partial \Phi(\sigma_t)}{\partial \overline{\eta}} = \frac{-1}{\Theta} \left[1 + \overline{\eta} r' \left(1 + \frac{1-\Omega}{\alpha}\right) \frac{\partial \sigma_{t+1}}{\partial \overline{\eta}}\right], \tag{G2}$$

where $\Theta \equiv \left[1 + \left(1 + \frac{1-\Omega}{\alpha}\right) \overline{\eta} r' \frac{\partial \sigma_{t+1}}{\partial b_t}\right] > 0$ from (D2).

Totally differentiating (C1) leads to the following conditions:

$$\frac{\partial \sigma_{t+1}}{\partial \bar{k}} = \frac{1}{A} \frac{-1}{\Delta^2} \frac{\partial \Delta}{\partial \bar{k}} \left(\frac{\Omega}{\alpha} \overline{\eta} r(\sigma_t) - b_t\right) < 0, \tag{G3}$$

$$\frac{\partial \sigma_{t+1}}{\partial \overline{\eta}} = \frac{1}{A} \frac{-1}{q(\sigma_{t+1})} < 0. \tag{G4}$$

Using (G3) and (G4), then we obtain the sign of (G1) and (G2) as follows:

$$\frac{\partial \Phi(\sigma_t)}{\partial \bar{k}} = \frac{1}{\Theta} \overline{\eta} r' \left(1 + \frac{1-\Omega}{\alpha}\right) \frac{1}{A} \frac{1}{\Delta^2} \frac{\partial \Delta}{\partial \bar{k}} \left(\frac{\Omega}{\alpha} \overline{\eta} r(\sigma_t) - b_t\right) > 0,$$

$$\frac{\partial \Phi(\sigma_t)}{\partial \overline{\eta}} = \frac{-1}{\Theta} \frac{1}{A} \overline{\eta} \left(1 + r(\sigma_{t+1}) + \frac{1-\Omega}{\alpha} r(\sigma_{t+1})\right) \frac{-q'(\sigma_{t+1})}{(q(\sigma_{t+1}))^2} < 0.$$

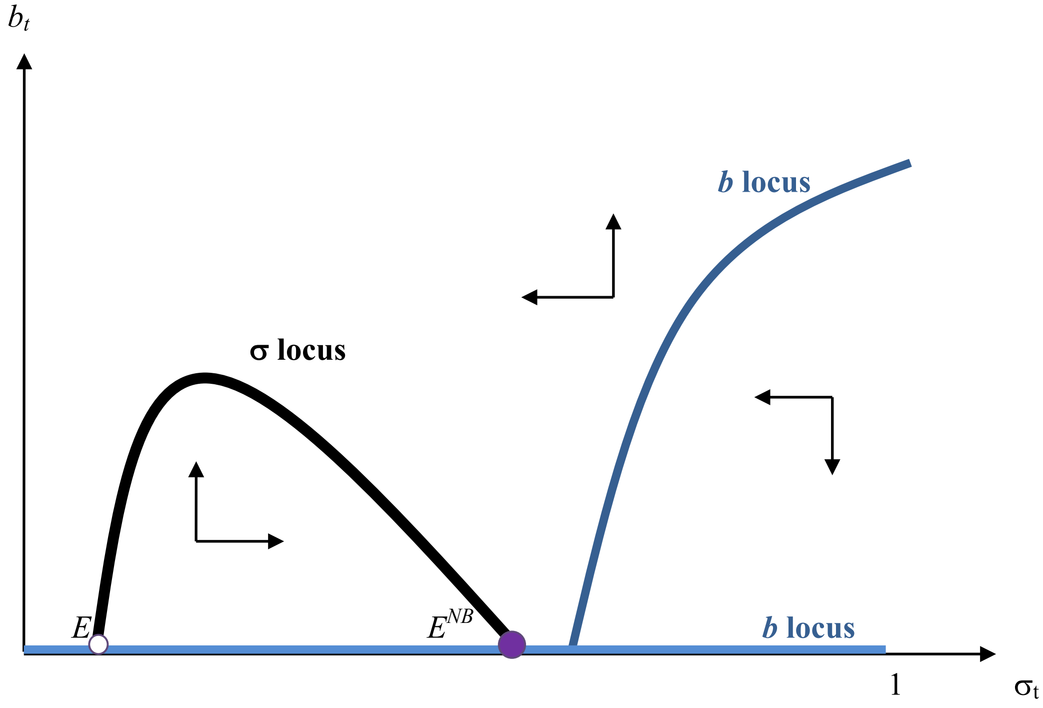

Figure 1. Phase diagram: (i) non-bubble regime

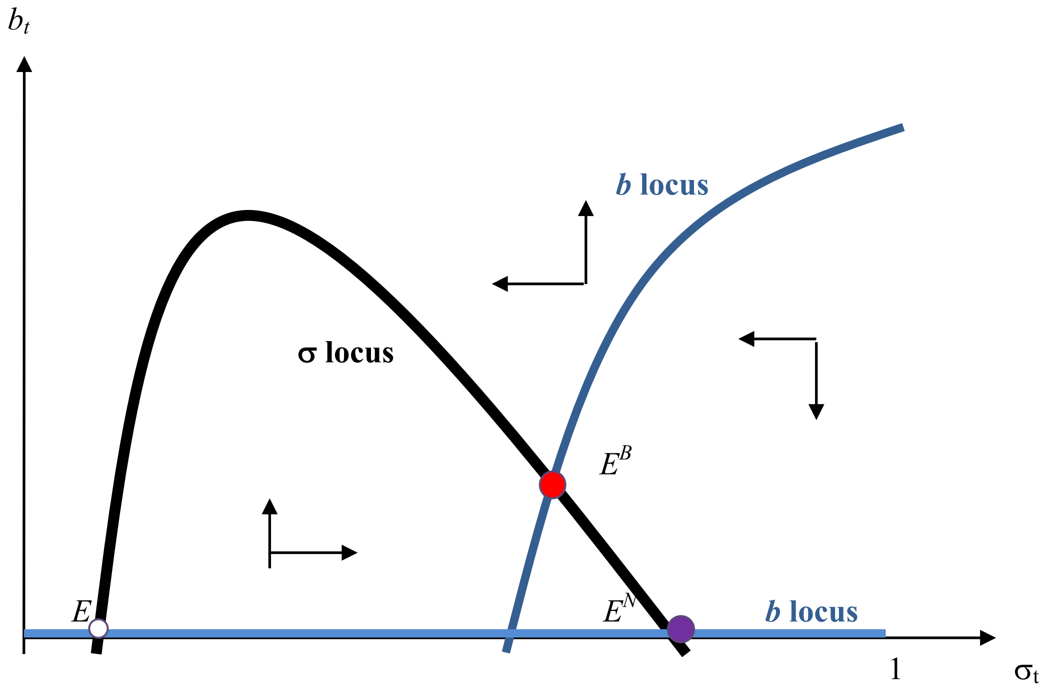

Figure 1. Phase diagram: (ii) bubble regime

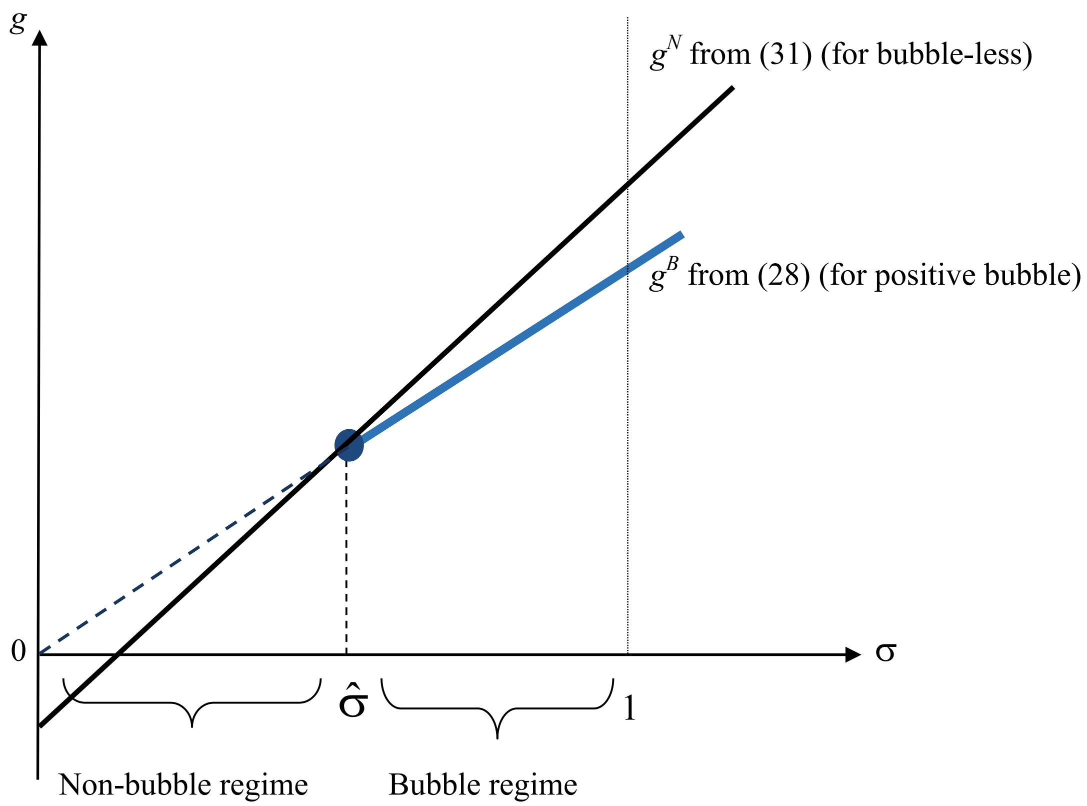

Figure 2. Employment and growth rate.

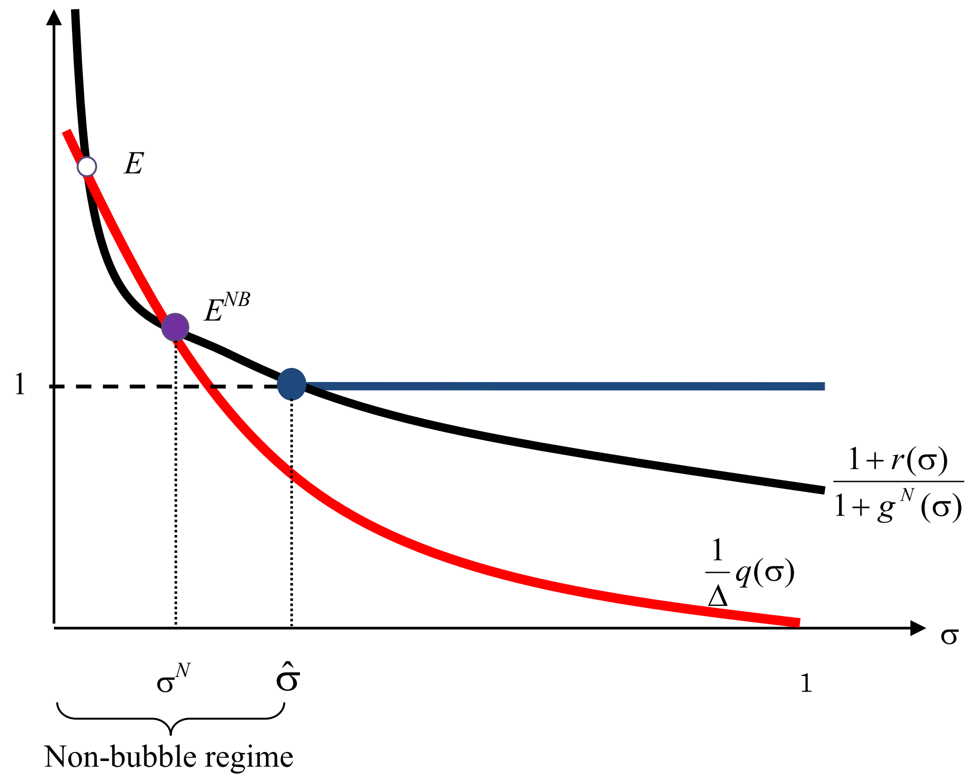

Figure 3 (a): The equilibrium of employment rate under non-bubble regime

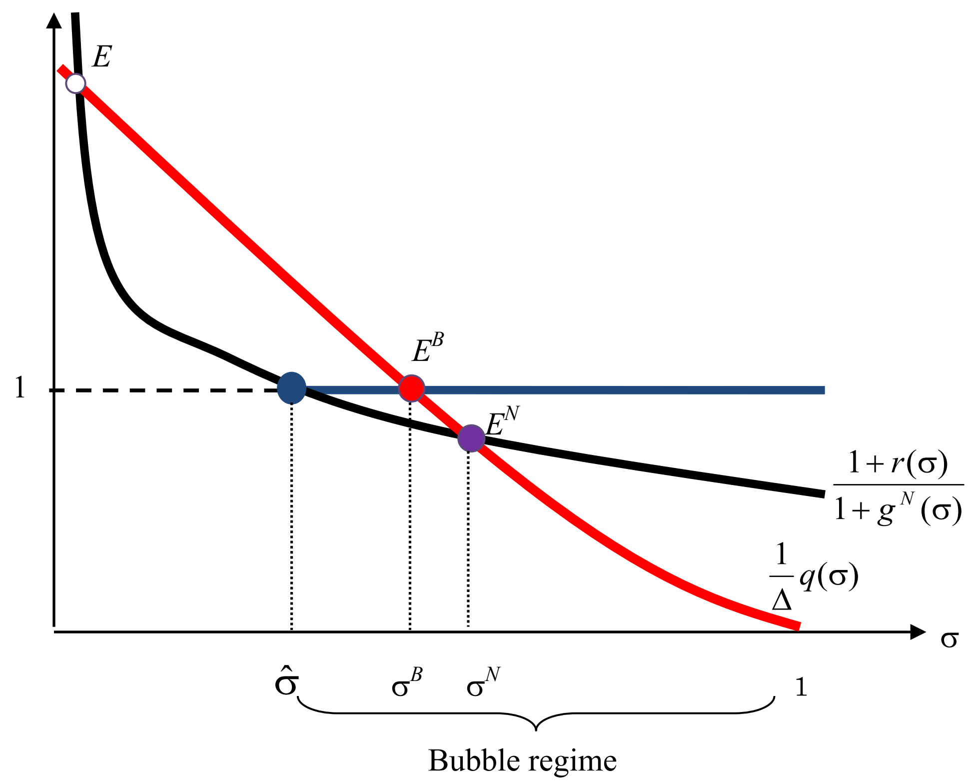

Figure 3 (b): The equilibrium of employment rates under bubble regime

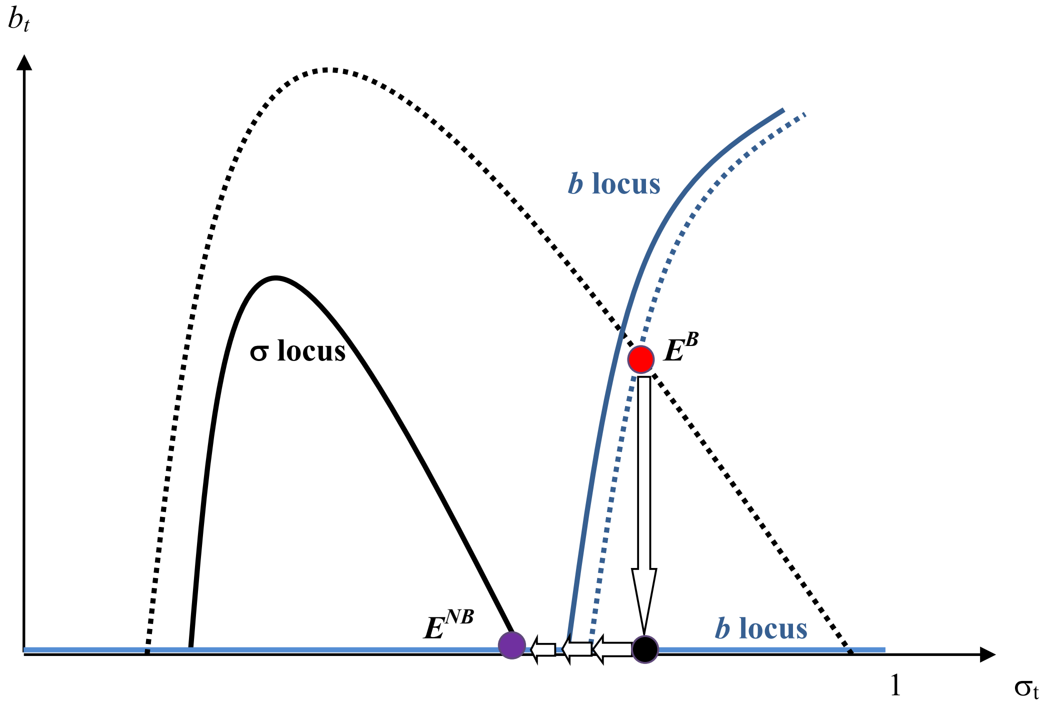

Figure 4 (a). The pattern of bubble bust

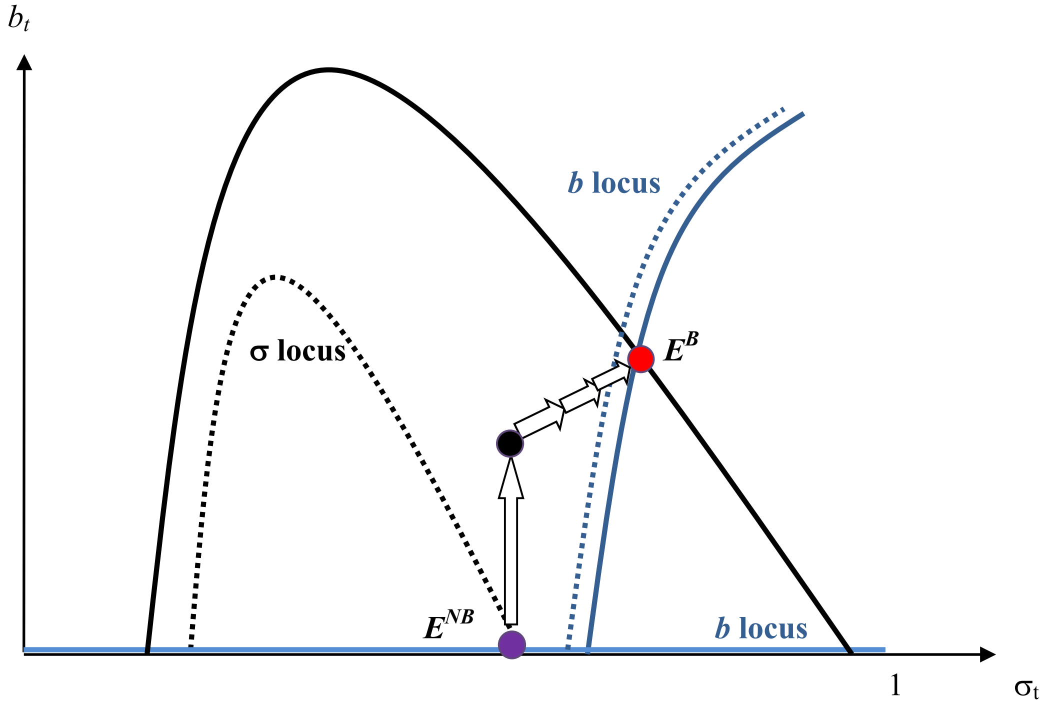

Figure 4 (b). The pattern of bubble boom

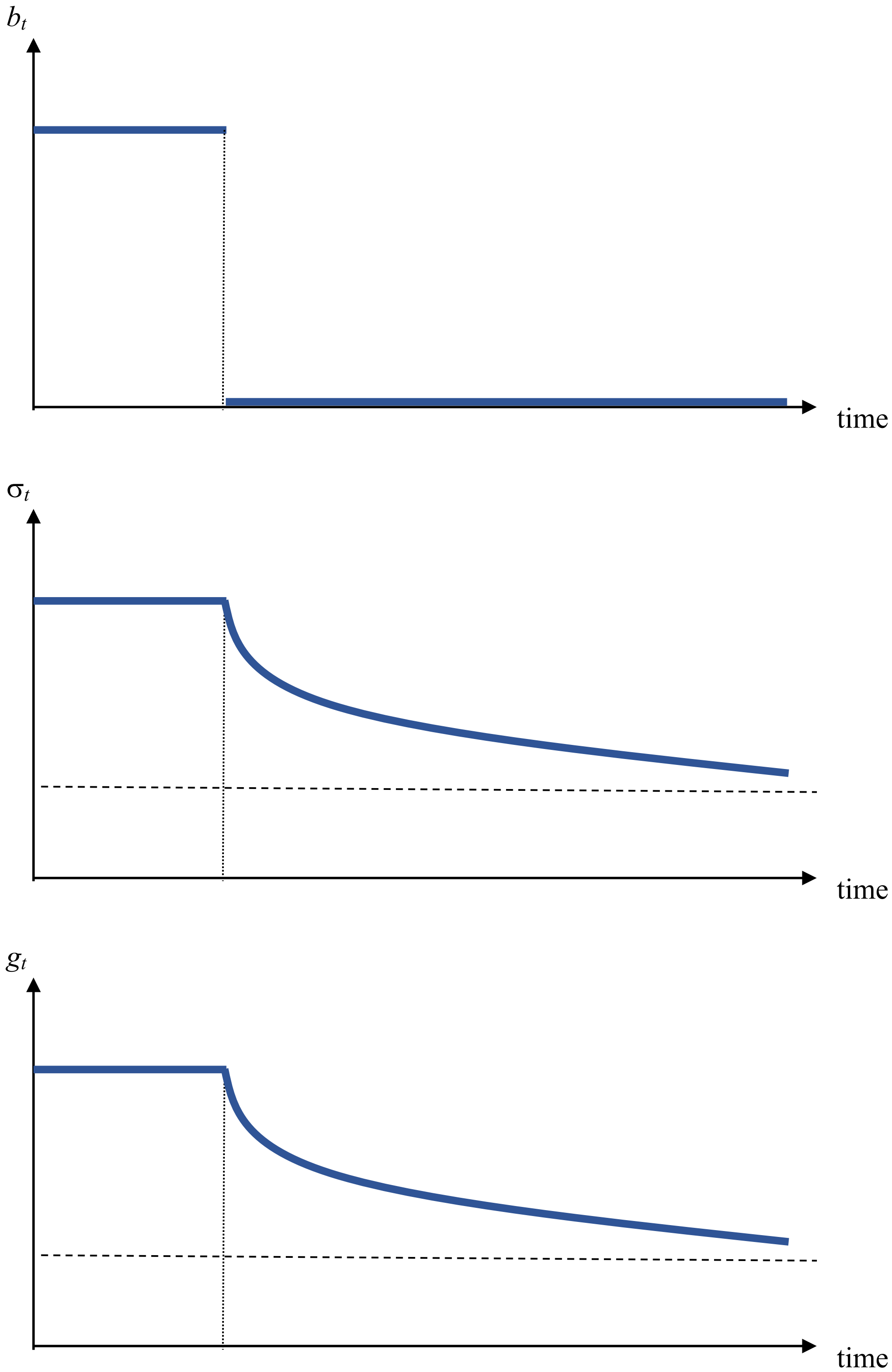

Figure 5. Dynamic paths after bubble bust

5. Asset bubbles, Financial Crisis, and Unemployment

5.1. Introduction

A bubble on an asset is defined as the deviation of the asset's market value from its fundamental value. Economic history has repeatedly witnessed severe financial crises accompanied by the collapse of asset prices in the modern monetary and financial systems. Before a financial crisis, asset prices often deviate upward from their fundamental values, and possibly lead to the higher output stimulating employment. When asset prices collapse, however, the output suddenly declines and the economy goes into a depression with unemployment expansion.[39] As such, a bubble bursting arguably causes a higher unemployment rate. Despite these historical observations regarding the collapse of asset prices and depressions, the impact of a bubble bursting on unemployment has not been fully investigated theoretically in macroeconomics although there has been a growing concern about the effect that the presence of asset bubbles has on economic growth recently. In this paper, we present a tractable overlapping-generations model with asset bubbles to demonstrate that a financial crisis triggered by a bubble bursting depresses an economy and expands unemployment.

In our model, a bubbly asset has a positive market value because selling the asset is a fund-raising method for those who draw sufficiently high productivity to initiate an investment project and purchasing the asset is a sole saving method for those who draw too low productivity to initiate a project. Our model is closely related to the model of Martin and Ventura (2012) who develop a tool to investigate how the occurrence of asset bubbles promotes capital accumulation and the bursting of bubbles causes depressions. As in Martin and Ventura's model, the youth who draw sufficiently high productivity shocks to become investors but face borrowing constraints issue the new bubbly assets to raise funds. Once they sell the new bubbly assets in the asset market, they do not have to purchase them back from the market. Accordingly, the youth have always incentives to issue the new bubbly assets and obtain more funds that cannot be acquired otherwise because of borrowing constraints.

Although the central role of asset bubbles in our model is similar to that of Martin and Ventura (2012), our model departs from theirs in some respects. First, we employ a continuous distribution with respect to idiosyncratic productivity shocks, whereas Martin and Ventura applies a binary distribution. The use of continuous productivity distribution significantly

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[39] Empirical studies such as Phelps (1999) and Fitoussi et al. (2000) provide evidence showing that a reduction in unemployment rates is accompanied by the growing asset prices.

simplifies the analysis. In particular, one can derive the productivity cutoff that divides agents into bubbly-asset holders and investors. Those who draw productivity shocks smaller than the cutoff purchase bubbly assets and those who draw productivity shocks greater than the cutoff become investors. Our model obtains a simple two-dimensional dynamical system with respect to capital and the cutoff, by which one can easily investigate the dynamic behavior of the system. It is impossible to analyze such a dynamical system with a binary productivity distribution. Second, we introduce labor market frictions. The investigation of the relationship between a bubble bursting and unemployment is a main theme in this paper. By introducing labor-market matching frictions in our tractable model along the same line as Bean and Pissarides (1993), we can demonstrate that a bubble bursting expands unemployment under mild parameter conditions, which is a new result in the literature that deals with asset bubbles ´a la Tirole (1985).[40]

The presence of asset bubbles corrects allocative inefficiency, relocating investment resources from low productive agents to high productive agents, and promotes capital accumulation if bubbles' crowding-out effect ´a la Tirole (1985) is relatively weak. As capital accumulates and output increases, the number of vacant positions increases because each firm acquires more funds to cover a fixed search cost. As a result, an unemployment rate decreases.[41] However, extrinsic uncertainty may burst asset bubbles and cause a self-fulfilling financial crisis, which is followed by the expansion of unemployment. The bubbly asset plays a financial intermediation role as pointed out by Mitsui and Watanabe (1989). As previously stated, however, the bubbly asset that is newly issued in each period is never withdrawn from the economy and investors never repay the funds raised by issuing the bubbly asset as in the model of Martin and Ventura (2012). This Ponzi game can be played because financial market imperfections render the market interest rate less than the economic growth rate in equilibrium when the bubbly asset is not present.

The literature of asset bubbles and economic growth has been renewably growing recently, in which the presence of asset bubbles promotes capital accumulation and economic

[40]For traditional models that address the relationship between economic growth and unemployment with labor market imperfections, see Aghion and Howitt (1994), Eriksson (1997), Caballero and Hammour (1996), and Haruyama and Leith (2010). See also Pissarides (2000) for the introduction to search friction models.

[41]This outcome gets along with many empirical studies that show a negative relationship between unemployment and economic growth (Ball and Moffitt, 2001; Muscatelli and Tirelli, 2001; Staiger et al., 2001; Tripier, 2006; Pissarides and Vallanti, 2007

growth.[42] In this renewably growing stream of literature, financial market imperfections and the productivity differences across agents are key factors in producing such a situation that asset bubbles enhance capital accumulation. Farhi and Tirole (2012), Martin and Ventura (2012), Carvalho et al. (2012), and Kunieda (2014) create such a situation by applying the overlapping generations framework of Samuelson (1958), Tirole (1985), or Blanchard (1985). To produce the same situation, Aoki and Nikolov (2015), Hirano et al. (2015), and Kunieda and Shibata (2016) develop dynamic general equilibrium models in which asset bubbles occur in equilibrium despite the assumption of infinitely lived agents and the presence of bubbles promotes economic growth through the mechanism similar to that found first by Mitsui and Watanabe (1989). Although all these studies consider asset bubbles ´a la Tirole (1985) and obtain the result that the presence of asset bubbles promote capital accumulation as in the current model, they do not investigate how unemployment rates are affected by the presence of asset bubbles. Miao et al. (2016) investigate the relationship between unemployment and stock market bubbles in an economy with labor market and financial market frictions. However, their definition of bubbles is totally different from ours: they essentially consider multiple equilibria of the fundamental values of an intrinsically useful asset.[43] The remainder of this paper is organized as follows. Section 2 develops the model and section 3 investigates the dynamic behavior in equilibrium and derives the relationship between the unemployment rate and capital accumulation. In section 4, the growth-promoting effect of asset bubbles is analyzed and section 5 derives a self-fulfilling financial crisis as a rational expectations equilibrium. Section 6 concludes the paper.

5.2. The model

The economy is represented in discrete time, ranging from time $t = 0$ to $t = \infty$, and it consists of overlapping generations: young and old agents. Each agent lives for two periods. The population of each generation is constant, which is given by L. Only young agents have an

[42]Researchers in the traditional literature on asset bubbles and economic growth have long discussed the growth effects of bubbles by applying the overlapping generations model. See Tirole (1985), Weil (1987), Grossman and Yanagawa (1993), King and Ferguson (1993), Futagami and Shibata (2000), Kunieda (2008), Mino (2008) and Matsuoka and Shibata (2012), among others. Regrettably, their results cannot explain the historical events in which severe economic depressions arguably follow the collapse of asset bubbles.

[43]Although Kocherlakota (2011) investigate the impact of the occurrence of asset bubbles on unemployment, he does not consider capital accumulation

opportunity to work, matching with a firm, so L is also the size of total labor force supplied in each period.

5.2.1. Final goods sector

In the final goods sector, many identical firms produce final goods with the same production technology. In addition to capital, one worker is necessary for a firm to produce the final goods. More concretely, workers and firms with vacant positions search for each other in the labor market. Firms that successfully match with a worker can operate their business. Firm i produce final goods $y_{i,t}$ at time t with a Cobb-Douglas production technology: $y_{i,t} = Az_{i,t}^{\alpha} l_{i,t}^{1-\alpha}$, where $\alpha \in (0,1)$ is a capital share of output, $z_{i,t}$ is capital, which depreciates in one period, $l_{i,t}$ is labor employed by firm i, and A is productivity of the technology. Because an operating firm hires only one worker eventually it holds that $l_{i,t} = 1$, and the production function is condensed as follows:

$$y_{i,t} = Az_{i,t}^{\alpha}. \tag{1}$$

Because the capital market is competitive, capital is paid its marginal product:

$$q_t = \alpha A z_{i,t}^{\alpha-1}, \tag{2}$$

where q_t is the capital price. Then, the remainder of output to be allocated between firm i and its worker is given by

$$\pi := y_{i,t} - q_t z_{i,t} = (1-\alpha) A z_t^{\alpha}. \tag{3}$$

The firm-specific index i is dropped because each firm employs the same amount of capital, facing the common capital price.

5.2.2. Agents

An agent born at time t exclusively derives her utility from consumption in old age, which is denoted by c_{t+1}^{ι}. Note that ι represents an agent's employment status: $\iota = e$ if employed and $\iota = u$ if unemployed, which is an outcome of job search in youth. Because she does not consume in the first period of her lifetime, she turns over all her income in youth to maximize her lifetime utility, $U_t^{\iota} = c_{t+1}^{\iota}$. In the first period, she is endowed with one unit of labor. A successful match with a firm enables her to work for the firm and earn a wage income, w_t. Otherwise, she receives an unemployment benefit, $\bar{\gamma}_t$, from the government. Because the government imposes a lump-sum tax, τ_t, on young agents to cover the unemployment benefit, the agent's net income in the first period is given by $\omega_t^t - \tau_t$ where $\omega_t^t = w_t$ if employed and $\omega_t^t = \bar{\gamma}_t$ if unemployed.

Following Martin and Ventura (2012) and Ikeda and Phan (2015), it is assumed that agents can issue new bubbly assets, which is intrinsically useless, to obtain extra funds in the first period although they face borrowing constraints. The agent's total funds available for saving are given by

$$s_t^t := \omega_t^t - \tau_t + b_t^N, \tag{4}$$

where b_t^N is new bubbly assets issued by the agent at time t.[44] To derive an equilibrium in which bubbly assets exist, we limit the ceiling of bubbly assets in amount that each agent can issue as follows:

$$b_t^N \leq \mu \tilde{b}_t \qquad \mu \in (0,1), \tag{5}$$

where $\tilde{b}_t$ is the average amount of bubbly assets per young agent that exist at the end of time t.[45] More concretely, $\tilde{b}_t$ satisfies $B_t = \tilde{b}_t L$, where B_t is the real value of the total bubbly asset at time t, which includes the newly issued bubbly asset at time t.[46] Agents are willing to raise new bubbly assets as many as possible, because once they obtain extra funds by issuing the assets, they do not have to repay for them. Therefore, the equality holds in inequality (5) in equilibrium.

There is no storage technology for the final goods, which are perishable in one period. Instead, agents have two saving methods: one is initiating an investment project and the other

[44]To understand the bubbly assets newly issued by agents but never redeemed, one can imagine the securitization of commercial loans. The recent financial innovation securitizes commercial loans, and an asset backed by the loans can be purchased and sold in the primary and secondary markets. In the process of securitization, asset holders may be unable to identify the fundamental value of the asset. In such a case, even though the fundamental value of the asset is actually zero, such a worthless asset would be traded in the financial market as far as participants in the market believe in the market value of the asset.

[45]The assumption regarding the limitation of the new issuance of bubbly assets is also imposed in Martin and Ventura (2012) and Ikeda and Phan (2015). In any case, one must impose the upper limit of the new issuance of bubbly asset; otherwise, the market for the bubbly asset cannot be sustainable. One may consider that the new issuance is regulated institutionally.

[46]In section 5.2.5, the formal definition for B_t is provided.

is purchasing bubbly assets. Agents that purchase one unit of bubbly assets at time t earn a (gross) return, r_{t+1}, at time $t + 1$, whereas agents that invest one unit of funds in a project at time t create Φ units of capital goods and sell them to firms at a price, q_{t+1}, at time $t + 1$; namely, they earn a return, $q_{t+1}\Phi$. Φ is productivity for capital production and varies across agents. When an agent is born, she receives an individual-specific shock, Φ. The support of Φ is $[0, \eta]$ where $\eta > 0$ and its cumulative distribution function is given by $G(\Phi)$, which is time-invariant and continuously differentiable on the support. Although Φ is an idiosyncratic shock, the realization of low productivity cannot be insured against because there is no insurance market for it. Φ is independent of the employment status. Note that when agents invest in a project, the shocks are already realized. Knowing their own productivity, they make a portfolio choice between investing in a project and purchasing bubbly assets to maximize their lifetime utility. As such, the individual-specific return is deterministic when they make a portfolio choice, which is given by $R_{t+1} = \max\{q_{t+1}\Phi, r_{t+1}\}$ and an agent's lifetime utility is given by

$$U_t^t = R_{t+1} s_t^t . \tag{6}$$

Define $\phi_t := r_{t+1}/q_{t+1}$. Then, a portfolio choice of an agent who draws productivity Φ is given by

$$k_t^t = \begin{cases} 0 & if \quad \Phi \leq \phi_t \\ s_t^t & if \quad \Phi > \phi_t \end{cases}, \tag{7}$$

and

$$b_t^t = \begin{cases} s_t^t & if \quad \Phi \leq \phi_t \\ 0 & if \quad \Phi > \phi_t \end{cases}, \tag{8}$$

As seen in Eqs. (7) and (8), agents who draw productivity smaller than ϕ_t purchase bubbly assets and agents who draw productivity greater than ϕ_t invest in a project.[47] Note that ϕ_t is a productivity cutoff that divides agents into investors and bubbly asset holders. The population of investors is $(1 - G(\phi_t))L$ and that of bubbly asset holders is $G(\phi_t)L$.

[47]In the current model, agents can issue new bubbly assets before the portfolio choice and the realization of individual-specific productivity shocks as presented in Eq. (4). For the trade timing in such market circumstances, we implicitly assume that a market maker is present in the asset market.

5.2.3. Government

The government runs a balanced budget to provide unemployment benefits for workers such that

$$\tau_t L = \bar{\gamma}_t u_t L, \tag{9}$$

where u_t is the unemployment rate. The left-hand side of Eq. (9) denotes the aggregate tax revenue and the right-hand side represents the total payments for unemployment benefits.

5.2.4. Labor market

We introduce labor-market matching frictions in the model along the same line as Bean and Pissarides (1993). Although the matching mechanism follows from the standard unemployment model (e.g., Diamond, 1982; Mortensen and Pissarides, 1999; Petrongolo and Pissarides; 1990), there is no time lag between a match of parties and a start of business operation in the current model.

5.2.4.1 Matching mechanism

Because workers and firms face matching frictions, unemployment occurs in equilibrium although each agent is born endowed with one unit of labor that she supplies inelasitically in youth. The number of successful matches are given by $F(L, \upsilon_t)$, which is a function of the population of workers, L, and the number of firms with vacancy, υ_t, where $0 \leq F(L, \upsilon_t) \leq \min\{L, \upsilon_t\}$ for $L \in [0, \infty)$ and $\upsilon_t \in [0, \infty)$, and $F(0, \upsilon_t) = 0$ and $F(L, 0) = 0$. The matching function $F(L, \upsilon_t)$ is continuously differentiable, concave, homogeneous of degree one, and increasing with respect to both L and υ_t. The tightness of the labor market is expressed by $\theta_t := \upsilon_t/L \in (0, \infty)$, which is considered as the jobs-to-applicants ratio, and the probability that a firm with vacancy matches with a worker is given by $F(L, \upsilon_t)/\upsilon_t = F(1/\theta_t, 1) =: f(\theta_t)$. It is assumed that $f(\theta_t)$ is continuously differentiable in $(0, \infty)$ where $f'(\theta_t) < 0$ for $\theta_t \in (0, \infty)$, $\lim_{\theta t \to 0} f(\theta_t) = 1$, and $\lim_{\theta t \to \infty} f(\theta_t) = 0$. Because the number of employment is equal to the number of successful matches, it follows that $(1 - u_t)L = F(L, \upsilon_t)$, which is rewritten as

$$1 - u_t = \theta_t f(\theta_t). \tag{10}$$

Eq. (10) shows the relationship between the unemployment rate and the labor market tightness.

Eq. (10) yields the unemployment rate, u_t, as a function of θ_t such that $u_t = u(\theta_t)$ where $u'(\theta_t) < 0$ because $\partial[\theta_t f(\theta_t)]/\partial\theta_t = \partial F(1, \theta_t)/\partial\theta_t > 0$. Therefore, Eq. (10) derives a negative relationship between the unemployment rate and the labor-market tightness, which is so-called the Beveridge curve.

A successful match enables a firm to produces the final goods. Because $f(\theta_t)$ is the probability that a firm matches with a worker at time t, the firm's expected profits, V_t, are given by

$$V_t = f(\theta_t)(\pi_t - w_t) - h, \tag{11}$$

where h is the search cost in the labor market that the firm incurs when searching for a worker.[48] Because the ceiling of $f(\theta_t)$ is 1, if the actual revenue $\pi_t - w_t$ less than h, no firms operate because the expected profits are negative. In other words, only if $\pi_t - w_t \geq h$, successful matches occur between workers and firms. We proceed our investigation for the case in which $\pi_t - w_t \geq h$ for a while unless otherwise stated. The free-entry condition for the final goods sector leads to zero profits of each firm. Accordingly, it follows that $V_t = 0$, or equivalently

$$\pi_t - w_t = \frac{h}{f(\theta_t)}. \tag{12}$$

5.2.4.2 Nash bargaining

The remainder of output after payments to capital is allotted between the firm and its worker. The shares to each are determined by maximizing the following Nash product with respect to the wage:

$$w_t = \arg\max_{w_t} (U_t^e - U_t^u)^\beta (\pi_t - w_t)^{1-\beta} = \{R_{t+1}(w_t - \bar{\gamma}_t)\}^\beta (\pi_t - w_t)^{1-\beta},$$

where $\beta \in (0, 1)$ is the worker's bargaining power and R_{t+1} is the return to saving, which was derived in section 5.2.2. From the Nash bargaining solution, it follows that

$$w_t = (1-\beta)\bar{\gamma}_t + \beta\pi_t. \tag{13}$$

The government policy regarding unemployment pays the unemployment benefits to

[48] The search cost would cover the recruitment activities such as job interviews and the evaluation of reference letters, which are done by using the firm's operating resources. One can consider that the search cost associated with these activities is an implicit opportunity cost that the firm incurs.

unemployed workers in such a way that $\bar{\gamma}_t = \gamma w_t$ where $\gamma \in [0, 1]$. We assume that when the firm and its worker are bargaining, the Nash product is maximized with $\bar{\gamma}_t$ given. The government eventually performs the policy in such a way that the benefit payment is proportional to the wage rate. Inserting Eq. (3) and $\bar{\gamma}_t = \gamma w_t$ in Eq. (13) yields

$$w_t = \Omega(1-\alpha)Az_t^{\alpha}, \tag{14}$$

where $\Omega := \beta/\{1 - (1 - \beta)\gamma\} \in (0, 1)$ is the worker's output share of π_t. Note from $\Omega = \beta/\{1 - (1 - \beta)\gamma\}$ that the larger outside option, γw_t, and the larger Nash bargaining power, β, lead to the greater worker's share. Substituting Eq. (3) and (14) in Eq. (12) yields

$$(1-\Omega)(1-\alpha)Az_t^{\alpha} = \frac{h}{f(\theta_t)}. \tag{15}$$

Thus far, we have investigated the model assuming that there are always operating firms. However, it is noted from Eq. (15) that given parameter values, if z_t is very small, firms cannot cover a search cost, h, because the upper limit of $f(\theta_t)$ is 1.

Proposition 1 *Define* $\bar{z} := [h/\{(1 - \Omega)(1 - \alpha)A\}]^{\frac{1}{\alpha}}$.

- *If* $z_t \leq \bar{z}$*, there are no operating firms in the economy at time t.*
- *If* $z_t > \bar{z}$*, there are operating firms in the economy at time t.*

Proof: See the Appendix.

In the first case of Proposition 1, the economy certainly breaks down at time t because no firms produce final goods at that time. In this case, no agents initiate an investment project at time $t - 1$, anticipating the breaking down. Moreover, no young agents at time t can purchase the bubbly asset because they do not earn the labor income at that time, and thus, the bubbly asset has no value at time t. Anticipating this, no young agents purchase the bubbly asset at time $t - 1$. Accordingly, the backward induction shows that the bubbly asset has no value even at time zero. Additionally, young agents at time $t - 1$ anticipate that they cannot obtain the returns from investment projects at time t. Given their anticipations, we can reasonably assume that young agents at time $t - 1$ do not supply their labor force at time $t - 1$ because they do not consume in the first period of their lifetime and are not necessarily benevolent. As a result, the economy breaks down at time $t - 1$. The backward induction, again, shows that the economy breaks down at time zero. In summary, if z_t becomes less than $\bar{z}$ at a certain point in time, it is highly likely that the economy is unsustainable for all $t \geq 0$ without the occurrence of production. In what follows, we assume away the first case of Proposition 1 and focus on the case in which $z_t > \bar{z}$ for all $t \geq 0$.

In the second case of Proposition 1, it follows from Eq. (15) that $\theta_t = v_t/L > 0$, which implies that there exist firms with vacancy at time t and the unemployment rate is less than one from Eq. (10).

5.2.5 Bubbly Asset

The bubbly asset is intrinsically useless. It is assumed that at time 0, there are identical old agents who hold the bubbly asset, M_{-1}, in total. Additionally, in each period, the bubbly asset is newly issued by young agents. Formally, for $t \geq 0$, we have a dynamic equation with respect to the nominal bubbly asset as follows:

$$M_t = M_{t-1} + M_t^N, \tag{16}$$

where M_t is the total nominal supply of the bubbly asset and M_t^N is the asset that is newly issued by young agents at time t. Multiplying both sides of Eq. (16) by the bubbly asset price, p_t, and defining the real value of the bubbly asset as $B_t = p_t M_t$ and $B_t^N = p_t M_t^N$, we obtain the dynamic equation of the real value of the bubbly asset as $B_t = (p_t/p_{t-1})B_{t-1} + B_t^N$, or equivalently

$$B_t = r_t B_{t-1} + B_t^N, \tag{17}$$

where $r_t := p_t/p_{t-1}$, which is the return to holding the bubbly asset.

5.3 Equilibrium

The equilibrium is characterized by the optimization conditions of the agents and firms, the outcomes of the Nash bargaining in the labor market, and the market clearing conditions for the bubbly asset and capital.

5.3.1. Market clearing conditions

B_t and B_t^N are the aggregations of $\tilde{b}_t$ and b_t^N over all agents, namely $B_t = \tilde{b}_t L$ and $B_t^N = b_t^N L$. Because the equality in inequality (5) holds in equilibrium, it follows that

$$B_t^N = \mu B_t. \tag{18}$$

Substituting Eq. (18) in Eq. (17) yields

$$B_t = \frac{r_t}{1-\mu} B_{t-1}. \tag{19}$$

In each period, the bubbly asset is purchased by less productive agents regardless of their employment status. Because the population of less productive agents who drew the productivity that is smaller than the cutoff, ϕ_t, and purchase the bubbly assets is $G(\phi_t)L$, the demand for the bubbly asset is given by

$$B_t^d = G(\phi_t)L[(1-u_t)(w_t - \tau_t + b_t^N) + u_t(\bar{\gamma}_t - \tau_t + b_t^N)]. \tag{20}$$

It follows that $B_t = B_t^d$ in equilibrium, and thus, the use of Eqs. (9), (14), and (18) rewrites Eq. (20) as follows:

$$B_t^d = \frac{\Omega(1-\alpha)Az_t^\alpha(1-u_t)G(\phi_t)L}{1-\mu G(\phi_t)}. \tag{21}$$

From Eqs. (2), (19), (21), and $\phi_{t-1} = r_t/q_t$, we obtain

$$\frac{G(\phi_t)}{1-\mu G(\phi_t)} z_t(1-u_t) = \frac{\alpha A \varphi_{t-1} G(\phi_{t-1})}{(1-\mu)(1-\mu G(\phi_{t-1}))} z_{t-1}^\alpha (1-u_{t-1}). \tag{22}$$

Capital at time t is produced by the agents who draw such high productivity that $\Phi > \phi_{t-1}$. Therefore, the aggregate capital is given by

$$Z_t := \int_{\phi_{t-1}}^{\eta} \Phi(k_{t-1}^e(1-u_{t-1})L + k_{t-1}^u u_{t-1}L)dG(\Phi). \tag{23}$$

The number of firms that successfully match with a worker at time t is $(1-u_t)L$, and thus, capital per operating firm, z_t, is given by $z_t = Z_t/\{(1 - u_t)L\}$. The use of Eqs. (4), (7), (9), (14), (18), and (21) rewrites Eq. (23) as follows:

$$z_t(1-u_t) = \frac{(1-\alpha)\Omega A H(\phi_{t-1})}{1-\mu G(\phi_{t-1})} z_{t-1}^\alpha (1-u_{t-1}), \tag{24}$$

where $H(\phi_{t-1}) := \int_{\phi_{t-1}}^{\eta} \Phi dG(\Phi)$.

5.3.2. Dynamical system

From Eqs. (10) and (15), we obtain the following equation:

$$1-u_t = \frac{h}{(1-\Omega)(1-\alpha)Az_t^{\alpha}} f^{-1}\left(\frac{h}{(1-\Omega)(1-\alpha)Az_t^{\alpha}}\right) =: \Psi(z_t). \tag{25}$$

Inserting this equation into Eq. (24) yields the dynamic equation with respect to z_t as follows:

$$z_t \Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi_{t-1})}{1-\mu G(\phi_{t-1})} z_{t-1}^{\alpha} \Psi(z_{t-1}). \tag{26}$$

Eqs. (22) and (24) yield the dynamic equation with respect to the cutoff ϕ_t :

$$\frac{(1-\alpha)\Omega G(\phi_t)}{1-\mu G(\phi_t)} = \frac{\alpha \phi_{t-1} G(\phi_{t-1})}{(1-\mu) H(\phi_{t-1})}. \tag{27}$$

Eqs. (26) and (27) can derive an autonomous dynamical system with respect z_t and ϕ_t . Noet that Eq. (27) is solely an autonomous difference equation with respect to ϕ_t . Because the cutoff, ϕ_t , is in [0, η] and because we focus on the case in which $z_t > \bar{z}$ for all $t \geq 0$ as discussed in the previous section, the domain of the dynamical system consisting of Eqs. (26) and (27) is given by $(\bar{z}, \infty) \times [0, \eta]$.

We assume that the initial total capital, Z_0, already exists at time 0. Then, the initial capital per operating firm, z_0, the initial labor-market tightness, θ_0, and the initial unemployment rate, u_0, are determined by Eqs. (10), (15), and $z_0(1 - u_0)L = Z_0$ simultaneously, which means that all these three variables are pre-determined at time 0. In contrast, the initial real value of bubbly asset, B_0, is not pre-determined because its price, p_0, can jump depending upon agents' self-fulfilling expectations. Accordingly, ϕ_0 is not pre-determined, either, because ϕ_t has a one-to-one relationship with B_t as seen in Eq. (21), given z_t and u_t. This means that ϕ_0 is also affected by agents' self-fulfilling expectations. Given $\{z_0, u_0, \theta_0, B_0\}$, the equilibrium sequences, $\{z_t, u_t, \theta_t, B_t, \phi_t\}_{t=0}^{\infty}$, are produced from Eqs. (10), (15), (21), (26), and (27), where $(z_t, \phi_t) \in (\bar{z}, \infty) \times [0, \eta]$ for all $t \geq 0$.

5.3.3. Steady states and stability

Proposition 2 *In the dynamical system consisting of Eqs. (26) and (27), there exist two (non-*

trivial) steady states: (z^*, ϕ^*) and (z^{**}, ϕ^{**}) such that

$$z^* = Q(\phi^*)^{\frac{1}{1-\alpha}}, \tag{28}$$

$$\frac{H(\phi^*)}{1-\mu G(\phi^*)} = \frac{\alpha\phi^*}{(1-\alpha)(1-\mu)\Omega}, \tag{29}$$

$$z^{**} = Q(0)^{\frac{1}{1-\alpha}}, \tag{30}$$

and

$$\phi^{**} = 0, \tag{31}$$

where $Q(x) = (1-\alpha)\Omega AH(x)/(1-\mu G(x))$.

Proof: See the Appendix.

Because ϕ^* > 0 and the unemployment rate is always less than one, Eq. (21) implies that the bubbly asset has a market value in the steady state given by (z^*, ϕ^*). So, we call this steady state a bubbly steady state. In contrast, in the steady state given by (z^{**}, ϕ^{**}), the bubbly asset has no market value and we call this steady state a bubbleless steady state. The linear approximation of the dynamical system around a steady state is computed from Eqs. (26) and (27) as follows:

$$\begin{pmatrix} z_t - \hat{z} \\ \phi_t - \hat{\phi} \end{pmatrix} = \begin{pmatrix} \kappa_1(z) & \dfrac{Q'(\hat{\phi})\hat{z}^{\alpha}\Psi(\hat{z})}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})} \\ 0 & \kappa_2(\hat{\phi}) \end{pmatrix} \begin{pmatrix} z_{t-1} - \hat{z} \\ \phi_{t-1} - \hat{\phi} \end{pmatrix}, \tag{32}$$

where ($\hat{z}$, $\hat{\phi}$)=(z^*, ϕ^*) or (z^{**}, ϕ^{**}). Note that $\kappa_1(\hat{z})$ and $\kappa_2(\hat{\phi})$ are the eigenvalues of the local dynamical system associated with Eq. (32).

Lemma 1 *The eigenvalues of the local dynamical system associated with Eq.(32) around the bubbly steady state, (z^*, ϕ^*), are given by*

$$\kappa_1(z^*) = \frac{\alpha\Psi(z^*) + z^*\Psi'(z^*)}{\Psi(z^*) + z^*\Psi'(z^*)},$$

and

$$\kappa_2(\phi^*) = \frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\left(1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi*)}{H(\phi^*)}\right).$$

The eigenvalues of the local dynamical system associated with Eq. (32) around the bubbleless steady state, (z^{**}, ϕ^{**}), are given by

$$\kappa_1(z^{**}) = \frac{\alpha\Psi(z^{**}) + z^{**}\Psi'(z^{**})}{\Psi(z^{**}) + z^*\Psi'(z^{**})},$$

And

$$\kappa_2(\phi^{**}) = 0.$$

Proof: See the Appendix.

Proposition 3 *In the dynamical system consisting of Eqs. (26) and (27), the bubbly steady state, (z^*, ϕ^*), is a saddle point and the bubbleless steady state, (z^{**}, ϕ^{**}), is totally stable.*

Proof: See the Appendix.

[Figure 1 around here]

Figure 1 provides a phase diagram that illustrates the dynamic behavior of the economy. Because ϕ_0 can jump and the bubbly steady state is a saddle point, the bubbly steady state is locally determinate. However, the bubbleless steady state is totally stable, and thus, any sequence of $\{z_t, \phi_t\}_{t=0}^{\infty}$ with (z_0, ϕ_0) $\in$ ($\bar{z}$, ∞) × (0, ϕ^*) that converges to (z^{**}, 0) is an equilibrium. This means that equilibrium is globally indeterminate. Because of indeterminacy of equilibrium, self-fulfilling financial crises are caused by extrinsic uncertainty as investigated in section 5. Note that any sequence of $\{z_t, \phi_t\}_{t=0}^{\infty}$ with (z_0, ϕ_0) $\in$ ($\bar{z}$, ∞)×(ϕ^*, η] cannot be an equilibrium because ϕ_t becomes greater than η or z_t becomes less than $\bar{z}$ in finite time.

5.3.4 Beveridge curve and capital accumulation

Eq. (10) can be rewritten as follows:

$$u_t = 1 - \theta_t f(\theta_t), \tag{33}$$

where $\partial u_t/\partial \theta_t < 0$. Eq. (33) is the Beveridge curve as stated in section 5.2.4. From Eq. (15), it follows that

$$\theta_t = f^{-1}\left(\frac{h}{(1-\Omega)(1-\alpha)Az_t^{\alpha}}\right), \tag{34}$$

which we call the job-creation condition following Pissarides (2000). From Eq.(34), it is straightforward to show that $\partial\theta_t/\partial z_t > 0$ because $f^{-1}(.)$ is a decreasing function. This means that capital accumulation promotes employment, rendering the labor market tighter. As capital accumulates, an economy moves down along the Beveridge curve from point A to point B in Figure 2 and the unemployment rate decreases.

[Figure 2 around here]

5.4 Capital accumulation, asset bubbles, and unemployment

The Beveridge curve given by Eq. (33) and the job-creation condition given by Eq. (34) demonstrates that capital accumulation decreases the unemployment rate. This means that if the presence of asset bubbles promotes capital accumulation, it decreases the unemployment rate. In this section, we investigate the effects that asset bubbles have on capital accumulation and the unemployment rate.

5.4.1. Comparison between bubbly and bubbleless steady states

Because the bubbly steady state is a saddle point and because the initial cutoff, ϕ_0, is non-predetermined, and initial capital is predetermined, the equilibrium in the neighborhood of the bubbly steady state is locally determinate. On the stable saddle path that converges to the bubbly steady state, the cutoff is constant, which is given by $\phi_t = \phi^*$, as illustrated in Figure 1, and the rational expectations equilibrium in the neighborhood of the bubbly steady state is given by the following equations:

$$\phi_t = \phi^* \tag{35}$$

and

$$z_t \Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi^*)}{1-\mu G(\phi^*)} z_{t-1}^{\alpha} \Psi(z_{t-1}) \tag{36}$$

By contrast, because the bubbleless steady state is totally stable, the equilibrium in the neighborhood of the bubbleless steady state is indeterminate, and there exist an uncountably infinite number of equilibrium trajectories around the bubbleless steady state. Under these circumstances, for the sake of investigating the growth-promoting effects of asset bubbles, we consider a particular rational expectations equilibrium in which agents anticipate no presence of asset bubbles for all $t \geq 0$, which is given by the following equations:

$$\phi_t = \phi^* (= 0) \tag{37}$$

and

$$z_t \Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi^{**})}{1-\mu G(\phi^{**})} z_{t-1}^{\alpha} \Psi(z_{t-1}). \tag{38}$$

Note from the right-hand sides of Eqs. (36) and (38) that the presence of asset bubbles promotes (impedes) capital accumulation if $H(\phi^*)/[1 - \mu G(\phi^*)]$ is greater (less) than $H(\phi^{**})/\{1 - \mu G(\phi^{**})\}$. To investigate whether $H(\phi^*)/[1 - \mu G(\phi^*)]$ is greater or less than $H(\phi^{**})/[1 - \mu G(\phi^{**})]$, consider the following function:

$$\Lambda(\phi) = \frac{H(\phi)}{1-\mu G(\phi)}, \tag{39}$$

which is used in the proof of Proposition 2 (in the Appendix). The first derivative of $\Lambda(\phi)$ is given by

$$\Lambda'(\phi) = G'(\phi) J(\phi) / [1-\mu G(\phi)]^2, \tag{40}$$

where $J(\phi) = \mu(1 - \mu G(\phi))[\Lambda(\phi) - \phi/\mu]$. As shown in the proof of Proposition 2, there exists $\phi = \bar{\phi} \in (0, \eta)$ such that for $\phi \in [0, \bar{\phi})$, we have $J(\phi) > 0$, and for $\phi \in (\bar{\phi}, \eta]$, we have $J(\phi) < 0$. From Eq. (39), it follows that $\Lambda(0) = H(0) > 0$, and $\Lambda(\eta) = 0$. Then, the configuration of $\Lambda(\phi)$ is obtained as in Figure 3. Note that $\bar{\phi}$ is given by the intersection of $\Lambda(\phi)$ with ϕ/μ. Moreover, ϕ^* is given by the intersection of $\Lambda(\phi)$ with $\Gamma(\phi) := \alpha\phi/[(1 - \alpha)(1 - \mu)\Omega]$. Figure 3 illustrates $\Lambda(\phi)$ and $\Gamma(\phi)$. Because $\Lambda(\phi)$ is inverted-U shaped, there exist a solution, $\tilde{\phi} > 0$, for $\Lambda(\phi) = H(0)$ as seen in Figure 3.

Proposition 4 *All parameter values being fixed, if $\alpha/[(1-\alpha)(1-\mu)\Omega] > (<) H(0)/\tilde{\phi}$, then capital more (less) accumulates in the bubbly steady state than in the bubbleless steady state.*

Proof. See the Appendix.

As can be seen in Figure 3, as the upper limit of the issuance of new bubbly assets is more relaxed, i.e., as μ increases, $\Lambda(\phi)$ shifts upward and $\Gamma(\phi)$ rotates counterclockwise. Therefore, as the upper limit of the issuance of new bubbly assets is more relaxed, it is more likely that capital more accumulates in the bubbly steady state than in the bubbleless steady state. In such a case, much issuance of the bubbly asset increases the market interest rate, r_t, and thus, excludes a larger number of less productive agents from production activities. Accordingly, more productive agents intensively use more production resources. As a result, they produces the final goods to a larger extent, and thus, capital accumulation is promoted.

Remark 1 below immediately follows from Proposition 4 because capital accumulation reduces the unemployment rate.

Remark 1 *All parameter values being fixed, if $\alpha/[(1-\alpha)(1-\mu)\Omega] > (<) H(0)/\tilde{\phi}$, the unemployment rate in the bubbly steady state, u^*, is less (greater) than in the bubbless steady state, u^{**}.*

All parameter values being constant, if $\alpha/[(1-\alpha)(1-\mu)\Omega] > H(0)/\tilde{\phi}$, u^* is less than u^{**}; however, β and γ, which represent the labor market conditions, have non-linear effects on the unemployment rate. This is because as Ω increases, capital accumulation is promoted through agents savings, whereas the firms' output share decreases and the decrease in the firms' output share causes a downward pressure on the number of vacant positions with a fixed search cost. In the next section, we numerically examine the effects that β and γ have on the unemployment rate assuming their plausible values.

[Figure 3 around here]

5.4.2. Numerical analysis

In this section, we numerically investigate the effects that labor market conditions, such as workers' Nash bargaining power, β, and the unemployment benefit ratio, γ, have on macroeconomic variables such as capital accumulation, unemployment rates, and the labor-market tightness (the jobs-to-applicants ratio) in both bubbly and bubbleless steady states.

5.4.2.1 Specification and paramerization

In doing the numerical analysis, the matching function is specified as $F(L, \upsilon_t) = L\upsilon_t\,(L^{\sigma}+\upsilon_t^{\sigma})^{-1/\sigma}$, following Den Haan et al. (2000). This matching function appropriately satisfies the conditions imposed in section 5.2.4. It is assumed that the individual-specific productivity shock, Φ, is uniformly distributed in [0, η]. Under these assumptions, each variable can be computed as in the following. In the bubbleless steady state, we obtain

$$z^{**} = \left(\frac{A\eta(1-\alpha)\Omega}{2}\right)^{\frac{1}{1-\alpha}},$$

$$u^{**} = 1 - \left(1 - \frac{1}{(A(1-\Omega)(1-\alpha)(z^{**})^{\alpha}/h)^{\sigma}}\right)^{\frac{1}{\sigma}},$$

and

$$\theta^{**} = \left(\left(\frac{A(1-\Omega)(1-\alpha)(z^{**})^{\alpha}}{h}\right)^{\sigma} - 1\right)^{\frac{1}{\sigma}},$$

where u^{**} and θ^{**} are respectively the unemployment rate and the labor-market tightness in the bubbleless steady state. Likewise, in the bubbly steady state, we obtain

$$z^{*} = \left(\frac{A\alpha\phi^{*}}{1-\mu}\right)^{\frac{1}{1-\alpha}},$$

$$u^{*} = 1 - \left(1 - \frac{1}{(A(1-\Omega)(1-\alpha)(z^{*})^{\alpha}/h)^{\sigma}}\right)^{\frac{1}{\sigma}},$$

and

$$\theta^* = \left(\left(\frac{A(1-\Omega)(1-\alpha)(z^*)^\alpha}{h} \right)^\sigma - 1 \right)^{\frac{1}{\sigma}},$$

where u^* and θ^* are respectively the unemployment rate and the labor-market tightness in the bubbly steady state, and ϕ^* is computed from Eq. (29) as

$$\phi^* = \frac{\eta\left(-\alpha + \sqrt{\alpha^2 + (1-\alpha)(1-\mu)\Omega[(1-\alpha)(1-\mu)\Omega - 2\alpha\mu]}\right)}{(1-\alpha)(1-\mu)\Omega - 2\alpha\mu}.$$

[Table 1 around here]

The parameters applied in the analysis are given in Table 1. Following Den Haan et al. (2000), we set $\alpha = 0.36$. We examine the effects of β (workers' Nash bargaining power) and γ (the unemployment benefit ratio) by varying β and γ. If β and/or γ are very close to 1, Ω is also close to 1. In this case, the economy becomes infeasible in the sense that production never occurs as clarified in Proposition 1 (in section 2) and the discussion that follows the proposition. Therefore, we must impose the upper ceiling of β and γ. DenHaan et al. (2000) examine the case in which the firm's Nash bargaining power is 0.50. Additionally, the 45%-80% of the average wage for the last six months is paid to the unemployed people in Japan for the unemployment benefit. Accordingly, we vary β from 0.40 to 0.60 and fix $\gamma = 0.80$ when examining the effect of β, and we vary γ from 0.60 to 0.87 and fix $\beta = 0.5$ when examining of the effect of γ.[49] We set h as a relatively low value, $h = 0.11$, such that the economy becomes feasible and production occurs in this analysis. We set η as a relatively high value, $\eta = 4$. This is because if η is small, the cutoff in the bubbly steady state, ϕ^*, is close to 0, and as a result, there appear only small differences between the bubbly and bubbleless steady states in capital, z, the unemployment rate, u, and the labor-market tightness, θ. However, in the Great Recession in 2009, the difference in the unemployment rate before and after the crisis is around 5% in the United States. To yield such a significant difference in the unemployment rate in the bubble bursting, a relatively high value of η is necessary. Regarding μ, we assume an aggressive issuance of the new bubbly asset by private agents and set $\mu = 0.7$. Regarding the remaining parameter values, A and σ, we set $A = 1.5$ and $\sigma = 4$ such that the average unemployment rates

[49] If we set γ greater than 0.87, the economy is infeasible.

are around 10% in the bubbleless steady state and around 2% in the bubbly steady state and the average jobs-to-applicants ratios (which is the average labor-market tightness) are around 1.44 in the bubbleless steady state and 2.05 in the bubbly steady state when varying the workers' Nash bargaining power, β.

Under these parameter values, the economy exhibits the case in which the presence of asset bubbles promotes capital accumulation, which we focus on in the current analysis.

5.4.2.2 Labor market conditions and macroeconomic variables

As seen in Figure 4, when varying β from 0.40 to 0.60, the workers' output share, Ω, increases and capital accumulation increases. This outcome is not surprising because as Ω also increases, the savings of young agents increases. The unemployment rate in the bubbly steady state, u^*, is always smaller than that in the bubbleless steady state, u^{**}, and the labor-market tightness in the bubbly steady state, θ^*, is always greater than that in the bubbleless steady state, θ^{**} because we focus on the case in which the presence of asset bubbles promotes capital accumulation. In both steady states, as β increases from 0.40 to 0.60, the labor-market tightness decreases and the unemployment rate increases. This outcome is not obvious because the workers' output share, Ω, has non-linear effects on these variables. As Ω increases, capital increases in both steady states through the workers' output share; however, the increase in Ω produces a downward pressure on the firms' output share, and thus, the firms post the smaller number of vacant positions given a fixed search cost. The effect of Ω on the unemployment rate (the labor-market tightness) in both steady states can be proven to be U-shaped (inverted U-shaped). In both steady states, the minimum unemployment rate is achieved around $\beta = 0.1$, which is an unrealistically small bargaining power in the advanced countries. When β changes from 0.40 to 0.60, the undesirable effect of Ω on the unemployment rate dominates the preferable effect and the unemployment rate increases. Moreover, as β increases, the difference in the unemployment rate between the bubbly and bubbleless steady states becomes wider although the difference in capital accumulation is relatively stable. Without asset bubbles, as β increased, the undesirable effect of Ω on the unemployment rate is accelerated. However, the presence of asset bubbles mitigates the acceleration of the undesirable effect.

As in the case of β, the increase in the unemployment benefit ratio, γ, increases the workers' output share, Ω, and thus, capital accumulation increases as seen in Figure 5. Although γ also has non-linear effects on the unemployment rate and the labor-market tightness in both steady states, one notes that the patterns of γ's effects on the macroeconomic variables are very similar to those of β. In particular, as γ increases from 0.60 to 0.87, the undesirable effect of Ω on the unemployment rate dominates the preferable effect. Without asset bubbles, the government policy that increases the unemployment benefit ratio, γ, would significantly increase the unemployment rate under the plausible value of γ. However, the presence of asset bubbles lessens this undesirable outcome: when asset bubbles occur, the increase in the unemployment rate is very small when γ varies from 0.60 to 0.87 relative to the case without asset bubbles.

[Figure 4 around here]

[Figure 5 around here]

5.5 Self-fufilling financial crisis

We consider a sunspot variable, ε_t, that follows a two-state Markov process, whose support is {0, 1} and transition probabilities are given by $Pr(\varepsilon_t = 1 | \varepsilon_{t-1} = 1) = \pi^a$ and $Pr(\varepsilon_t = 0 | \varepsilon_{t-1} = 0) = \pi^b$ where πa and $\pi^b \in (0, 1]$. Denote the history of sunspot events until time t by $\varepsilon^t = \{\varepsilon_0, \varepsilon_1, ..., \varepsilon_t\}$. The sunspot events are common across agents in each generation, being independent of idiosyncratic productivity shocks. The market price of the bubbly asset is subject to the sunspot variable, so we denote $p_t = p_t(\varepsilon_t)$. When determining the cutoff, ϕ_{t-1}, agents have rational expectations regarding future sunspot events given the sunspot event, ε_{t-1} at time $t-1$, and thus, we denote $\phi_{t-1} = \phi_{t-1}(\varepsilon_{t-1})$. Note that $\phi_{t-1}(\varepsilon_{t-1})$ becomes a deterministic variable when ε_{t-1} is realized although it is a stochastic variable before the realization of ε_{t-1}.

5.5.1 Cutoffs in the stationary states

The cutoff, $\phi_{t-1}(\varepsilon_{t-1})$, is no longer equal to r_t/q_t because the individual-specific return is a random variable. The market price of the bubbly asset is affected by the sunspot variable, so the individual-specific return, R_{t+1}, is a function of ε_{t+1}, given ε^t. Then, R_{t+1} is denoted by $R_{t+1}(\varepsilon_{t+1})$ and obtained as follows:

$$R_{t+1}(\varepsilon_{t+1}) = \begin{cases} p_{t+1}(\varepsilon_{t+1}) / p_t(\varepsilon_t) & if \quad \Phi \le \phi_t(\varepsilon_t) \\ q_{t+1}(\varepsilon^t)\Phi & if \quad \Phi > \phi_t(\varepsilon_t) \end{cases},$$

Note that $q_{t+1} = \alpha z_{t+1}^{\alpha-1}$ depends upon the sunspot history, ε^t, because capital at time $t+1$ is determined at time *t*. Given the sunspot event, ε_t, an agent at time *t* chooses a portfolio to maximize her expected lifetime utility:

$$E_t[U_t^t \mid \varepsilon_t] = s_t^t E_t[R_{t+1}(\varepsilon_{t+1}) \mid \varepsilon_t].$$

$E_t[R_{t+1}(\varepsilon_{t+1}) \mid \varepsilon_t]$ is given by

$$E_t[R_{t+1}(\varepsilon_{t+1}) \mid \varepsilon_t = 1] = \begin{cases} \rho_{t+1}^a & if \quad \Phi \le \phi_t(\varepsilon_t = 1) \\ q_{t+1}(\varepsilon_t = 1, \varepsilon^{t-1})\Phi & if \quad \Phi > \phi_t(\varepsilon_t = 1) \end{cases},$$

and

$$E_t[R_{t+1}(\varepsilon_{t+1}) \mid \varepsilon_t = 0] = \begin{cases} \rho_{t+1}^b & if \quad \Phi \le \phi_t(\varepsilon_t = 0) \\ q_{t+1}(\varepsilon_t = 0, \varepsilon^{t-1})\Phi & if \quad \Phi > \phi_t(\varepsilon_t = 0) \end{cases},$$

where $\rho_{t+1}^a = \{\pi^a p_{t+1}(1) + (1 - \pi^a) p_{t+1}(0)\}/p_t(1)$ and $\rho_{t+1}^b = \{\pi^b p_{t+1}(0) + (1 - \pi^b) p_{t+1}(1)\}/p_t(0)$. From these two equations, we obtain the cutoffs depending upon the sunspot realizations such as $\phi_t^a := \phi_t(\varepsilon_t = 1) = \rho_{t+1}^a / q_{t+1}(\varepsilon_t = 1, \varepsilon^{t-1})$ and $\phi_t^b := \phi_t(\varepsilon_t = 0) = \rho_{t+1}^b / q_{t+1}(\varepsilon_t = 0, \varepsilon^{t-1})$. Because the return of holding the one unit of the bubbly asset is given by $p_t(\varepsilon_t)/p_{t-1}(\varepsilon_{t-1})$, Eq. (19) is rewritten as

$$B_t(\varepsilon_t) = \frac{p_t(\varepsilon_t)}{(1-\mu) p_{t-1}(\varepsilon_{t-1})} B_{t-1}(\varepsilon_{t-1}), \tag{41}$$

where $B_t(\varepsilon_t) = p_t(\varepsilon_t) M_t$. Given the sunspot event, ε_{t-1}, taking the expectation of both sides of Eq. (41) yields

$$E_{t-1}[B_t(\varepsilon_t) \mid \varepsilon_{t-1}] = \frac{E_{t-1}[p_t(\varepsilon_t) \mid \varepsilon_{t-1}]}{(1-\mu) p_{t-1}(\varepsilon_{t-1})} B_{t-1}(\varepsilon_{t-1}).$$

Depending upon the realization of ε_{t-1}, this equation can be rewritten as

$$\pi^a B_t(1) + (1 - \pi^a) B_t(0) = \frac{\rho_t^a}{1-\mu} B_{t-1}(1), \tag{42}$$

and

$$\pi^b B_t(1) + (1 - \pi^b) B_t(0) = \frac{\rho_t^b}{1-\mu} B_{t-1}(0), \tag{43}$$

Inserting Eq. (21) respectively into Eqs. (42) and (43) yields

$$Az_t^{\alpha}(1-u_t)\left[\pi^a \frac{G(\phi_t^a)}{1-\mu G(\phi_t^a)}+(1-\pi^a)\frac{G(\phi_t^b)}{1-\mu G(\phi_t^b)}\right] = \frac{\rho_t^a}{1-\mu}\times\frac{Az_{t-1}^{\alpha}(1-u_{t-1})G(\phi_{t-1}^a)}{1-\mu G(\phi_{t-1}^a)}, \tag{44}$$

and

$$Az_t^{\alpha}(1-u_t)\left[\pi^b \frac{G(\phi_t^b)}{1-\mu G(\phi_t^b)}+(1-\pi^b)\frac{G(\phi_t^a)}{1-\mu G(\phi_t^a)}\right] = \frac{\rho_t^b}{1-\mu}\times\frac{Az_{t-1}^{\alpha}(1-u_{t-1})G(\phi_{t-1}^b)}{1-\mu G(\phi_{t-1}^b)}, \tag{45}$$

In what follows, our analysis focuses on a stationary rational expectations equilibrium with sunspots, such that $\phi^a := \phi_t^a = \phi_{t-1}^a$ and $\phi^b := \phi_t^b = \phi_{t-1}^b$.[50] By using Eqs. (24), (44) and (45) with $\phi^a = \rho_t^a/q_t(\varepsilon^{t-1})$ and $\phi^b = \rho_t^b/q_t(\varepsilon^{t-1})$, we obtain the following two equations:

$$\pi^a \frac{G(\phi^a)}{1-\mu G(\phi^a)}+(1-\pi^a)\frac{G(\phi^b)}{1-\mu G(\phi^b)} = \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)} \tag{46}$$

and

$$\pi^b \frac{G(\phi^b)}{1-\mu G(\phi^b)}+(1-\pi^b)\frac{G(\phi^a)}{1-\mu G(\phi^a)} = \frac{\alpha\phi^b G(\phi^b)}{(1-\mu)(1-\alpha)\Omega H(\phi^b)}. \tag{47}$$

We assume that $\phi^a > \phi^b$. Because the bubbly asset is freely disposable, $B_t \geq 0$ and thus $\phi_t \geq 0$ for all $t \geq 0$ from Eq. (21).

Lemma 2 *Suppose that $\pi^a \in (0, 1)$ and $0 \leq \phi^a < \phi^b$. Then, if there exists a rational expectations equilibrium with the two-state sunspot variable that satisfies Eqs. (46) and (47), it must follow that $\phi^b = 0$ and $\pi^b = 1$.*

Proof. See the Appendix.

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[50]To be accurate, z_t and u_t in Eqs. (44) and (45) depend on the sunspot history, ε^{t-1}. Therefore, we should have explicitly written as $z_t(\varepsilon_{t-1} = 1, \varepsilon^{t-2})$ and $u_t(\varepsilon_{t-1} = 1, \varepsilon^{t-2})$ in Eq. (44) and $z_t(\varepsilon_{t-1} = 0, \varepsilon^{t-2})$ and $u_t(\varepsilon_{t-1} = 0, \varepsilon^{t-2})$ in Eq. (45); however, we use simple notations to save a space.

Proposition 5 *There exists a rational expectations equilibrium with the two state sunspot variable that satisfies Eqs. (46) and (47) such that* $\phi^b = 0$ *with* $\pi^b = 1$ *and* $\phi^a \in (0, \phi^*)$ *with* $\pi^a \in (0, 1)$.

Proof. See the Appendix.

The state given by ϕ^a is bubbly whereas the state given by ϕ^b is bubbleless. Proposition 5 implies that once asset bubbles burst caused by self-fulfilling expectations, the bubbly asset never has a market value after the bursting. This outcome is obtained because the bubbly asset is freely disposable and because as demonstrated in the previous section, the steady state, ϕ^*, in the dynamical system (27) is unstable and the steady state $\phi^{**} = 0$ is stable. As noted from Eq. (26), capital accumulation in each state is given by

$$z_t \Psi(z_t) = \frac{(1-\alpha)\Omega A H(\phi^a)}{1-\mu G(\phi^a)} z_{t-1}^{\alpha} \Psi(z_{t-1}). \tag{48}$$

and

$$z_t \Psi(z_t) = (1-\alpha)\Omega A H(0) z_{t-1}^{\alpha} \Psi(z_{t-1}), \tag{49}$$

respectively. Figure 6 illustrates the case in which the presence of asset bubbles promotes capital accumulation, i.e., $H(\phi^a)/[1-\mu G(\phi^a)] > H(0)$. In this case, Eq. (48) is located in the upper place relative to Eq. (49).

[Figure 6 around here]

Now we assume that $\varepsilon_0 = 1$, meaning that asset bubbles are present at time 0. In this case, capital accumulates over time if $z_0 < z^a$ where $z^a = Q(\phi^a)1/(1-\alpha)$ as seen in Figure 4. However, once asset bubbles burst at a certain time, say, $t = \hat{t}$, capital begins to decrease if $z_{\hat{t}} > z^b$ where $z^b = Q(\phi^b)1/(1-\alpha)$, and accordingly the unemployment rate begins to increase following Eq. (26).

5.6 Conclusion

An overlapping-generation model is presented in which the presence asset bubbles ´a la Tirole (1985) promotes capital accumulation under mild parameter conditions. In a financially constrained economy, although the presence of asset bubbles correct allocative inefficiency by excluding less productive agents from production activities, only the second best outcome can be attained as clarified by Bewley (1980). This consequence can be easily verified in our model by observing that not only agents who draw the highest productivity shock but also agents who draw relatively low productivity shocks engage in capital production when asset bubbles are present. Therefore, the unemployment rate when asset bubbles occur is not lowest relative to that in the first best outcome, which means that government policy is necessary for the economy to be Pareto-improved even though the presence of asset bubbles reduces the unemployment rate. The analysis of such government policy is beyond the scope of the current paper and left for future research.

Appendix

Proof of Proposition 1

If $z_t \leq \bar{z}$, no firms with vacancy can cover a search cost, h, at time t even though the matching probability is equal to one. Therefore, no firms can operate at time t. If $z_t > \bar{z}$, given the matching probability $f(\theta_t) \in (0, 1)$, firms with vacancy will cover a search cost, h, and firms that successfully match with a worker operate their business at time t. □

Proof of Proposition 2

From Eq. (25), $\phi^{**} = 0$ is obviously a steady state of the dynamical system because $G(\phi^{**}) = 0$. It is noted from Eq. (25) that ϕ^{*} is a candidate of another steady state. Therefore, all we must show is that ϕ^{*} is uniquely determined. Define $\Lambda(\phi) := H(\phi)/(1 - \mu G(\phi))$ and $\Gamma(\phi) := \phi\alpha/[(1 - \alpha)(1 - \mu)\Omega]$. Note that $\Lambda(\phi)$ is the left-hand of Eq. (28) and $\Gamma(\phi)$ is the right-hand. $\Gamma(\phi)$ is linear with respect to ϕ with a positive slope and passes through the origin. To investigate the configuration of $\Lambda(\phi)$, define a function such that $J(\phi) := \mu H(\phi) - \phi(1 - \mu G(\phi))$. Because $J'(\phi)(\phi) = \mu G(\phi) - 1 < 0$, $J(\phi)$ is monotonically decreasing. Additionally, $J(0) = \mu H(0) > 0$ and $J(\eta) = -h(1 - \mu) < 0$. Therefore, $J(\phi) = 0$ has a unique solution $\phi = \bar{\phi} \in (0, \eta)$ such that for $\phi \in [0, \bar{\phi})$, it follows that $J(\phi) > 0$ and for $\phi \in (\bar{\phi}, \eta]$, it follows that $J(\phi) < 0$. Because $\Lambda'(\phi)(\phi) = G'(\phi)J(\phi)/[1 - \mu G(\phi)]^2$, $\Lambda'(\phi)$ is increasing in $\phi \in [0, \bar{\phi})$ and decreasing in $\phi \in (\bar{\phi}, \eta]$. Moreover, $\Lambda(0) = H(0) > 0$ and $\Lambda(\eta) = 0$. As such, the configurations of $\Gamma(\phi)$ and $\Lambda(\phi)$ confirm the uniqueness of ϕ^{*} in Eq. (27). □

Proof of Lemma1

Because $(1 - \alpha)\Omega A H(\hat{\phi})\hat{z}^{\alpha-1}/(1 - \mu G(\hat{\phi})) = 1$, the linearization of Eq. (26) around the steady state, $(\hat{z}, \hat{\phi})$, yields

$$z_t - \hat{z} = \frac{\alpha\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}(z_{t-1} - \hat{z}) + \frac{\hat{z}^{\alpha}\Psi(\hat{z})Q'(\phi)}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}(\phi_{t-1} - \hat{\phi}), \tag{A.1}$$

where $Q(x) = (1 - \alpha)\Omega H(x)/(1 - \mu G(x))$. Eq. (A.1) yields $\kappa_1(\hat{z})$ as follows:

$$\kappa_1(\hat{z}) = \frac{\alpha\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})}{\Psi(\hat{z}) + \hat{z}\Psi'(\hat{z})},$$

where $\hat{z} = z^*$ or z^{**}. The linearization of Eq. (25) around the steady state yields

$$\phi_{t-1} - \hat{\phi} = \left(\frac{\alpha(1-\mu G(\hat{\phi}))}{(1-\mu)(1-\alpha)\Omega H(\hat{\phi})}\right)\left(\frac{G(\hat{\phi})(1-\mu G(\hat{\phi}))}{G'(\hat{\phi})}\right) \times \left[1 + \frac{\hat{\phi}G'(\hat{\phi})}{G(\phi)} + \frac{\hat{\phi}^2 G'(\hat{\phi})}{H(\hat{\phi})}\right](\phi_{t-1} - \hat{\phi}). \tag{A.2}$$

When $\hat{\phi} = \phi^*$, Eq. (A.2) can be rewritten as

$$\phi_{t-1} - \phi^* = \left(\frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\right)\left[1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right](\phi_{t-1} - \phi^*), \tag{A.3}$$

because $\alpha(1 - \mu G(\phi^*))/[(1 - \mu)(1 - \alpha)\Omega H(\phi^*)] = 1/\phi^*$. Therefore, we have

$$\kappa_2(\phi^*) = \frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\left(1 + \frac{\phi^* G'(\phi^*)}{G(\phi^*)} + \frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right).$$

When $\hat{\phi} = \phi^{**} = 0$, Eq. (A.2) can be rewritten as

$$\phi_t - \phi^{**} = 0(\phi_{t-1} - \phi^{**}). \tag{A.4}$$

Therefore, we have

$$\kappa_2(\phi^{**}) = 0. \ \square$$

Proof of Proposition 3

Obviously, it follows that $|\kappa_1(z^{**})| < 1$ and $|\kappa_2(\phi^{**})| < 1$. Therefore, the bubbleless steady state, (z^{**}, ϕ^{**}), is totally stable. Because $|\kappa_1(z^*)| < 1$, all we must show is $|\kappa_2(\phi^*)| > 1$. To show this, define

$$\Theta(\phi) := \Gamma(\phi) - \Lambda(\phi),$$

where $\Gamma(\phi)$ and $\Lambda(\phi)$ are defined in the proof of Proposition 2. As shown in the proof of Proposition 2, $\Theta(\phi) = 0$ has a unique solution, which is $\phi = \phi^*$. Therefore, the fact that $\Theta(0) < 0$ and $\Theta(\eta) > 0$ implies that $\Theta'(\phi^*)(\phi^*) > 0$, or equivalently,

$$\frac{\alpha}{(1-\alpha)(1-\mu)\Omega}-\frac{G'(\phi^*)[\mu H(\phi^*)-\phi^*(1-\mu G(\phi^*))]}{(1-\mu G(\phi^*))^2}>0. \tag{B.1}$$

The use of Eq. (29) rewrites Eq. (B.1) as

$$\frac{H(\phi^*)}{\phi^*(1-\mu G(\phi^*))}-\frac{G'(\phi^*)[\mu H(\phi^*)-\phi^*(1-\mu G(\phi^*))]}{(1-\mu G(\phi^*))^2}>0. \tag{B.2}$$

Furthermore, Eq. (B.2) can be computed as

$$(1-\mu G(\phi^*))\left(1+\frac{\phi^* G'(\phi^*)}{H(\phi^*)}\right)-\phi^*\mu G'(\phi^*)>0,$$

which can be transformed into

$$\frac{G(\phi^*)(1-\mu G(\phi^*))}{\phi^* G'(\phi^*)}\left(1+\frac{\phi^* G'(\phi^*)}{G(\phi^*)}+\frac{(\phi^*)^2 G'(\phi^*)}{H(\phi^*)}\right)>1. \tag{B.3}$$

The left-hand side of Eq. (B.3) is $\kappa_2(\phi^*)$, and thus, the bubbly steady state is a saddle point. □

Proof of Proposition 4

From Figure 4, if $\alpha/[(1-\alpha)(1-\mu)\Omega]>(<) H(0)/\tilde{\phi}$, it follows that $\Lambda(\phi^*)>(<)\Lambda(0)=H(0)$, and thus, $Q(\phi^*)>(<)Q(0)$. From the last and Proposition 2, we obtain the desired conclusion.

□

Proof of lemma 2

The proof is done by contradiction. Suppose that $\pi^b \in (0, 1)$. From Eq. (47), we have

$$\pi^b\left[-\frac{G(\phi^b)}{1-\mu G(\phi^b)}+\frac{G(\phi^a)}{1-\mu G(\phi^a)}\right]=-\frac{\alpha\phi^b G(\phi^b)}{(1-\mu)(1-\alpha)\Omega H(\phi^b)}+\frac{G(\phi^a)}{1-\mu G(\phi^a)}. \tag{C.1}$$

Because $\pi^b \in (0, 1)$ and $\phi^b < \phi^a$, it follows from Eq. (C.1) that

$$\frac{G(\phi^b)}{1-\mu G(\phi^b)}<\frac{\alpha\phi^b G(\phi^b)}{(1-\mu)(1-\alpha)\Omega H(\phi^b)}. \tag{C.2}$$

Likewise, from Eq. (46), it follows that

$$\frac{G(\phi^a)}{1-\mu G(\phi^a)}>\frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)}. \tag{C.3}$$

From the configurations of $\Lambda(\phi)$ and $\Gamma(\phi)$ (which are defined in the proof of Proposition 2), for $\phi \in (\phi^*, \eta]$, it holds that $\Lambda(\phi) < \Gamma(\phi)$, and for $\phi \in [0, \phi^*)$, it holds that $\Lambda(\phi) > \Gamma(\phi)$. Therefore, we obtain $\phi^* < \phi^b$. Because $\phi^b < \phi^a$, we have $\Lambda(\phi^a) < \Gamma(\phi^a)$, or equivalently,

$$\frac{G(\phi^a)}{1-\mu G(\phi^a)} < \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)}. \tag{C.4}$$

Eq. (C.4) contradicts Eq. (C.3). Therefore, it must follow that $\pi^b = 1$. When $\pi^b = 1$, Eq. (47) yields $\phi^b = \phi^*$ or $\phi^b = 0$. If $\phi^b = \phi^*$, we have $\phi^* < \phi^a$. However, $\phi^* < \phi^a$ again leads a contradiction. Hence, $\phi^b = 0$. □

Proof of Proposition 5

From Lemma 2, it must hold that $\pi^b = 1$ and $\phi^b = 0$. In this case, Eq. (46) can be rewritten as

$$\pi^a \frac{G(\phi^a)}{1-\mu G(\phi^a)} = \frac{\alpha\phi^a G(\phi^a)}{(1-\mu)(1-\alpha)\Omega H(\phi^a)}. \tag{D.1}$$

From Eq. (D.1), we obtain $\Lambda(\phi^a) > \Gamma(\phi^a)$ for $\pi^a \in (0, 1)$, and thus, $\phi^a \in (0, \phi^*)$ with $\pi^a \in (0, 1)$. □

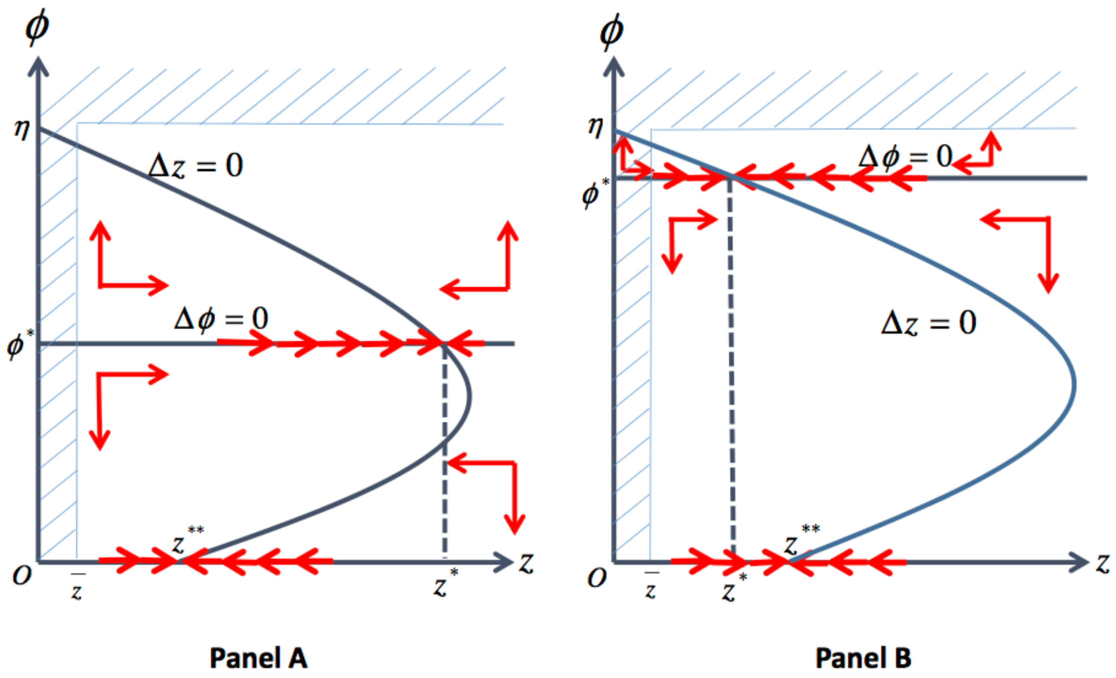

Fig. 1. Phase diagrams. *Notes:* **Panel A** is the case in which the presence of bubbles promotes capital accumulation and **Panel B** is the case in which the presence of bubbles impedes capital accumulation. The shadow area is *not* a domain of the dynamical system.

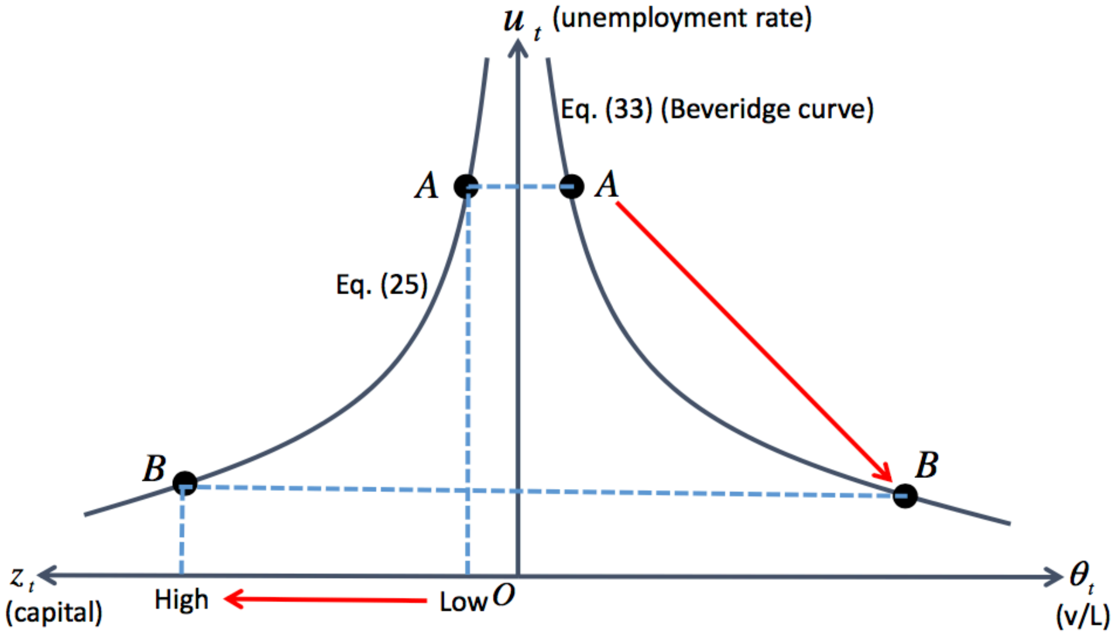

Fig. 2. Beveridge curve and capital accumulation. *Notes:* Capital accumulation is low at point A and high at point B. Capital accumulation promotes employment, rendering the labor market tighter.

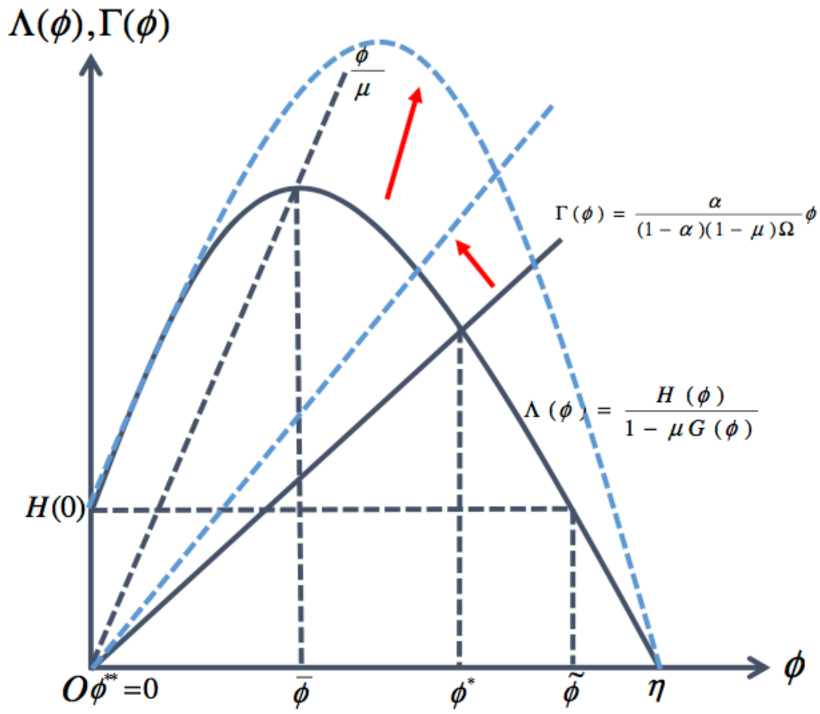

Fig. 3. Comparison between bubbly and bubbleless steady states. *Notes:* If $\Lambda(\phi^*) > H(0)$, capital accumulates more in the bubbly steady state, and if $\Lambda(\phi^*) < H(0)$, capital accumulates less in the bubbly steady state than in the bubbleless steady state. As μ increases, $\Lambda(\phi)$ shifts upward and $\Gamma(\phi)$ rotates counterclockwise.

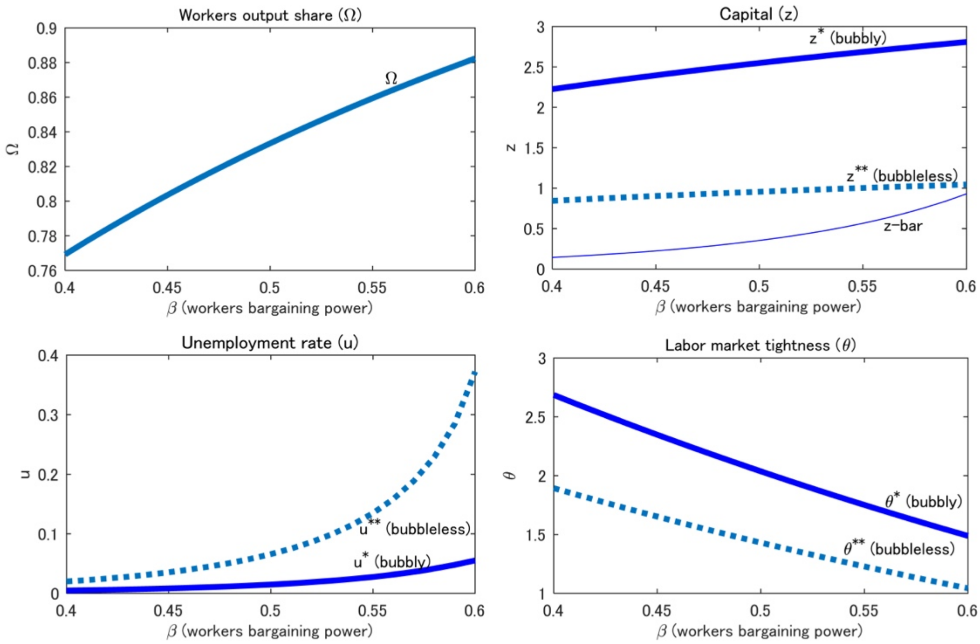

Fig. 4. Effects of workers' bargaining power. *Notes:* z-bar means the lower limit of capital, $\bar{z}$, defined in Proposition 1.

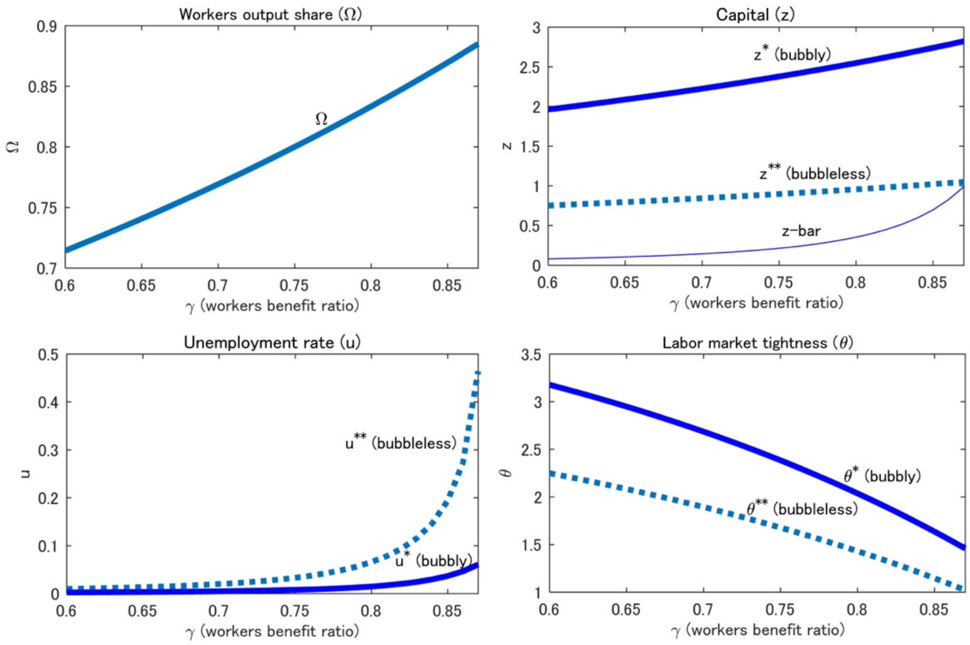

Fig. 5. Effects of the unemployment benefit ratio. *Notes:* z-bar means the lower limit of capital, $\bar{z}$, defined in Proposition 1.

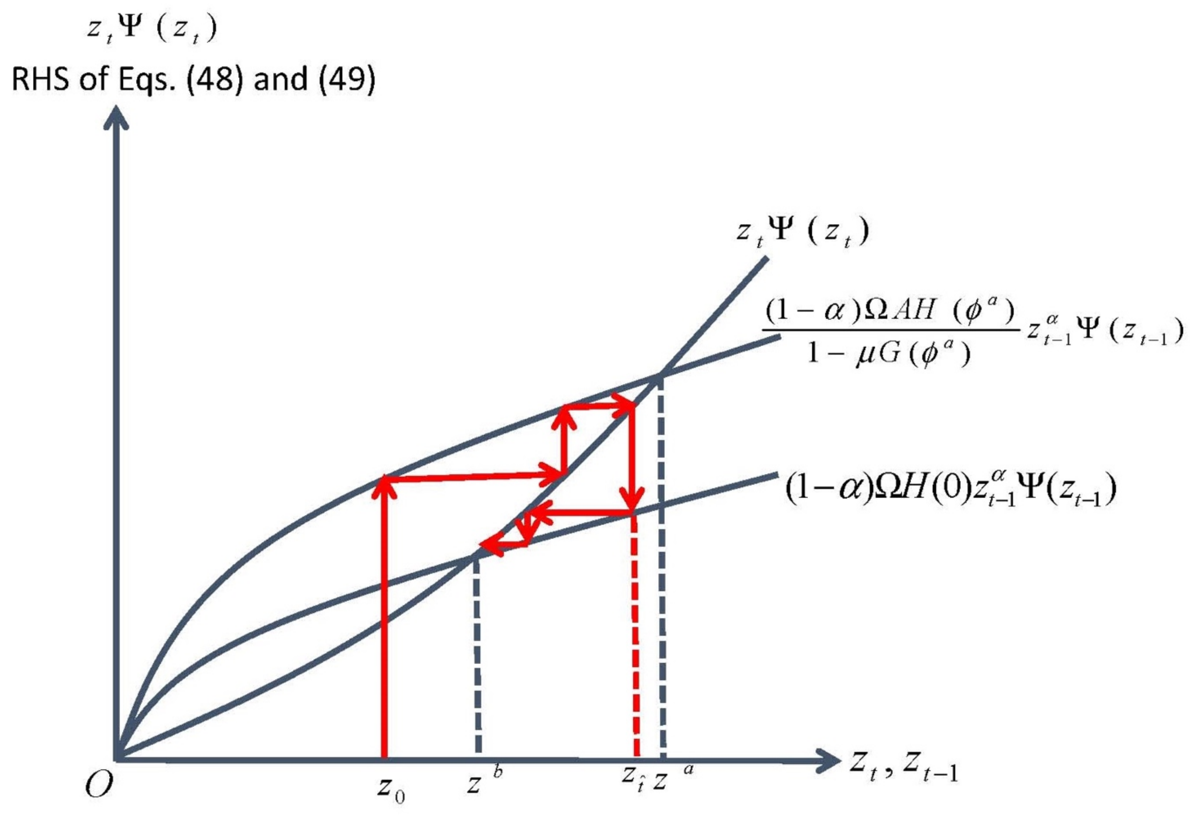

Fig. 6. Bubble bursting and financial crisis. *Notes:* This is the case in which $H(\phi^a)/[1-\mu G(\phi^a)] > H(0)$. Asset bubbles burst at time $\hat{t}$ and capital begins to decrease.

Table 1: Parameterization

$\alpha = 0.36$	$\beta = 0.50$ (Fig. 5)	$\gamma = 0.80$ (Fig. 4)	$\eta = 4$
$\sigma = 4$	$\mu = 0.7$	$A = 1.5$	$h = 0.11$

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