Distortions of incentives in voting and reporting behaviors

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投票・報告における誘引の歪み

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1 Introduction

Expressing one's opinions and findings is important for social decisions, managing organization and so on. If the players' preferences are not aligned or expressing ones' opinions and findings are costly, there are distortions in incentive to express a certain message (i.e., player has an incentive not to tell the true information) and it becomes an issue to achieve a socially optimal decision. Eliciting the true information from the players is a central problem in the studies of mechanism design and organizational economics.

In this paper, we study how incentives of telling the truth are distorted in some organizations and mechanisms that often used in real world.

First, consider a situation that a person complains about some issue. If he makes a complaint about the issue, it may be improved, whereas making a complaint is costly for him while overlooking is costless. In the context of this study, improving the issue is considered as a public good and making a complaint is considered as a contribution to the public good. Chapter 2 considers such situation as a problem of private provision of a discrete public good with a binary contribution. That is, when a certain number of persons (threshold) contributes a public good, it is provided, otherwise not. This chapter examines the property of equilibria in a large population. We show that the limit probability that the public good is provided is irrelevant to the preferences distribution, which shows that such system is meaningless.

In chapter 2, since sending a message is costly, only most zealous players to provide a public good send messages and thus, the outcome is irrelevant to the citizens' preferences. However, if there is a surrogate who sends a message on behalf of the citizens, the tragic result may change.

In chapter 3, we consider an initiative process based on a signature collection where there is a campaigner who gathers signatures. In this setting, the campaigner is a surrogate of citizens. If signing and not signing are costless, unlikely in the previous model, the number of collected signatures is increasing in the ratio of supporters of the campaign. In this case, however, the campaigner's incentive to campaign becomes an issue since the campaigner can gather many signatures by asking many people to sign even when the campaign is supported by only a small ratio of people. Thus, if the signature requirement is too low, too many campaigns can meet the requirement. On the other hand, since campaigning is costly, with a too high requirement, almost all campaigners give up campaigning.

Chapter 3 examines the optimal requirement to balance the tradeoff. This is the minimum requirement to the campaigner gathers the required signatures for popular laws while give up gathering the required signatures for unpopular law. We show that when the difference between popular and unpopular laws is large in the number of collected signatures, a low requirement is needed for the purpose. We also show that the difference is large if either uncertainty in citizens' preference distribution is high or, citizens have a keen interest on whether the law is enacted (the importance of the law is high). Therefore, in such case, the optimal requirement is low.

To summarize, chapter 2 and 3 study systems dealing with citizens' request for improvement and examine an optimal system to elicit the true information about their preferences from the people. This type of question is also seen in another situation. In an organization, managers cannot always find problems about the organization, so they delegate their subordinates to find the problems and ask him to report the problem if he finds it. Here the subordinate's information is whether he knows the problem. However, in the real world, the subordinates may not report the problems that they find even when they may be punished for not reporting. There should be incentives of not reporting the problem.

In chapter 4, we study workers' incentives for reporting problems within an OLG organization consisting of a subordinate and a manager. In this paper, we assume that in each period, the subordinate proceeds to be a manager and the manager retires in the next period. Subordinates have responsibility to report the problem and managers have responsibility to solve the reported problem. We also assume that the manager's responsibility is heavier when the problem is reported than the case of being not reported. Then, if the subordinate reports and the manager does not solve the reported problem, the subordinate will have a responsibility for solving the reported problem in the next period. On the other hand, if the subordinate conceals the problem and, so does the next subordinate, since the problem has not reported, the responsibility of the subordinate will be reduced. This mechanism creates subordinates' incentive of not reporting. We study the properties of equilibria and show that this incentive has a strategic complementarity, which implies that not reporting of the problem is realized as a firm's culture (Kreps, 1990).

We also see that the timing when the problem is reported. To consider this, we assume that the size of the problem, which determines the size of punishment and the cost of solving the problem, evolves as time proceeds. We show that when the growth rate of the size decreases as time proceeds, there exists a period such that before the period, no player reports, and thus the report is delayed. On the contrary, if the growth rate of the size increases, there exist a period such that after the period, no player reports. In the latter case, if the size of the problem increases, after a period, the problem will be ignored until accidents occur even when the managers could solve if it is reported.

2 Binary contribution to a discrete public good in a large population[1]

2.1 Introduction

This study examines the private provision of a discrete public good problem with non-refunded binary contribution developed by Palfrey and Rosenthal (1984).[2] We investigate the asymptotic properties of this model when the population is finite but sufficiently large.

This model can apply to the following situations. Consider the situation that a citizen complains about, say, a TV program, a collection of books in a library, or a government's policy implementation. For example, the BBC receives about 250,000 complaints every year,[3] while the American Library Association reports lists of books that are frequently requested to be removed from libraries.[4] If a sufficiently large number of citizens complain about such issues, the parties concerned would make a response and may modify the issues. In the context of this study, the modification of the issue is considered to be a discrete public good, while the act of complaining is a binary contribution.[5]

Similarly, in some countries, if the required number of signatures to a petition are gathered, citizens can request that a law be enacted. Signing or not signing is again a binary choice and enacting the law is a public good. For example, US government has a online platform,"We the People" to citizens make a petition and if the petition collects 100,000 signatures, US government reviews and responds

[1] The author is deeply grateful to Eiichi Miyagawa and Yoshikatsu Tatamitani, who provided carefully considered feedback and valuable comments. The author is also indebted to the many people who commented on this work. Their comments significantly contributed to improving this manuscript. Any errors in this paper are the author's own. This work was supported by JSPS KAKENHI grant number 14J05350.

[2] This game is also called a threshold game.

[3] `http://www.bbc.co.uk/complaints/handle-complaint/`.

[4] `http://www.ala.org/bbooks/frequentlychallengedbooks/top10`.

[5] Liang (2013) considers consumer complaints and analyzes information transmission when the number of consumers is 2. Prendergast (2002) also considers consumer complaints as a tool of resolving agency problem.

the petition.[6]

In either case, the population is considered to be large. Therefore, considering an infinite population approximates the real world. Moreover, the costs of complaining and signing cannot be recovered. This is the reason why we consider non-refunded contribution.

In this setting, we consider the symmetric Bayesian Nash equilibrium (symmetric BNE) to investigate the probability that the discrete public good is provided when the population goes to infinity. We provide the necessary and sufficient condition that there exists a symmetric equilibrium in which each player contributes a positive probability with a sufficiently large population. Let h be the threshold level, $\overline{V}$ be the maximum valuation, and $\underline{C}$ be the minimum cost of contribution among citizens. We call a player whose value and contribution cost are respectively $\overline{V}$ and $\underline{C}$ the most zealous player.

If $h > 1$, the act of no player contributing is a trivial equilibrium. In the non-trivial equilibrium, we show that as the population goes to infinity, only the most zealous players contribute and the distribution of the number of contributors converges to a Poisson distribution with parameter x, which solves $\overline{V}e^{-x}x^{(h-1)}/(h-1)! = \underline{C}$. The intuition is as follows. Note that the probability that a given player contributes converges to zero as the population goes to infinity. In this case, the number of contributors follows a Poisson distribution. Then, $e^{-x}x^{(h-1)}/(h-1)!$ is the probability that the number of contributors is $h-1$. Hence, $e^{-x}x^{(h-1)}/(h-1)!$ is the probability that a given player is pivotal. Therefore, $\overline{V}e^{-x}x^{(h-1)}/(h-1)!$ and $\underline{C}$ are respectively the expected utility and cost of the most zealous player's contribution. In the non-trivial equilibrium, these values coincide. If the most zealous player's cost exceeds his expected utility, no player wishes to contribute. This is a trivial equilibrium (i.e., a contradiction). If the most zealous player's expected utility exceeds his cost, since the population goes to infinity, an infinite number of players contribute. This fact implies that the probability that the number of contributors is $h-1$ goes to zero and, in turn, the expected utility of the most zealous player's contribution goes to zero, which, again, is a contradiction. Thus, we have $\overline{V}e^{-x}x^{(h-1)}/(h-1)! = \underline{C}$.

If the threshold is one, the symmetric BNE is unique. If the minimum cost is sufficiently small, the act of no player contributing is not an equilibrium. In particular, if the maximum valuation is infinity or the minimum cost is 0, in any symmetric equilibrium, the probability that the public good is provided goes to 1 as the population goes to infinity. When the threshold is more than one, if the minimum cost is sufficiently small, there are exactly two equilibria in which each player contributes

[6] `https://petitions.whitehouse.gov`.

a positive probability. These equilibria differ in the expected number of contributors. We call the equilibrium with the smaller expected number of contributors the *low contributing equilibrium* and that with the larger expected number of contributors the *high contributing equilibrium*.

We show that with a large population, in low contributing equilibria, the probability that a given player contributes is decreasing in $\overline{V}/\underline{C}$. On the contrary, in high contributing equilibria, the probability that a given player contributes is increasing in $\overline{V}/\underline{C}$. We also consider the threshold effect. We show that in both high contributing and low contributing equilibria, the probability that a given player contributes is increasing at the threshold. On the contrary, in high contributing equilibria, the probability that the public good is provided is decreasing at the threshold.

These comparative static results can be applied to design optimal complaint management systems. Consider a public organization. Some citizens are dissatisfied with its decisions, systems, and institutions. However, the people currently working in the organization may be unable to grasp all its decisions, as some were made a long time ago for example. In another case, such decisions are made by a decentralized party. Such decisions cause complaints and are requested to be modified. However, how should the organization handle such complaints?

For example, consider a collection of books in a public library. Typically, these books are collected by librarians. Some citizens may complain that the library's collection includes unsuitable books and request their removal.

We consider a system such that if an issue causes complaints and the number of complaining citizens is larger than h, the decision is modified. Since the government cannot grasp the issue, setting $h = 0$ may be impossible. In the case that setting $h = 0$ is impossible, if the modification is socially desirable, $h = 1$ is optimal in the following two senses. First, since setting $h = 1$ implies the highest probability of a sufficient number of citizens complaining. Problematic issues are thus easily exposed. Second, setting $h = 1$ also implies the lowest number of complaining citizens since this number is increasing at the threshold. Typically, handling a large number of complaints is costly. Thus, fewer complaining citizens is more desirable. On the contrary, if the modification is socially undesirable, rejecting all complaints is optimal. In summary, the optimal policy is either $h = 1$ or rejecting all complaints. That is, the number of complaining citizens does not matter. Indeed, as a real-world example, the BBC has previously stated that the number of complaints does not make a difference.[7] Our result may explain this policy.

[7] `http://www.bbc.co.uk/complaints/handle-complaint/`.

Previous studies have assumed that players simultaneously decide whether to contribute. However, each player may also observe the behavior of others before deciding whether to contribute. For example, consider an anti-governmental demonstration. A group of campaigners are campaigning to repeal a policy. They count the number of citizens who have already participated in the demonstration and announce this figure publicly, which inevitably affects the future decisions of other possible participants. As another example, at the website of US government's platform to citizen make a petition, we can view on-going status of collected signatures of each petition. This also creates a dynamic situation.

We consider the case. Let h be the threshold. In the static model, if $h > 1$, the act of no player contributing is necessarily an equilibrium. Indeed, if h is more than the entire number of opportunities to announce the number of contributors, the act of no player contributing is still a symmetric perfect Bayesian equilibrium. However, we show that if h is less than the entire number of opportunities, as the population goes to infinity, there is a positive infimum in the probability that at least h players contribute in any symmetric perfect Bayesian equilibrium. Moreover, if the minimum cost among players goes to zero, the probability that at least h players contribute converges to 1 in any symmetric perfect Bayesian equilibrium.

Recall that when $h = 1$, even in the static model, the act of no player contributing is not an equilibrium with sufficiently low cost. Suppose that there are two chances to contribute and $h = 2$. Suppose also that one player contributed in the first stage. Then, in the second stage, it is the same situation as $h = 1$. One player's contribution is sufficient to provide the public good. Therefore, in the second stage, the act of no player contributing is not an equilibrium. Then, in turn, in the first stage, since at least one player contributes, at least one player contributes in the second stage and thus the public good is provided. This situation is the same as the case for $h = 1$. Thus, the act of no player contributing is not an equilibrium. This discussion can be generalized for the case of a finite number of chances to contribute.

The remainder of this paper consists of the following sections. Section 3.2 reviews the related literature. Section 2.2 describes our model. The results of the equilibrium analysis are presented in Section 2.3, where we calculate the probability that the public good is provided in an infinitely large population. Section 2.4 addresses the dynamic provision model. Section 2.5 considers threshold uncertainty before our conclusions are finally drawn. Omitted proofs are presented in the Appendix.

2.1.1 Related Literature

This section reviews the literature on the private provision of discrete public goods. To the best of the author's knowledge, Palfrey and Rosenthal (1984) was the first study to analyze this problem, showing both efficient and inefficient equilibria. Harrington (2001), Xu (2001), and Bergstrom (2012b) all find that when the threshold level is one, as the population goes to infinity, the probability of the provision of public goods converges to a constant positive number. Xu (2002) analyzes how an increase at the threshold level influences the probability of the provision of a public good by dividing the effects into the threshold effect (direct effect) and strategic effect (indirect effect), showing that the effect of a threshold increase is not necessarily negative. That is, increasing at the threshold can raise the probability of the provision of a public good. In contrast to these authors, who deal with identical preferences for public goods, Menezes, Monteiro and Temimi (2001) analyze the case that preferences are private information and continuous contribution. They show that if the provision cost is sufficiently high, the unique equilibrium is non-contributing. Barbieri and Malueg (2008) also consider the continuous contribution case and study the efficiency of equilibria.

The present study examines the case that both players' preferences and the contribution cost are private information. In addition, we consider the decision to be binary (i.e., contribute to public goods or not (Palfrey and Rosenthal, 1984)) rather than continuous contribution (Menezes, Monteiro and Temimi, 2001). Binary contribution model is similar to voting model. Related to this paper, as in our analysis, Taylor and Yildirim (2010,b) show that the limiting distribution of the number of votes converges to a Poission distribution.

Many studies have examined the dynamic contribution of discrete public goods. Bliss and Nalebuff (1984) and Bilodeau and Slivinski (1996) consider the case that public goods are provided in a single provision. Marx and Matthews (2000) consider a general case with continuous contribution in a repeated game setting, showing that an efficient allocation is achieved. However, Marx and Matthews (2000)'s model, in contrast to that presented in our study, needs the belief that a punishment ensues if one does not contribute. Gradstein (1992) considers the dynamic provision of public goods with a binary contribution in a large population, finding that although a contribution delay occurs in a finite population, such a delay disappears for an infinite population, although the equilibrium remains inefficient. Yildirim (2006) also considers the dynamic provision of public goods with a binary contribution. In Yildirim's setting, different from our setting, each player decides whether to contribute a public good in each period and if a sufficient number of public goods are provided, players are rewarded. In contrast to our result, in that setting, an equilibrium in which no one contributes is not excluded. Bag and Roy (2008) consider the dynamic provision of public goods with a continuous contribution under incomplete information and verify the effect of announcement.

While the above studies consider repeated forms of simultaneous games, Bergstrom (2012a) considers provision by randomly arriving players, while Admati and Perry (1991) deal with continuous alternative contributions in the two-person case, showing that a Pareto-efficient allocation is not completed without commitment under certain conditions. Bag and Roy (2011) consider the world of incomplete information in the setting that the order of contribution is exogenously given.

Bagnoli and Lipman (1989) consider a dynamic mechanism to provide multiple discrete public goods where in each stage, players contribute to a public good and if a threshold level is achieved, a public good is provided. They show that the mechanism implements core allocations.

In a recent study, Iijima and Kasahara (2016) consider a general model of a continuous time dynamic contribution game with complete information and stochastic noise that allows a gradual adjustment to the contribution and show the uniqueness of a Nash equilibrium. This study relates to ours by showing that multiple equilibria in a static game disappear in a dynamic extension.

We also study threshold uncertainty in line with Nitzan and Romano (1990), McBride (2006), and Barbieri and Malueg (2010), who analyze the problem of providing discrete public goods under such uncertainty. Nitzan and Romano (1990) and McBride (2006) assume a common value and cost for public goods, while Barbieri and Malueg (2010) consider a private-value model of a subscription game (contribution is refunded) and show the existence and uniqueness of the equilibrium.

2.2 Model

There are $n \in \mathbb{N}$ players who consider whether to contribute a public good. If $h \in \{0,, \ldots, n\}$ players contribute, the public good is provided.

Let $N = \{1, 2, \ldots, n\}$ be the set of players. Player $i \in N$ chooses whether to contribute; if he contributes, he incurs contribution cost c_i. Costs have a continuous distribution. We assume that each player's cost is private information and i.i.d. The cost distribution function has density g such that $g(c_i) > 0,\ \forall c_i \in (\underline{C}, \overline{C}), \underline{C} \geqslant 0, \overline{C} \in (0, \infty)$.

The utility of player i from the provided public good is v_i, where v_i is the valuation of the policy and density is $f(v_i) > 0, \forall v_i \in (\underline{V}, \overline{V})$, where $\underline{V} \leqslant 0$ and $\overline{V} > 0$. The distribution of each player's valuation is i.i.d. The valuation and cost of each player are allowed to be correlated. For each cost c, let the conditional density function of valuation v be $f(v \mid c)$ and $f(v \mid c) > 0$ for all v, c.

2.3 Static Model

This section considers only the symmetric BNE. First, we assume $\underline{C} < \overline{V}$ since if $\underline{C} \geqslant \overline{V}$, no one contributes. For each v, c, interim expected utility is

$$q(v,c)v\Psi(h-1) + (1-q(v,c))v\Psi(h) - cq(v,c) = q(v,c)[\{\Psi(h-1) - \Psi(h)\}v - c] + v\Psi(h),$$

where $q(v,c)$ is an ex-ante strategy that if the citizen observes his type v, c, he complains with a probability of $q(v,c)$. $\Psi(h)$ is the probability that the number of contributions by others is no more than h and is defined as[8]

$$\Psi(h) = \sum_{k=h}^{n-1} \left[\binom{n-1}{k} \left(\Pr(q(v,c)=1)\right)^k \left(\Pr(q(v,c)=0)\right)^{n-1-k} \right].$$

Then, his best response is

$$\begin{aligned} q(v,c) = 1 \ \text{ if } \frac{v}{c}[\Psi(h-1) - \Psi(h)] \geqslant 1, \\ q(v,c) = 0 \ \text{ if } \frac{v}{c}[\Psi(h-1) - \Psi(h)] \leqslant 1. \end{aligned}$$

This strategy is called the *cutoff strategy*. We show that this strategy profile is an equilibrium. We set $y_{n,h}$ to satisfy $y_{n,h} = 1/[\Psi(h-1) - \Psi(h)]$. This value is called the *cutoff point*. If $v/c < y_{n,h}$, then $q(v,c) = 0$, and if $v/c > y_{n,h}$, then $q(v,c) = 1$. Firstly, we assume the existence of the cutoff point. Then,

$$\Psi(h-1) - \Psi(h) = \left[\binom{n-1}{h-1} \left(\int_{y_{n,h} < v/c} f(v \mid c) g(c) \, dv \, dc \right)^{h-1} \left(\int_{y_{n,h} > v/c} f(v \mid c) g(c) \, dv \, dc \right)^{n-h} \right].$$

[8] The term $\binom{n}{r}$ denotes the binomial coefficient, that is $\binom{n}{r} = n!/[(n-r)!r!]$.

By putting $P(y_{n,h}) = \int_{y_{n,h}<v/c} f(v \mid c)g(c)\, dv\, dc$, the above equation becomes

$$\Psi(h-1) - \Psi(h) = \frac{(n-1)!}{(h-1)!(n-h)!}[P(y_{n,h})]^{h-1}[1-P(y_{n,h})]^{n-h}.$$

Therefore, the best response is

$$q(v,c) = 1 \ \text{ if } \frac{v}{c} > \frac{(h-1)!(n-h)!}{(n-1)!}\frac{1}{[P(y_{n,h})]^{h-1}[1-P(y_{n,h})]^{n-h}},$$
$$q(v,c) = 0 \ \text{ if } \frac{v}{c} < \frac{(h-1)!(n-h)!}{(n-1)!}\frac{1}{[P(y_{n,h})]^{h-1}[1-P(y_{n,h})]^{n-h}}.$$

That is, $y_{n,h}$ is a solution for $y_{n,h}\frac{(n-1)!}{(h-1)!(n-h)!}[P(y_{n,h})]^{h-1}[1-P(y_{n,h})]^{n-h} = 1$. Define function $\Gamma_{n,h}$ as

$$\Gamma_{n,h}(y) := \frac{1}{[P(y)]^{h-1}[1-P(y)]^{n-h}}\frac{(h-1)!(n-h)!}{(n-1)!}. \tag{2.1}$$

Then, cutoff point $y_{n,h}$ is a fixed point of $\Gamma_{n,h}$. If a fixed point exists, a cutoff point that satisfies the equilibrium condition exists. Therefore, the cutoff strategy is an equilibrium.

2.3.1 Large Population

If $h > 1$, the act of no player contributing is a trivial symmetric equilibrium. Consider the case of no fixed point of $\Gamma_{n,h}$. Then, $\Gamma_{n,h}(y') > y, \forall y' \in [W, \overline{V}/\underline{C}]$, where $W = \min\{0, \underline{V}/\overline{C}\}$. Hence, when there is no fixed point of $\Gamma_{n,h}$, no one contributes. The following lemma proves the sufficient condition that the act of no player contributing is the unique equilibrium.

Lemma 2.1. *If the following inequality is established, the act of no player contributing in any symmetric BNE:*

$$\frac{\overline{V}}{\underline{C}} < \frac{(n-1)^{n-1}}{(h-1)^{h-1}(n-h)^{n-h}}\frac{(h-1)!(n-h)!}{(n-1)!}. \tag{2.2}$$

We consider the case that $\underline{C} > 0$ and $\overline{V} < \infty$.

Firstly, we characterize the condition that $\Gamma_{n,h}$ has a fixed point. At the threshold, (2.2) is almost a necessary and sufficient condition that ensures that $\Gamma_{n,h}$ has no fixed point.

Lemma 2.2. *Suppose that $h \in \mathbb{N}$. Then, $\Gamma_{n,h}$ has a fixed point with sufficiently large n if*

$$\frac{\overline{V}}{\underline{C}} > e^{h-1}\frac{(h-2)!}{(h-1)^{h-2}}.$$

On the contrary, if

$$\frac{\overline{V}}{\underline{C}} < e^{h-1}\frac{(h-2)!}{(h-1)^{h-2}},$$

there exists $\bar{n} \in \mathbb{N}$ such that for each $n \geqslant \bar{n}$, $\Gamma_{n,h}$ has no fixed point.

If a fixed point of $\Gamma_{n,h}$, namely $y_{n,h}$ exists,

$$y_{n,h} = \frac{1}{[P(y_{n,h})]^{h-1}[1-P(y_{n,h})]^{n-h}}\frac{(h-1)!(n-h)!}{(n-1)!}. \tag{2.3}$$

This equation characterizes the probability that a given player contributes. In any equilibrium, we can show that $\sup_n nP(y_{n,h}) < \infty$. Then, we have $P(y_{n,h}) \to 0$ and $y_{n,h} \to \overline{V}/\underline{C}$. We also find that there exists a subsequence that $nP(y_{n,h})$ converges to a real number. Suppose that $nP(y_{n,h}) \to x$, then $[1-P(y_{n,h})]^n \to 1/e^x$. Therefore, the equilibrium condition becomes

$$\frac{\overline{V}}{\underline{C}} = \frac{(h-1)!e^x}{x^{h-1}}. \tag{2.4}$$

Therefore, the solution of (2.4) characterizes the convergence of $nP(y_{n,h})$. We now have

Proposition 2.1. *Suppose that h is fixed, $\overline{V} < \infty$ and $\underline{C} > 0$.*

(i) Suppose that

$$\frac{\overline{V}}{\underline{C}} > e^{h-1}\frac{(h-2)!}{(h-1)^{h-2}}.$$

Then, in a sequence of symmetric BNEs where a given player contributes with a positive probability, an x solves (2.4) *such that the probability that a given player contributes converges to x. Moreover, the distribution of the number of contributors converges to the Poisson distribution with parameter x.*

Conversely, for each solution to (2.4)*, namely x, there exists a sequence of symmetric BNEs such that the distribution of the number of contributors converges to a Poisson distribution with parameter x. In particular, if $h \geqslant 2$, the number of solutions to* (2.4) *is two.*

(ii) Suppose that

$$\frac{\overline{V}}{\underline{C}} < e^{h-1}\frac{(h-2)!}{(h-1)^{h-2}}.$$

The unique symmetric BNE is the act of no player contributing.

Since the number of contributors follows a Poisson distribution, the probability that a public good is provided converges to $1 - \sum_{\ell=0}^{h-1} \frac{x^\ell}{\ell!} e^{-x}$.

Note that if h is sufficiently large, $\sum_{\ell=0}^{h-1} \frac{x^\ell}{\ell!} \to e^x$ and the probability that the number of contributors is less than h goes to one. In other words, the probability that the public good is provided goes to zero.

When $h = 1$, this problem reduces to the best-shot public good. In this case, in contrast to the case of $h \geqslant 2$, the act of no player contributing is not an equilibrium when $\overline{V} > \underline{C}$.

Corollary 2.1 (Harrington (2001); Xu (2001); Bergstrom (2012b))**.** *Suppose that* $h = 1$, $\overline{V} < \infty$ *and* $\underline{C} > 0$.

(i) If $\overline{V} > \underline{C}$, *in the symmetric BNE, the probability that the public good is provided converges to* $1 - \underline{C}/\overline{V}$.

(ii) If $\overline{V} < \underline{C}$, *the unique symmetric BNE is the act of no player contributing.*

2.3.2 Unbounded Valuation

This section considers the case that $\overline{V} = \infty$ or $\underline{C} = 0$. We show that the probability that the public good is provided converges to 1 or 0. First, we consider the case that $h = 1$. In this case, we obtain

Corollary 2.2. *Suppose that* $h = 1$ *and* $\overline{V} = \infty$ *or* $\underline{C} = 0$ *hold. In the symmetric BNE, the probability that the public good is provided converges to* 1.

The probability that the public good is provided converges to 1 or 0.

Corollary 2.3. *Suppose that* $\overline{V} = \infty$ *or* $\underline{C} = 0$ *hold. In the symmetric BNE, the probability that the public good is provided converges to* 1 *or* 0.

If the valuation per unit cost is unbounded, the equilibrium in which a given player contributes with a positive probability may be unique.

Example 2.1. Suppose that $h = 2$, $g(c)$, and $f(v \mid c)$ are uniformly distributed on $(0, 1)$.

Then, the probability that a given player contributes is

$$P(y_{n,2}) = \int_0^{1/y_{n,2}} \int_{c y_{n,2}}^{1} dv\, dc = \int_0^{1/y_{n,2}} (1 - c y_{n,2})\, dc = \frac{1}{2 y_{n,2}}.$$

From the equilibrium condition and the above equation, the probability that a given player does not contribute is

$$1 - P(y_{n,2}) = \left(\frac{2}{n-1}\right)^{\frac{1}{n-2}}.$$

Thus, an equilibrium exists and $y_{n,2} \to \infty$. Rearranging the equilibrium condition yields

$$[1 - P(y_{n,2})]^n = \frac{[1 - P(y_{n,2})]^2}{(n-1) P(y_{n,2}) y_{n,2}} = \frac{[1 - P(y_{n,2})]^2}{(n-1)/2}.$$

The LHS of the above equation is the probability that no one contributes. When the population diverges to infinity, according to the above equation, we find that $\lim_{n\to\infty}[1 - P(y_{n,2})]^n = 0$.

On the contrary, rearranging the equilibrium condition also yields

$$(n-1)[1 - P(y_{n,2})]^{n-2} P(y_{n,2}) = \frac{1}{y_{n,2}}.$$

By rearranging the above equation, we obtain

$$n[1 - P(y_{n,2})]^{n-1} P(y_{n,2}) = \frac{n}{n-1} \frac{1 - P(y_{n,2})}{y_{n,2}}. \tag{2.5}$$

The LHS of the above equation is the probability that only one player contributes, which, if the population diverges to infinity, converges to 0.

Hence, the probability that at least two players contribute is one. Therefore, the probability that the public good is provided converges to one when the population diverges to infinity. △

Although this example shows that the probability that the public good is provided converges to one, this is not the general case, even when $h = 2$. The following example shows the case that the probability that the public good is provided converges to zero.

Example 2.2. Suppose that $h = 2$, $g(c) = c$ on $[0, \sqrt{2}]$, and $f(v \mid c)$ is a uniform distribution on $[0, 1]$.

The probability that a given player contributes, that is, $P(y_{n,2})$, is calculated as

$$P(y_{n,2}) = \int_0^{1/y_{n,2}} \int_{cy_{n,2}}^{1} dvc\, dc = \int_0^{1/y_{n,2}} (c - c^2 y_{n,2})\, dc = \frac{1}{6[y_{n,2}]^2}. \tag{2.6}$$

From the equilibrium condition, we obtain

$$[1 - P(y_{n,2})]^{n-2} = \frac{6y_{n,2}}{(n-1)}. \tag{2.7}$$

By using these equations, we obtain

$$\left(1 - \frac{1}{6[y_{n,2}]^2}\right)^{n-2} = \frac{6y_{n,2}}{(n-1)}. \tag{2.8}$$

Let $y_{n,2} = \varepsilon_n(n-1)/6$ for the positive real number ε_n. Then, by rearranging (2.8), we have

$$T_n(\varepsilon_n) := \varepsilon_n^2(1 - \varepsilon_n^{1/(n-2)}) = \frac{6}{(n-1)^2}. \tag{2.9}$$

To show the existence of $y_{n,2}$ that satisfies (2.8), it is sufficient to show that (2.9) has a solution. Note that when $T_n(1) = T_n(0) = 0$. Moreover,

$$T_n'(\varepsilon) = 2\varepsilon(1 - \varepsilon^{1/(n-2)}) - \frac{1}{n-2}\varepsilon^{1+1/(n-2)}.$$

Therefore, for $\varepsilon \in (0, 1)$, according to the Taylor expansion, $T_n(\varepsilon) = -(1-\varepsilon)[2\varepsilon'(1 - (\varepsilon')^{1/(n-2)}) - \frac{1}{n-1}\varepsilon^{1+1/(n-2)}]$ for some $\varepsilon' \in (\varepsilon, 1)$. We now consider $A_n(\varepsilon) := T_n(\varepsilon)(n-1)^2$. Then,

$$A_n(\varepsilon) = -(1-\varepsilon)\left[2(\varepsilon')(1 - (\varepsilon')^{1/(n-2)})(n-1)^2 - \frac{n-1}{n-2}(n-1)(\varepsilon')^{1+1/(n-2)}\right].$$

From the Taylor expansion, $(\varepsilon')^{1/(n-2)} = 1 - (1-\varepsilon')/(1-n)[\varepsilon'']^{1/(n-2)-1}$, where $\varepsilon'' \in (\varepsilon', 1)$.

$$A_n(\varepsilon)/(n-1) = -(1-\varepsilon)\varepsilon'\left[2[\varepsilon'']^{1/(n-2)}\left(1 - \frac{\varepsilon'}{\varepsilon''}\right) - (\varepsilon')^{1/(n-2)}\right].$$

Since both $[\varepsilon']^{1/(n-2)}$ and $[\varepsilon'']^{1/(n-2)}$ converge to 1 as $n \to \infty$, $A_n(\varepsilon)/(n-1)$ is positive with ε sufficiently near 1. Thus, $A_n/(n-1)$ does not converge to 0. Therefore, $A_n(\varepsilon) > 6$ for sufficiently large n. This finding implies that for sufficiently large n, $T_n(\varepsilon) > 6/(n-1)^2 > T_n(1)$. Hence, a point in $(\varepsilon, 1)$ that satisfies (2.9) exists.

This finding also implies that there exists $y_{n,2}$ that satisfies (2.8) and $y_{n,2} = \frac{\varepsilon}{6(n-1)}$ for some $\varepsilon \in (0, 1)$. We now calculate $\lim_{n\to\infty}[1 - P(y_{n,2})]^n$. Since $P(y_{n,2}) = \frac{6}{(n-1)^2}$, $nP(y_{n,2}) \to 0$. Therefore, $\lim_{n\to\infty}[1 - P(y_{n,2})]^n = 1$. That is, the probability that no one contributes converges to one. Hence, we conclude that there is an equilibrium such that the probability that the public good is provided converges to zero. △

2.3.3 Comparative Statics

Based on the case of a finite number of players, Xu (2002) discusses the effect of increases at the threshold, stating that the threshold effect can be either positive or negative. Indeed, Xu shows an example that has two equilibria with the following properties. In one equilibrium, the probabilities that both a public good is provided and a given player contributes decrease when the threshold increases. In another equilibrium, however, the probabilities that both a public good is provided and a given player contributes increase when the threshold increases. In Xu's example, the number of players is three. This section thus considers the threshold effect with a large number of players.

If $h \geqslant 2$ and

$$\frac{\overline{V}}{\underline{C}} > e^{h-1}\frac{(h-2)!}{(h-1)^{h-2}}$$

holds, as shown in Proposition 2.1, there exist three equilibria with sufficiently large n. Let $\bar{y} = \overline{V}/\underline{C}$. By taking the logarithm of (2.4), we have

$$(h-1)\ln x_h - x_h = \ln(h-1)! - \ln \bar{y} \tag{2.10}$$

for some h. Let $T_h(x) = (h-1)\ln x - x$ and $S(h) = \ln(h-1)! - \ln \bar{y}$. Note that T_h is a concave single peaked function of x and is maximized when $x = h - 1$. If $h \geqslant 2$, according to Proposition 2.1, the number of x_h that satisfies (2.10) is two. Let x_h^L and x_h^H be the two solutions and $x_h^L < x_h^H$. Then, we find that $x_h^L < h - 1 < x_h^H$. Note that x is the expected number of contributors in the equilibrium. We call the equilibrium in which the expected number of contributors converges to x_h^H (resp. x_h^L) the *high contributing equilibrium* (resp. *low contributing equilibrium*). When $h = 1$, since $x_1 > 0 = h - 1$, there is no low contributing equilibrium.

Increases at the threshold

Note that $S(h) - S(h-1) = \ln(h-1)$ and $T_h(x) - T_{h-1}(x) = \ln x$. Therefore, $S(h) - S(h-1) > T_h(x) - T_{h-1}(x)$ if and only if $h - 1 > x$. Since $T_h(x_h^L) = S(h)$ and $x_h^L < h-1$, $T_{h-1}(x_h^L) > S(h-1)$. In the same way, we have $T_{h-1}(x_h^H) < S(h-1)$. Since $T_{h-1}(x)$ is decreasing when $x > h-1$ and x_{h-1}^H is the greatest solution of $T_{h-1}(x) = S(h-1)$, $x_{h-1}^H < x_h^H$. Since $x_h^L < x_h^H$, $T_{h-1}(x_h^L) > S(h-1)$ and $T_{h-1}(x)$ is increasing when $x < h-1$, we have $x_{h-1}^L < x_h^L$. Therefore, for each $i \in \{1, 2\}$, x_h^i is increasing in h.

Proposition 2.2. *In each equilibrium, the expected number of contributors is increasing at the threshold level.*

Consider the probability that the public good is provided. We focus on the high contributing equilibrium, in which the expected number of contributors is x_h^H.

Proposition 2.3. *Consider the high contributing equilibrium. Then, the probability that the public good is provided is decreasing at the threshold level.*

Increasing in the maximum valuation

This section considers the case of increasing in $\bar{y}$. Let $S_h(\bar{y}) = \ln(h-1)! - \ln \bar{y}$. Note that $S_h(\bar{y})$ is decreasing in $\bar{y}$. Let $x_h^L(\bar{y})$ and $x_h^H(\bar{y})$ be the numbers that satisfy $T_h(x_h^L(\bar{y})) = S_h(\bar{y}) = T_h(x_h^H(\bar{y}))$ and $x_h^L(\bar{y}) < x_h^H(\bar{y})$. Since T_h is concave, $T_h(x)$ is increasing when $x \leqslant h-1$ and decreasing when $x \geqslant h-1$. Let $y > \bar{y}$. Note that $T_h(x_h^L(\bar{y})) > S_h(y)$. Since $x_h^1(\bar{y}) \leqslant h-1$ and $T_h(x)$ is increasing, $x_h^L(y) < x_h^L(\bar{y})$. In the same way, we find that $x_h^H(y) > x_h^H(\bar{y})$.

2.4 Dynamic Provision Model

This section considers the case that campaigners provoke players to contribute by announcing the number of players who have already contributed. First, we consider a two-stage game. In the first stage, players decide whether to contribute. In the second stage, with this knowledge about the number of previous contributors, the players who did not contribute in the first stage decide whether to contribute. If the threshold number of players contribute, the public good is provided, otherwise not. In the previous section, if $h \geqslant 2$, the act of no player contributing was a trivial equilibrium. However, in the dynamic model, the act of no player contributing is no longer an equilibrium.

Consider the case that $h = 2$ and a two-stage game. We show that the strategy profile that no one contributes in the equilibrium path is not an equilibrium. Suppose, by contradiction, that the act of no player contributing in either stage of equilibrium path is an equilibrium. Consider the behavior of players in the second stage. Suppose that a player contributes in the first stage. Then, in the second stage, if at least one player contributes, the public good is provided. Therefore, this is the same situation as the static version of our game analyzed in the previous section. According to Corollary 2.1, if $\overline{V}$ is sufficiently large, the strategy profile that no one contributes is not an equilibrium. Thus, if one player contributes in the first stage, the public good is provided with a positive probability. When $\overline{V}$ is sufficiently large, the probability approaches 1. That is, in the second stage, there is an incentive to contribute. Thus, when $\overline{V}$ is sufficiently large, in the first stage, one player's contribution is sufficient to provide the public good. This is also the same situation as in the static game when $h = 1$; thus, the act of no player contributing is not an equilibrium.

The above discussion does not apply when $h = 3$ in our two-stage game, however. Consider the behavior in the second stage. If two players contribute in the first stage, as in the above discussion, the act of no player contributing is not an equilibrium. However, if the number of contributors in the first stage is less than one, the act of no player contributing is still an equilibrium. Therefore, even if a player contributes in the first stage, the probability that the public good is provided is 0. Thus, no player has an incentive to deviate from the strategy profile that no one contributes in the first stage. Therefore, the number of contributors in the first stage is 0; thus, the act of no player contributing in the second stage is still an equilibrium.

This observation predicts that a player contributes with a positive probability in any equilibrium path if and only if the number of stages is greater than or equal to the threshold, h.

We now formalize the above discussion for a finite stage game. Our T-stage game is described as follows. Each player decides whether to contribute in each stage. If the number of contributors is no less than h in stage T, the public good is provided in stage T. In each stage, each player is informed of the number of contributors in the previous stages. Each player's contribution is permanent and he can contribute only once. Let $\delta \in (0, 1)$ be a discount factor common to all players. Suppose T and h are finite numbers such that $T \geqslant h$. We first note that there exists a symmetric perfect Bayesian equilibrium (PBE).

Proposition 2.4. *Consider the T-stage model. Then, there exists a symmetric PBE.*

We calculate the probability that the public good is provided. We prove the following proposition by using mathematical induction.

Proposition 2.5. *Consider the T-stage game. When $T \geqslant h$, in any symmetric PBE, the limiting probability that the public good is provided is greater than* $1 - h\frac{\underline{C}}{\delta^{T-1}\overline{V}}$.

Note that in the low contributing equilibrium of the static case, the probability that the public good is provided is $1 - \sum_{\ell=0}^{h-1} \frac{(x)^\ell}{\ell!} e^{-x}$, and $x < h-1$. Therefore, regardless of the value of $\underline{C}/\overline{V}$, the probability that the public good is provided is strictly less than 1. Indeed, Example 4.2 shows an equilibrium where $\underline{C} = 0$ and the probability that the public good is provided converges to 0. However, in the dynamic case, if $\underline{C}/\overline{V}$ falls, the probability that the public good is provided arbitrarily approaches 1.

On the contrary, when $T < h$, the act of no player contributing in either stage is still an equilibrium.

Proposition 2.6. *Consider the T-stage game. When $T < h$, there is a symmetric PBE where no one contributes on the equilibrium path.*

2.5 Threshold Uncertainty

The previous sections considered the case that if the threshold number of players contribute, the public good is necessarily provided. However, in the real world, this is not always true. In this section, we consider the case of an uncertain threshold. In this regard, we slightly generalize our model of simultaneous moves. For each $\ell \in \mathbb{N}$, let $\Omega(\ell)$ be the probability that the public good is provided when the number of contributors is ℓ. We suppose that Ω is nondecreasing in the number of players who contribute and there exists $\bar{n}$ such that for each $n \in \mathbb{N}$, $\Omega(n) \leqslant \Omega(\bar{n})$. Let $\psi_n(\ell)$ be the probability that only ℓ players contribute. Then, the best response is

$$q(v, c) = 1 \text{ if } \frac{v}{c} \geqslant \frac{1}{\sum_{\ell=1}^{\bar{n}} \psi(\ell-1)[\Omega(\ell) - \Omega(\ell-1)]},$$
$$q(v, c) = 0 \text{ if } \frac{v}{c} \leqslant \frac{1}{\sum_{\ell=1}^{\bar{n}} \psi(\ell-1)[\Omega(\ell) - \Omega(\ell-1)]}.$$

As in the basic model, as the population goes to infinity, the proportion of players who contribute converges to 0. Then, as the population goes to infinity, from the Poisson law of small numbers, there exists x such that

$$\frac{\underline{C}}{\overline{V}} e^x = \sum_{\ell=1}^{\bar{n}} \frac{x^{\ell-1}}{(\ell-1)!}[\Omega(\ell) - \Omega(\ell-1)]$$

and x is the expected number of contributors at the limit.

Proposition 2.7. *There is an equilibrium in which no one contributes for sufficiently large population if and only if*

$$\frac{1}{\Omega(1) - \Omega(0)} \geqslant \frac{\overline{V}}{\underline{C}}.$$

There is an equilibrium in which a given player contributes with a positive probability if and only if there is an x such that

$$\frac{\overline{V}}{\underline{C}} = \frac{1}{\sum_{\ell=1}^{\bar{n}} \frac{x^{\ell-1}}{(\ell-1)!} e^{-x}[\Omega(\ell) - \Omega(\ell-1)]}.$$

Moreover, the distribution of the number of contributors converges to a Poisson distribution with parameter x.

2.6 Concluding Remarks

This study analyzes a private provision of a discrete public good model and calculates the asymptotic probability that the public good is provided, which depends on the threshold, maximum valuation of the public good, and minimum contribution cost among players. We show the monotonicity of the expected number of contributors at the threshold. We also find that the probability that the public good is provided decreases as the threshold increases. When the threshold number is above two, there is an equilibrium in which no one contributes. However, if several opportunities to make a contribution exist, and the number of opportunities is above the threshold level, we can calculate a lower bound of the probability that the public good is provided that larger than zero.

In our model, we assume that each player knows preferences distribution, whereas it may not be adequate since if there are sufficiently many population, by the law of large number, the government knows players' preferences. In this case, without using such systems, the government can deal with the issue. Considering uncertainty in players' preference distribution is left for the future works.

Appendix

2.A Omitted Proofs

Proof of Lemma 2.1. To consider the case of $h \neq 1$, we verify whether the equilibrium in which a player contributes exists by using the shape of function $\Gamma_{n,h}$. Since $P(\overline{V}/\underline{C}) = 0$, we find that $\lim_{y \to \overline{V}/\underline{C}} \Gamma_{n,h}(y) = \infty$. Differentiating $\Gamma_{n,h}(y)$ by y yields

$$\frac{\mathrm{d}\Gamma_{n,h}(y)}{\mathrm{d}y} = \frac{[P(y)]^{h-2}[1-P(y)]^{n-h-1}P'(y)[(n-1)P(y)] - (h-1)}{([1-P(y)]^{n-h}[P(y)]^{h-1})^2} \frac{(h-1)!(n-h)!}{(n-1)!}.$$

Since $P(y)$ is decreasing, if $(n-1)P(y) - (h-1)$ is positive, $\Gamma_{n,h}(y)$ is increasing, while if it is negative, $\Gamma_{n,h}(y)$ is decreasing. From $P(\overline{V}/\underline{C}) = 0$ and $h \geqslant 1$, $\Gamma_{n,h}(y)$ is single-dipped. The abscissa of the vertex of graph $\Gamma_{n,h}$ is y, which satisfies $(n-1)P(y) - (h-1) = 0$. Then, the coordinate of the vertex is

$$(y, \Gamma_{n,h}(y)) = \left(P^{-1}\left(\frac{n-h}{n-1}\right), \frac{[(h-1)!(n-h)!]/[(n-1)!]}{[(h-1)/(n-1)]^{h-1}[(n-h)/(n-1)]^{n-h}}.\right)$$

A larger h makes a larger abscissa $P^{-1}(h-1/n-1)$.[9] We thus establish a sufficient condition that no one contributes in the equilibrium.

If inequality (2.2) holds, for any y, y',

$$y < \overline{V}/\underline{C} < \Gamma_{n,h}(y'). \tag{2.11}$$

According to the best response, $q(v,c) = 0$ is the dominant strategy. Therefore, no one contributes in the equilibrium. □

[9] Palfrey and Rosenthal (1984) show this property under complete information with a mixed strategy.

Proof of Lemma 2.2. Recall that for each y,

$$\Gamma_{n,h}(y) \geqslant \frac{(n-1)^{n-1}}{(h-1)^{h-1}(n-h)^{n-h}} \frac{(h-1)!(n-h)!}{(n-1)!}.$$

Let

$$A(n,h) := \frac{(n-1)^{n-1}}{(h-1)^{h-1}(n-h)^{n-h}} \frac{(h-1)!(n-h)!}{(n-1)!}.$$

Computing the ratio between $A(n,h)$ and $A(n,h-1)$ yields

$$\frac{A(n,h)}{A(n,h-1)} = \frac{(h-2)^{h-2}(n-h+1)^{n-h+1}}{(h-1)^{h-1}(n-h)^{n-h}} \frac{(h-1)!(n-h)!}{(h-2)!(n-h+1)!} = \left(\frac{h-2}{h-1}\right)^{h-2} \left(\frac{n-h+1}{n-h}\right)^{n-h}.$$

Then, taking a limit of it yields

$$\lim_{n\to\infty} \frac{A(n,h)}{A(n,h-1)} = \left(\frac{h-2}{h-1}\right)^{h-2} e.$$

Note that $A(n,1) = 1$ for each n. Then, $\lim_{n\to\infty} A(n,2) = e$. By induction, for each h, $A(n,h)$ converges to $e^{h-1}\frac{(h-2)!}{(h-1)^{h-2}}$. Let $A(h)$ be the convergent.

Suppose that $\overline{V}/\underline{C} < A(h)$; then, with sufficiently large n, $\Gamma_{n,h} \geqslant A(h)$ and thus, there is no fixed point with a sufficiently large population.

Suppose that $\overline{V}/\underline{C} > A(h)$. Recall that $\Gamma_{n,h}$ is minimized at $P^{-1}((n-h)/(n-1))$. Note that $\lim_{n\to\infty} \Gamma_{n,h}(P^{-1}((n-h)/(n-1))) = A(h)$, and $P^{-1}((n-h)/(n-1)) \to \overline{V}/\underline{C}$. Note also that $\lim_{n\to\infty} \Gamma_{n,h}(0) = \infty$. Thus, the intermediate value theorem implies that a fixed point of $\Gamma_{n,h}$ exists with sufficiently large n. □

Proof of Proposition 2.1. (i) Rearranging the equilibrium condition yields

$$(1-P(y_{n,h}))^{n-h} = \frac{1}{y_{n,h}[P(y_{n,h})]^{h-1}} \frac{(h-1)!(n-h)!}{(n-1)!}.$$

Suppose that each $\Gamma_{n,h}$ has a fixed point. Without loss of generality, we assume that $P(y_{n,h})(n-1) \to x \in \mathbb{R}_+ \cup \{\infty\}$. Then, $(1-P(y_{n,h}))^{n-h} \to 1/e^x$ and

$$\frac{1}{y_{n,h}[P(y_{n,h})]^{h-1}} \frac{(h-1)!(n-h)!}{(n-1)!} \to \frac{(h-1)!}{\overline{y}} \frac{1}{x^{h-1}}.$$

Thus, x satisfies

$$\frac{1}{e^x} = \frac{(h-1)!}{\overline{y}} \frac{1}{x^{h-1}}.$$

By taking the logarithm, we have

$$(h-1)\ln x - x = \ln(h-1)! - \ln \overline{y}.$$

Suppose that $x = 0$. Then, the LHS diverges to $-\infty$, whereas the RHS is finite (i.e., a contradiction). Suppose that $x = \infty$. Then, the LHS diverges to $-\infty$, which, again, is a contradiction. Thus, x is a positive real number, which implies that $\{P(y_n)(n-1)\}_n$ is a bounded sequence. Therefore, it has a convergent. Without loss of generality, we assume that $\{P(y_n)(n-1)\}_n$ converges. Let $T(x) := (h-1)\ln x - x$, which is concave and maximized at $x = h-1$. Note that $\lim_{x\to 0} T(x) = \lim_{x\to\infty} T(x) = -\infty$. Therefore, if $(h-1)\ln(h-1) - (h-1) > \ln(h-1)! - \ln \overline{y}$, equation $T(x) = \ln(h-1)! - \ln \overline{y}$ has exactly two solutions when $h > 1$ and has exactly one solution when $h = 1$. Rearranging the condition yields

$$\overline{y} > e^{h-1} \frac{(h-2)!}{(h-1)^{h-2}}.$$

This condition is always satisfied when $\Gamma_{n,h}$ has a fixed point with sufficiently large n. We now calculate the probability that the public good is provided. The probability that ℓ players contribute to the public good is

$$\frac{n!}{\ell!(n-\ell)!}[P(y_{n,h})]^{\ell}[1 - P(y_{n,h})]^{n-\ell}.$$

Since $(n-1)P(y_{n,h}) \to x$, the probability that the number of contributors is ℓ converges to

$$\lim_{n\to\infty} \frac{n!}{\ell!(n-\ell)!}[P(y_{n,h})]^{\ell}[1 - P(y_{n,h})]^{n-\ell} = \frac{x^{\ell}}{\ell!} \frac{1}{e^x}.$$

To show the latter part of (i), let x be the solution of (2.4). Take an $\varepsilon > 0$ sufficiently small such that

$$T(x+\varepsilon) > \ln(h-1)! - \ln \overline{y} > T(x-\varepsilon) \text{ or}$$
$$T(x+\varepsilon) < \ln(h-1)! - \ln \overline{y} < T(x-\varepsilon).$$

Define $\overline{y}_n$ and $\underline{y}_n$ such that

$$n\Pr(\overline{y}_n \leqslant v/c) = x+\varepsilon$$
$$n\Pr(\underline{y}_n \leqslant v/c) = x-\varepsilon$$

According to the definition of T and given that both $\overline{y}_n$ and $\underline{y}_n$ converge to $\overline{y}$ with sufficiently large n, we have either of the following:

$$\Gamma_{n,h}(\overline{y}_n) > \overline{y}_n \text{ and } \Gamma_{n,h}(\underline{y}_n) < \underline{y}_n \text{ or}$$
$$\Gamma_{n,h}(\overline{y}_n) < \overline{y}_n \text{ and } \Gamma_{n,h}(\underline{y}_n) > \underline{y}_n.$$

Therefore, according to the intermediate theorem, fixed point $y_n^* \in (\overline{y}_n, \underline{y}_n)$ exists. From this definition, with sufficiently large n, $nP(y_n^*) \in (x-\varepsilon, x+\varepsilon)$. Then, letting $\varepsilon \to 0$ implies that $nP(y_n^*) \to x$, which concludes the proof.

(ii) This case is a corollary of Lemma 2.2. □

Proof of Corollary 2.3. Since the case for $h = 1$ is shown in Corollary 2.2, consider the case for $h \geqslant 2$. As in the case of $\overline{V} < \infty$ or $\underline{C} > 0$, the fixed point of $\Gamma_{n,h}$ characterizes the equilibria. If $\Gamma_{n,h}$ has no fixed point, the probability that the public good is provided is 0. Let $A = \{n : \exists y \text{ s.t. } \Gamma_{n,h}(y) = y\}$. Suppose that $|A| < \infty$. This implies that there exists $\bar{n}$ such that for each $n > \bar{n}$ and $\Gamma_{n,h}(y) \neq y$ for each y.

Suppose that $|A| = \infty$. Note that $\{nP(y_{n,h})\}_{n\in A}$ has a convergent subsequence or subsequence that diverges to infinity. Without loss of generality, we assume that $\{nP(y_{n,h})\}_{n\in A}$ converges to $x \in \mathbb{R}_+ \cup \{\infty\}$. If $x = \infty$, the sequence diverges to infinity.

Suppose that $x < (0, \infty)$. Since $\{nP(y_{n,h})\}_{n\in A}$ converges, $P(y_{n,h}) \to 0$. Therefore, according to the

Poisson law of small numbers, $(1 - P(y_{n,h}))^{n-h} \to 1/e^x$. Therefore, from the definition of $y_{n,h}$,

$$\lim_{n\to\infty} y_{n,h} = \lim_{n\to\infty} \frac{1}{[P(y_{n,h})]^{h-1}[1 - P(y_{n,h})]^{n-h}} \frac{(h-1)!(n-h)!}{(n-1)!}$$
$$= \frac{e^x}{x^{h-1}}(h-1)!.$$

On the contrary, since $P(y_{n,h}) \to 0$, $y_{n,h} \to \overline{V}/\underline{C} = \infty$; thus, x cannot in $(0, \infty)$. Therefore, $x = 0$ or $x = \infty$. Note that the limit of the probability that the number of contributors is less than h is $\sum_{\ell=0}^{h-1} \frac{x^\ell}{\ell!} \frac{1}{e^x}$. If $x = 0$, $\sum_{\ell=0}^{h-1} \frac{x^\ell}{\ell!} \frac{1}{e^x} = 1$, which implies that the probability that the public good is provided converges to 0. If $x = \infty$, $\sum_{\ell=0}^{h-1} \frac{x^\ell}{\ell!} \frac{1}{e^x} = 0$, which implies that the probability that the public good is provided converges to 1. □

Proof of Proposition 2.3. Let π_h be the probability that the public good is provided when the threshold is h, that is,

$$\pi_h = 1 - \sum_{\ell=0}^{h-1} \frac{(x_h^H)^\ell}{\ell!} \frac{1}{e^{x_h^H}}.$$

Note that x_h^H satisfies $\frac{(h-1)! e^{x_h^H}}{(x_h^H)^{h-1}} = \bar{y}$. Therefore,

$$\pi_{h+1} - \pi_h = \frac{1}{\bar{y}} \Big[1 + (h-1)(x_h^H)^{-1} + (h-1)(h-2)(x_h^H)^{-2} + \cdots + (h-1)!(x_h^H)^{-h+1}$$
$$-1 - h(x_{h+1}^H)^{-1} - h(h-1)(x_{h+1}^H)^{-2} - \cdots - h!(x_{h+1}^H)^{-h+1} - h!(x_{h+1}^H)^{-h} \Big].$$

To prove $\pi_{h+1} < \pi_h$, we prepare

Lemma 2.3. *For each* $h \geqslant 1$, $(h-1)x_{h+1}^H < h x_h^H$.

Proof of Lemma 2.3. Note that when $h = 1$, this inequality is always satisfied. Consider the case of $h \geqslant 2$. To show this, consider $T_{h+1}((h/(h-1))x_h^H) - S(h+1)$. By using $T_h(x_h^H) - S(h) = 0$, this is calculated as

$$\begin{aligned} T_{h+1}((h/(h-1))x_h^H) - S(h+1) &= h\ln(h/(h-1))x_h^H - (h/(h-1))x_h^H - \ln h! + \ln \bar{y} \\ &= h\ln h - h\ln(h-1) + h\ln x_h^H - x_h^H - \frac{1}{h-1}x_h^H - \ln h! + \ln \bar{y} \\ &= (h-1)\ln h - h\ln(h-1) + \ln x_h^H - \ln(h-1) - \frac{1}{h-1}x_h^H \\ &= (h-1)\ln h - (h+1)\ln(h-1) + \ln x_h^H - \frac{1}{h-1}x_h^H. \end{aligned}$$

Since $x_h^H > h-1$, for each $h \geqslant 2$, $\ln x_h^H - \frac{1}{h-1}x_h^H < \ln(h-1) - \frac{1}{h-1}(h-1)$. Thus,

$$T_{h+1}((h/(h-1))x_h^H) - S(h+1) < (h-1)\ln h - h\ln(h-1) - 1.$$

Note that $(h-1)\ln h - h\ln(h-1)$ is decreasing in h. This is because

$$(h-1)\ln h - h\ln(h-1) - [h\ln(h+1) - (h+1)\ln h] = h(2\ln h - \ln(h+1) - \ln(h-1)] \leqslant 0.$$

Note also that $\ln 2 - 2\ln 1 - 1 = \ln 2 - 1 < 0$. Thus, $T_{h+1}((h/(h-1))x_h^H) - S(h+1)$. Note that since $x_h^H > h-1$, $(h/(h-1))x_h^H > h$. Therefore, $T_{h+1}((h/(h-1))x_h^H)$ is decreasing. Since $T_{h+1}(x_{h+1}^H) - S(h+1) = 0$, $x_{h+1}^H < (h/(h-1))x_h^H$. □

From Lemma 2.3, we have $(h-1)x_{h+1}^H < hx_h^H$. This also implies that for each $\ell < h$, $(\ell-1)x_{h+1}^H < \ell x_h^H$. This implies that $\pi_{h+1} < \pi_h$. □

Proof of Proposition 2.4. Let $\ell^\tau = (0, \ell_1, \ldots, \ell_{\tau-1})$ be the history of contributors in the previous periods before stage τ. Let $p_{n,\tau}(\ell^\tau)$ be the conditional probability that a given player contributes in stage τ when the history of contributors is ℓ^τ. Let $\rho = (p_{n,\tau}(\ell^\tau))_{\tau\in\{1,\ldots,T\},\ell_t\in\{\ell_{t-1},\ldots,n\}^\tau} \in [0,1]^M$ be given, where M is some integer.

Consider player i's behavior in stage τ. We consider the equilibrium such that if players are indifferent between contributing and not contributing, they do not contribute. Let $R_{n,\tau}(\ell)$ be the probability that the public good is provided when ℓ players contributed at the beginning of stage τ. Let $\psi_{n,\tau,\ell^\tau}(\ell - \ell_{\tau-1})$ be the probability that $\ell - \ell_{\tau-1}$ players contribute in stage τ when the history of the number of contributors is ℓ^τ. Then, player i's expected utility of contributing in stage τ is

$$\left[\sum_{\ell=\ell_{\tau-1}}^{h-1} \psi_{n,\tau,\ell^\tau}(\ell-\ell_{\tau-1})R_{n,\tau+1}(\ell+1)+\sum_{\ell\geqslant h}\psi_{n,\tau,\ell^\tau}(\ell-\ell_{\tau-1})\right]\delta^{T-\tau}v_i - c_i.$$

Let $S_\tau(\ell^\tau)$ be the set of available continuation strategies after stage τ when the history of contributors is ℓ^τ. Since the numbers of players and stages are finite, there exists $M \in \mathbb{N}$ such that $S_\tau(\ell^\tau) = [0,1]^M$. Let $q(t,\ell' \mid s)$ be the conditional probability that the player contributes after being induced by strategy $s \in S_\tau(\ell)$ in period t when the number of contributors is $\ell' \geqslant \ell$. Note that for each $s \in S_\tau(\ell^\tau)$, $s = (q(t,\ell' \mid s))_{t\geqslant\tau,\ell'\geqslant\ell_{\tau-1}}$.

For each $s \in S_\tau(\ell^\tau)$, let $P(s)$ be the probability that the public good is provided and $Q_t(s)$ be the probability that player i contributes in stage t after being induced by strategy s and given ρ. Note that $P(s)$ and $Q_t(s)$ are the sums and products of the elements of s and ρ.

Then, the expected utility of employing strategy s is

$$P(s)\delta^{T-\tau}v_i - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}c_i.$$

Let

$$A_\tau = \left[\sum_{\ell=\ell_{\tau-1}}^{h-1} \psi_{n,\tau,\ell^\tau}(\ell-\ell_{\tau-1})R_{n,\tau+1}(\ell+1)+\sum_{\ell\geqslant h}\psi_{n,\tau,\ell^\tau}(\ell-\ell_{\tau-1})\right].$$

Then, player i contributes in stage τ only if for each $s \in S_\tau(\ell)$,

$$(A_\tau - P(s))\delta^{T-\tau}\frac{v_i}{c_i} \geqslant 1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau} > 0. \tag{2.12}$$

The following set is the set of valuations at which player i who has not contributed contributes in stage τ.

$$VC_\tau(\ell^\tau) := \left\{\frac{v_i}{c_i} \in Y : (A_\tau - P(s))\delta^{T-\tau}\frac{v_i}{c_i} \geqslant 1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau} \text{ for all } s \in S_\tau(\ell^\tau)\right\},$$

where $Y = [\min\{\underline{V}/\underline{C}, \underline{V}/\overline{C}\}, \overline{V}/\underline{C}]$. We show that $VC_\tau(\ell^\tau)$ is a closed interval or empty set.

Note that if $|A_\tau - P(s)|$ is sufficiently small, since v_i/c_i is bounded, no v_i/c_i satisfies (2.12).

Therefore, there exists $\varepsilon > 0$ such that for some s, if $|A_\tau - P(s)| < \varepsilon$, each player does not contribute. In this case, $VC_\tau(\ell^\tau) = \emptyset$.

If $A_\tau - P(s) \geqslant 0$ for some $s \in S_\tau(\ell^\tau)$ and $A_\tau - P(s) \leqslant 0$ for some s, each player does not contribute and thus, $VC_\tau(\ell^\tau) = \emptyset$.

If $A_\tau - P(s) \geqslant \varepsilon$ for each $s \in S_\tau(\ell^\tau)$, player i contributes in stage τ if and only if

$$\frac{v_i}{c_i} \geqslant \sup_{s \in S_\tau(\ell^\tau)} \frac{1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}}{\delta^{T-\tau}(A_\tau - P(s))}.$$

If $A_\tau - P(s) \leqslant -\varepsilon$ for each $s \in S_\tau(\ell^\tau)$, player i contributes in stage τ if and only if

$$\frac{v_i}{c_i} \leqslant \inf_{s \in S_\tau(\ell^\tau)} \frac{1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}}{\delta^{T-\tau}(A_\tau - P(s))}.$$

In each case, $VC_\tau(\ell^\tau)$ is a closed interval.

Using, $VC_\tau(\ell^\tau)$ the probability that a given remaining player contributes is

$$p_{n,\tau}(\ell^\tau) = \frac{\int\int_{v/c \in VC_\tau(\ell) \setminus [\bigcup_{t<\tau} VC_t(\ell^t)]} f(v \mid c)g(c)\, dv\, dc}{\int\int_{v/c \in Y \setminus [\bigcup_{t<\tau} VC_t(\ell^t)]} f(v \mid c)g(c)\, dv\, dc}, \tag{2.13}$$

and this is the equilibrium condition for the probability that a given player contributes in stage τ when the number of contributors is ℓ. Note that since $A_\tau < 1$ and $P(s) < 1$, $(\delta - 1, 1 - \delta) \not\subset VC_t(\ell^t)$ for each t and ℓ^t. This implies that in any equilibrium path, there is a player who does not contribute with a positive probability and thus, $Y \setminus [\bigcup_{t<\tau} VC_t(\ell^t)] \neq \emptyset$. Therefore, the RHS of (2.13) is well defined. This fact also implies that in any off equilibrium path, there is a player who contributes, however, this player is now nonactive and thus, it brings no effect on the belief to the remaining players.

We now show that the RHS of (2.13) is continuous in ρ. Note that since $\min_{s \in S_\tau(\ell)} 1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau} > 0$, $A_\tau - P(s) \to 0$ for some s, $\int\int_{VC_\tau(\ell^\tau)} f(v \mid c)g(c)\, dv\, dc \to 0$. Thus, $p_{n,\tau}(\ell)$ is continuous around $A_\tau - P(s) = 0$ for some s. Suppose that there exists $\varepsilon > 0$ such that $A_\tau - P(s) \geqslant \varepsilon$ for each s. Since A_τ, $P(s)$ and $Q_t(s)$ are the sums and products of the elements of s, they are continuous in s. Since $S_\tau(\ell^\tau)$ is finite (and thus, compact),

$$\sup_{s \in S_\tau(\ell^\tau)} \frac{1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}}{\delta^{T-\tau}(A_\tau - P(s))} = \max_{s \in S_\tau(\ell^\tau)} \frac{1 - \sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}}{\delta^{T-\tau}(A_\tau - P(s))}. \tag{2.14}$$

Then, according to the maximum theorem, the above equation is continuous in ρ. In the same way, if there exists $\varepsilon > 0$ such that $A_\tau - P(s) \leqslant -\varepsilon$ for each s,

$$\inf_{s\in S_\tau(\ell^\tau)} \frac{1-\sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}}{\delta^{T-\tau}(A_\tau - P(s))} = \min_{s\in S_\tau(\ell^\tau)} \frac{1-\sum_{t=\tau+1}^{T} Q_t(s)\delta^{t-\tau}}{\delta^{T-\tau}(A_\tau - P(s))}$$

is continuous in ρ.

Since $VC_\tau(\ell^\tau)$ is an interval and its endpoints are continuous in ρ, $p_{n,\tau}(\ell)$ is also continuous in ρ.

Then, $(p_{n,\tau}(\ell^\tau))$ is a continuous function of ρ and $\rho = (p_{n,\tau}(\ell^\tau))$. Therefore, according to Brouwer's fixed point theorem, a fixed point exists. The fixed point $(p_{n,\tau}(\ell^\tau))_{\tau\in\{1,\ldots,T\},\ell^\tau}$ is an equilibrium. □

Proof of Proposition 2.5. First, when $T = 1 \geqslant h = 1$, according to Corollary 2.1, the probability converges to $1 - \underline{C}/\overline{V}$. Second, we show the following.

Lemma 2.4. *When $T = m \geqslant 1$ and $h = 1$, as population goes to infinity, the limit of the probability that at least one player contributes is greater than $1 - \underline{C}/\delta^{T-1}\overline{V}$.*

Proof. Consider player i's behavior in the first stage, in which his expected utility of contributing is $\delta^{T-1}v_i - c_i$, while that of not contributing in any stage is

$$[1 - \Psi_{n,1}(0) + \Psi_{n,1}(0)R_{n,2}^{-i}(0)]\delta^{T-1}v_i,$$

where $\Psi_{n,1}(0)$ is the probability that no one contributes in the first stage and $R_{n,2}^{-i}(0)$ is the probability that another player contributes after the first stage when no one contributes in the first stage. Therefore, player i contributes only if

$$\delta^{T-1}\frac{v_i}{c_i} \geqslant \frac{1}{\Psi_{n,1}(0)(1 - R_{n,2}^{-i}(0))}.$$

Therefore, in the second stage, if no player contributes in the first stage, each player updates his information such that for each $j \in N$,

$$\delta^{T-1}\frac{v_j}{c_j} \leqslant \frac{1}{\Psi_{n,1}(0)(1 - R_{n,2}^{-i}(0))}.$$

Let $p_{n,\tau}$ be the probability that a given player contributes in stage τ in the equilibrium. Since $p_{n,\tau}$ is a bounded sequence and has a convergent subsequence, we assume that $p_{n,\tau} \to x_\tau$. If $x_\tau > 0$ for some

τ, $\Psi_{n,\tau}(0) \to 0$; that is, at least one player contributes in stage $\tau \in \{1, \dots, T\}$. Thus, the probability that the public good is provided converges to 1.

Suppose that $x_\tau = 0$ for each τ. This implies that for each $\varepsilon > 0$, for each player i such that $v_i/c_i < \overline{V}/\underline{C} - \varepsilon$ prefers not contributing to contributing in each stage τ for sufficiently large n. Therefore, at the limit,

$$\delta^{T-1}\frac{\overline{V}}{\underline{C}} \leqslant \frac{1}{\lim_{n\to\infty}\Psi_{n,1}(0)(1-R^{-i}_{n,2}(0))}.$$

Rearranging the above inequality yields

$$\lim_{n\to\infty} 1-\Psi_{n,1}(0)(1-R^{-i}_{n,2}(0)) \geqslant 1-\frac{\underline{C}}{\delta^{T-1}\overline{V}}.$$

The RHS is the probability that at least one player contributes in this game, which concludes the proof. □

We consider the case that $T \geqslant h$. To show this, as an induction assumption, we suppose that for each T' and h such that $h \leqslant m$, $T' < T$ and $m \leqslant T$, the probability that the public good is provided is greater than $1 - m\underline{C}/[\delta^{T'-1}\overline{V}]$. We now consider the case that $h = m + 1$.

Consider player i's behavior in the first stage. As in the proof of the case that $h = 2$, we consider a contributor in the first stage. Let $R_{n,\tau}(\ell)$ be the probability that the public good is provided when ℓ players have contributed at the beginning of stage τ. Let $R^{-i}_{n,\tau}(\ell)$ also be the probability that the public good is provided when ℓ players have contributed at the beginning of stage τ but player i does not contribute in either stage.

Since player i contributes in stage τ only when his expected payoff of contributing in stage τ is higher than that of not contributing in either stage, we have

$$\begin{aligned}&\left[\sum_{\ell=\ell_{\tau-1}}^{h-1}\psi_{n,\tau,\ell^\tau}(\ell)R_{n,\tau+1,\ell^\tau}(\ell_\tau+\ell+1)+\sum_{\ell\geqslant h}\psi_{n,\tau,\ell^\tau}(\ell)\right]\delta^{T-\tau}v_i-c_i\\ \geqslant&\left[\sum_{\ell=0}^{h-1}\psi_{n,\tau,\ell^\tau}(\ell)R^{-i}_{n,\tau+1,\ell^\tau}(\ell_\tau+\ell+1)+\sum_{\ell\geqslant h}\psi_{n,\tau,\ell^\tau}(\ell)\right]\delta^{T-\tau}v_i.\end{aligned}$$

Lemma 2.5. *For each τ and ℓ, $p_{n,\tau}(\ell^\tau) \to 0$.*

Proof. Suppose that $\limsup_{n\to\infty} p_{n,\tau}(\ell^\tau) > 0$ for some τ and ℓ^τ. Then, since h is finite, according the law of large numbers, $\sum_{\ell \geqslant h} \psi_{n,\tau,\ell^\tau}(\ell) \to 1$. Therefore,

$$\sum_{\ell=\ell_{\tau-1}}^{h-1} \psi_{n,\tau,\ell^\tau}(\ell) R_{n,\tau+1,\ell^\tau}(\ell_\tau + \ell + 1) + \sum_{\ell \geqslant h} \psi_{n,\tau,\ell^\tau}(\ell) \to 1$$
$$\sum_{\ell=0}^{h-1} \psi_{n,\tau,\ell^\tau}(\ell) R^{-i}_{n,\tau+1,\ell^\tau}(\ell_\tau + \ell + 1) + \sum_{\ell \geqslant h} \psi_{n,\tau,\ell^\tau}(\ell) \to 1,$$

which implies that for each v_i and c_i,

$$\lim_{n\to\infty} \left[\sum_{\ell=\ell_{\tau-1}}^{h-1} \psi_{n,\tau,\ell^\tau}(\ell) R_{n,\tau+1,\ell^\tau}(\ell_\tau + \ell + 1) + \sum_{\ell \geqslant h} \psi_{n,\tau,\ell^\tau}(\ell) \right] \delta^{T-\tau} v_i - c_i$$
$$< \lim_{n\to\infty} \left[\sum_{\ell=0}^{h-1} \psi_{n,\tau,\ell^\tau}(\ell) R^{-i}_{n,\tau+1,\ell^\tau}(\ell_\tau + \ell + 1) + \sum_{\ell \geqslant h} \psi_{n,\tau,\ell^\tau}(\ell) \right] \delta^{T-\tau} v_i.$$

Thus, $p_{n,\tau}(\ell^\tau) = 0$ for sufficiently large n, which is a contradiction. □

This lemma implies that for each $\varepsilon > 0$, for each player i such that $v_i/c_i < \overline{V}/\underline{C} - \varepsilon$ prefers not contributing to contributing in each stage τ for sufficiently large n. This implies

$$\left[\sum_{\ell=0}^{h-1} \psi_{n,1}(\ell) R_{n,2}(\ell+1) + \sum_{\ell \geqslant h} \psi_{n,1}(\ell) \right] - \frac{c_i}{\delta^{T-1} v_i} \leqslant \left[\sum_{\ell=0}^{h-1} \psi_{n,1}(\ell) R^{-i}_{n,2}(\ell) + \sum_{\ell \geqslant h} \psi_{n,1}(\ell) \right], \qquad (2.15)$$

for each $v/c \leqslant \overline{V}/\underline{C} - \varepsilon$. Recall that for each $\varepsilon > 0$, there exists $n \in \mathbb{N}$ such that player i does not contribute when $v_i/c_i < \overline{V}/\underline{C} - \varepsilon$. Therefore, we can see the second stage as the first stage of the $T-1$-stage game and the supremum of each player's v/c is $\overline{V}/\underline{C}$ when $n \to \infty$. According to the induction assumption, for each $\ell \geqslant 0$, $\liminf_{n\to\infty} R_{n,2}(\ell+1) \geqslant 1 - (h-\ell-1)\underline{C}/[\delta^{T-2}\overline{V}] \geqslant 1 - (h-1)\underline{C}/[\delta^{T-1}\overline{V}]$. Thus, from (2.15), we have

$$1 - h\frac{\underline{C}}{\delta^{T-1}\overline{V}} \leqslant \lim_{n\to\infty} \left[\sum_{\ell=0}^{h-1} \psi_{n,1}(\ell) R^{-i}_{n,2}(\ell) + \sum_{\ell \geqslant h} \psi_{n,1}(\ell) \right]$$

Note that $R^{-i}_{n,1}(0) = \lim_{n\to\infty} \sum_{\ell=0}^{h-1} \psi_{n,1}(\ell) R^{-i}_{n,2}(\ell) + \sum_{\ell \geqslant h} \psi_{n,1}(\ell)$. Further, $R_{n,1}(0)$ is the ex-ante probability that the public good is provided. Thus, it is sufficient to show that $\lim_{n\to\infty} R_{n,1}(0) = R^{-i}_{n,1}(0)$.

We now show that $\lim_{n\to\infty} R^{-i}_{n,\tau}(\ell) = \lim_{n\to\infty} R_{n,\tau}(\ell)$ for each τ and ℓ.

Let $p_{n,\tau}(\ell)$ be the probability that a given player contributes in stage τ when ℓ players contribute before stage τ. Let $\psi_{n,\tau,\ell}(\ell'-\ell) = \binom{n-\ell-1}{\ell'-\ell}[p_{n,\tau}(\ell)]^{\ell'-\ell}[1-p_{n,\tau}(\ell)]^{n-1-\ell'}$. This is the probability that exactly $\ell'-\ell$ players contribute in stage τ. Now, $R_{n,\tau}(\ell)$ and $R^{-i}_{n,\tau}(\ell)$ are calculated as

$$
\begin{aligned}
R_{n,\tau}(\ell) &= \sum_{\ell'=\ell}^{h-1} \psi_{n,\tau,\ell}(\ell'-\ell)[p_{n,\tau}(\ell)R_{n,\tau+1}(\ell'+1) + (1-p_{n,\tau}(\ell))R_{n,\tau+1}(\ell')] \\
&\quad + \sum_{\ell'=h} (1-p_{n,\tau}(\ell))\psi_{n,\tau,\ell'}(\ell'-\ell) + p_{n,\tau}(\ell)\psi_{n,\tau,\ell'}(\ell'-\ell-1) \\
R_{n,T}(\ell) &= \sum_{\ell'=h} (1-p_{n,T}(\ell))\psi_{n,T,\ell'}(\ell'-\ell) + p_{n,T}(\ell)\psi_{n,T,\ell'}(\ell'-\ell-1) \\
R^{-i}_{n,\tau}(\ell) &= \sum_{\ell'=\ell}^{h-1} \psi_{n,\tau,\ell}(\ell'-\ell)R^{-i}_{n,\tau+1}(\ell') + \sum_{\ell'=h} \psi_{n,\tau,\ell'}(\ell'-\ell) \\
R^{-i}_{n,T}(\ell) &= \sum_{\ell'=h} \psi_{n,T,\ell'}(\ell'-\ell).
\end{aligned}
$$

When $\tau = T$, since $\lim_{n\to\infty} p_{n,T}(\ell) = 0$. Then, $R^{-i}_{n,T}(\ell) = R_{n,T}(\ell)$.

From the induction assumption, suppose that for each $\tau > k$, $\lim_{n\to\infty} R^{-i}_{n,\tau}(\ell) = \lim_{n\to\infty} R_{n,\tau}(\ell)$. Consider the case that $\tau = k$. Since $\lim_{n\to\infty} p_{n,k}(\ell) = 0$,

$$
\lim_{n\to\infty} R_{n,k}(\ell) = \lim_{n\to\infty} \sum_{\ell'=\ell}^{h-1} \psi_{k,\ell}(\ell'-\ell)[R_{n,k+1}(\ell')] + \sum_{\ell'=h} \psi_{\ell'}(\ell'-\ell).
$$

According to the induction assumption, we have $\lim_{n\to\infty} R_{n,k+1}(\ell') = \lim_{n\to\infty} R^{-i}_{n,k+1}(\ell')$. □

Proof of Proposition 2.6. Consider the following strategy: Let ℓ_t be the number of contributors at period t. If $\ell_\tau < h-(T-\tau+1)$ for each $\tau < t$, then each player does not contribute at period t. Otherwise, there exists $t' < t$ such that $\ell_{t'} \geqslant h-(T-t'+1)$. If $\ell_{t'} \geqslant h$, each player has nothing to do. Otherwise, each player plays a symmetric PBE of the game with period $T-t'$ with threshold being $h-(T-t'+1)$. The existence is guaranteed by Proposition 2.4. In this case we assume that each player has the initial belief for other players.

Consider the path followed by this strategy profile. In the first period, since $T > h$, $h-(T-1+1) > 0$. Thus, no one contributes (i.e., the number of contributors is zero). Since $h-(T-t+1)$ is increasing in t, in each period, no one contributes. Then, the public good is not provided on path.

We now check this strategy profile is a PBE. Consider a path such that $\ell_\tau < h-(T-\tau+1)$ for each $\tau < t$. If a player deviates to contribute at this period, $\ell_t = \ell_{t-1}+1$. However, since $\ell_{t-1} < h-(T-(t-1)+1)$, $\ell_t = \ell_{t-1}+1 < h-(T-t+1)$ is satisfied and thus, in period $t+1$, no one contributes. This fact implies that after period $t+1$, no one contributes. Then, the total number of contributors failed to reach the threshold and thus, the public good is also not provided. This implies that the contributing player's payoff is strictly worse off. Therefore, the strategy is an equilibrium.

Consider the other case: there exists $t' < t$ such that $\ell_{t'} \geqslant h - (T - t' + 1)$. If $t - 1 = t'$, since each player has the initial belief, the strategy is a PBE by definition. After the period, the strategy is also a PBE by the definition. □

Proof of Proposition 2.7. As in the basic model, let y_n be the cutoff point; hence, a given player contributes if and only if $v_i/c_i > y_n$. Let $P(y_n) = \int_{y_n < v/c} f(v \mid c) g(c)\, dv\, dc$ and then,

$$\psi_n(\ell) = \binom{n-1}{\ell} (P(y_n))^{\ell} (1 - P(y_n))^{n-1-\ell}.$$

Let $\widetilde{\Gamma}_n(y_n)$ satisfy

$$\widetilde{\Gamma}_n(y_n) = \frac{1}{\sum_{\ell=1}^{\bar{n}} \psi(\ell - 1)[\Omega(\ell) - \Omega(\ell - 1)]}.$$

Then, the equilibrium cutoff point is $y_n = \Gamma_n(y_n) := \min\{G(y_n), \overline{V}/\underline{C}\}$. Since $\Gamma_n(y_n)$ is a continuous function, according to Brouwer's fixed point theorem, a y_n satisfies the above condition.

We first show that $\liminf_n y_n \geqslant \overline{V}/\underline{C}$. If not, since $P(y_n) > 0$, for each ℓ, $\limsup_n \psi_n(\ell) = 0$. Therefore, $\widetilde{\Gamma}_n(y_n) \to \infty$ and thus, with sufficiently large n, $y_n \geqslant \overline{V}/\underline{C}$, which is a contradiction. Therefore, $P(y_n) \to 0$. We next show that $nP(y_n)$ is bounded above. To show this, suppose, by contradiction, that there is an equilibrium sequence that $nP(y_n) \to \infty$. Let $x_n = nP(y_n)$. Then, $\psi_n(\ell) \approx \frac{1}{\ell!}(x_n)^{\ell} e^{-x_n}$ and thus, $\psi_n(\ell) \to 0$. This implies that with sufficiently large n, $y_n > \overline{V}/\underline{C}$ and thus $P(y_n) = 0$, which contradicts to $nP(y_n) \to \infty$. Without loss of generality, $nP(y_n) \to x \geqslant 0$. Then, $\lim_n \psi_n(\ell) = \frac{1}{\ell!}(x)^{\ell} e^{-x}$. Thus, since $\lim_n y_n = \overline{V}/\underline{C}$, at the limit,

$$\frac{\overline{V}}{\underline{C}} = \min\left\{\frac{1}{\sum_{\ell=1}^{\bar{n}} \frac{x^{\ell-1}}{(\ell-1)!} e^{-x}[\Omega(\ell) - \Omega(\ell - 1)]}, \frac{\overline{V}}{\underline{C}}\right\}.$$

Note that x is the expected number of contributors. Therefore, there is an equilibrium in which no one contributes at the limit if and only if

$$\frac{1}{\Omega(1) - \Omega(0)} \geqslant \frac{\overline{V}}{\underline{C}}.$$

In the same way, there is an equilibrium in which a given player contributes with a positive probability if and only if there is an x such that

$$\frac{\overline{V}}{\underline{C}} = \frac{1}{\sum_{\ell=1}^{\bar{n}} \frac{x^{\ell-1}}{(\ell-1)!} e^{-x} [\Omega(\ell) - \Omega(\ell - 1)]}.$$

Since $nP(y_n) \to x$, the distribution of the number of contributors converges to a Poisson distribution with parameter x. □

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3 Optimal signature requirements for initiatives[1]

3.1 Introduction

In addition to traditional representative democracy as a means of social decision making, some subnational regions and countries (e.g., many countries in Europe, several states in the United States, and local governments in Japan) adopt forms of direct democracy such as initiatives and referendums.[2] Under these systems, they can request new laws to be enacted or preexisting ones to be repealed if campaigners gather signatures from a predetermined proportion of citizens. Typically, when the required signatures are collected, a citizens' referendum is held to decide whether to approve the proposal.

The primary purpose of the signature requirement is to prevent citizens from overusing initiatives.[3] With a too low requirement, many proposals gather the required number of signatures, although most of them will be rejected in the referendum or legislature's decision stage. Indeed, in Switzerland, which needs 100,000 signatures for constitutional initiatives (about 2% of the registered voters), about 150 laws gathered the required signatures from 1893 to 2003, whereas only 15 laws were accepted in citizens' referenda.[4][5] This observation suggests that many unpopular laws gather the required

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[2] Initiatives are requests by ordinary citizens to enact a new law, and referendums are requests to repeal a preexisting law (Matsusaka, 2005).

[3] Ellis (2003), p.44.

[4] Kaufmann and Waters (2004).

[5] As another example, in Japan, where the signature requirement is also 2% of the registered voters, only 10% of petitions that gather the required signatures are approved (Ministry of Internal Affairs and Communication of Japan, 2014).

signatures, which may be a social cost as it requires holding a referendum on a law that will be rejected. However, with a too high requirement, no campaigner can gather the required number of signatures as it is too costly, and thus initiative will not used.[6] The optimal requirement should balance the tradeoff.

By considering the tradeoff, this study examines the optimal signature requirement and its properties. We characterize the optimal signature requirement by citizens' preference distribution and the social cost of holding a referendum. The preference distribution and social costs are different place to place, and so is the optimal requirement that balances the tradeoff. Indeed, in the real world, while the requirement is about 2% in Switzerland and Japan,[7], in US states, it ranges from 2% to 15%. Moreover, different requirements are also imposed on different types of laws. In US states, signature requirements for statutes are typically higher than those for constitutional amendments. We also study which types of countries or laws need high signature requirements for initiatives.

In the present study, we construct a three-stage model of the initiative system based on signature collection. There are n citizens and one campaigner. In stage 1, the campaigner chooses the size of a campaign that determines the probability that a given citizen has a chance to sign. In stage 2, each citizen meets the campaigner with a positive probability determined by the size of the campaign. Citizens who meet a campaigner decide whether to sign. If λn signatures are gathered (where $0 < \lambda < 1$), then the game moves to stage 3, where the citizens' referendum decides whether to enact the law.

Each citizen has a valuation for the law that is known to himself but not to others. The distribution of citizens' valuations for the law has several states that determine the number of supporters for the law. We focus on a case in which there are only two states: good and bad. In the good state, more than half of the citizens are favorable toward the law, while in the bad state, more than half of the voters are opposed. The true state is assumed to be unknown to any citizen.

We assume that the campaigner profits from not only enacting the law but also gathering the required signatures. As an example, it is a feeling of fulfillment. Interest groups may consider that collecting sufficient signatures publicizes their activities, which also becomes the motivation. As another example, if the campaigner employs a signature-gathering firm for the campaign, the profit of

[6] Indeed, Arnold and Freier (2015) empirically show the negative relation between signature requirements and citizens' initiative use.

[7] In Japan, the requirement for requests of enacting new bylaws is 2%, whereas that for recalling heads of the local government and members of municipal councils is 1/3 of the registered voters.

gathering the required signatures is a reputation concern. This is because, if the firm fails to gather required signature, even when the law is unpopular, since no one knows the true valuation distribution, some potential campaigners see the firm as a bad firm, which loses a future profit. In reality, many campaign employs signature-gathering firms.[8] These stories justify our assumption.

Now we consider the campaigner's and citizens' behaviors. No matter which state of the law has been realized, as each citizen knows his valuation, the citizen votes for the law if and only if he supports the law. Therefore, if the required signatures are gathered, the law is enacted in the referendum stage if and only if over half of the voters approve the law to be enacted.

In the signing stage, on the other hand, even though both signing and not signing are costless, the condition that the required signatures are gathered is somewhat different from the voting stage. This is because the campaigner can control the number of citizens who meet the campaigner, which may also partially control the number of gathered signatures. Therefore, if the signature requirement is sufficiently small, even when the law is opposed by most citizens, the campaign can collect the required signatures.

Therefore, we need to find a signature requirement so that the campaigner gathers signatures only for the law with the good state but gives up gathering signatures for the law with the bad state. If signing and not signing are costless, the citizen signs if and only if he is favorable toward the law. Then, the law of the bad state requires a large-scale campaign to gather the required signatures, which is more costly for the campaigner. Therefore, with the requirement being in a certain range, the campaigner gives up gathering the required signatures for the law with the bad state and tries to gather signatures for the law with the good state, which is socially desirable. Thus, we can find the optimal requirement in the range. Consider the minimum value of the range. Taking into account the campaigner's cost, the value is the optimal requirement.

Based on the above discussion, the gap in the number of gathered signatures is large when the gap in the number of supporters between the good and bad states is large. Therefore, in this case, a lower requirement is needed for the campaigner to give up gathering signatures for the law with the bad state. This implies that the optimal requirement is low when the gap in the number of supporters between the good and bad states is large. Such a situation is likely when the uncertainty in the distribution of the citizens' valuation is large. This result may suggest that the optimal requirement is high for the issues where citizens have solid opinions. Our result may also suggest that the optimal requirement is low

[8] Ellis (2002, 2003).

for countries in which citizens' preferences are diverse and high requirement is optimal for countries with relatively homogeneous citizens.

Among US states, the requirement is low (less or equal to 8%) in California, Colorado, Idaho, Florida, Massachusetts, Missouri, North Dakota, Oregon, Washington, etc. On the other hand, the requirement is high (over or equal to 10%) in Alaska, Arizona, Maine, Mississippi, Nevada, Utah, Wyoming, etc. From this observation, in relatively populous states (excluding Arizona), the requirement is low, while in less populous states (excluding Idaho, North Dakota, and Oregon), the requirement is high. The population is sometimes used as an index of uncertainty in citizens' preferences.[9] Our result may partly explain this fact.

We extend our basic model to allow continuous state case. We first show that the citizens' welfare is an inverted V shape of the requirements. Therefore, when the campaigner's profit from the enacted law is sufficiently large, there exists a requirement where the campaigner makes campaign size the socially optimal level. On the other hand, when the campaigner's profit from the enacted law is low, the requirement where campaigning and not campaigning is indifferent for the campaigner is optimal. We also show the similar results to our basic model. Under some condition, when the variance of the popularity of the law increases, the optimal requirement declines.

As an application of characterization result, we consider a reform of signature-gathering process; banning paid campaigners. Banning paid campaigners is an often suggested but rarely realized reform.[10] This reform is aimed at preventing well-financed campaigner gathering too much signatures for their favorite law. Employing paid campaigners makes it easy to collect much signatures by increasing efficiency of the campaign. Then, unpopular law may be more likely to get qualified. Our question is whether banning paid petitioners improves the welfare when the requirement is optimally chosen. When employing paid campaigners is banned, cost of campaigning increases and then, campaigns are rarely conducted. Therefore, the optimal requirement declines, which may increases the possibility that unpopular laws are qualified. We understand banning paid petitioners is a decline in efficiency of campaign, that is an increase in campaigning cost. We show that an increase in the campaigning cost reduces the citizens welfare.

As another interpretation of banning paid petitioner is a fall in the profit of gathering the required signatures. This is a source of inefficiency of campaign since the campaigner has an incentive to

[9] For example, see Matsusaka and McCarty (2001).
[10] See for example Ellis (2002, 2003); Hoesly (2005).

campaign for unpopular laws. Indeed, if there is no such profit, unpopular laws cannot qualified. On the other hand, if the regulation is incomplete, such profits may remain. A small reduction of the profit of gathering the required signatures has the same effect of increases in the cost, which implies that it reduces the welfare. We conclude that incomplete regulation reduces the citizens' welfare.

The above discussion depends on the assumption that both signing and not signing are costless. However, in reality, signing is costly. More importantly, as citizens are asked to sign in front of campaigners, some citizens may incur a cost to refuse a signature. These costs may be small compared to the importance of the law at issue, but so is the effect of each citizen's action.

In this case, with a sufficiently large population, the signature-gathering stage may not work well. As the population goes to infinity, the probability of a citizen's action being pivotal converges to zero. Thus, if the population is large enough, a citizen's decision making is determined solely by the cost of signing and that of refusing to sign. That is, a citizen signs if and only if refusing is more costly than signing. Therefore, the required signatures are gathered if and only if λ is less than the proportion of citizens who meet the campaigner and for whom the refusal cost exceeds the signing cost. Therefore, citizens' valuation for the law has nothing to do with the outcome of the signing stage in the limit.

However, if the population is finite, citizens' valuation for the law still has a relation with the outcome of the signing stage.

As in the case of costless signing and not signing, we focus on the gap in the probability that a given citizen signs for the law in either the good or the bad state. We show that this gap is large if either (1) the difference of supporters among the realizable states is large, (2) the difference between the costs of signing and refusing a signature is small, or (3) the absolute value of citizens' valuation for the law is large. Case (1) is the same result as obtained in the case of costless signing and not signing. Cases (2) and (3) are specific to the costly interaction case. The reason for case (2) is as follows. Note that if signing and not signing are costly, in addition to the valuation for the law, each citizen also considers the difference between the costs of deciding whether to sign or not. If the difference is small, each citizen can decide whether to sign more freely, which enlarges the gap in the gathered signatures between popular and unpopular laws. Case (2) suggests another reform that prevents bad laws from gathering the required signatures without raising the signature requirement, that is, reducing the cost of signing and refusing a signature. The reason for case (3) is similar to that of case (2): If the absolute value of citizens' valuation for enacting the law is large, then citizens are likely to have greater interest in whether to enact the law or not. Then, each citizen takes the decision of whether to or not sign more seriously, even when it is costly. This case is more likely for important laws such as constitutions rather than ordinal policies, which are typically imposed higher requirements in US states. Our study may suggest that lower requirement should be imposed for such laws.

3.2 Related Literature

To the best of the author's knowledge, no existing literature studies signature-gathering campaigns theoretically. As theoretical studies of initiatives, Gerber (1996) and Matsusaka and McCarty (2001) consider theoretical models of initiatives in which the law is determined in equilibrium in a dynamic game between the legislature and an interest group. Many other studies such as Besley and Coate (2008) and Gregor and Smith (2012) also consider theoretical models of initiatives. However, these works do not describe the process of signature collection explicitly. They assume that the interest group can always collect enough signatures by paying a fixed cost. The present paper introduces the process of signature collection explicitly into the game, in which the cost of collecting sufficient signatures is determined endogenously.

As empirical studies of signature requirements, Matsusaka and McCarty (2001) and Arnold and Freier (2015) provide empirical evidence that higher requirement has a negative effect on citizens' initiative use.

Signing is similar to voting in majority voting (see Ledyard, 1981, 1984, Palfrey and Rosenthal, 1983, 1985, Bögers, 2004, and Krishna and Morgan, 2015 for reviews of this problem). If voting is costless, then only the majority-supported law is enacted. Krishna and Morgan (2015) show that if voting is costly, the result of majority voting maximizes a utilitarian social welfare. Related to their results, this study also shows that under some assumptions, more signatures are gathered if enacting the law improves a social welfare, whereas fewer signatures are gathered if the law reduces a social welfare. This result also relates to that of probabilistic voting in which two candidates compete for an office by setting their platform, and each voter has an ideological bias toward the candidates, which is independent of the voter's valuation for the platform.[11] In this model, both candidates choose the policy that maximizes a utilitarian social welfare, as both attempt to convince swing voters (i.e., voters who have less bias) to vote for them. In our model, the costs of signing and refusing a signature are similar to the ideological bias for candidates in the probabilistic voting model.[12]

[11] For example, see Lindbeck and Weibull (1987), Persson and Tabellini (1999), and so on.

[12] It is also known that in a costly voting, there is an equivalent probabilistic voting model that presents similar results

Our model is also similar to the private provision of a discrete public good (Palfrey and Rosenthal, 1984), as signing is considered a contribution to a discrete public good. The cost of signing is considered as a contribution cost, and the cost of refusing a signature is considered as a warm-glow to the campaign.[13] The main difference between this model and ours is that we introduce a referendum stage after the required signatures are gathered and the campaigner's decision stage.

An initiative based on signature collection is similar to a referendum with a turnout threshold, in which the voting outcome is valid only if the voter turnout exceeds a predetermined threshold. If the turnout falls short of the threshold, the status quo is maintained (see Aguiar-Conraria and Magalhães, 2010; Hizen and Shinmyo, 2011).

3.3 Model

There are many citizens and one campaigner. Let N be the set of citizens. The game is described as follows:

1. The campaigner decides the size of the campaign $m \in [0, 1)$. Setting $m = 0$ indicates that the campaigner does not conduct a campaign.

2. A given citizen has a chance to sign (meet the campaigner) with probability $m \in (0, 1)$. This probability is referred to as *meeting probability*. A citizen who meets a campaigner decides whether to sign or not.

3. If over a fraction λ [$\lambda \in (0, 1)$] of citizens sign, a petition goes to a citizens' referendum. In the referendum, all citizens decide to vote for or against the law. Voting is assumed to be costless. The law is enacted if and only if a majority of citizens vote for the law.

Each citizen i has a valuation for the enacted law, which is denoted by $v_i \in \mathbb{R}$. This value is distributed by an absolutely continuous distribution function V_π. Here, π is the unknown parameter that describes the popularity of the law, referred to as the *state* of the law. Let Π be the set of realizable states and $p(\pi)$ be the probability that $\pi \in \Pi$ is realized. Assume that $\int |v| \, dV_\pi(v) < \infty$ for each π. We further assume that each V_π has the same support $(\underline{V}, \overline{V})$.

to those of the costly voting model (Kamada and Kojima, 2013).

[13] Hindriks and Pancs (2002) and Makris (2009) consider this problem of allowing warm-glow contributors and investigate the group-size effect and free-riding incentives.

The campaigner has a valuation $v_C > 0$ for enacting the law and $\beta \geqslant 0$ for gathering the required signatures. An example of the utility of gathering the required signatures, β, is the feeling of fulfillment. As another example, if the campaigner employs a signature-gathering firm for the campaign,[14] the firm may care for its reputation. That is, if the firm fails to gather the required signatures, the firm may be seen as a bad firm as the popularity of the law is not directly observable by others. This motivates the firm to gather the required signatures even when the law is unpopular. The cost of campaigning with the meeting probability being m is $C(m)$, where C is strictly increasing, convex, and $C(0) = 0$. We further assume that $C(m)$ is twice continuously differentiable at each $m > 0$. Moreover, to simplify the discussion, we avoid a corner solution $m = 1$ by assuming that $C'(1)$ is sufficiently large.[15] The cost of the campaign is allowed to be discontinuous at zero, which may imply that campaigning needs a fixed cost.[16] For example, there may be a deposit for the campaign and a cost of looking for cooperators, volunteers, paid petitioners, and so on.[17] Lastly, we assume that holding a referendum costs $K > 0$.

3.4 Benchmark Case

As a benchmark case, we assume that both signing and not signing are costless. For simplicity, we further assume that the set of citizens is a continuum on $(\underline{V}, \overline{V})$. We consider the case that $\Pi = \{g, b\}$, where $1 - V_g(0) > 1/2 > 1 - V_b(0)$. Then, the law that gathers sufficient signatures is enacted if and only if the realized state is g. In the signing stage, as both signing and not signing are costless, and each citizen knows his valuation for the law, the citizen signs if and only if $v_i > 0$. Then, $m[1 - V_\pi(0)]$ is the ratio of citizens who sign. The expected utility of campaigner is given by

$$U(m, \lambda) = \begin{cases} -C(m) & \text{if} \quad m < \frac{\lambda}{1-V_g(0)} \\ (v_C + \beta)p - C(m) & \text{if} \quad m \in \left[\frac{\lambda}{1-V_g(0)}, \frac{\lambda}{1-V_b(0)}\right) \\ v_C p + \beta - C(m) & \text{if} \quad m \geqslant \frac{\lambda}{1-V_b(0)}, \end{cases}$$

[14] In the US, it is common for a special interest group to employ a firm to gather signatures (see, for example, Ellis, 2002, 2003).

[15] For example, if $C(m) = am/(1 - m) + b, a > 0.b \geqslant 0$, $C'(1) = \infty$.

[16] Fixed cost is defined by $\lim_{m \to 0} C(m)$.

[17] For example, some states in the US require a deposit to conduct a campaign for initiatives (See http://www.iandrinstitute.org/docs/A_Comparison_of_Statewide_IandR_Processes.pdf).

where $p = p(g)$. Therefore, the optimal campaign size for the campaigner is

$$m^* = \begin{cases} 0 & \text{if} \quad \begin{cases} 0 \geqslant (v_C + \beta)p - C\left(\frac{\lambda}{1-V_g(0)}\right) \\ 0 \geqslant v_C p + \beta - C\left(\frac{\lambda}{1-V_b(0)}\right) \end{cases} \\ \frac{\lambda}{1-V_g(0)} & \text{if} \quad \begin{cases} (v_C + \beta)p - C\left(\frac{\lambda}{1-V_g(0)}\right) \geqslant 0 \\ C\left(\frac{\lambda}{1-V_b(0)}\right) - C\left(\frac{\lambda}{1-V_g(0)}\right) \geqslant (1-p)\beta \end{cases} \\ \frac{\lambda}{1-V_b(0)} & \text{if} \quad \begin{cases} v_C p + \beta - C\left(\frac{\lambda}{1-V_b(0)}\right) \geqslant 0 \\ (1-p)\beta \geqslant C\left(\frac{\lambda}{1-V_b(0)}\right) - C\left(\frac{\lambda}{1-V_g(0)}\right). \end{cases} \end{cases}$$

We now consider the optimal requirement to maximize social welfare. Let $\bar{v}_\pi = \int v \, dV_\pi(v)$ be the social welfare from the law. Assume that $\bar{v}_g - K > 0 > \bar{v}_b$. Then, it is socially optimal to enforce the campaigner to choose $m^* = \lambda/[1 - V_g(0)]$. To do so, the requirement λ needs to satisfy

$$(v_C + \beta)p - C\left(\frac{\lambda}{1 - V_g(0)}\right) \geqslant 0, \tag{3.1}$$

$$C\left(\frac{\lambda}{1 - V_b(0)}\right) - C\left(\frac{\lambda}{1 - V_g(0)}\right) \geqslant (1-p)\beta. \tag{3.2}$$

The constraint (3.1) is individual rationality and (3.2) is incentive compatibility. We consider the minimum λ that satisfies the above conditions as the optimal requirement. If we consider the campaigner's utility into social welfare and if the weight on the campaigner is sufficiently small, such λ is optimal. Putting a small weight on the campaigner may be justified as the campaigner is only a small portion of the citizens.

The first condition is satisfied when λ is sufficiently small since $\sigma_g^{-1}(\lambda) = \lambda/[1 - V_g(K/p)]$. Differentiating $C\left(\frac{\lambda}{1-V_b(0)}\right) - C\left(\frac{\lambda}{1-V_g(0)}\right)$ by λ yields the following:

$$\frac{C'\left(\frac{\lambda}{1-V_b(0)}\right)}{1 - V_b(0)} - \frac{C'\left(\frac{\lambda}{1-V_g(0)}\right)}{1 - V_g(0)}.$$

Note that the above equation is positive since C is an increasing convex function and $1 - V_g(0) > 1 - V_b(0)$. Therefore, $C\left(\frac{\lambda}{1-V_b(0)}\right) - C\left(\frac{\lambda}{1-V_g(0)}\right)$ is increasing in λ. Let λ^* satisfy $C\left(\frac{\lambda^*}{1-V_b(0)}\right) - C\left(\frac{\lambda^*}{1-V_g(0)}\right) = (1-p)\beta$. Based on the above discussion, this value λ^* is uniquely determined if it exists. If λ^* exists and $(v_C + \beta)p - C\left(\frac{\lambda^*}{1-V_g(0)}\right) \geqslant 0$, then, the first best action is enforceable. However, if λ^* does not exist or $(v_C + \beta)p - C\left(\frac{\lambda^*}{1-V_g(0)}\right) < 0$, the campaigner gives up gathering signatures. In this case, we consider the second best action. We consider two cases: (1) $p\bar{v}_g > K$. Here, it is optimal to hold the referendum at any time. Thus, $\lambda = 0$ is the second best solution. (2) $p\bar{v}_g < K$. Here, holding a referendum is too costly for the law. Therefore, $\lambda = 1$ is the second best solution.

In each case, the signature-gathering campaign does not make any sense. In case (1), if the government can hold a referendum by itself, such laws cannot be sent to the signing stage. Case (2) presents the same situation in which initiatives are not allowed. The following summarizes this discussion.

Result 3.1. Let λ^* solve $C\left(\frac{\lambda^*}{1-V_b(0)}\right) - C\left(\frac{\lambda^*}{1-V_g(0)}\right) = (1-p)\beta$ if it exists. Then, if λ^* exists and $(v_C + \beta)p - C\left(\frac{\lambda^*}{1-V_g(0)}\right) \geqslant 0$, the optimal requirement is λ^*. Otherwise, (1) if $p\bar{v}_g > K$, it is optimal to hold a referendum without a signature-gathering campaign, or (2) if $p\bar{v}_g < K$, not allowing a signature-gathering campaign is optimal.

Now, we consider the question of which types of laws need high requirement. To answer this question, we see the effect of $1 - V_g(0)$ and $1 - V_b(0)$ on the optimal requirement. To simplify the notation, let $g = 1 - V_g(0)$ and $b = 1 - V_b(0)$, which are the ratios of the supporters of the law for each state. As Proposition 3.1 shows, if it is an interior solution, the optimal requirement satisfies $C(\lambda^*(g,b)/b) - C(\lambda^*(g,b)/g) = (1-p)\beta$. By differentiating both sides of this equation, we can easily check the following results.

Result 3.2. For each g, b, such that $\bar{v}_g > K > \bar{v}_b$ and $g > 1/2 > b$, $\frac{\partial \lambda^*}{\partial g} < 0$ and $\frac{\partial \lambda^*}{\partial b} > 0$.

This result indicates that the optimal requirement λ^* increases as $|g - b|$ decreases. This is because, as g and b approach each other, the number of gathered signatures in each state also approach each other. Thus, higher requirement is needed for the campaigner to give up gathering the required signatures for the bad state.

3.5 Continuum states of the law

In the previous section, we assume that the set of possible states of the law Π contains only two states. This section considers that Π contains more elements; Π is a continuum. We assume that $\Pi = (0,1)$ and p is the probability measure (cumulative distribution function) on Π.[18] We also assume

[18] We can show the following results if Π is an open interval $(\underline{\pi}, \overline{\pi}) \subset (0,1)$ that contains 1/2.

that p is twice continuously differentiable at the support. To simplify the notation, we assume that $\pi = 1 - V_\pi(0)$. Then, the expected utility of campaigner is given by

$$U(m,\lambda) = \begin{cases} -C(m) & \text{if} \quad m \leqslant \lambda \\ (v_C + \beta)\int_{\lambda/m}^{1} dp(\pi) - C(m) & \text{if} \quad m \in (\lambda, 2\lambda] \\ (v_C + \beta)P + \beta \int_{\lambda/m}^{1/2} dp(\pi) - C(m) & \text{if} \quad m > 2\lambda, \end{cases}$$

where $P = \int_{1/2} dp(\pi)$. Note that for each $\lambda > 0$, $U(m,\lambda)$ is continuous at each $m > 0$. Note also that $U(0,\lambda) > U(m,\lambda)$ for each $m < \lambda$. Therefore, the optimal meeting probability m is one of the following: (i) $m = 0$ or (ii) $m \in [\lambda, 1]$ that maximizes $U(m,\lambda)$. Let $m^*(\lambda)$ be the global optimum and $m^{**}(\lambda)$ be the solution to problem (ii). Since $U(m,\lambda)$ is continuous in m, $m^*(\lambda)$ and $m^{**}(\lambda)$ exist for each λ.

To determine the campaigner's action uniquely, we assume the following.

Assumption 3.1. *If $U(m,\lambda) = U(m',\lambda) \geqslant U(m'',\lambda)$ for each $m'' \in [0,1]$ and for some $m, m' \in [0,1]$ such that $m > m'$, the campaigner takes m.*

The following assumption ensures that there is a requirement level under which the campaigner has an incentive to conduct a campaign.

Assumption 3.2. $\lim_{m\to 0} C(m) < v_C P + \beta$.

To see this, consider $m = \sqrt{\lambda}$. Note that $\lim_{\lambda\to 0} U(\sqrt{\lambda},\lambda) = v_C P + \beta - \lim_{\lambda\to 0} C(\sqrt{\lambda}) > 0$. Thus, for some $\lambda > 0$, campaigning is optimal for the campaigner.

To characterize how the chosen meeting probability varies as the requirement λ increases, we show the following.

Lemma 3.1. *Under Assumptions 3.1 and 3.2,*

(1) there exists $\bar{\lambda}$ such that for each $\lambda > \bar{\lambda}$, $m^*(\lambda) = 0$ and each $\lambda \leqslant \bar{\lambda}$, $m^*(\lambda) = m^{**}(\lambda)$.

(2) $\lambda/m^*(\lambda)$ is nondecreasing in λ.

By using Lemma 3.1, we can calculate the optimal signature requirement to maximize social welfare. To simplify the discussion, unlike in the previous subsection, we ignore the campaigner's utility. Furthermore, we also assume the following.

Assumption 3.3. *The social welfare from the law $\bar{v}_\pi$ is continuous and increasing in π and $\bar{v}_1 > K > \bar{v}_{1/2}$.*

Assumption 3.4. *$p''(\pi)\pi + p'(\pi) \geqslant 0$ for each $\pi \in [0, 1]$.*

Assumption 3.3 ensures the existence of the optimal threshold of the state. Calculating the expected social welfare yields

$$\mathrm{SW}(\lambda) = \begin{cases} 0 & \text{if} \quad m^*(\lambda) = 0 \\ \int_{\lambda/m^*(\lambda)} [\bar{v}_\pi - K] \, dp(\pi) & \text{if} \quad m^*(\lambda) \leqslant 2\lambda \\ \int_{1/2} \bar{v}_\pi \, dp(\pi) - \int_{\lambda/m^*(\lambda)}^{1} K \, dp(\pi) & \text{if} \quad m^*(\lambda) > 2\lambda. \end{cases}$$

Then, by Assumption 3.3, the first best is setting λ so that $\bar{v}_{\lambda/m^*(\lambda)} = K$. Let π_K satisfy $\bar{v}_{\pi_K} = K$. Assumption 3.4 makes the characterization of the optimal requirement easy as seen later.[19]

We now characterize the optimal requirement. Consider the case that there exists λ such that $\lambda/m^*(\lambda) = \pi_K$. Then, setting λ that satisfies $\lambda/m^*(\lambda) = \pi_K$ is optimal. If not, we consider the second best. Let λ_K^* satisfy $m^{**}(\lambda_K^*) = \pi_K/\lambda_K^*$ if it exists. As setting $\lambda/m^*(\lambda) = \pi_K$ is not feasible, $U(m^{**}(\lambda_K^*), \lambda_K^*) < 0$ or there is no such λ_K^*. The following lemma shows that the latter case is impossible.

Lemma 3.2. *Under Assumption 3.4, there exists λ_K^* such that $m^{**}(\lambda_K^*) = \pi_K/\lambda_K^*$.*

Therefore, we consider the former case, that is, $U(m^{**}(\lambda_K^*), \lambda_K^*) < 0$. Since $\lambda/m^{**}(\lambda)$ is increasing in λ and by Lemma 3.1, there exists $\bar{\lambda}$ such that $U(m^{**}(\bar{\lambda}), \bar{\lambda}) = 0$ and $\pi_K > \bar{\lambda}/m^{**}(\bar{\lambda})$. If $\lambda < \bar{\lambda}$, with larger λ, $\lambda/m^{**}(\lambda)$ is smaller, and therefore $\mathrm{SW}(\lambda)$ is increasing in λ. If $\mathrm{SW}(\bar{\lambda}) > 0$, then $\lambda = \bar{\lambda}$ is optimal. This is a standard way in the contract theory that extracts all profit from the campaigner. If $\mathrm{SW}(\bar{\lambda}) \leqslant 0$, not allowing the signature-gathering campaign is optimal.

3.5.1 Comparative statics

Now, we consider the question of when is the requirement λ large. To answer this question, we compare two probability measures on distribution p and $\widehat{p}$ that satisfy the following conditions: (i) p and $\widehat{p}$ has the same mean and (ii) p' crosses $\widehat{p}'$ exactly twice; that is, there exists $\pi^1, \pi^2 \in (0, 1)$ such that $\pi^1 < \pi^2$ for each $\pi \in (0, 1)$ if $\pi \in (\pi^1, \pi^2)$, $p'(\pi) > \widehat{p}'(\pi)$, and $p'(\pi) \leqslant \widehat{p}'(\pi)$ otherwise.

This condition implies that $\widehat{p}$ puts heavier weight on the tails. Indeed, one can show that $\widehat{p}$ is a mean preserving spread of p, which implies that the variance of $\widehat{p}$ is higher than that of p. One can also

[19] For example, if the probability measure has nondecreasing density, Assumption 3.4 is satisfied.

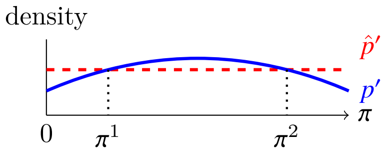


Figure 3.1: Example of density functions satisfying conditions (i) and (ii).

show that $\widehat{p}$ crosses p exactly once; that is, there exists $\widehat{\pi} \in (\pi^1, \pi^2)$ such that if $\pi < \widehat{\pi}$, $\widehat{p}(\pi) > p(\pi)$, and $p(\pi) \geqslant \widehat{p}(\pi)$ otherwise. Figure 3.1 illustrates an example.

Let λ^* and $\widehat{\lambda}^*$ be the optimal requirement for p and $\widehat{p}$, respectively, and let m^* and $\widehat{m}^*$ be the best response for p and $\widehat{p}$, respectively. In the case that $\Pi = \{g, b\}$, we have shown that with more extreme states of the law, less requirement is needed. We can show a similar result under some conditions.

Proposition 3.1. *Suppose that Assumptions 3.1, 3.2, 3.3, and 3.4 hold. Moreover, suppose that allowing a signature-gathering campaign is optimal. Then,*

(1) suppose that $\pi_K \in (\pi^1, \pi^2)$, *and* $\lambda^*/m^*(\lambda) = \widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) = \pi_K$. *Then,* $\lambda^* \geqslant \widehat{\lambda}^*$.

(2) assume $\widehat{\pi} \geqslant 1/2$, *and suppose that* $\lambda^*/m^*(\lambda) < \widehat{\pi}$ *and* $\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) < \widehat{\pi}$. *Then,* $\lambda^* \geqslant \widehat{\lambda}^*$.

(3) assume $\widehat{\pi} \geqslant 1/2$, *and suppose that* $\lambda^*/m^*(\lambda) > \widehat{\pi}$ *and* $\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) > \widehat{\pi}$. *Then,* $\lambda^* \leqslant \widehat{\lambda}^*$.

The logic underlying this proposition is different from the two states case. Consider the case that the government can enforce the campaigner to set m that satisfies $\lambda/m^*(\lambda) = \pi_K$. If $\pi_K \in (\pi^1, \pi^2)$, the law faces relatively moderate opinion. If the probability measure is p, the density around π_K is higher than if the probability measure is $\widehat{p}$, and the campaigner makes much more effort to meet the citizens. Thus, for p, a higher requirement is needed.

Consider the case that the government set λ to satisfy $U(m, \lambda) = 0$. In this case, $\lambda/m^*(\lambda) < \pi_K$. If $\lambda/m < \widehat{\pi}$, the campaigner's expected payoff is higher when the probability measure is p. Therefore, the government can reduce the campaigner's payoff by raising λ. By doing this, as shown in Lemma 3.1, λ/m becomes higher, which improves citizens' welfare. If $\lambda/m > \widehat{\pi}$, the converse holds; the campaigner's expected payoff is higher when the probability measure is $\widehat{p}$. Therefore, in this case, raising the requirement is optimal when p changes to $\widehat{p}$. However, if $\pi_K \leqslant \widehat{\pi}$, the latter case is impossible since $\lambda^*/m^*(\lambda^*) \leqslant \pi_K$ for each optimal requirement λ. Even when $\pi_K > \widehat{\pi}$, if π_K is sufficiently close to $\widehat{\pi}$, the latter case hardly holds. Now, it is a question of when the above condition is likely to hold. We say that $\widehat{p}$ is a *median preserving spread* of p if $\widehat{x}$ is a median of p and $\widehat{p}$.[20]

[20] See Malamud and Torojani (2009) for the formal definition and properties.

Assume that $\widehat{p}$ is a median preserving spread of p. Then, if the median of p is greater than $1/2$ and K is sufficiently small,[21] $\widehat{\pi} > \pi_K$ holds, and thus we can say that $\lambda > \widehat{\lambda}$.

On the contrary, as seen above, if the median of p is sufficiently small, β is sufficiently large, and the difference, $\sup_x |p(x) - \widehat{p}(x)|$, is so small that $\widehat{\pi} < \lambda^*/m^*(\lambda^*), \widehat{\pi} < \widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*)$, and the converse relation holds: $\widehat{\lambda} > \lambda$.

3.5.2 Banning paid petitioners

In the literature, as a reform plan of the initiative process, banning of paid petitioners has been proposed as paid petitioners enable wealthy interest groups to gather the required signatures easily.[22] However, banning paid petitioners would make an initiative campaign too costly, and, thus, few campaigns would be conducted. To cope with this problem, there is a need for lowering the requirement, which will increase initiative campaigns. The question is whether banning paid petitioners and setting the requirement optimally improves the social welfare. To answer this, let the cost function be $kC(m)$. The parameter k represents the efficiency of the campaign; small k implies that the campaign is efficient. Banning paid petitioners implies a decline in the efficiency of the campaign, that is, in our term, an increase in k. Thus, for this purpose, we see the effect of k.

Let $m_k^*(\lambda)$ be the best response of the campaigner to the requirement when the efficiency parameter of the cost function is k. Let λ_k^* be the socially optimal threshold. Consider the case that $\lambda_k^*/m^*(\lambda_k^*) = \pi_K$. In this case, since the first best is achieved, the social welfare cannot be improved. Consider the case that $\lambda_k^*/m^*(\lambda_k^*) < \pi_K$. Especially, focus on the case that $\lambda_k^*/m^*(\lambda_k^*) \neq 1/2$.[23] In this case, we show that an increase in k reduces the social welfare.

Proposition 3.2. *Suppose that Assumptions 3.1, 3.2, 3.3, and 3.4 hold, allowing the signature-gathering campaign to be optimal, and* $\lambda_k^*/m^*(\lambda_k^*) < \pi_K$.

(1) Suppose that $U(m^*(\lambda_k^*), \lambda_k^*) > U(2\lambda_k^*, \lambda_k^*)$. *Then,* $\frac{d\lambda_k^*/m_k^*(\lambda_k^*)}{dk} \leqslant 0$ *if and only if* $\left[\frac{C'(m)}{C(m)/m}\right]' \geqslant 0$ *at* $m = m_k^*(\lambda_k^*)$.

(2) Suppose that $m^*(\lambda_k^*) = 2\lambda_k^*$ *and* $U(m, \lambda_k^*) < U(2\lambda_k^*, \lambda_k^*)$ *for each* $m \neq 2\lambda_k^*$, $\frac{d\lambda_k^*/m_k^*(\lambda_k^*)}{dk} = 0$.

Note that $\frac{C'(m)}{C(m)/m}$ is the ratio of the marginal cost to the average cost. An intuitive explanation of (1) is the following. The requirement λ is determined by zero profit condition, and λ is decreasing in the

[21] In many US states, the referendum cost K is considered small as referendums are held at the same time as elections.
[22] See, for example, Ellis (2002) and Hoesly (2005).
[23] If $\lambda_k^*/m^*(\lambda_k^*) = 1/2$, as the second statement of the following proposition shows, $\frac{d\lambda_k^*/m_k^*(\lambda_k^*)}{dk} = 0$.

average cost. The campaign size m is determined by the first order condition, and m is decreasing in the marginal cost. Therefore, λ/m is the fraction of decreasing functions of the average and marginal cost. A decline in efficiency of the campaign decreases campaign size m, which decreases both of the average and marginal cost. Therefore, if the fraction of the marginal and average costs is increasing in m, a decline in efficiency of the campaign increases the fraction of the average and marginal costs. Since λ/m is a decreasing relation of it, a decline in efficiency decreases λ/m.

We verify when $\frac{C'(m)}{C(m)/m}$ is increasing. At a glance, since $C \geqslant 0$, $C' \geqslant 0$ and $C'' \geqslant 0$, this inequality is satisfied if $C(m)/m > C'(m)$; that is, the average cost exceeds the marginal cost. If there is a fixed cost to start a campaign, this condition is likely to be satisfied. As a concrete example, increasing linear functions and increasing convex quadratic functions satisfy the condition. As another example, $C(m) = a\frac{m}{1-m} + b$, $a > 0, b > 0$ also satisfies the condition.

In each case, therefore, a decline in the efficiency of a campaign decreases the social welfare.

To conclude, we interpret the banning of paid petitioners to be a decline in the efficiency of the campaign. We have examined two cases: When the first best is achieved, banning paid petitioners cannot improve the social welfare. Moreover, if the campaigner gathers the required signatures over the socially optimal level [i.e., $\lambda_k^*/m_k^*(\lambda_k^*) < \pi_K$], banning paid petitioners may reduce the social welfare (depending on the shape of the cost function).

On the other hand, as another interpretation, banning paid petitioner is also considered to be a decline in the motivation of gathering the required signatures, namely β. This is because, β can be considered as a reputation concern for signature-gathering firms. However, this case is similar to the decline in the efficiency of campaigns. If $v_C = 0$, the profit function of the campaigner is $(v_C + \beta)P + \beta\left[\int_{\lambda/m}^{1/2} dp(\pi)\right] - C(m)$, which is equivalent to $\left[P + \int_{\lambda/m}^{1/2} dp(\pi)\right] - (1/\beta)[C(m) - v_C]$. Therefore, a decline in β is equivalent to a decline in the efficiency of the campaign.

Corollary 3.1. *Suppose that Assumptions 3.1, 3.2, 3.3, and 3.4 hold, allowing the signature-gathering campaign to be optimal, and* $\lambda_k^*/m^*(\lambda_k^*) < \pi_K$.

Suppose that $U(m^*(\lambda^*), \lambda^*) > U(2\lambda^*, \lambda^*)$. *Then,* $\frac{d\lambda^*/m^*(\lambda^*)}{d\beta} \geqslant 0$ *if and only if* $\left[\frac{C'(m)}{(C(m)-v_C)/m}\right]' \geqslant 0$ *at* $m = m^*(\lambda^*)$.

Therefore, if v_C is sufficiently small and fixed cost is sufficiently high, a small decline in β reduces the citizens' welfare. The motivation of gathering required signature is a cause of inefficiency of signature gathering campaign. However, when the requirement is optimally chosen, an incomplete reduction of the motivation may reduce the citizens' welfare.

3.6 Costly Interaction with the Campaigner

In the previous section, we assume that both signing and not signing are costless. However, in the real world, both signing and refusing a signature may be costly. This section considers the real-world case. Precisely, we assume that when a given citizen meets the campaigner, the citizen incurs a cost c_i^s if he signs, whereas the citizen also incurs a cost c_i^r if he refuses to sign. Let $c_i = c_i^s - c_i^r$. The value c_i is distributed by a differentiable distribution function, $H(\cdot)$. Assume that the derivative of H is bounded. Furthermore, while in the previous section, we assume that the set of citizens is a continuum, in this section, we assume that the set of citizens is $N = \{1, \ldots, n\}$ to consider the cost of signing and not signing. Without loss of generality, we assume that n is an odd number.

3.6.1 Equilibrium in the signing stage

This section considers the equilibrium of the signing and referendum stages. Let $m > 0$ be fixed. We first consider the referendum stage. Let $D_{n,\pi}(v_i)$ be the belief of citizen i at the signing stage about the probability that the given law is enacted in the referendum when the realized state is π. As each citizen's valuation for the law is a private value and voting is costless, each citizen votes for the law if and only if it is favorable for that citizen. Therefore,[24]

$$D_{n,\pi}(v_i) = \begin{cases} D^+_{n,\pi} := \sum_{k \geqslant (n-1)/2} \binom{n-1}{k}[1 - V_\pi(0)]^k [V_\pi(0)]^{n-k-1} & \text{if} \quad v_i > 0 \\ D^-_{n,\pi} := \sum_{k \geqslant (n-1)/2+1} \binom{n-1}{k}[1 - V_\pi(0)]^k [V_\pi(0)]^{n-k-1} & \text{if} \quad v_i \leqslant 0. \end{cases}$$

A given citizen signs if and only if[25]

$$\sum_{\pi \in \Pi} p(\pi)[D_{n,\pi}(v_i)\Pr(\text{number of signatures by others} \geqslant h-1)]v_i - c_i^s >$$
$$\sum_{\pi \in \Pi} p(\pi)[D_{n,\pi}(v_i)\Pr(\text{number of signatures by others} \geqslant h)]v_i - c_i^r.$$

Equivalently, a given citizen signs if and only if

$$\sum_{\pi \in \Pi} p(\pi)[D_{n,\pi}(v_i)\Pr(\text{number of signatures by others} = h-1)]v_i > (c_i^s - c_i^r).$$

[24] Without loss of generality, we assume that the citizen whose valuation for the law is 0 votes against the law.
[25] Without loss of generality, we also assume that if signing and not signing are indifferent, the citizen does not sign.

Let the probability that a given citizen signs when state π is realized be σ_π. Then,

$$\sigma_{\pi'} = F^\alpha_{\pi'}(\sigma) := m \int H\left(E[D_{n,\pi}(v_i)\binom{n-1}{h-1}\sigma_\pi^{h-1}(1-\sigma_\pi)^{n-h}]v_i\right)\, dV_{\pi'}(v_i),$$

where α summarizes the parameters. Since $F^\alpha := (F^\alpha_\pi)_{\pi\in\Pi}$ is a continuous function of $(\sigma_\pi) \in [0,1]^{|\Pi|}$ and $F^\alpha : [0,1]^{|\Pi|} \to [0,1]^{|\Pi|}$, it has a fixed point $\sigma^*_\pi(\alpha)$, which constructs a symmetric perfect Bayesian Nash equilibrium (PBE).

The following proposition shows that if the population goes to infinity, the limiting probability that a given citizen signs is irrelevant to the citizens' valuation for the law.

Proposition 3.3. *Consider a symmetric PBE. Let $\sigma_\pi(n)$ be the probability that a given citizen signs when the population is n. Then, for each $\pi \in \Pi$, $\lim_{n\to\infty} \sigma_\pi(n) = mH(0)$.*

Proposition 3.3 suggests that the citizens' valuation for the law is irrelevant to the outcome of the signature-gathering process. By this result, if the population is large enough, not allowing the initiative to conduct the signature-gathering campaign is optimal. However, this result only holds in the limit. The following section considers a finite population case.

3.6.2 Signature-gathering campaign in a finite population

This section considers a finite population and shows the relation between the realized preference distribution and number of gathered signatures. To see detailed properties, we specify the set of potential distributions of valuations, Π. We assume the following.

Assumption 3.5. *For each $\pi \in \Pi$, there exists a continuously differentiable distribution Φ that satisfies (1) symmetric at* 0 *and (2) $V'_\pi(v)/\Phi'(v)$ is monotone in v.*[26]

Under Assumption 3.5, as the likelihood ratio dominance implies first-order stochastic dominance, $1 - V_\pi(0) > 1/2$ and $\int v\, dV_\pi(v) > 0$ if $V'_\pi(v)/\Phi'(v)$ is nondecreasing and if $V'_\pi(v)/\Phi'(v)$ is nonincreasing, the converses hold. Let $\Pi_+ := \{\pi \in \Pi : 1 - V_\pi(0) > 1/2\}$ and $\Pi_- := \{\pi \in \Pi : 1 - V_\pi(0) < 1/2\}$. As discussed above, under Assumption 3.5, $\Pi = \Pi_+ \cup \Pi_-$.

To simplify the discussion, we assume that $D^+_{n,\pi} = D^-_{n,\pi} = 1$ for each $\pi \in \Pi_+$ and $D^+_{n,\pi} = D^-_{n,\pi} = 0$ for each $\pi \in \Pi_-$. This assumption is justified when the population is sufficiently large as an

[26] Formally, a distribution function F is symmetric at v_0 if $F'(v_0 + v) = F'(v_0 - v)$ for each v.

approximation.[27] Then, there exists χ such that citizen i signs if and only if $\chi v_i > c_i$, where χ is defined as

$$\chi = G_m(\sigma) := \sum_{\pi \in \Pi_+} \left[p(\pi) \binom{n-1}{h-1} \sigma_{\pi'}^{h-1} (1 - \sigma_{\pi'})^{n-h} \right].$$

Then, the probability that a given citizen signs is

$$\sigma_\pi(\chi) = \Gamma_\pi(\chi, m) = m \int H(\chi v_i)\ dV_{\pi'}(v_i).$$

Note that $\sum_{\pi \in \Pi_+} \left[p(\pi) \binom{n-1}{h-1} \sigma_{\pi'}^{h-1} (1 - \sigma_{\pi'})^{n-h} \right]$ is bounded above by $\Sigma := \binom{n-1}{h-1}$, $G(\sigma(\cdot)) : [0, \Sigma] \to [0, \Sigma]$ is a continuous self mapping of χ. Therefore it has a fixed point, which constructs an equilibrium. The following lemma shows the basic property of the equilibrium in the signing stage, which is used for comparative statics.

Lemma 3.3. *Assume Assumption 3.5 and $D^+_{n,\pi} = D^-_{n,\pi} = 1$ for each $\pi \in \Pi_+$ and $D^+_{n,\pi} = D^-_{n,\pi} = 0$ for each $\pi \in \Pi_-$. Suppose that H has a probability density function that is symmetric at 0. Then, in any symmetric PBE, $\frac{\partial \Gamma_\pi}{\partial \chi}(\chi, m) \geqslant 0$ for each χ, m if $\pi \in \Pi_+$ and otherwise $\frac{\partial \Gamma_\pi}{\partial \chi}(\chi, m) \leqslant 0$ for each χ, m. Moreover, for the probability that a given citizen signs, σ^α_π, $\sigma^\alpha_\pi > mH(0)$ if and only if $\pi \in \Pi_+$.*

3.6.3 Gaps in gathered signatures

We now go back to the campaigner's problem. To do this, we summarize the equilibrium of the signing stage. Let $\sigma^*_\pi(m, h)$ be the probability that a given citizen signs in the equilibrium of the signing stage. In the signing stage, the equilibrium may not be unique. In this case, we employ the equilibrium in which the cutoff point is maximized among the equilibrium cutoff points. Let us call this equilibrium *the maximum equilibrium*. The following lemma states that the maximum equilibrium exists.

Lemma 3.4. *Suppose that each citizen considers that $D^+_{n,\pi} = D^-_{n,\pi} = 1$ for each $\pi \in \Pi_+$ and $D^+_{n,\pi} = D^-_{n,\pi} = 0$ for each $\pi \in \Pi_-$. Then, the maximum equilibrium exists for each m^*.*

[27] Note that even when n is sufficiently large so that $|D^+_{n,\pi} - 1| < \varepsilon$ for each $\pi \in \Pi_+$ and $|D^+_{n,\pi}| < \varepsilon$ for each $\pi \in \Pi_-$, $|\sigma_\pi(m) - mH(0)| > \eta$ for some $\eta > \varepsilon$. This is because while $D^+_{n,\pi}$ is sufficiently close to its convergent if $V_\pi(0) \neq 1/2$, $|\sigma_\pi(m) - mH(0)|$ cannot be so small if $|\bar{v}_\pi|$ is large enough.

Now, we can define the expected payoff of the campaigner as follows.

$$U(m,h) = E\left[[v_C D_{n,\pi} + \beta] \sum_{k=h} \binom{n}{k} [\sigma^*_\pi(m,h)]^k [1-\sigma^*_\pi(m,h)]^{n-k} \right] - C_n(m).$$

To ensure the existence of the maximizer of $U(m, h)$, we assume that the set of choosable m is finite.[28]

Then, if the population is sufficiently large,

$$U(m,h) \approx \sum_{\pi\in\Pi_+} \left[[v_C p(\pi) + \beta] \sum_{k=h} \binom{n}{k} [\sigma^*_\pi(m,h)]^k [1-\sigma^*_\pi(m,h)]^{n-k} \right] - C_n(m).$$

As in section 3.4, we will show that if the gaps in the gathered signatures between the good states and bad states is large, a less signature requirement is sufficient to prevent a law with bad state from gathering the required signatures. To see this, consider the case that $\Pi = \{g, b\}$ where $g \in \Pi_+$ and $b \notin \Pi_+$. Let $m^*_\pi(\lambda)$ be the minimum meeting probability that satisfies $\sigma^*_\pi(m^*_\pi(\lambda), \lambda n) > \lambda$.[29]

As seen in section 3.4, to prevent a law with bad state from gathering the required signatures, we need set λ to satisfy

$$C(m^*_b(\lambda)) - C(m^*_g(\lambda)) \geqslant (1-p)\beta.$$

Section 3.4 shows that the optimal signature requirement is lower when the gap in the number of signatures is large. This also holds for the costly interaction model. To see this, we consider two sets of states $\Pi = \{g, b\}, \Pi' = \{g', b'\}$ such that $\sigma^*_g(m,h) > \sigma^*_{g'}(m,h) > mH(0) > \sigma^*_{b'}(m,h) > \sigma^*_b(m,h)$ for each m, h. Then, it easy to show that $m^*_g(\lambda) \leqslant m^*_{g'}(\lambda) \leqslant m^*_{b'}(\lambda) \leqslant m^*_b(\lambda)$. Thus,

$$C(m^*_{b'}(\lambda)) - C(m^*_{g'}(\lambda)) \geqslant C(m^*_b(\lambda)) - C(m^*_g(\lambda)) \geqslant (1-p)\beta.$$

This implies that the minimum signature-requirement to prevent the law with bad state from gathering the required signatures is lower for Π than for Π'.

Now, it is a question of when is the gap in the number of gathered signatures large. The following

[28] Note that in the signing stage, there might be multiple equilibria. In this case, the cutoff point may not be continuous in m. One may think that if the cutoff point is upper semicontinuous in m, the payoff function is upper semicontinuous in m and it has the maximum. However, this may not be right. Note that while for each $\pi \in \Pi_+$, $\Gamma_\pi(\chi)$ is increasing in π, for each $\pi \notin \Pi_+$, $\Gamma_\pi(\chi)$ is decreasing in π (see the proof of Lemma 3.3). This is because the payoff function is increasing in $\Gamma_\pi(\chi)$ for not only $\pi \in \Pi_+$ but also $\pi \notin \Pi_+$. Then, the upper semicontinuity of the payoff function is not guaranteed.

[29] Since the set of choosable m is finite, it is well defined.

propositions provide sufficient conditions.

Proposition 3.4. *Assume assumptions of Lemma 3.3. Suppose that the cost of signing $c = c^s - c^r$ is written as $c = kc^*$ for some $k > 0$, where c^* is distributed by H. Suppose also that $v = \iota v^*$ for some $\iota > 0$, where v^* is distributed by V_π when $\pi \in \Pi$ is realized. Then, as $p(\pi)$ for each $\pi \in \Pi_+$ increases, k decreases or ι increases,*

1. *For each $\pi \in \Pi_+$, $\sigma^*_\pi(m, h)$ increases.*
2. *For each $\pi \in \Pi_-$, $\sigma^*_\pi(m, h)$ decreases.*

In this proposition, k represents the relative size of cost and ι represents the importance, that is, with higher ι, the benefit (loss if $v < 0$) from enacting the law is large.

To see a more specific property, we consider the case that $\Pi_a = \{g(a), b(a)\}$. We assume that $V'_{g(a)}(v) = V'_{b(a)}(-v) = \varphi(v, a)$ for each v. Let $\Phi(v, a)$ be the function that satisfies $\frac{\partial \Phi}{\partial v}(v, a) = \varphi(v, a)$ for each v, a. We further assume that for each $a > a'$, $\Phi(v, a)$ first-order stochastically dominates $\Phi(v, a')$. The size of a represents the importance of enacting the law. With larger a, the difference from the expected social welfare of enacting the law is large. Related to this, we can show the following.

Proposition 3.5. *Assume the Assumptions of Lemma 3.3. Then, σ_g is increasing in a, while σ_b is decreasing in a.*

To summarize, based on the above discussions, we can state that when the law with $\pi \in \Pi_-$, it is more likely to gather the required signatures; that is, a higher signature requirement is needed. Consider the following cases: (1) large β, (2) large k, (3) low $\sum_{\pi \in \Pi_+} p(\pi)$, (4) small ι, and (5) small a.

The first case is trivial. The cause of the remaining cases is attributable to the citizens' behavior. Large k implies that the difference between the cost of signing and refusing a signature is large. With large k, citizens are less sensitive to their valuation for the law as writing and refusing a signature are more costly. With low $\sum_{\pi \in \Pi_+} p(\pi)$, citizens are also less sensitive to their valuation as the possibility that the law is enacted is low. This is similar to the cases of small ι and a. Small ι and a imply that the law is less important. As discussed above, for such laws, a higher requirement is optimal. On the contrary, for more important laws, a lower requirement is optimal.

Our results suggest another policy to prevent the law from gathering the required signatures when $\pi \in \Pi_-$ without raising the requirement: reducing the cost k. As shown above, by reducing the costs, the number of gathered signatures when $\pi \in \Pi_+$ increases while the number of gathered signatures when $\pi \in \Pi_-$ decreases. As a result, it increases the probability that good laws are enacted without increasing the probability that the bad laws gather the required signatures.

3.6.4 Numerical example

Example 3.1. Let $\Pi = \{g, b\}$. We consider the following two types of distributions:

1. Density is given by

$$dV_g(v) = \begin{cases} \frac{1}{2(a+2)}v + \frac{1}{(a+2)} & \text{if } v \in (-2, a) \\ -\frac{1}{2(-a+2)}v + \frac{1}{(-a+2)} & \text{if } v \in [a, 2). \end{cases}$$

$\bar{v}_g = a/3$, $1 - V_g(0) = \frac{1}{2-a}$.

2. Density is given by

$$dV_b(v) = \begin{cases} \frac{1}{2(-a+2)}v + \frac{1}{(-a+2)} & \text{if } v \in (-2, -a) \\ -\frac{1}{2(a+2)}v + \frac{1}{(a+2)} & \text{if } v \in [-a, 2). \end{cases}$$

$\bar{v}_b = -a/3$, $1 - V_b(0) = \frac{1}{2+a}$.

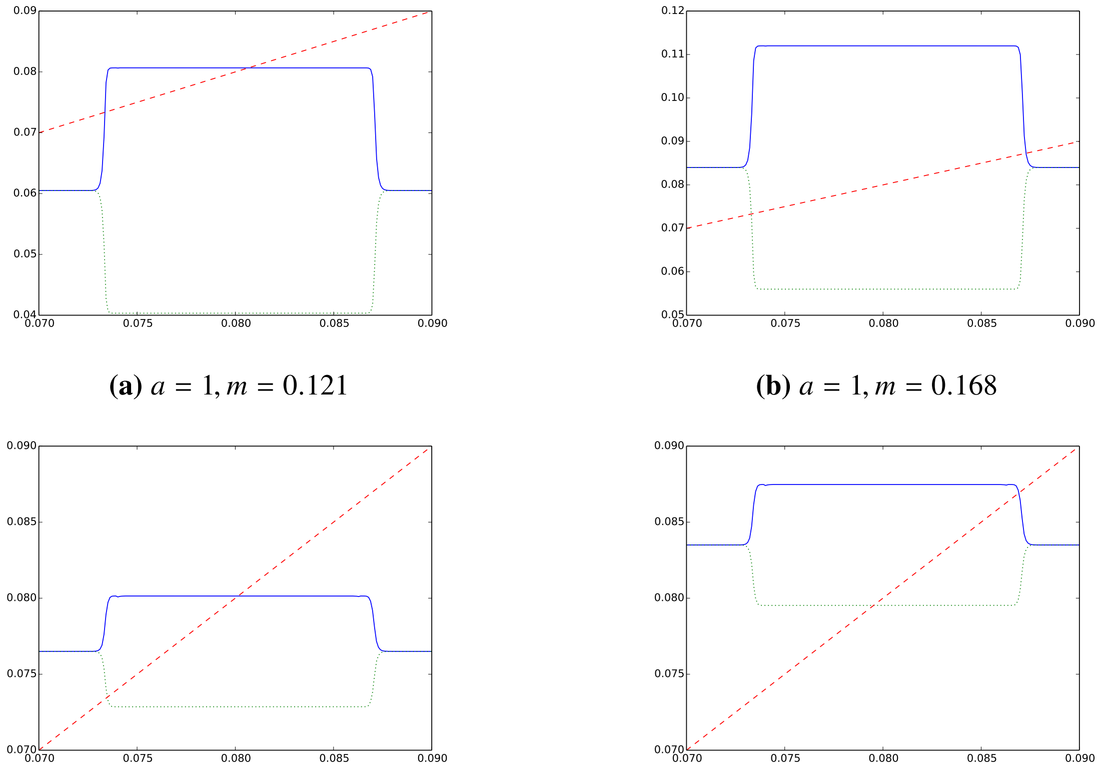
We assume that H is a uniform distribution on $(-c, c)$ for some $c > 0$. The signature-requirement $\lambda = 0.08$ and the population $n = 100,000$.

Figure 3.1 shows $\Gamma_g(G_m(\sigma))$ and $\Gamma_b(G_m(\sigma))$. As each citizen cares only if $\pi = g$, the equilibrium probability that a given citizen signs satisfies $\sigma_g^* = \Gamma_g(G_m(\sigma_g^*))$ and $\sigma_b^* = \Gamma_b(G_m(\sigma_g^*))$. In each figure, the solid line shows $\Gamma_g(G_m(\sigma))$, the dotted line shows $\Gamma_b(G_m(\sigma))$, and dashed line shows the 45-degree line. The intersection of the solid and dashed lines shows the fixed point σ_g^*. If there are two or more fixed points in the maximum equilibrium, we employ the largest one as σ_g^*.

Figure 3.1a shows the case for $a = 1$ and $m = 0.121$, which is the minimal value of m that the campaigner gathers the requirement when $\pi = g$. In this case, there are multiple equilibria.

Figure 3.1b shows the case for $a = 1$ and $m = 0.168$, which is the minimal value of m for satisfying the requirement even when $\pi = b$.

Figures 3.1c and 3.1d consider the case for $a = 0.1$. Each figure, respectively, shows the case for $m = 0.153$ and $m = 0.167$, which are the minimal requirements for gathering the required signatures,



(a) $a = 1, m = 0.121$ **(b)** $a = 1, m = 0.168$

(c) $a = 0.1, m = 0.153$ **(d)** $a = 0.1, m = 0.167$

Figure 3.1: Graphs of $\Gamma_m(G_m(\sigma), \pi)$ in Example 1.

respectively, for $\pi = g$ and $\pi = b$. Through these figures, we can see that with small a, the gap in the minimum requirement of m between $\pi = g$ and $\pi = b$ is small.

3.7 Discussion

3.7.1 Asymmetric cost distribution

In the case of costly interaction with campaigner, we assume that H' is symmetric at 0. This section considers a case that the assumption is violated. To see the case, we consider the following distribution.

$$H'(a) = \begin{cases} \delta\psi(a) & \text{if } a > 0 \\ (1-\delta)\psi(|a|) & \text{if } a \leqslant 0, \end{cases}$$

where $\int_0^\infty \psi(a) = 1$. Smaller δ implies that cost of refusing a signature is more likely to be the greater issue for citizens than the cost of signing. When $\delta = 1/2$, H' is symmetric at 0. Assume the assumptions of Lemma 3.5. Then, as in Lemma 3.5, we define

$$\Gamma_\pi(\chi,\delta) = m\int_0^\infty [\delta\Psi(\chi v) + (1-\delta)]V_\pi(v)\,dv + m\int^0 (1-\delta)[1-\Psi(-\chi v)]V_\pi(v)\,dv.$$

In an equilibrium of the signing stage, the following equations hold:

$$\begin{aligned}
\chi_a = G_a(\chi_a) &= p\binom{n-1}{h-1}[\sigma_g(\chi_a)]^{h-1}[1-\sigma_g(\chi_a)]^{n-h},\\
\sigma_g(\chi_a) = \Gamma_g(\chi,a) &= m\int H\,(\chi v)\,\varphi(v,a)\,dv,\\
\sigma_b(\chi_a) = \Gamma_b(\chi,a) &= m\int H\,(\chi v)\,\varphi(-v,a)\,dv.
\end{aligned}$$

We see the effect of an increase in δ. Differentiating Γ by δ yields the following.

$$\begin{aligned}
\frac{\partial\Gamma_g(\chi,\delta)}{\partial\delta} &= m\int_0^\infty [\Psi(\chi v)-1]\varphi(v,a)\,dv + m\int^0 [\Psi(-\chi v)-1]\varphi(v,a)\,dv\\
&= m\int_0^\infty [\Psi(\chi v)-1][\varphi(v,a)+\varphi(-v,a)]\,dv < 0.
\end{aligned}$$

In the same way, we can show that $\frac{\partial\Gamma_g(\chi,\delta)}{\partial\delta} = \frac{\partial\Gamma_b(\chi,\delta)}{\partial\delta}$.

Second, we consider the indirect effect via cutoff point χ, which is given by

$$\frac{\partial\Gamma_g(\chi,\delta)}{\partial\chi} = \delta m\int_0^\infty \psi(\chi v)v\varphi(v)\,dv + (1-\delta)m\int^0 \psi(-\chi v)v\varphi(v)\,dv.$$

When $\delta = 1/2$, $\frac{\partial\Gamma_g(\chi,\delta)}{\partial\chi} > 0$. In the same way, we can show that $\frac{\partial\Gamma_b(\chi,\delta)}{\partial\chi} < 0$ when $\delta = 1/2$. We now see the effect on gathered signatures. If $\delta = 1/2$, the derivatives are given by

$$\frac{d\sigma_g}{d\delta} = \frac{\partial\Gamma_g/\partial\delta}{1-\frac{\partial\Gamma_g}{\partial\chi}G'(\chi)},\qquad \frac{d\sigma_b}{d\delta} = \frac{\frac{\partial\Gamma_g}{\partial\delta} - 2\frac{\partial\Gamma_g}{\partial\delta}G'(\chi)\frac{\partial\Gamma_g}{\partial\chi}}{1-\frac{\partial\Gamma_g}{\partial\chi}G'(\chi)}.$$

Then $\frac{d\sigma_g}{d\delta} > \frac{d\sigma_b}{d\delta}$ if and only if $G'(\chi) < 0$. We can easily to show that $G'(\chi) < 0$ if and only if $\sigma_g > (h-1)/(n-1)$. This condition is satisfied when the campaigner gathers the required signatures for the law with good state, which is always the case when the campaigner campaigns. Therefore, in equilibrium, $0 > \frac{d\sigma_g}{d\delta} > \frac{d\sigma_b}{d\delta}$ holds. Thus, if $\delta > 1/2$ and is it sufficiently close to $1/2$, the gap between σ_g and σ_b increases. Conversely if $\delta < 1/2$, the gap between σ_g and σ_b decreases. Therefore,

$\delta < 1/2$ is a more problematic case to force the campaigner to take the first best action.

3.8 Conclusion

To summarize, this study has examined an initiative process based on a signature-gathering campaign and explored the optimal signature requirements. As a benchmark case, we considered the situation in which the law has two states, namely good and bad. In this case, the optimal requirement is high when the gap in the popularity between good and bad states is large, which may imply that for countries where uncertainty in preferences of citizens is large, the optimal requirement is low. This result partly holds for the case in which there are uncountable states.

On the other hand, if signing and refusing a signature are costly and the population is infinite, which are common in the real world, the differences in the gathered signatures between laws vanish, which implies that the signature requirement does not make sense. Therefore, reducing costs of both signing and refusing a signature is needed. As an example of such policy, establishing an online signature-gathering system is considered. In addition, with an online signature-gathering campaign, the campaigning cost may also reduce, which will need higher requirement.

When the population is finite with small costs, unlike the above case, the result for the benchmark case holds even when both signing and not signing are costly. In addition, we show that in the case of costly interaction with the campaigner, a low requirement is optimal for more important laws.

This study has several shortcomings. One is that in our costly interaction model, the case of continuum states is not analyzed. Our discussion also depends on costless voting in the referendum stage. If the voting is costly, a somewhat different outcome is obtained.[30] Moreover, since the signing stage reveals the preference distribution of citizens, it may also affect the voting behavior.[31] This study also ignored the type of law campaigned, which may be an issue of interest in the spatial voting theory. These shortcomings are left to future research to address.

[30] See Ledyard (1984); Krishna and Morgan (2015).

[31] Taylor and Yildirim (2010) verify that information about citizens' preference distribution affects the voting behavior, and they show that much information may reduce the social welfare.

Appendix

3.A Omitted proofs

Proof of Lemma 3.1. (1) We show the first part of the lemma. Note that $U(m^{**}(\lambda), \lambda)$ is decreasing in λ, since with larger λ, it needs larger m to gather the required signatures. Moreover, since U is continuous in m and λ, by the maximum theorem, $U(m^{**}(\lambda), \lambda)$ is continuous in λ.

Note also by Assumption 3.2, $U(m^{**}(\lambda), \lambda) > 0$ for some λ. If $\lambda = 1$, even when $m = 1$, the probability that the required signatures are gathered is zero. Therefore, $U(m^{**}(1), 1) = -C(1) < 0$.

These facts imply that there exists $\bar{\lambda} \in (0, 1)$, such that for each $\lambda \geqslant \bar{\lambda}$, $U(m^{**}(\lambda), \lambda) \leqslant 0$, and for each $\lambda < \bar{\lambda}$, $U(m^{**}(\lambda), \lambda) > 0$. Then, $m^{*}(\lambda) = 0$ for each $\lambda > \bar{\lambda}$, and $m^{*}(\lambda) > 0$ for each $\lambda < \bar{\lambda}$.

(2) We now prove the second part of the lemma. Since the objective function of the campaigner may not be (quasi or pseudo) concave, it may have several local optimal m. Let $M(\lambda)$ be the set of locally optimal meeting probabilities m that can be the $m^{**}(\lambda)$ that is the chosen candidate except for 0. Note that by the first-order condition, for each $m \in M(\lambda)$, it satisfies one of the following:[32]

$$
\begin{aligned}
&(v_C + \beta)p'(\lambda/m)\lambda/m^2 = C'(m) \text{ if } \lambda < m < 2\lambda,\\
&\beta p'(\lambda/m)\lambda/m^2 = C'(m) \text{ if } 2\lambda < m,\\
&m = 2\lambda,\\
&m = \lambda.
\end{aligned}
$$

Step 1. For each $m(\lambda) \in M(\lambda)$ that satisfies the first-order condition, that is, $(v_C+\beta)p'(\lambda/m)\lambda/m^2 = C'(m)$ or $\beta p'(\lambda/m)\lambda/m^2 = C'(m)$, $\lambda/m(\lambda)$ is increasing in λ.

Let $m \in M(\lambda)$. (a) First consider the case that there exists $\varepsilon > 0$ such that m is the unique local optimum in $(m - \varepsilon, m + \varepsilon)$. Consider the case that $\lambda/m < 1/2$ and write $m = m_b(\lambda)$. By the implicit

[32] Note that $m = 1$ is excluded by the assumption. Note also that $m \leqslant \lambda$ is dominated by $m = 0$.

function theorem, $m_b(\lambda)$ is continuously differentiable at λ. By the definition, the derivative of m_b is given by

$$\beta\frac{1}{(m_b)^2}\left[\left\{p''\left(\frac{\lambda}{m_b}\right)\frac{\lambda}{m_b}+p'\left(\frac{\lambda}{m_b}\right)\right\}-\left\{p''\left(\frac{\lambda}{m_b}\right)\frac{\lambda}{m_b}+2p'\left(\frac{\lambda}{m_b}\right)\right\}\frac{\lambda m_b'}{m_b}\right]=C''(m_b)m_b'. \tag{A1}$$

For the local optimum, the second-order condition

$$0<\beta\left\{p''\left(\frac{\lambda}{m_b}\right)\frac{\lambda}{m_b}+2p'\left(\frac{\lambda}{m_b}\right)\right\}\frac{\lambda}{(m_b)^3}+C''(m_b) \tag{A2}$$

must be satisfied. We have two cases.

(a-1) Consider the case that $p''\left(\frac{\lambda}{m_b}\right)\frac{\lambda}{m_b}+p'\left(\frac{\lambda}{m_b}\right)>0$. By equation (A1) and the second-order condition, we can show that $m_b'=\frac{\partial m_b(\lambda)}{\partial\lambda}>0$. By (A1) and since $p'\geqslant 0$, we also have

$$\beta\left[p''\left(\frac{\lambda}{m_b}\right)\frac{\lambda}{m_b}+2p'\left(\frac{\lambda}{m_b}\right)\right]\frac{1}{(m_b)^2}\left(1-\frac{\lambda m_b'}{m_b}\right)>C''(m_b)m_b'>0,$$

and thus $m_b>\lambda m_b'$. This implies that $\lambda/m_b(\lambda)$ is increasing in λ.

(a-2) Consider the case that $p''\left(\frac{\lambda}{m_b}\right)\frac{\lambda}{m_b}+p'\left(\frac{\lambda}{m_b}\right)\leqslant 0$. Then, by (A1), we have $m_b'(\lambda)\leqslant 0$, and thus $\lambda/m_b(\lambda)$ is increasing in λ.

Proof for the case of $\lambda/m>1/2$ is completely parallel.

(b) Consider the case that m is not a unique local optimum in $(m-\varepsilon, m+\varepsilon)$ for each $\varepsilon>0$. Then, since the utility function is continuous in $m>0$, there is a continuum of local maximum points. Let the supremum of such point be $\bar{m}_b(\lambda)$. By Assumption 3.1, it is a candidate of the chosen meeting probability. Since $m=1$ is not optimal, $\bar{m}_b(\lambda)<1$. Thus, for each $m>\bar{m}_b(\lambda)$, the first-order condition does not hold. This implies that the second-order condition (A2) holds at $m_b=\bar{m}_b(\lambda)$. Therefore, by the implicit function theorem, $\bar{m}_b(\lambda)$ is continuously differentiable, and $\lambda/\bar{m}_b(\lambda)$ is also increasing in λ.

Step 2. $\lambda/m^{**}(\lambda)$ is increasing in λ.

We now prove that $\lambda/m^{**}(\lambda)$ is increasing in λ. Note that $M^*(\lambda)$ is divided into a finite set of closed intervals, $\mathcal{M}^*(\lambda)$. For each $I\in\mathcal{M}^*(\lambda)$, let $m_I^{***}(\lambda)=\sup I$. Then, for each $I, I'\in\mathcal{M}^*(\lambda)$, there exists $\varepsilon>0$ such that $|m_I^{***}(\lambda)-m_{I'}^{***}(\lambda)|>\varepsilon$. Let $m^{**}(\lambda)=m_{I^*}^{***}(\lambda)$ for some $I^*\in\mathcal{M}^*(\lambda)$.

(a) Consider the case that $U(m^{**}(\lambda),\lambda)>U(m_I^{***},\lambda)$ for each $I\in\mathcal{M}^*(\lambda)/\{I^*\}$. By the continuity of U, for each $\varepsilon>0$, there exists $\delta>0$ such that for each $\lambda'\in(\lambda-\delta,\lambda+\delta)$, $m^{**}(\lambda')\in(m^{**}(\lambda)-$

$\varepsilon, m^{**}(\lambda)+\varepsilon)$. Therefore, as shown in the above, $\partial(\lambda/m^{**}(\lambda))/\partial\lambda > 0$.

(b) Consider the case that $U(m^{**}(\lambda),\lambda) = U(m_I^{***}(\lambda),\lambda)$ for some $I \in \mathcal{M}^*(\lambda)/I^*$. Let $m_I^{***}(\lambda)$ be the minimum one that satisfies the above equality. By the assumption, $m^{**}(\lambda) > m_I^{***}(\lambda)$. To simplify the notation, let $m = m^{**}(\lambda)$ and $m' = m_I^{***}(\lambda)$.

(b-1) Assume that $\lambda/m' > \lambda/m > 1/2$. Then, by the first-order condition,

$$(v_C+\beta)p'(\lambda/m)\lambda/m = mC'(m) > m'C'(m') = (v_C+\beta)p'(\lambda/m')\lambda/m',$$
$$\text{and thus } (v_C+\beta)p'(\lambda/m)(1/m) > (v_C+\beta)p'(\lambda/m')(1/m').$$

Note that by the envelope theorem, $\frac{dU(m(\lambda),\lambda)}{d\lambda} = -(v_C+\beta)p'(\lambda/m)(1/m)$. Therefore, for each $\lambda' > \lambda$ that is sufficiently near λ, $U(m',\lambda') > U(m,\lambda')$. Thus, $m^{**}(\lambda')$ is in neighborhood of $m' = m_I^{***}(\lambda)$. Since $m^{**}(\lambda) > m_I^{***}(\lambda)$, $\lambda/m^{**}(\lambda)$ is also increasing at λ. For the case of $\lambda/m' > 1/2 > \lambda/m$ and $1/2 > \lambda/m' > \lambda/m$, the proof is parallel.

(b-2) Consider the case that $\lambda/m' = 1/2 > \lambda/m$. In this case, we write $m = m_b(\lambda)$. By this condition, $U(m',\lambda) - U(m,\lambda) = 0$, that is,

$$C(m_b(\lambda)) - C(2\lambda) - \beta\int_{\lambda/m_b(\lambda)}^{1/2} dp(\pi) = 0.$$

We show that the difference $U(m',\lambda) - U(m_b(\lambda),\lambda)$ is increasing in λ. To show this, we note that by the envelope theorem and the first-order condition, $\frac{\partial U(m_b(\lambda),\lambda)}{\partial\lambda} = -\beta p'(\lambda/m_b(\lambda))(1/m_b(\lambda)) = -m_b(\lambda)/\lambda C'(m_b(\lambda))$. Since $m_b(\lambda) > 2\lambda$ and $\frac{dU(m',\lambda)}{d\lambda} = \frac{dU(2\lambda,\lambda)}{d\lambda} = 2C'(2\lambda)$, $\frac{dU(m',\lambda)}{d\lambda} > \frac{dU(m_b(\lambda),\lambda)}{d\lambda}$

Therefore, for each $\lambda' > \lambda$ such that $m_b(\lambda)$ exists, $U(m',\lambda') - U(m_b(\lambda'),\lambda') > 0$. Thus, $\lambda'/m' = 1/2 > \lambda'/m_b(\lambda') > \lambda/m_b(\lambda)$.

(b-3) Consider the case that $\lambda/m' > 1/2 = \lambda/m$. For the case that $\lambda/m = 1/2$, $\frac{dU(2\lambda,\lambda)}{d\lambda} = -2C'(2\lambda)$. On the other hand, by the envelope theorem and the first-order condition, $\frac{dU(m'(\lambda),\lambda)}{d\lambda} = -(v_C + \beta)p'(\lambda/m')(1/m') = -m'/\lambda C'(m')$. Since $m' < 2\lambda$, $\frac{dU(m'(\lambda),\lambda)}{d\lambda} > -2C'(m') > -2C'(2\lambda) = \frac{dU(2\lambda,\lambda)}{d\lambda}$. Therefore, for $\lambda' > \lambda$, m' that satisfies $\lambda'/m' > 1/2$ is optimal. Thus, we can conclude that as λ increases, $\lambda/m^{**}(\lambda)$ increases.

(b-4) Consider the case that $m' = \lambda$. Then, $U(m',\lambda) = -C(\lambda)$. Note that $\frac{dU(\lambda,\lambda)}{d\lambda} = -C'(\lambda)$.

(b-4-1) $\lambda/m > 1/2$. Then, as in case (b-3), $\frac{dU(m',\lambda)}{d\lambda} - \frac{dU(m,\lambda)}{d\lambda} > 0$.

(b-4-2) $\lambda/m = 1/2$. Then, since $U(m',\lambda) - U(m,\lambda) = C(2\lambda) - C(\lambda)$, by the convexity of C, $\frac{dU(m',\lambda)}{d\lambda} - \frac{dU(m,\lambda)}{d\lambda} > 0$.

(b-4-3) $\lambda/m < 1/2$. As in case (b-2), $\frac{dU(m',\lambda)}{d\lambda} - \frac{dU(m,\lambda)}{d\lambda} > 0$.

The above discussion concludes the proof of Step 2. By (1) and Step 2, we conclude that $\lambda/m^*(\lambda)$ is increasing in λ. □

Proof of Lemma 3.2. For the proof, we provide the following claim.

Claim 3.1. *Under Assumption 3.4, the number of m that satisfies the first-order condition is at most one, and the second-order condition is always satisfied.*

Proof of Claim 3.1. By the assumption, $p'(\lambda/m)\lambda/m^2$ is weakly decreasing in m. Therefore, since $C'(m)$ is strictly increasing, the number of m that satisfies the first-order condition is at most one. Since $p' \geqslant 0$, we can easily check the second-order condition under the assumption. □

Suppose by contradiction that there is no λ_K^*. We first show that there is λ such that $(v_C + \beta)p'(\pi_K)\pi_K = \lambda/\pi_K C'(\lambda/\pi_K)$. Suppose not. Consider the case that $(v_C+\beta)p'(\pi_K)\pi_K > \lambda/\pi_K C'(\lambda/\pi_K)$ for some λ. Note that as $\lambda \to \pi_K$, $C'(\lambda/\pi_K) \to \infty$. Then, there exists λ that satisfies the first-order condition, which is a contradiction. Then, $(v_C + \beta)p'(\pi_K)\pi_K < \lambda/\pi_K C'(\lambda/\pi_K)$ for each λ. By $\lambda \to 0$, the left-hand side goes to 0, a contradiction. Therefore, there exists λ such that $(v_C + \beta)p'(\pi_K)\pi_K = \lambda/\pi_K C'(\lambda/\pi_K)$. Then, in this case, as no other m except for $m = \lambda/\pi_K$ satisfies the first-order condition, $m^{**}(\lambda) = \lambda/\pi_K$ or $m^{**}(\lambda) = 2\lambda$. By assumption, the latter case holds. Note that by Assumption 3.4, $p'(\lambda/m)\lambda/m$ is decreasing. Furthermore, since $mC'(m)$ is increasing, $(v_C + \beta)p'(1/2)(1/2) < 2\lambda C'(2\lambda)$. This implies that $m = 2\lambda$ is not local optimum as reducing m improves the campaigner's utility, which is a contradiction. □

Proof of Proposition 3.1. Let m^{**} and $\widehat{m}^{**}$ be the optimal meeting probability except for 0. We can easily show that $\lambda^*/m^*(\lambda^*) \leqslant \pi_K$ and $\widehat{\lambda}^*/\widehat{m}^*(\lambda^*) \leqslant \pi_K$. Therefore, we have the following four cases.

(1) $\lambda^*/m^*(\lambda^*) = \widehat{\lambda}^*/\widehat{m}^*(\lambda^*) = \pi_K$. Then,

$$(v_C + \beta)p'(\pi_K)\pi_K = m^*(\lambda^*)C'(m^*(\lambda^*)),$$
$$(v_C + \beta)\widehat{p}'(\pi_K)\pi_K = \widehat{m}^*(\widehat{\lambda}^*)C'(\widehat{m}^*(\widehat{\lambda}^*)).$$

Since $\pi_K \in (\pi^1, \widehat{\pi}) \subset (\pi^1, \pi^2)$, $p'(\pi_K) > \widehat{p}'(\pi_k)$. Therefore, $m^*(\lambda^*)C'(m^*(\lambda^*)) > \widehat{m}^*(\widehat{\lambda}^*)C'(\widehat{m}^*(\widehat{\lambda}^*))$, and thus $m^*(\lambda^*) > \widehat{m}^*(\widehat{\lambda}^*)$. Since $\lambda^*/m^*(\lambda^*) = \widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) = \pi_K$, $\lambda^* > \widehat{\lambda}^*$.

(2) $\lambda^*/m^*(\lambda^*) < \widehat{\pi}$ and $\widehat{\lambda}^*/\widehat{m}^*(\lambda^*) < \widehat{\pi}$. Then, $U(m^*(\lambda^*), \lambda^*) = \widehat{U}(\widehat{m}^*(\widehat{\lambda}^*), \widehat{\lambda}^*) = 0$, where $\widehat{U}$ is the expected utility of the campaigner when the probability measure is $\widehat{p}$.

If $\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) > 1/2$, the expected utility is given by

$$U(m,\lambda) = (v_C + \beta)\int_{\lambda/m}^{1} dp(\pi) - C(m),$$
$$\widehat{U}(m,\lambda) = (v_C + \beta)\int_{\lambda/m}^{1} d\widehat{p}(\pi) - C(m).$$

Since $\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) < \widehat{\pi}$, $1 - p(\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*)) > 1 - \widehat{p}(\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*))$. Therefore, $U(m^*(\widehat{\lambda}^*), \widehat{\lambda}^*) \geqslant U(\widehat{m}^*(\widehat{\lambda}^*), \widehat{\lambda}^*) > \widehat{U}(\widehat{m}^*(\widehat{\lambda}^*), \lambda^*)$. Since $U(m^*(\lambda^*), \lambda^*) = 0$ and $U(m^*(\lambda), \lambda)$ is decreasing in λ, $\lambda^* > \widehat{\lambda}^*$.

Consider the case that $\widehat{\lambda}^*/\widehat{m}^*(\widehat{\lambda}^*) < 1/2$. Then, the expected utility is given by

$$U(m,\lambda) = (v_C + \beta)\int_{1/2}^{1} dp(\pi) + \beta\int_{\lambda/m}^{1/2} dp(\pi) - C(m),$$
$$\widehat{U}(m,\lambda) = (v_C + \beta)\int_{1/2}^{1} d\widehat{p}(\pi) + \beta\int_{\lambda/m}^{1/2} d\widehat{p}(\pi) - C(m).$$

Since $\widehat{\pi} \geqslant 1/2$, we also have $U(\widehat{m}^*(\widehat{\lambda}^*), \widehat{\lambda}^*) \geqslant \widehat{U}(\widehat{m}^*(\widehat{\lambda}^*), \widehat{\lambda}^*)$. Thus, we can show that $\lambda^* \geqslant \widehat{\lambda}^*$.

(3) $\lambda^*/m^*(\lambda^*) > \widehat{\pi}$ and $\widehat{\lambda}^*/\widehat{m}^*(\lambda^*) > \widehat{\pi}$. Since $p(\lambda^*/m^*(\lambda^*)) > \widehat{p}(\lambda^*/m^*(\lambda^*))$, as the same logic in case (2), we can show that $\lambda^* \leqslant \widehat{\lambda}^*$. □

Proof of Proposition 3.2. (1) Consider the case that $U(m^*(\lambda_k^*), \lambda_k^*) > U(2\lambda_k^*, \lambda_k^*)$. Then, $m^*(\lambda_k^*)$ is the interior solution, and thus it satisfies the first-order condition. Furthermore, since $\lambda_k^*/m^*(\lambda_k^*) < \pi_K$, $U(m^*(\lambda_k^*), \lambda_k^*) = 0$. We consider the case that $\lambda_k^*/m^*(\lambda_k^*) < 1/2$ (The following discussion also holds in the case of $\lambda_k^*/m^*(\lambda_k^*) > 1/2$). Therefore, there exists $\varepsilon > 0$, and for each $k \in (k-\varepsilon, k+\varepsilon)$ and each $\lambda \in (\lambda - \varepsilon, \lambda + \varepsilon)$,

$$(v_C + \beta)P + \beta\int_{\lambda_k^*/m_k^*(\lambda_k^*)}^{1/2} dp(\pi) - kC(m_k^*(\lambda_k^*)) = 0, \tag{A3}$$

$$\beta p'(\lambda/m_k^*(\lambda))\frac{\lambda}{(m_k^*(\lambda))^2} = kC'(m_k^*(\lambda)). \tag{A4}$$

Note that λ_k^* is determined by (A3) and $m_k^*(\lambda)$ is determined by (A4). With a small change in k, the same condition is maintained. Thus, by differentiating both sides of (A3) by k and (A4),

$$\begin{aligned} &-\beta p'(\lambda_k^*/m^*(\lambda_k^*))\left(\frac{\frac{\partial \lambda_k^*}{\partial k}}{m^*(\lambda_k^*)} - \frac{\lambda_k^*}{(m^*(\lambda_k^*))^2}\frac{d[m_k^*(\lambda_k^*)]}{dk}\right) \\ &= C(m_k^*(\lambda_k^*)) + kC'(m_k^*(\lambda_k^*))\frac{d[m_k^*(\lambda_k^*)]}{dk} \end{aligned} \tag{A5}$$

$$m_k(\lambda_k^*) = -\frac{\partial \lambda_k^*}{\partial k}\beta p'(\lambda_k^*/m^*(\lambda_k^*))\frac{1}{C(m_k^*(\lambda_k^*))}, \tag{A6}$$

where

$$\frac{d[m_k^*(\lambda_k^*)]}{dk} = \frac{\partial m_k^*}{\partial k}(\lambda_k^*) + \frac{\partial m_k^*}{\partial \lambda}(\lambda_k^*)\frac{d\lambda_k^*}{dk}.$$

Note that by equation (A4) and Assumption 3.1, $\frac{\partial m_k^*}{\partial k}(\lambda) < 0$ for each λ. Substituting (A6) into (A4) yields

$$\frac{\partial \lambda_k^*}{\partial k}kC'(m_k^*(\lambda_k^*)) = -\frac{\lambda_k^*}{m_k^*(\lambda_k^*)}C(m_k^*(\lambda_k^*)).$$

Abusing a notation, we write $p'' = p''(\lambda_k^*/m_k^*(\lambda_k^*))$, $p' = p'(\lambda_k^*/m_k^*(\lambda_k^*))$, $m_k^*(\lambda_k^*) = m_k^*$, $C = C(m_k^*(\lambda_k^*))$, $C' = C'(m_k^*(\lambda_k^*))$, and $C'' = C''(m_k^*(\lambda_k^*))$.

Then by the above equations and calculating $\frac{\partial m_k^*}{\partial k}$ and $\frac{\partial m_k^*}{\partial \lambda}$ using the first-order condition,

$$\begin{aligned} C(m_k^*(\lambda_k^*)) + kC'(m_k^*(\lambda_k^*))\frac{d[m_k^*(\lambda_k^*)]}{dk} &= \left[1 - \frac{k(C')^2/C + \beta[p''\lambda_k^*/m_k^* + p']\lambda_k^*/(m_k^*)^3}{kC'' + \beta[p''\lambda_k^*/m_k^* + 2p']\lambda_k^*/(m_k^*)^3}\right]C \\ &= \left[1 - \frac{k(C')^2/C - kC'/m_k^* + \beta[p''\lambda_k^*/m_k^* + 2p']\lambda_k^*/(m_k^*)^3}{kC'' + \beta[p''\lambda_k^*/m_k^* + 2p']\lambda_k^*/(m_k^*)^3}\right]C. \end{aligned}$$

The second equality follows from the first-order condition. By this equation, since $m < 1$, $p' > 0$, and $C'' > 0$, the following equation

$$C(m_k^*(\lambda_k^*)) + kC'(m_k^*(\lambda_k^*))\frac{d[m_k^*(\lambda_k^*)]}{dk}$$

is positive if and only if $C'' > (C')^2/C - C'/m_k^*$.

Then, by this and equation (A5), we have

$$\frac{d\lambda_k^*/m_k^*(\lambda_k^*)}{dk} = \left(\frac{\frac{\partial \lambda_k^*}{\partial k}}{m_k^*(\lambda_k^*)} - \frac{\lambda_k^*}{(m_k^*(\lambda_k^*))^2} \frac{\partial m_k^*(\lambda)}{\partial k} \right) < 0,$$

if and only if $C'' > (C')^2/C - C'/m_k^*$. This inequality is equivalent to $\left[\frac{C'(m)}{C(m)/m}\right]' > 0$ at $m = m_k^*$.

(2) Consider the case that $U(m, \lambda_k^*) < U(2\lambda_k^*, \lambda_k^*)$ for each $m \neq \lambda_k^*$. Then, $m = \lambda_k^*$ is unique optimal. Let M be the set of m that satisfies the first-order condition. As shown in Claim 3.1, M is single, and thus it is continuous at the parameters. Let $m_k^{***}(\lambda_k^*)$ be the unique element of M. Then, $U(m_k^{***}(\lambda_k^*), \lambda_k^*) < U(2\lambda_k^*, \lambda_k^*)$. Therefore, with a small change in k and λ_k^*, $m = 2\lambda_k^*$ is unique optimal. Thus, $\lambda_k^*/m_k^*(\lambda_k^*) = 1/2$ for a small change in k, and, consequently, $\frac{d\lambda_k^*/m_k^*(\lambda_k^*)}{dk} = 0$. □

Proof of Proposition 3.3. In each equilibrium,

$$\sigma_\pi(n) = m \int H \left(D_n(v_i) \binom{n-1}{h-1} (\sigma_\pi(n))^{h-1} (1 - \sigma_\pi(n))^{n-h-1} v_i \right) dV_\pi(v_i).$$

Since for each $v \in \mathbb{R}$, $H(\binom{n-1}{h-1}(\sigma_\pi(n))^{h-1}(1-\sigma_\pi(n))^{n-h-1}v) \to H(0)$ pointwisely, as $n \to \infty$, and H is bounded above, the bounded convergence theorem implies

$$\begin{aligned}
\lim_{n\to\infty} \sigma_\pi(n) &= \lim_{n\to\infty} m \int H \left(D_n(v_i) \binom{n-1}{h-1} (\sigma_\pi(n))^{h-1} (1 - \sigma_\pi(n))^{n-h-1} v_i \right) dV_\pi(v_i) \\
&= m \int \lim_{n\to\infty} H \left(D_n(v_i) \binom{n-1}{h-1} (\sigma_\pi(n))^{h-1} (1 - \sigma_\pi(n))^{n-h-1} v_i \right) dV_\pi(v_i) \\
&= mH(0).
\end{aligned}$$

□

Proof of Lemma 3.3. We first note that for each H, $\Gamma_\pi(0, m) = mH(0)$. The derivative of $\Gamma_\pi(\chi, m)$ is given by

$$\frac{\partial \Gamma_\pi}{\partial \chi}(\chi, m) = m \int H'(\chi v_i) v_i \, dV_\pi(v_i).$$

First note that $\frac{\partial \Gamma_\pi}{\partial \chi}(0, m) = H'(0) \int v \, dV_\pi(v)$.

Consider the case that $\pi \in \Pi_+$. We show that $\frac{\partial \Gamma_\pi}{\partial \chi}(\chi, m) \geqslant 0$ for each χ, m. Since H' is symmetric at 0, $H'(\chi v_i)v_i = -H'(-\chi v_i)v_i$ for each $v_i \geqslant 0$. Then, by Assumption 3.5, there exists a 0-symmetric distribution Φ such that $V_\pi'(v)/\Phi'(v)$ is monotone. Since $1 - V_\pi(0) > 1/2 = 1 - \Phi(0)$, $V_\pi'(v)/\Phi'(v)$ is nondecreasing. If not, by the monotonicity condition, $V_\pi'(v)/\Phi'(v)$ is nonincreasing, which implies that Φ first-order stochastically dominates V_π. Then, $1 - V_\pi(0) \leqslant 1 - \Phi(0) = 1/2$, which is a contradiction. Therefore, for each $v > 0$,

$$\frac{V_\pi'(v)}{\Phi'(v)} \geqslant \frac{V_\pi'(-v)}{\Phi'(-v)}.$$

Since Φ is symmetric at 0, $\Phi'(v) = \Phi'(-v)$, which implies that $V_\pi'(v) \geqslant V_\pi'(-v)$. Then, $\frac{\partial \Gamma_m}{\partial \chi}(\chi, \pi) \geqslant 0$, which implies that $\Gamma_m(\chi, \pi) > mH(0)$ for each $\chi > 0$. Thus, in equilibrium, $\sigma_\pi > mH(0)$. In the same way, we can show that if $\pi \in \Pi_-$, $\sigma_\pi < mH(0)$. □

Proof of Lemma 3.4. Let χ_α be the equilibrium, where α is some parameter. Then, it satisfies

$$\chi_\alpha = G_\alpha(\chi_\alpha) := \sum_{\pi' \in \Pi_+} \left[p(\pi') \binom{n-1}{h-1} [\sigma_{\pi'}(\chi_\alpha)]^{h-1} [1 - \sigma_{\pi'}(\chi_\alpha)]^{n-h} \right].$$

$$\sigma_\pi(\chi) = \Gamma_\pi(\chi, m) = m \int H\left(\chi v_i / k\right)\, dV_\pi(v_i).$$

Since G_α is a continuous self-map, it admits a fixed point. Moreover, since G_α is continuous, the set of fixed points is closed and thus compact. Therefore, the maximum fixed point exists. □

Proof of Proposition 3.4. We first consider the case that for each $\pi \in \Pi_+$, $p(\pi)$ increases. Then, $G_\alpha(\chi_\alpha)$ also increases for each χ_α. Therefore, the maximum fixed point also increases. If $\pi \in \Pi_+$, as see in the proof of Lemma 3.3, $\sigma_\pi(\chi_\alpha)$ also increases, while if $\pi \in \Pi_-$, $\sigma_\pi(\chi_\alpha)$ decreases.

Second, we consider the case that k increases.

(1) Suppose that $\frac{dG_k((\Gamma_\pi(\chi_k, m))_\pi)}{d\chi} < 1$. We first show that χ_k is continuously differentiable at k. By an abuse of notation, we write $G_k(\chi) = G_k((\Gamma_\pi(\chi, m))_\pi)$. By the implicit function theorem, there exists $\delta > 0$ such that for each $k' \in (k - \delta, k + \delta)$, the solution to $G_{k'}(\chi) = \chi$ is uniquely determined. Let $\widehat{\chi}(k')$ be the solution. Note that $\chi_{k'}$ is the maximum χ that satisfies $G_{k'}(\chi) = \chi$. Note also that $G_{k'}$ is continuous in k'. Since we consider the maximum equilibrium, for each $\chi > \chi_k$, $G_k(\chi) < \chi$. Then, by letting $k' \to k$, $\chi_{k'}$ can be arbitrarily close to χ_k, since if not, there is χ that satisfies $G_k(\chi) = \chi$ and $\chi > \chi_k$. Therefore, $\chi_{k'} = \widehat{\chi}(k')$ for each $k' \in (k - \delta, k + \delta)$. The implicit function theorem also implies that χ_k is continuously differentiable by k at k .

Differentiating σ_π by k yields the following:

$$\frac{d\sigma_\pi(\chi_\alpha)}{dk} = -m \int H'(\chi_\alpha v/k)\frac{v}{k}\, dV_\pi(v) \left[\frac{1}{k}\chi_\alpha - \frac{d\chi_\alpha}{dk}\right]$$
$$\frac{d\chi_\alpha}{dk} = \sum_{\pi' \in \Pi_+} X_{\pi'} \frac{\partial \sigma_{\pi'}(\chi_\alpha)}{\partial k},$$

where

$$X_{\pi'} = \left[p(\pi')\binom{n-1}{h-1}[h-1-(n-1)\sigma_{\pi'}(\chi_\alpha)][\sigma_{\pi'}(\chi_\alpha)]^{h-2}[1-\sigma_{\pi'}(\chi_\alpha)]^{n-h-1}\right].$$

Let $A_\pi = m\int H'(\chi_\alpha v/k)\frac{v}{k}\, dV_\pi(v)$. By calculating the derivative, $\frac{d\chi_\alpha}{dk} = \frac{-\sum_{\pi' \in \Pi_+} X_{\pi'}A_{\pi'}}{1-\sum_{\pi' \in \Pi_+} X_{\pi'}A_{\pi'}}\frac{1}{k}\chi_\alpha$.
Note that

$$\frac{dG_k((\Gamma_\pi(\chi_k,\alpha))_\pi)}{d\chi} = \sum_{\pi' \in \Pi_+} A_{\pi'}X_{\pi'} < 1.$$

We have two cases. Consider the case that $\sum_{\pi' \in \Pi_+} A_{\pi'}X_{\pi'} > 0$. Since $-\sum_{\pi' \in \Pi_+} A_{\pi'}X_{\pi'} > -1$, $\frac{\partial \chi_\alpha}{\partial k} < 0$, and thus $\frac{1}{k}\chi_\alpha - \frac{\partial \chi_\alpha}{\partial k} > 0$.

Next, consider the case that $\sum_{\pi' \in \Pi_+} A_{\pi'}X_{\pi'} \leqslant 0$. Then, $\frac{\partial \chi_\alpha}{\partial k} < -\frac{1}{k}\chi_\alpha$. Therefore, $\frac{1}{k}\chi_\alpha - \frac{\partial \chi_\alpha}{\partial k} > 0$. Note that as seen in the proof of Lemma 3.3, $A_\pi < 0$ if and only if $\pi \in \Pi_+$. Thus, in each case, $\frac{\partial \sigma_\pi(\chi_\alpha)}{\partial k} > 0$ if and only if $\pi \in \Pi_-$.

(2) Consider the case that $\frac{dG_k((\Gamma_\pi(\chi_k,\alpha))_\pi)}{d\chi} \geqslant 1$. Since for each $\pi \in \Pi_+$, $m\int H(\chi v)\, dV_\pi(v)$ is increasing in χ, $m\int H((\chi/k)v)\, dV_\pi(v) > m\int H((\chi'/k')v)\, dV_\pi(v)$ if and only if $\chi/k > \chi'/k'$. Therefore, it is sufficient to show that χ_k/k is decreasing at k.

Note that

$$\frac{\partial \Gamma_\pi}{\partial k}(\chi,\alpha) = -\chi m \int H'(\chi v/k)\frac{v}{k^2}\, dV_\pi(v) = -\frac{\chi}{k}A_\pi.$$

Thus, since $\sum_{\pi' \in \Pi_+} A_{\pi'}X_{\pi'} \geqslant 1$,

$$\frac{\partial G_k}{\partial k} = -\sum_{\pi' \in \Pi_+} \frac{\chi}{k}A_{\pi'}X_{\pi'} < 0.$$

Note that since we consider the maximum equilibrium, $G_k(\chi) < \chi$ for each $\chi > \chi_k$. Therefore, for each $k' > k$ and each $\chi > \chi_k$, $G_{k'}(\chi) < \chi$. Thus, $\chi_{k'} < \chi_k$. On the other hand, since for each

$k < k'$, $G_{k'}(\chi_k) > \chi_k$, there exists χ such that $G_{k'}(\chi) = \chi$ and $\chi > \chi_k$. Since we consider the maximum equilibrium, $\chi_{k'} > \chi_k$, which concludes the proof.

The proof of the case for the effect of ι is omitted since it is completely symmetric to the case of the effect of k.

□

Proof of Proposition 3.5. Note that

$$\begin{aligned}
\chi_a = G_a(\chi_a) = p\binom{n-1}{h-1}[\sigma_g(\chi_a)]^{h-1}[1-\sigma_g(\chi_a)]^{n-h},\\
\sigma_g(\chi_a) = \Gamma_g(\chi, a) = m\int H\,(\chi v)\,\varphi(v,a)\,dv,\\
\sigma_b(\chi_a) = \Gamma_b(\chi, a) = m\int H\,(\chi v)\,\varphi(-v,a)\,dv.
\end{aligned}$$

By the assumption, for each χ, $\Gamma_g(\chi, a)$ is increasing in a while $\Gamma_b(\chi, a)$ is decreasing in a.

Suppose that $\frac{dG_a(\Gamma_g(\chi_a,a))}{d\chi} < 1$. Then, as in the proof of Proposition 3.4, χ and σ_π are differentiable at a. Differentiating σ_π by a yields the following:

$$\begin{aligned}
\frac{d\sigma_\pi}{da} &= \frac{\partial\Gamma_\pi(\chi,a)}{\partial a} + \frac{\partial\Gamma_\pi(\chi,a)}{\partial\chi}\frac{\partial\chi}{\partial a},\\
\frac{d\chi}{da} &= p\binom{n-1}{h-1}[h-1-(n-1)\sigma_g(\chi_\alpha)][\sigma_g(\chi_\alpha)]^{h-2}[1-\sigma_g(\chi_\alpha)]^{n-h-1}\frac{\partial\sigma_g}{\partial a}.
\end{aligned}$$

Consider the case of $\pi = g$. Let $A_\pi = \frac{\partial\Gamma_\pi(\chi,a)}{\partial a}$, $B_\pi = \frac{\partial\Gamma_\pi(\chi,a)}{\partial\chi}$ and

$$X = p\binom{n-1}{h-1}[h-1-(n-1)\sigma_g(\chi_\alpha)][\sigma_g(\chi_\alpha)]^{h-2}[1-\sigma_g(\chi_\alpha)]^{n-h-1}.$$

Then, $\frac{\partial\sigma_g}{\partial a} = \frac{A_g}{1-B_gX}$. Note that $A_g > 0$. Note also that $\frac{dG_a(\Gamma_g(\chi_a,a))}{d\chi} = B_gX < 1$. Therefore, $\frac{\partial\sigma_g}{\partial a} > 0$.

Next, consider the case of $\pi = b$. Then, $\frac{\partial\sigma_b}{\partial a} = A_b + B_b\frac{A_gX}{1-B_gX}$. Note that $A_b < 0$. Note also that since H' is symmetric at 0,

$$\begin{aligned}
B_g &= m\int H'(\chi v)v\varphi(v,a)\,dv\\
&= -m\int H'(\chi(-v))(-v)\varphi(v,a)\,dv\\
&= -m\int H'(\chi(v))(v)\varphi(-v,a)\,dv = -B_b.
\end{aligned}$$

Therefore, $\frac{\partial \sigma_b}{\partial a} = A_b - \frac{A_g B_g X}{1 - B_g X}$. If $B_g X > 0$, since $A_g > 0$, $\frac{\partial \sigma_b}{\partial a} < 0$.

Consider the case that $B_g X < 0$. By integration by parts, we have

$$\begin{aligned}
A_g &= m \int H(\chi v) \frac{\partial \varphi(v, a)}{\partial a} \, dv \\
&= m \left[H(\chi v) \frac{\partial \Phi(v, a)}{\partial a} \right]_{-\infty}^{\infty} - m \int H'(\chi v) \chi \frac{\partial \Phi(v, a)}{\partial a} \, dv \\
&= -m \int H'(\chi v) \chi \frac{\partial \Phi(v, a)}{\partial a} \, dv.
\end{aligned}$$

On the other hand,

$$\begin{aligned}
A_b &= m \int H(\chi v) \frac{\partial \varphi(-v, a)}{\partial a} \, dv = m \int H(-\chi v) \frac{\partial \varphi(v, a)}{\partial a} \, dv \\
&= m \left[H(\chi v) \frac{\partial \Phi(v, a)}{\partial a} \right]_{-\infty}^{\infty} + m \int H'(-\chi v) \chi \frac{\partial \Phi(v, a)}{\partial a} \, dv \\
&= m \int H'(\chi v) \chi \frac{\partial \Phi(v, a)}{\partial a} \, dv = -A_g.
\end{aligned}$$

Therefore, $\frac{\partial \sigma_b}{\partial a} = A_b \frac{1}{1 - B_g X} < 0$.

Suppose that $\frac{dG_a(\Gamma_g(\chi_a, a))}{d\chi} \geqslant 1$. Since $\frac{\partial \Gamma_g}{\partial a}(\chi, a) > 0$, as in the proof of Proposition 3.4, we can show that χ_a is increasing at a. By this fact and the facts that $\Gamma_g(\chi, a)$ is increasing and $\Gamma_b(\chi, a)$ is decreasing χ, a, the statement of the proposition immediately follows. □

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4 Concealments of Problems: An Incentive of Reducing the Responsibility[1]

4.1 Introduction

Firms sometime have problems for their products and management systems. If these problems are ignored, they would cause accidents that harm consumers. Some of problems are growing over time and harm from the accidents gets larger as time proceeds.

As an example case, consider a mass food poisoning that caused by a Japanese dairy company, Snow Brand Milk Products Co. in 2000. It left 14,780 people ill. The direct cause was that toxic material that produced when a production line stopped by power outage sent to the next level of production line. As an organizational cause, it is considered that their crisis management has some problems. For example, no policy is placed to prevent toxic material from being mixed when a production line is stopped.[2] In this case, as time proceeds, the harm form such an accidents could get larger. This is because as the scale of plants gets larger, the harm from such a food poisoning also gets larger.

To prevent such accidents, such problems should be solved in their early. To do so, workers should report them as soon as they notice the problems. Then, do workers report them to their superior? In the above example case, failure in the crisis management are not resolved until the accident occurs. It is suspected that no one made an issue of it. However, it is also doubtful that no one noticed the failure. Indeed, workers at the plant told that the plant's manual had been ignored for years.[3] Thus, it

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[2] Failure Knowledge Database, `http://www.sozogaku.com/fkd/en/cfen/CA1000622.html`.

[3] "Snow Brand pays the price", The Japan Times, July 12, 2000.

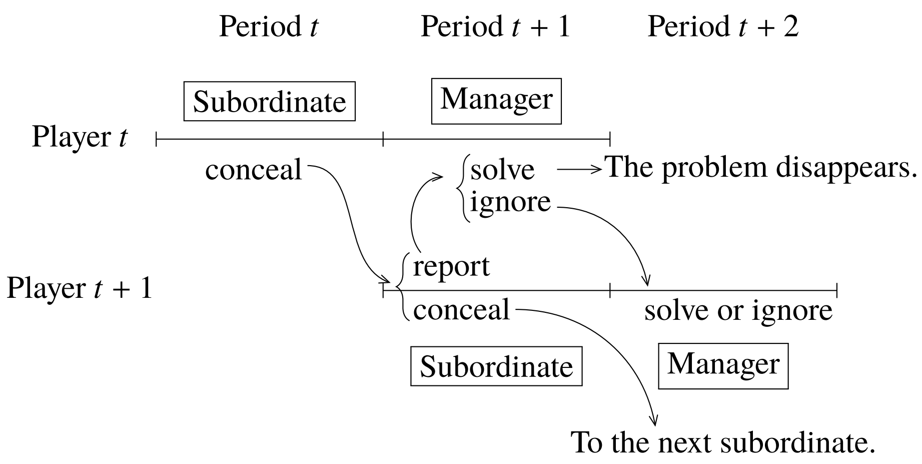

Figure 4.1: Timing of the game

is also suspected that workers have noticed the failure but not reported.

Let us consider the workers' incentive to conceal such problem even when they detect it. If they report the problems, it is possible that the reported problems are ignored. In that case, if one of the workers promotes to be a manager, he would be in charge of dealing with the reported problem. Moreover, if the reported problem remains unsolved, it is considered as a systematic illegal action, which causes harsh criticism for the firm and the manager when accident occurs. Then, workers may think that they want to avoid such trouble, which may incite a worker to conceal the problem. This paper formalizes this idea.

We consider an overlapping-generation organization that consists of two kinds of workers: a subordinate and a manager. Each worker lives for two periods: he works as a subordinate in his first period and works as a manager in his second period. In each period, the subordinate is in charge of investigating whether there is a problem and reporting it to the manager when detect it. Reporting the problem is assumed to be costless. The manager is in charge of solving the reported problem, which is costly for the manager. The cost of solving the problem depends on the manager's ability. Figure 4.1 illustrates the timing of the game.

An unsolved problem may cause accidents and in which case, only the worker in that period may be punished. If the manager does not solved the reported problem and the unsolved problem causes accidents,[4] he is exposed to harsh criticism. On the other hand, the problem have not reported when it causes accidents, criticism for the manager is softened but the subordinate is also punished for not reporting or detecting. The sizes of punishments (criticism) and the cost of solving the problem are

[4] In other case, the problem is detected by an outsider, which also causes blames for the workers.

determined by the scale of the problem. We assume that the sizes of punishments and the cost of solving a problem are proportional to the scale. The scale varies over time. For instance, consider the mass food poisoning case that we present in the beginning of this section. In this case, solving the problem corresponds to introducing a guidelines that prevent such accidents and educating workers. Such acts are more costly when the scale of plants is larger. The size of harm also depends on the scale of plants. Sizes of punishments are also larger as the size of harm gets larger. Therefore, sizes of punishments are larger when the scale of plants is larger.

We now consider the motivation of not reporting the problem in this model. Note that even if the subordinate reports the problem, when the manager's ability is too low, he ignores the reported problem. If the reported problem remains unsolved in the next period, the subordinate is responsible for solving it in the next period as a manager, which is costly for him. On the contrary, if he does not report the problem, the next subordinate is in charge of detecting and reporting the problem. The next subordinate may fail to detect the unreported problem or conceal it. Then, the manager in the next period is free from the responsibility for solving. This possibility of successful reduction of the responsibility is the motivation of concealment.

The motivation of reducing responsibility has some interesting features. First, each of reporting and concealing has complementarity, that is a subordinate is more likely to report when others report. This is because, since if the next subordinate reports, even when the subordinate conceals, the problem is reported in the next period. Then, the responsibility is not reduced. Therefore, incentive to conceal is reduced. This property suggests that the possibility of multiple equilibria. That is, both of the strategy profiles where all subordinates report and all subordinates conceal can be equilibria with the same parameter. In this case, as Kreps (1990) discusses, corporate culture has important role to determine which equilibrium is realized. That is, if "reporting" is a culture of the firm, each player believes that the others report and thus "reporting" is an equilibrium. The culture "reporting" is realized as an equilibrium. In the same way, the culture "concealment" is also realized as an equilibrium.

We can also say that if the scale of problem is increasing, concealments are more likely to occur. To see this, note that the benefit of reporting is reducing the punishment of not reporting in the present period, while the cost of reporting is the responsibility of solving the problem in the next period. If the scale is increasing, the responsibility in the next period is heavier, which implies that the cost of reporting is higher.

We see the relation between the incentive to reduce the responsibility and the size of punishment.

Note that the cost of reporting is the responsibility of solving the reported problem. Then, increasing in the size of punishment has two kinds of effects. One is increasing in the probability that the manager solves the problem. This decreases the cost of reporting. The other is decreasing in the expected utility when the reported problem remains unsolved. This increases the cost of reporting the problem since the motivation of concealment comes from reducing the future responsibility. Therefore, if the first effect is small while the second effect is large, increasing in the size of punishment makes the incentive of concealment be larger.

Increasing in the punishment does not always increase the incentive of concealment. If the punishment is sufficiently severe, an increasing in the size of punishment makes the incentive of concealment decrease. The intuition is as follows. If the punishment is sufficiently severe, the probability that the present manager solves the problem is high. If the probability is high enough, even if the responsibility in the next period becomes heavier, since the possibility of facing the unsolved reported problem is low, the effect on the cost of reporting, that is, the reported manager's expected utility is also low.

In our model, the incentive of concealment comes from the motivation of reducing the responsibility. Why do players want to reduce the responsibility? This is because dealing with the problem is costly and thus each player wants not to be responsible for solving the problem. Thus, one may consider that if there is a sufficient reward for solving the problem, this incentive would disappear. We consider introducing a reward to managers for solving problems. As long as the expected utility of the manager facing a reported problem is negative, there remains an incentive to reduce the responsibility and thus, the equilibria have the same features in the previous analysis. However, if the expected utility of managers of facing the reported problem is positive, while the incentive for reducing the responsibility disappears, another incentive of concealment arises: the subordinate has an incentive to seek the reward for solving the problem. To obtain the reward, the subordinate conceals the problem in the present period and solves it when he is a manager. In this case, the problem is more likely to be solved, but the solution is delayed.

The previous analyses assume that the manager can solve the problem certainly. However, in the real world, they may fail to solve the problem. In this case, even if the manager tries to solve problem, it is possible that the reported problem remains unsolved and the subordinate faces it as the manager in the next period. In our basic model, if the punishment is sufficiently severe, severer punishment makes the incentive of concealment small. However, in this case, we show that if the solving probability is sufficiently small, the incentive of concealment increases as punishment becomes severer.

The previous analyses also assume that the retired manager is not blamed. However, in the real world, if a player have ignored the problem when he is a manager and his negligence is detected later, even if he is retired, he would be punished. Considering this, if the scale growing rate is higher, the future punishment is heavier. Thus, if the scale is increasing and the growth rate is sufficiently high, the incentive of concealment is small. On the other hand, as in the basic model, if the scale is decreasing and the shrinkage rate is sufficiently high, since it implies the future responsibility gets smaller, the incentive of concealment is also small. Therefore, concealments are most likely to happen when scale growing rate is modest.

The remainder of this study consists of the following sections. The next subsection reviews the related researches. Section 4.2 describes our basic model. Section 4.3 summarizes the behaviors of players. Section 4.4 characterizes perfect Bayesian equilibria and section 4.5 performs comparative statics. Section 4.6 extends our basic model. Subsection 4.6.1 considers the case of introducing the rewards for managers. Subsection 4.6.2 considers the case when managers' managements of the problems may fail to solve the problem. Subsection 4.6.3 considers the case when retired managers may also be punished. All omitted proofs are delegated to the appendix.

4.1.1 Related Literature

As a research of concealment, Kerton and Bodell (1995) study firms' concealment of information about bad property of their products. In their setting, the motivation of concealment is profiting from asymmetric information between the firm and consumers by hiding the information about the products.

Our model is different from inspection games (Dresher, 1962; Maschler, 1966). The major difference between the standard inspection game and ours is that detectors may also be responsible for solving the reported problem. In the standard inspection game, inspector's responsibility is only detecting the problem.

Our study deals with workers' incentive to report problems. As a similar situation, whistleblowing is considered. In the literature of corporate governance, many papers study the motivation of whistleblowing and its consequence (For example, see a survey of literature, Dasgupta and Kesharwani, 2010 and empirical works, Bowen et al., 2010; Dyck et al., 2010).

As costs of whistleblowing, many papers consider reputational effect, revenge from the company, etc. For example, in the literature of corruption, a corrupting agent can threaten the monitor with retaliating and it motivates the monitor not to report the corruption. Chassang and Padró i Miquel

(2014) consider such situation and explore anti-corruption mechanisms. Unlike these motivations, in this paper, we assume that reporting brings no direct cost mentioned above. One of our contributions is showing that even when reporting itself is not costly and brings no such friction between the company, workers may conceal the detected problem.

Another incentive of not reporting arises when the report is non-verifiable. Consider a situation that a principal tries to make an agent effort. In case of not detecting of the agent's negligence is rewarded, since the principal cannot observes whether the agent efforts, the monitor has an incentive to misreport. Rahman (2012) proposes a contract to motivating the monitor to report truthfully.

Our study relates to the researches of crime economics (Becker, 1968). In the literature of crime economics, increasing in punishment is less costly than that in probability of conviction. However, in our model, since punishment for managers distorts subordinates' incentive of reporting, heavier punishment may cause concealment of problems. Therefore, it may increase the probability of accidents.

Our model is also relate a problem of dynamic contribution to a single public good. This is because, solving and reporting a problem is considered as kinds of contributions to a public good. As examples of such study, Bliss and Nalebuff (1984) and Bilodeau and Slivinski (1996) consider such problem with continuous time and simultaneous decision making. The models of Bolle (2011) and Bergstrom (2012) consider private provision of a public good with sequential decision making. In Bolle's setting, in each period t, only player t can contribute to the public good with discrete time. Bergstrom assumes that contribution can be done by random arriving players in continuous time.

In our model, unlike these papers, to provide a public good (solving the problem), two steps of contributions are needed: reporting and solving. In the standard model of private provision of a public good, the motivation of not contributing comes from free-riding incentives: a player does not contribute since others would do. In our model, each subordinate conceals since the manager does not solve.

4.2 Model

There are two players in each period. Player $t \in \mathbb{Z}$ lives for two periods; period t and $t+1$. In period t, player t works as a subordinate and is promoted to a manager in period $t+1$. We assume that a problem arises at some period. No other problems are caused after the period. We assume that until

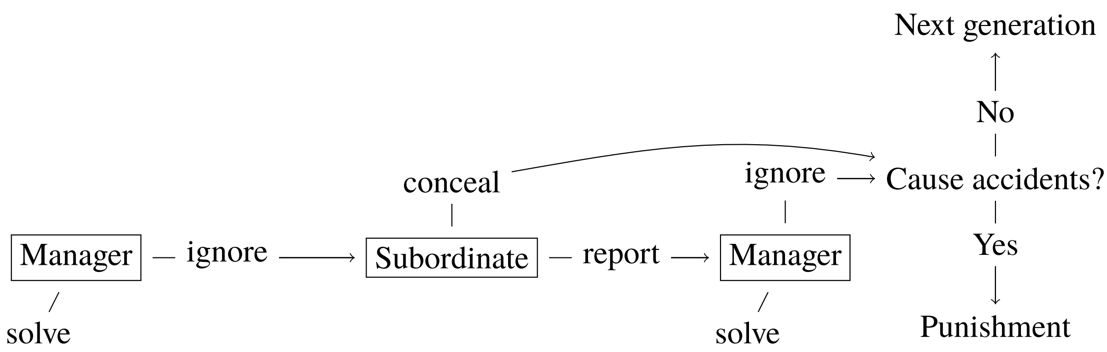

Figure 4.1: Flow of decisions in each period

a player detects or is reported the problem, he does not know that there is a problem. If a player knows the problem, he learns the period that the problem arises. The period that the problem arises is normalized to be 0. We assume that once the problem is reported, each player knows that the problem is reported.

In the beginning of each period, the subordinate detects the problem with probability $q^S \in [0, 1]$ and the manager detects the problem with probability $q^M \in [0, 1]$. If the manager has detected the problem, he decides whether to solve the problem. If the subordinate detects the problem which has not solved, he decides whether to report the problem. If the problem is reported, the manager decides whether to solve the problem. That is, the flow of decisions at each period is

1. If the manager has detected the problem (as a manager or when he is a subordinate), he decides whether to solve the problem.

2. If the problem has not solved and not reported, the subordinate who detects the problem decides whether to report the problem.

3. If the problem is reported, the manager decides whether to solve the problem.

Figure 4.1 illustrates the decision flow. When the problem is reported or solved, they are known to each player. Once the problem is reported, stage 1 and 2 are skipped. Once the problem is solved, since no problem occurs after, each player has nothing to do.

Let the scale of the problem at period t be denoted by s_t. If a subordinate detects a problem, he knows the sequence of the scales of the problem. We assume that the sequence of scales $(s_t)_{t\in\mathbb{N}}$ is exogenously given. The reward for solving the problem to the manager is denoted by $b^M s_t$ and that of reporting the problem for the subordinate is denoted by $b^S s_t$. The cost of solving the problem for the manager in period t is $c_t s_t$, where c_t is private information of the manager. Each player learns the parameter c_t after he promotes to a manager at period t. For each t, c_t follows a distribution function F independently. We assume that the support of F is an interval in $\mathbb{R}_{++}$.

If the problem is unsolved, the problem causes an accident with probability $p \in (0, 1)$.[5] If the problem causes an accident at period t, only the players in period t are blamed and the manger is forced to solve the problem. If the problem is unreported, the punishment incurs disutility $d^{M,U} s_t$ for the manager and $d^S s_t$ for the subordinate. If the problem is reported in the past and the manager does not solved, the subordinate is not punished. On the other hand, the manager is punished and the disutility from the punishment for the reported manager is $d^{M,R} s_t$. We assume that $d^{M,R} > d^{M,U}$. We also assume that even if the unsolved problem that has been reported in period $\tau < t$ causes an accident at period t, the manager at period t is punished. Finally, let $\delta \in (0, 1)$ denote the common discount factor.

4.3 Behavior of managers and subordinates

This section considers the behaviors of managers and subordinates, and summarizes their best responses.

4.3.1 Managers' behavior

We consider the manager's problem. Suppose that the problem is reported. The expected utility of solving the problem is $b^M s_t - c_t s_t$, while the expected utility of ignoring the problem is $-p(d^{M,R} s_t + c_t s_t)$. Therefore, the manager solves the reported problem if and only if

$$\frac{b^M + p d^{M,R}}{1 - p} \geqslant c_t.$$

Suppose that the manager detects (or detected in the previous period but did not report) the problem. Then, before the decision of the subordinate, the manager decides whether to solve the problem. Let r_t be the probability that the subordinate in period t reports the problem when he detects the problem.

[5] In other case, the problem is detected by an outsider with a probability, which also causes blames for the subordinate and the manager in the period.

Then, the expected utility of ignoring the problem is

$$-(1-q^S r_t)p(d^{M,U}s_t + c_t s_t) + q^S r_t \max\{b^M s_t - c_t s_t, -p(d^{M,R}s_t + c_t s_t)\}.$$

Therefore, the manager solves the problem if and only if

$$\frac{b^M + pd^{M,U}}{1-p} \geqslant c_t.$$

Since we have assumed that $d^{M,R} > d^{M,U}$, the manager is more likely to solve the problem when the problem is reported.

4.3.2 Subordinates' behavior

Since we have characterized the behaviors of managers, we consider the behaviors of subordinates. If the problem has been reported or the subordinate fails to detect the problem, he has nothing to do. Suppose that the subordinate detects the unreported problem.

We first consider the belief of the subordinate about his manager's cost. Let $I(r_{t-1})$ be the updated subordinate's belief about that the manager ignores the reported problem when the subordinate detects an unreported problem. If the problem has been solved or reported, the subordinate learns it and he has nothing to do. Therefore, when the subordinate decides whether to report, he learns that the manager does not solve the problem in the first stage of the period. This implies that the manager detected and ignores the problem or the manager does not detect it. Then, I is defined as

$$I(r_{t-1}) := \frac{(q^S(1-r_{t-1}) + (1-q^S))\left[1 - F\left(\frac{b^M + pd^{M,R}}{1-p}\right)\right]}{(q^S(1-r_{t-1}) + (1-q^S)q^M)\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1-q^S)(1-q^M)}$$

Since the problem happens in period 0, manager 0 cannot know the problem when he was a subordinate. Therefore, if the manager knows the problem, he found it after he proceeds to a manager. Therefore the belief of subordinate 0 about his manager is

$$I_0 := \frac{1 - F\left(\frac{b^M + pd^{M,R}}{1-p}\right)}{q^M\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1-q^M)}.$$

Note that $I_0 = I_0(1)$. Thus, without loss of generality, we assume that $r_{-1} = 1$.

Next, we consider the subordinate's expected utility of being promoted to be a manager. Let D^R and D^U satisfy

$$D^R := \mathbb{E}_c\left[\max\{-p(d^{M,R}+c), b^M - c\}\right], \quad D^U := \mathbb{E}_c\left[\max\{-p(d^{M,U}+c), b^M - c\}\right].$$

Then, $D^R s_{t+1}$ is the expected utility of the manager in the next period when the problem is reported, while $D^U s_{t+1}$ is that when the problem is unreported.

Summing up these elements, the expected utility of reporting the problem is

$$\delta(1-p)I(r_{t-1})D^R s_{t+1} + b^S s_t.$$

On the other hand, the expected utility of not reporting the problem is

$$\delta(1-p)\left[(1-q^S r_{t+1})D^U s_{t+1} + q^S r_{t+1} D^R s_{t+1}\right] - pd^S s_t.$$

Then, the necessary and sufficient condition for reporting the problem is characterized as

Lemma 4.1. *The subordinate in period $t > 0$ reports the problem if and only if*

$$\varphi_{r_{t-1},r_{t+1}}(t) := \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S \geqslant 0.$$

Intuitively, $\varphi_{r_{t-1},r_{t+1}}$ is the net payoff of reporting the problem. First, pd^S is the cost of not reporting in the present period. The other terms are the net payoff of reporting. Consider a gain from reporting. Suppose that reward b^M is sufficiently low so that $D^R < 0$. If the subordinate reports and the manager solves, the problem disappears. Then, the utility of facing a reported problem, D^R disappears. Note that $1 - I(r_{t-1})$ is the probability that the manager solves when the subordinate reports. Then, the gain from reporting is $-(1 - I(r_{t-1}))D^R$. There is also a loss from reporting. If the subordinate reports but the manager ignores, the subordinate will face a reported problem and obtain utility D^R. On the other hand, if the subordinate conceals and the next subordinate does not report, the subordinate will obtain D^U. Note that $D^U > D^R$. Reporting changes D^U into D^R. If the next subordinate does not report, by concealing, the subordinate can earn $D^U - D^R$, which is called *a gain form reducing the responsibility*. Note also that $1 - q^S r_{t+1}$ is the probability that the next subordinate does not report. Thus, $(1 - q^S r_{t+1})(D^U - D^R)$ is the expected loss from reporting.

We concentrate on the case that s_{t+1}/s_t is monotone in t. To guarantee the monotonicity of s_{t+1}/s_t, we prepare some sufficient conditions. At first, suppose that s_t has a continuous extension $s : \mathbb{R}_+ \to \mathbb{R}_+$. If $s(t)$ is log-concave (resp. log-convex) function of t, s_{t+1}/s_t is decreasing (resp. increasing) in t. For example, the density function of a normal distribution function is log-concave. The following fact shows other sufficient conditions.

Fact 4.1. *Consider an increasing function G such that $G(0) = 0$. Suppose that $s_{t+1} = G(s_t)$ for each $t \in \mathbb{N}$. Then, if G is (strictly) concave, $G(s)/s$ is (strictly) decreasing and if G is (strictly) convex, $G(s)/s$ is (strictly) increasing in s. Thus, if $G(s) > s$ for each s and G is strictly concave, s_{t+1}/s_t is strictly decreasing.*

4.4 Equilibrium of the basic model

This section characterizes on pure strategy perfect Bayesian equilibria For notational simplicity, "subordinate reports" implies that the subordinate takes the strategy where he reports the problem when he detects it. In the same way, "subordinate does not report" and "subordinate conceals" imply that the subordinate takes the strategy where he does not report the problem even if he detects it.

As a basic model, we assume that $b^M = 0$. This implies that $D^R < 0$ and $D^U < 0$. We first investigate the property of incentive to report. Recall that $D^R < 0$, $D^U > D^R$ and $I(r_{t-1})$ is decreasing in r_{t-1}. The following lemma shows the property of φ. Although the proof of this lemma is obvious from these facts and the definition of φ, it has an insight about subordinates' incentive.

Lemma 4.2 (Complementarity of reporting)**.** *Suppose that $D^R < 0$. Then, for each $t \in \mathbb{N}$, $\varphi_{r,r'}(t)$ is increasing in $r \in [0, 1]$ and $r' \in [0, 1]$.*

This lemma implies that a subordinate tends to report when the others report, that is, reporting (and thus concealing) are complementary. By Lemma 4.1 and 4.2, we obtain two equilibria and the necessary and sufficient conditions for their existences.

Lemma 4.3. *There is a perfect Bayesian equilibrium where the subordinate in each period reports if and only if for each $t \in \mathbb{N}$, $\varphi_{1,1}(t) \geqslant 0$.*

Lemma 4.4. *There is a perfect Bayesian equilibrium where the subordinate in each period does not report if and only if for each $t \geqslant 1$, $\varphi_{0,0}(t) \leqslant 0$ and $\varphi_{1,0}(0) \leqslant 0$.*

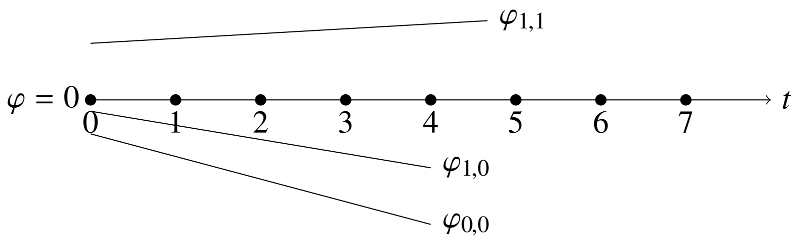

Figure 4.1: Multiple equilibria

Remark 4.1 (Multiple equilibria)**.** Note that existence conditions for the above equilibria are not exclusive each other. Thus there can be multiple equilibria with the same parameter. Figure 4.1 illustrates an example. If $\varphi_{1,1}(t) \geqslant 0$, $\varphi_{0,0}(t) \leqslant 0$ for each $t \in \mathbb{N}$, and $\varphi_{1,0}(0) \leqslant 0$ are hold, both of the strategy profile where all subordinates report and that where all subordinates conceal are equilibria. In this case, if each player believes that others report, he reports and if he believes that others conceal, he conceals. This suggest that corporate culture plays a role to determine which equilibrium is realized (Kreps, 1990). For example, consider a firm that has a culture such that even when a subordinate detects a problem, he conceals it. A subordinate in a firm believes that others conceal and thus he conceals. As a result, each subordinate conceals and the culture is realized as an equilibrium. On the other hand, if "reporting" is a culture, it is also realized as an equilibrium.

Note that if $q^S = 1$, while $\varphi_{1,1} > 0$ and $\varphi_{0,1} > 0$, $\varphi_{0,0}$ and $\varphi_{1,0}$ could be negative. Therefore, if q^S is large, multiple equilibria are likely to occur. △

We now explore the other equilibria. We focus on the case that the scale growth rate s_{t+1}/s_t is monotone.

First, we consider the case that s_{t+1}/s_t is nondecreasing in t, that is scale growth rate is nondecreasing over time. Let $t^{1,0} \in \mathbb{N}$ be the time such that $\varphi_{1,0}(t^{1,0}) > 0 > \varphi_{1,0}(t^{1,0} + 1)$. Since $\varphi_{1,0}$ is monotone, if $t^{1,0}$ exists, it is determined uniquely. In this case, in addition to the equilibria that appeared in Lemmata 4.3 and 4.4, the following strategy profile is an equilibrium.

Lemma 4.5. *Suppose that $b^M = 0$, and s_{t+1}/s_t is nondecreasing in t. Suppose also that $t^{1,0}$ exists. Then, there is a PBE such that for each $t \leqslant t^{1,0}$, subordinate at period t reports and for each $t > t^{1,0}$, subordinate at period t conceals the problem.*

We have shown that there can be three PBEs with pure strategy. The following theorem shows that no other pure strategy PBE.

Theorem 4.1. *Suppose that $b^M = 0$, s_{t+1}/s_t is nondecreasing in t. Suppose also that there is no $t \in \mathbb{N}$ such that $\varphi_{1,0}(t) = 0$. Then, there is no pure strategy PBE other than the following strategy profiles.*

1. *The subordinate in each period reports the problem.*

2. *The subordinate in each period conceals the problem.*

3. *For each $t \leqslant t^{1,0}$, subordinate at period t reports and for each $t > t^{1,0}$, subordinate at period t conceals the problem.*

When s_{t+1}/s_t is nonincreasing, the equilibria are characterized as in the previous case.

Corollary 4.1. *Suppose that $b^M = 0$. Suppose also that s_{t+1}/s_t is nonincreasing in t and there is no $t \in \mathbb{N}$ such that $\varphi_{0,1}(t) = 0$. Let $t^{0,1} \in \mathbb{N}$ be the time such that $\varphi_{0,1}(t^{0,1}) < 0 < \varphi_{0,1}(t^{0,1} + 1)$. Then, there are at most three PBEs with pure strategy:*

1. *The subordinate in each period reports the problem.*

2. *The subordinate in each period conceals the problem.*

3. *For each $t \leqslant t^{0,1}$, the subordinate in period t conceals and for each $t > t^{0,1}$, the subordinate in period t reports the problem.*

In either case, the equilibrium results in a tragedy. Consider the case of increasing scales. When the scale growth rate is nondecreasing, if the problem is not detected until the threshold period, the problem will never be reported. Thus, the problem grows until the accidents occurs, which causes greater accidents. When the scale growth rate is nonincreasing, until the threshold period, the problem remains unreported. This results in higher probability of accidents, greater harm for consumers and higher solving costs.

4.4.1 Welfare

In this section, we compute the welfare. Let $L^R(s)$(resp. $L^U(s)$) be the total loss of citizens other than the personnels of the firm when the problem is reported (resp. unreported), an accident occurs and the scale is s. Both of $L^R(s)$ and $L^U(s)$ include compensation for the accident by the firm. The amount of $L^R(s)$ and $L^U(s)$ can differ, which represents that the amount of compensation can differ when the problem is reported or not. To simplify the notation,

$$\bar{c}^R = \int^{\frac{pd^{M,R}}{1-p}} cdF(c)s_t, \quad \bar{c}^U = \int^{\frac{pd^{M,U}}{1-p}} cdF(c)s_t, \quad \bar{c} = \int cdF(c)s_t$$
$$F^R = F\left(\frac{pd^{M,R}}{1-p}\right), \quad F^U = F\left(\frac{pd^{M,U}}{1-p}\right).$$

Consider an equilibrium where each subordinate before (including) period $t^{1,0}$ reports and each subordinate after the period conceals. In this case, the social welfare $SW_{RCt^{1,0}}$ is calculated as

$$\begin{aligned} SW_{RCt^{1,0}} = &- \sum_{t=0}^{t^{1,0}} \left[(1-p)(1-q^S)(1-q^M F^U)\delta\right]^t \left[q^S SW^R(t) + (1-q^S)\widetilde{B}_t\right] \\ &- \left[(1-p)(1-q^S)(1-q^M F^U)\delta\right]^{t^{1,0}+1} \Bigg[\widetilde{B}_{t^{1,0}+1} \\ &+ (1-p)(1-q^M F^U)\delta \sum_{\tau=t^{1,0}+2}^{\infty} [(1-p)(1-q^M F^U)\delta]^{\tau-t^{1,0}-2} B_\tau\Bigg], \end{aligned}$$

where

$$\begin{aligned} SW^R(t) &= \sum_{\tau=t}^{\infty} [(1-F^R)(1-p)\delta]^{\tau-t} A_\tau \\ A_\tau &= (1-p)\bar{c}^R s_\tau + p(1-F^R)\left(L(s_\tau), +d^{M,R}s_\tau\right) + p\bar{c}s_t, \\ \widetilde{B}_\tau &= q^M(1-p)\bar{c}^U s_\tau + p(1-q^M F^U)((d^{M,U}+d^S)s_\tau + L(s_t)) + p\bar{c}s_\tau, \\ B_\tau &= q^*(1-p)\bar{c}^U s_\tau + p(1-q^* F^U)((d^{M,U}+d^S)s_\tau + L(s_t)) + p\bar{c}s_\tau, \\ q^* &= 1-(1-q^M)(1-q^S). \end{aligned}$$

We consider the effect of a delay in $t^{1,0}$, that is, compare the welfare when $t^{1,0} = t^*$ and $t^{1,0} = t^* + 1$. Then, by calculation,

$$\begin{aligned} &SW_{RCt^*+1} > SW_{RCt^*} \iff \\ &q^S \widetilde{B}_{t^*+1} + (1-q^M F^U)(1-p)\delta(B_{t^*+2} - (1-q^S)\widetilde{B}_{t^*+2}) \\ &+ q^S[(1-q^M F^U)(1-p)\delta]^2 \sum_{\tau=t^*+3} [(1-q^* F^U)(1-p)\delta]^{\tau-t^*-3} B_\tau - q^S SW^R(t^*+1) > 0. \end{aligned}$$

Observing this, since $d^{M,R}$ appears only in $SW^R(t^*+1)$, sufficiently large $d^{M,R}$ may induce that $SW_{RCt^*+1} < SW_{RCt^*}$. However, in reality, such punishment would be prohibited. The total amount of punishment would be no more than the loss of the citizens by the accidents. To represent such idea, we assume that

Assumption 4.1. *1. For each s,* $L^R(s) + d^{M,R}s = L^U(s) + (d^{M,U} + d^S)s = SD(s) > 0.$

2. For each s, $L^R(s) \geqslant 0$ *and* $L^U(s) \geqslant 0$.

The term $SD(s)$ represents the social damage from the accidents with scale s. The first condition implies that either the problem is reported or not, the social damage from the accidents keeps constant. If the punishment is only compensation for the accidents, this condition is satisfied. The second condition implies that citizens cannot benefit from the accidents. If they are negative, the firm compensate too much for the accidents.

Under the assumption, we can show that

Proposition 4.1. *Under Assumption 4.1,* $SW_{RCt^*+1} > SW_{RCt^*}$.

Therefore, such type of equilibrium is inefficient. As in the same way, we can show that one time concealment reduces the social welfare. Therefore, no concealment is socially optimal.

4.5 Comparative statics

In this section, we assume that s_{t+1}/s_t is nondecreasing in t. We focus on strategy profile 3 of Theorem 4.1, that is there exists t^* such that subordinate $t < t^{1,0}$ reports and subordinate $t > t^{1,0}$ conceals the detected problem. As Theorem 4.1 shows, $\varphi_{1,0}$ characterizes the equilibrium. Note that

$$\varphi_{1,0}(t) = \delta(1-p)\left[I(1)D^R - D^U\right]\frac{s_{t+1}}{s_t} + b^S + pd^S. \tag{4.1}$$

If $\varphi_{1,0}(t)$ increases for each t, $t^{1,0}$ also increases, which decreases concealments.

4.5.1 Discount factor

Consider the effect of discount factor δ. If $I(1)D^R - D^U \geqslant 0$, $\varphi_{1,0}(t) > 0$. Therefore, no one conceals. Thus, if someone conceals, $I(1)D^R - D^U < 0$. In this case, increasing in δ decreases $\varphi_{1,0}$, and thus, increasing concealments. This suggests that more farsighted subordinate is more likely to conceal the problem. In another interpretation, consider δ as the probability that the subordinate promotes to be a manager. Then, it also suggests that subordinate who is more likely to promote is more likely to conceal.

4.5.2 Punishment and reward for subordinates

We consider increasings in punishment and reward for subordinates. Since they appear only in the last two terms in (4.1), their increasings directly increase $\varphi_{1,0}$. That is, increasings in punishment and reward for subordinates decrease concealment.

4.5.3 Punishment for reported managers

Consider an increasing in punishment for the manager when he does not solve the reported problem, i.e., $d^{M,R}$. Since $d^{M,R}$ appears in D^R and $I(1)$, the effect is not obvious. Differentiating $\varphi_{1,0}$ by $d^{M,R}$ gives

$$\frac{\partial \varphi_{1,0}(t)}{\partial d^{M,R}} = \delta(1-p)\left[\frac{\partial I(1)}{\partial d^{M,R}} D^R + I(1)\frac{\partial D^R}{\partial d^{M,R}}\right]\frac{s_{t+1}}{s_t},$$

where

$$\frac{\partial I(1)}{\partial d^{M,R}} = -\frac{p}{1-p}\left[\frac{f\left(\frac{pd^{M,R}}{1-p}\right)}{q^M\left[1-F\left(\frac{pd^{M,U}}{1-p}\right)\right]+(1-q^M)}\right] < 0,$$
$$\frac{\partial D^R}{\partial d^{M,R}} = -p[1-F(pd^{M,R}/(1-p))] < 0.$$

Since $D^R < 0$, we have the following observation.

Observation 4.1. Suppose that $f(pd^{M,R}/(1-p))$ is sufficiently small but $1-F(d^{M,R}/(1-p))$ is not so small. Then, then an increase in punishment for managers decreases $\varphi_{1,0}(s)$.

Recall that I is the probability that the manager ignores the problem. Intuitively, if $f(pd^{M,R}/(1-p))$ is sufficiently small, the decrease in I that caused by an increase in $d^{M,R}$, is small. On the other hand, suppose that I itself is not small. Since the manager is likely to ignore the reported problem, if the subordinate reports the problem, he is likely to face the problem when he is the manager. The increase in punishment for reported manager decreases the reported manager's expected utility. Therefore, it increases the incentive to conceal the problem.

Remark 4.2. The conditions in observation that $\varphi_{1,0}$ is decreasing in the size of punishment may not hold when $d^{M,R}$ is sufficiently large. To see this, note that

$$\frac{\partial^2 D^R}{\partial (d^{M,R})^2} = pf(pd^{M,R}/(1-p))] > 0.$$

Suppose that f is nondecreasing. Then, the second order derivative of $\varphi_{1,0}$ is

$$\frac{\partial^2 \varphi_{1,0}(t)}{\partial (d^{M,R})^2} = \delta(1-p)\left[\frac{\partial^2 I(1)}{\partial d^{M,R}} D^R + 2\frac{\partial I(1)}{\partial d^{M,R}}\frac{\partial D^R}{\partial d^{M,R}} + \frac{\partial^2 D^R}{\partial (d^{M,R})^2} I(1)\right]\frac{s_{t+1}}{s_t} > 0.$$

Thus, $\varphi_{1,0}(t)$ is a convex function of $d^{M,R}$ for each t. This implies that even if $\varphi_{1,0}$ is decreasing when $d^{M,R}$ is small, if $d^{M,R}$ is sufficiently large, $\varphi_{1,0}(t)$ is increasing in $d^{M,R}$. This is because, since manager can solve the problem with probability 1, when $d^{M,R}$ is sufficiently large, the probability that the reported problem is solved is 1. This result depends on the assumption that each manager is able to solve the problem with probability 1. We revisit this problem in section 4.6.2. △

Example 4.1. Suppose that parameter c is distributed uniformly on $(0, \bar{c})$. Then,

$$\frac{\partial I(1)}{\partial d^{M,R}} D^R + I(1)\frac{\partial D^R}{\partial d^{M,R}} = \begin{cases} \frac{3p^2 d^{M,R}}{(1-p)\bar{c}}\left(1 - \frac{1}{2}\frac{pd^{M,R}}{(1-p)\bar{c}}\right) - p\left(\frac{1-2p}{1-p}\right) & \text{if } \frac{pd^{M,R}}{1-p} \leqslant \bar{c} \\ 0 & \text{otherwise.} \end{cases}$$

Therefore, if $p > 1/2$, $\varphi_{1,0}$ is increasing. If $p < 1/2$, when $d^{M,R} = 0$, $\frac{\partial \varphi_{1,0}(t)}{\partial s^M} < 0$ and when $pd^{M,R}/(1-p) = \bar{c}$, $\frac{\partial \varphi_{1,0}(s)}{\partial s^M} > 0$. Since the uniform distribution has nondecreasing density, $\varphi_{1,0}$ is convex function of $d^{M,R}$. Therefore, there exists $\bar{d}$ such that for each $d^{M,R} < \bar{d}$, $\varphi_{1,0}$ is decreasing and for each $d^{M,R} > \bar{d}$, $\varphi_{1,0}$ is increasing in $d^{M,R}$. △

4.5.4 Punishment for unreported managers

We consider an increasing in $d^{M,U}$, that is punishment for managers when the problem is unreported. Recall that the net benefit of reporting, φ is given by

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1 - I(r_{t-1}))D^R - (1 - q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Recall also that we have assumed that $d^{M,U} < d^{M,R}$. By increasing $d^{M,U}$, gain from reducing the responsibility, $D^U - D^R$ gets close to 0, which implies that if $d^{M,U} = d^{M,R}$,

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R\right]\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Since $D^R < 0$, $\varphi_{r_{t-1},r_{t+1}}(t) > 0$ and therefore no concealment occurs. The intuition is as follows. The incentive for concealment comes from transferring the responsibility to the next subordinate. That is, if the problem is unreported, the responsibility of the manager is reduced. However, if $d^{M,R} = d^{M,U}$, whether the problem is reported or not, the manager receives the same punishment. Subordinates cannot reduce the responsibility.

4.5.5 Scale growth rate

We consider two problems P, P' that are identified by the sequences of scales. That is, $P = (s_t)$, $P' = (s'_t)$. Suppose that no other parameters differ. Note that corresponding φ and φ' are given by

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S,$$
$$\varphi'_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s'_{t+1}}{s'_t} + b^S + pd^S.$$

Assume that $\{s_t\}$ is a strictly increasing sequence and $\{s'_t\}$ is a strictly decreasing sequence. Then, we have $s_{t+1}/s_t > 1 > s'_{t'+1}/s'_{t'}$ for each $t, t' \in \mathbb{N}$. Assume that the coefficient of s_{t+1}/s_t is negative. Then, in this case, $\varphi'_{r_{t-1},r_{t+1}}(t) > \varphi_{r_{t'-1},r_{t'+1}}(t')$ for each $t, t' \in \mathbb{N}$. Consider the case that for problem P, there is no PBE such that each player does not report the problem. This and Proposition 4.4 imply that for some $t' \in \mathbb{N}$, $\varphi_{0,0}(t') > 0$. Then, $\varphi'_{0,0}(t) > 0$ for each $t \in \mathbb{N}$. Since $\varphi'_{r_{t-1},r_{t+1}}(t') > \varphi'_{0,0}(t) > 0$ for each $t, t' \in \mathbb{N}$, the unique equilibrium is the strategy profile such that each player reports. This shows

Theorem 4.2. *Consider two problems P, P'. Suppose that problem P has increasing scales and P' has decreasing scales. Then, if there is no equilibrium such that no one reports in problem P, then, in problem P', the unique equilibrium is the strategy profile such that each player reports. Conversely, there is no equilibrium such that no one conceal in problem P', then in problem P, the unique equilibrium is the strategy profile such that no one reports the problem.*

This proposition implies that concealment happens more likely in problems with increasing scales.

The intuition is as follows. If the scale is increasing, punishment in the next period is heavier than that in the present period. If subordinate conceals the problem, he is likely to be punished in the present period as a subordinate, but not in the next period. On the other hand, if he reports the problem, he is likely to be punished in the next period as a manager. If the scale is decreasing, punishment in the next period is less greater in the present period. Therefore, the motive for reducing the punishment for manager is more greater in problems with increasing scale.

4.6 Extensions

4.6.1 Introducing rewards for the managers

In the later sections, without mentions, we assume that s_{t+1}/s_t is strictly increasing. We consider the case that $D^R < 0$. Then, since the relation $\varphi_{0,0} < \varphi_{r,r'} < \varphi_{1,1}$ still holds, equilibrium is characterized as Theorem 4.1.

We consider the case that $D^R > 0$. Then, the motivation of concealing the problem is different from the case for $D^R < 0$. If a subordinate reports the problem, since solving the problem brings high reward, the problem would be solved. On the other hand, if he conceals the problem and solves it by himself in the next period, he could earn the reward. This incentive for the seeking the reward is another motivation of concealment for the case that $D^R > 0$. We consider the features of this motivation.

Equilibria

First, we characterize equilibria. Recall that $\varphi_{r_{t-1},r_{t+1}}$ is written as

$$\varphi_{r_{t-1},r_{t+1}}(t) = \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{s_{t+1}}{s_t} + b^S + pd^S$$

Then, $\varphi_{r_{t-1},r_{t+1}}(t)$ is increasing in I. Note that $I(r_{t-1})$ is decreasing in r_{t-1}. Thus, $\varphi_{r_{t-1},r_{t+1}}(t)$ is decreasing in r_{t-1}. This implies that $\varphi_{0,1}(t) \geqslant \varphi_{0,r_{t+1}}(t) \geqslant \varphi_{r_{t-1},r_{t+1}}(t) \geqslant \varphi_{1,0}(t)$ for each $r_{t-1}, r_{t+1} \in [0,1]$ and $t \in \mathbb{N}$. Note that $D^U > D^R$ and $1 > I(r)$. Thus, the net gain from the future responsibility by reporitng is negative.

When $D^R > 0$, we have the following two cases: $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t)$ and $\varphi_{0,0}(t) \geqslant \varphi_{1,1}(t)$.

Case 1. Consider the case that $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t)$ for each $t \in \mathbb{N}$. Then, the equilibria has the following feature.

Lemma 4.6. *Suppose that $\varphi_{1,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$. Then, in any pure strategy PBE, for each $t \in \mathbb{N}$ such that $\varphi_{0,0}(s_t) < 0$, subordinate t conceals the problem.*

In some case, we can characterize pure strategy PBE.

Proposition 4.2. *Suppose that $\varphi_{1,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$ and there is no $t' \in \mathbb{N}$ such that $\varphi_{1,0}(t') = 0$. If $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| \leqslant 1$, the unique pure strategy PBE is the following strategy: subordinate t reports if $\varphi_{1,0}(t) > 0$ and subordinate t conceals the problem if $\varphi_{1,0}(t) < 0$.*

On the other hand, in general, there is no pure strategy PBE.

Proposition 4.3. *Suppose that $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$ and there is no $t' \in \mathbb{N}$ such that $\varphi_{1,0}(t') = 0$. Suppose also that $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| \geqslant 2$. Then, there is no pure strategy PBE.*

However, in the range of mixed strategy, we have an equilibrium.

Theorem 4.3. *Suppose that $\varphi_{0,0}(t) \leqslant \varphi_{1,1}(t)$. Suppose that $\varphi_{1,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$ and there is no $t \in \mathbb{N}$ such that $\varphi_{1,0}(t) = 0$. Then, there exists a PBE such that for each t such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.*

Case 2. Suppose that $\varphi_{0,0}(t) \geqslant \varphi_{1,1}(t)$. In this case, the equilibria have the following features.

Lemma 4.7. *Suppose that $\varphi_{0,0}(t) \geqslant \varphi_{1,1}(t)$. Also suppose that $\varphi_{0,0}(t) \leqslant 0$ for some $t \in \mathbb{N}$. Then, in PBE with pure strategy,*

1. *for each t such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.*
2. *Suppose that $|\{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}| \geqslant 2$. Then, for each $t', t'-1 \in \{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}$, if subordinate t' reports, subordinate $t'-1$ conceals the problem. If subordinate t' conceals, subordinate $t'-1$ reports the problem.*

The feature 1 is common to the previous case, but feature 2 is specific in the present case. The following proposition shows the existence of equilibrium that has these features.

Theorem 4.4 (Existence of alternate generations equilibrium)**.** *Suppose that $\varphi_{0,0}(t) \geqslant \varphi_{1,1}(t)$ and $\varphi_{0,0}(t) < 0$ for some $t \in \mathbb{N}$. Also suppose that $|\{t : \varphi_{0,0}(s_t) > 0 > \varphi_{1,1}(t)\}| \geqslant 2$.*

Then, there is a PBE with pure strategy that satisfies

1. *for each t such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.*

2. *Suppose that $|\{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}| \geqslant 2$. Then, for each $t', t'-1 \in \{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}$, if subordinate t' reports, subordinate $t'-1$ conceals the problem. If subordinate t' conceals, subordinate $t'-1$ reports the problem.*

The reason for the alternation of action plans of subordinates is as follows. Since $D^R > 0$, the subordinate seeks the reward for solving the problem. Therefore, the subordinate wants to conceal the problem when the manager is likely to solve the problem. If his manager takes the strategy that he conceals the problem when he is a subordinate, the probability that he knows the problem is higher than if he takes the strategy that he reports the problem when he detects it. This implies that the manager's c is high enough since he knows the problem but does not solve it. Therefore, the incentive for concealment is weaker when the manager conceals than that when the manager reports. This implies that the strength of the incentive for concealment alternates in each generation.

Remark 4.3. Note that since there is at most one problem, the actual actions, reporting and not reporting do not alternate. △

Comparative statics

In each case, $\varphi_{0,0}$ plays an important role to determine the start timing of concealment. Recall that the function $\varphi_{0,0}$ is written as

$$\varphi_{0,0}(t) = \delta(1-p)(I(0)D^R - D^U)\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

We consider the effect of increasing punishment and reward. The following proposition shows the effect of the punishment for the managers' neglect.

Proposition 4.4. *If $D^R > 0$, increasing in $d^{M,R}$ decreases $\varphi_{0,0}(t)$.*

The intuition is as follows. Consider $D^R > 0$. In this case, if the manager ignores the problem, the subordinate can earn award by solving the problem. Then, he wants not the manager to solve it when the subordinate reports it. On the other hand, by concealing the problem, the subordinate can solve it as a manager in the next period. Note that an increasing in $d^{M,R}$ increases the probability that the manager solves the problem. This encourages the subordinate to conceal the problem.

Consider the effect of increasing in reward for managers, b^M.

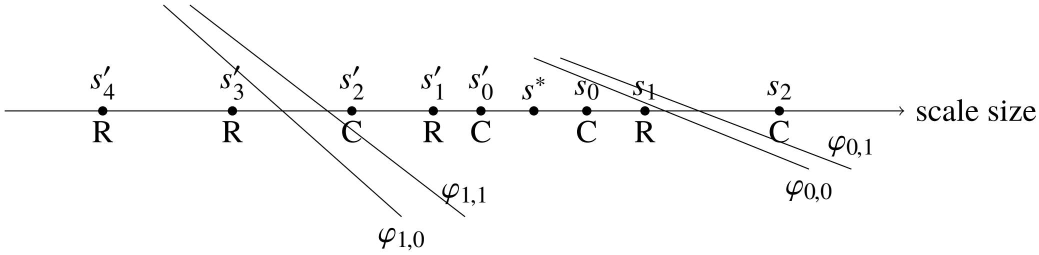

Figure 4.1: Illustration of Example 4.2

Proposition 4.5. *Suppose that $f(x)/(1-F(x))$ is increasing in x. Let $w^R = (b^M + pd^{M,R})/(1-p)$ and $w^U = (b^M + pd^{M,U})/(1-p)$. Then,*

(1) If $F(w^U) > 1/2$ and $D^R > 0$, increasing in b^M decreases $\varphi_{0,0}(t)$ for each t.

(2) If $F(w^R)(1 - F(w^R)) > F(w^U)$ and $D^R < 0$, increasing in b^M increases $\varphi_{0,0}(t)$ for each t.

Note that if b^M is sufficiently high, the conditions $F(w^U) > 1/2$ and $D^R > 0$ are satisfied. Thus, when b^M is sufficiently high, $\varphi_{0,0}(t)$ is decreasing in b^M. The intuition is straightforward. With sufficiently large b^M, the benefit from solving a problem is large enough, which also implies that low probability of facing the problem. Since the benefit is large, the incentive for seeking rewards is also large. Both of the effects strengthen the incentive to conceal. On the other hand, if $D^R < 0$, subordinates want not to face the problem. Therefore, the effect of reducing the probability of facing the problem reduces the incentive to conceal.

We also perform a comparative static for the scale of a problem. As in the benchmark model, if $D^R < 0$, since the relation $\varphi_{0,0} < \varphi_{r,r'} < \varphi_{1,1}$ for each $r, r' \in [0, 1]$ still holds, the statement of Proposition 4.2 also holds. However, if $D^R > 0$, the relation $\varphi_{0,0} < \varphi_{r,r'} < \varphi_{1,1}$ is violated, but if $\varphi_{0,0} \leqslant \varphi_{1,1}$, the statement of Theorem 4.2 still holds in the range of pure strategy.

Theorem 4.5. *Consider two problems $P = (s_t)_{t=0}^{\infty}$, $P' = (s'_t)_{t=0}^{\infty}$ that satisfy $s_{t+1} > s_t$ and $s'_{t+1} < s'_t$ for each $t \in \mathbb{N}$. Suppose also that $\varphi_{0,0} \leqslant \varphi_{1,1}$. Then, if there is no PBE such that no one reports in problem P, then, in problem P', the unique pure strategy PBE is the strategy profile such that each player reports. Conversely, there is no PBE such that no one conceals in problem P', then in problem P, the unique pure strategy PBE is the strategy profile such that no one reports the problem.*

If $\varphi_{0,0} > \varphi_{1,1}$, a statement like Theorem 4.5 may be violated. Example 4.2 shows the case.

Example 4.2. Consider φ, $(s_t)_{t=0}^{\infty}$, and $(s'_t)_{t=0}^{\infty}$ that are drawn in figure 4.1. Suppose that there is an increasing function G such that $(s_t)_{t=0}^{\infty}$, and $(s'_t)_{t=0}^{\infty}$ are generated by G, that is $s_{t+1} = G(s_t)$ and

$s'_{t+1} = G(s'_t)$. Function G is drawn in figure 4.2. Assume that there exists s^* such that $G(s^*) = s^*$ (see figure 4.2). In this example, if the initial value $s'_0 < s^*$, the scale is decreasing, while if $s_0 > s^*$, the scale is increasing. In this case, not that since φ is written as

$$\varphi_{r_{t-1},r_{t+1}}(t) := \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{G(s_t)}{s_t} + b^S + pd^S,$$

$\varphi_{r_{t-1},r_{t+1}}(t)$ is depend only on s_t. Therefore, consider $\varphi_{r_{t-1},r_{t+1}}(t)$ as a function of scales, that is

$$\varphi_{r_{t-1},r_{t+1}}(s) := \delta(1-p)\left[-(1-I(r_{t-1}))D^R - (1-q^S r_{t+1})(D^U - D^R)\right]\frac{G(s)}{s} + b^S + pd^S.$$

Then, problems $(s_t)_{t=0}^{\infty}$, and $(s'_t)_{t=0}^{\infty}$ have the same $\varphi_{r_{t-1},r_{t+1}}$.

Figure 4.1 shows a PBE. The term R indicates that the subordinate at the point reports in PBE and C indicates that the subordinate at the point conceals. We show that the action profiles are PBEs. Consider $\{s_t\}$. Note that for each $t \geqslant 2$, since $\varphi_{r,r'}(s_t) < 0$, concealing is the dominant strategy. Consider $t = 0$. As seen early in this section, since subordinate 0 is the first person who can detect the problem, the manager 0 does not detect the problem when he is a subordinate. Thus, this situation is the same one that subordinate -1 takes a strategy that reports the problem when he detects. Since $\varphi_{1,r}(s_0) < 0$, the best response of subordinate 0 is concealing the problem. Then, since $r_0 = r_2 = 0$ and $\varphi_{0,0}(s_1) > 0$, $r_1 = 1$.

Consider problem $(s'_t)_{t=0}^{\infty}$. Since for each $t \geqslant 3$, $\varphi_{r,r'}(s'_t) > 0$, reporting the problem is the dominant strategy. Consider $t = 0$. As shown in above, since $\varphi_{1,r}(s'_0) < 0$, the best response of subordinate 0 is concealing. Since $r_0 = 0$ and $\varphi_{0,r}(s'_1) > 0$, $r_1 = 1$. Since $r_1 = r_3 = 1$ and $\varphi_{1,1}(s'_2) < 0$, $r_2 = 0$.

Obviously, the PBE in increasing scale is not “all conceal”. However, in decreasing scale, this PBE is not “all report”. △

4.6.2 Possibility of failure in solving the problem

In the previous section, we assume that managers who facing the reported problem have only two choices; solve or ignore. This section assumes that managers can chooses the probability that the problem is solved. To simplifies discussion, we assume that if the manager fails to solve the problem, the problem is inherited the next manager. In this case, his effort of trying to solve the problem has no influence on the cost function of the next manager. We also assume that even if the problem causes

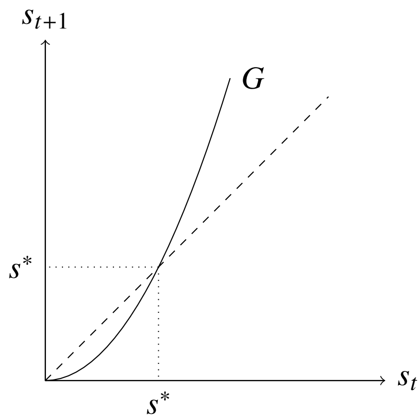

Figure 4.2: Illustration of function G in Example 4.2.

accidents, the manager in the period is not forced to solve the problem.

Let ρ be the probability that the problem is solved, which managers can choose. Manager t's cost function to achieve ρ is given by $c_t \chi(\rho) s$, where c_t is private information of the manager. Suppose that c_t is independently and identically distributed by a cumulative distribution function F on $(0, \bar{c})$, where $\bar{c} \in \mathbb{R}_+$. As in the basic model, we assume that each player learns c after he promotes to a manager. The function $\chi : [0, \bar{\rho}) \to \mathbb{R}_+$ is a strictly convex, differentiable, strictly increasing, and satisfies $\chi(0) = 0$ and $\lim_{\rho\to\bar{\rho}} \chi(\rho) = \lim_{\rho\to\bar{\rho}} \chi'(\rho) = \infty$. The term $\bar{\rho} \leqslant 1$ is the supremum of achievable probability. By this assumption, χ' is strictly increasing and thus, has the strictly increasing inverse function.

We assume that if the manager tries to solve the problem, that is he chooses $\rho > 0$, it is known to the firm's personnel that the manager knows the problem, but the level of ρ is unverifiable to any others. We assume that punishment for the manager is irrelevant to the level of ρ. Let $\widetilde{D}^R$ be the reported manager's expected utility, $\widetilde{D}^U$ be the unreported manager's utility and $\widetilde{I}$ be the probability that the reported manager ignores.[6] Then, as in the basic model, we show

Lemma 4.8. *There exist a function* $\widetilde{I} : [0, 1] \to [0, 1]$ *and constants* $\widetilde{D}^U$ *and* $\widetilde{D}^R$ *such that subordinate* t *reports if and only if*

$$\widetilde{\varphi}_{r_{t-1},r_{t+1}}(t) := \delta \left[(\widetilde{I}(r_{t-1}) - 1)\widetilde{D}^R - (1-p)(1 - q^S r_{t+1})(\widetilde{D}^U - \widetilde{D}^R) \right] \frac{s_{t+1}}{s_t}$$

$$+ b^S + pd^S \geqslant 0.$$

[6] For the detail definition, see appendix.

In this case, the shape of function φ has the same characteristics of that in the previous sections 4.4. The major differences arise in the effects of increasing in the sizes of punishments and rewards for managers.

We consider the effect of $d^{M,R}$. As in the basic model, we consider $\widetilde{\varphi}_{1,0}$, which determines the equilibrium when $\widetilde{D}^R < 0$. In our basic model, as shown in example 4.1, we have the case that if $d^{M,R}$ is sufficiently small, $\widetilde{\varphi}_{1,0}(s)$ is decreasing in $d^{M,R}$, while when $d^{M,R}$ is large, it is increasing. However, in this model, the latter may not hold. Indeed, we show

Theorem 4.6. *Suppose that the support of F is $(0, \bar{c})$. (1) Suppose that the supremum of the solving probability $\bar{\rho} < 1 - p$. Then, there exists $\bar{d}$ such that for each $d^{M,R} > \bar{d}$, $\widetilde{\varphi}_{1,0}(t)$ is decreasing in $d^{M,R}$.*

(2) Suppose that the supremum of the solving probability $\bar{\rho} > 1 - p$. Then, there exists $\bar{d}$ such that for each $d^{M,R} > \bar{d}$, $\varphi_{1,0}(t)$ is increasing in $d^{M,R}$.

The intuition is as follows. Note that

$$\widetilde{\varphi}_{1,0}(t) := \delta \left[-(1 - \widetilde{I}(1))\widetilde{D}^R - (1 - p)(\widetilde{D}^U - \widetilde{D}^R) \right] \frac{s_{t+1}}{s_t} + b^S + pd^S \geqslant 0.$$

Note also that $1 - \widetilde{I}(1)$ is the probability that the manager solves the problem, which converges to $\bar{\rho}$ as $d^{M,R}$ goes to infinity. By increasing in $d^{M,R}$, which implies decreasing in $\widetilde{D}^R$, it increasing the merit of reporting if the manager solves the problem. Therefore, the benefit increases by $\bar{\rho}$. On the other hand, if the subordinate does not report, with probability $1 - p$, the problem is unreported. Recall that if the problem remains unreported, the subordinate can obtain the gain from reducing the responsibility $\widetilde{D}^U - \widetilde{D}^R$. Thus, by increasing in $d^{M,R}$, it increases the gain from reducing the responsibility by $1 - p$. Therefore, if $\bar{\rho} < 1 - p$, the increasing in the net benefit of reporting is negative, and thus, $\widetilde{\varphi}_{1,0}$ decreases.

Under some additional conditions, we can also say that $\widetilde{\varphi}_{1,0}$ is decreasing.

Proposition 4.6. *Suppose that $q^M = d^{M,U} = 0$ and $\widetilde{D}^R < 0$. Suppose also that $\chi(\rho) = \rho/(\bar{\rho} - \rho)$. Then, if $\bar{\rho} < 1 - p$, $\widetilde{\varphi}_{1,0}$ is decreasing in $d^{M,R}$.*

4.6.3 After retirement blames

In our basic model, we assume that managers are not punished after their retirement. This section relaxes this assumption. Suppose that each player lives at most three periods. He works as subordinate in his first period, works as manager in his second period and retires in his third period. We assume that the retired manager lives at most one period. The probability that the manager lives after his retirement is μ. We also suppose that the support of F is $(0, \bar{c})$ and F has differentiable density. If he ignores a problem, his negligence is detected when the problem causes accidents. When his negligence is detected, he is punished. Let $d^R s_t$ be the disutility of punishment for the living retired manager in period t. We assume that the scale s_t is determined by a transition function G, that is $s_t = G(s_{t-1})$ for each $t \in \mathbb{N}$. Let $\widehat{D}^R$ be the reported manager's utility, $\widehat{D}^U$ be the unreported manager's utility and $\widehat{I}$ be the probability that the reported manager ignores.[7] Then, as in the basic model, we can define the incentive to report, $\widehat{\varphi}$. However, since each manager considers his retired period, whether to solve the problem is affected by the scale in his retired period. Thus, the expected utility of facing a reported problem, $\widehat{D}^R$ and the probability that manager ignores the reported problem, $\widehat{I}$ is a function of the scale.[8] Then, we show

Lemma 4.9. *Suppose that $G(s)/s$ and f are well defined on $\mathbb{R}_+$ and $\sup_{s \in \mathbb{R}_+} G(s)/s < \infty$. Then, there exists functions $\widehat{I} : [0,1] \times \mathbb{R} \to \mathbb{R}$ and $\widehat{D}^R : \mathbb{R} \to \mathbb{R}$, and a constant $\widehat{D}^U$ such that subordinate in period t reports the problem if and only if $\widehat{\varphi}_{r_{t-1}, r_{t+1}}(t) \geqslant 0$, where*

$$\widehat{\varphi}_{r_{t-1}, r_{t+1}}(t) = \delta(1-p)\left[-\{1 - \widehat{I}(r_{t-1}, s_t)\}\widehat{D}^R(s_t) - (1 - q^S r_{t+1})(\widehat{D}^U - \widehat{D}^R(s_t))\right]\frac{G(s_t)}{s_t}$$

$$+ b^S + p d^S.$$

In this case, since $\widehat{\varphi}_{r_{t-1}, r_{t+1}}(t)$ is written as a function of s_t, we write $\widehat{\varphi}_{r_{t-1}, r_{t+1}}$ as a function of s, that is $\widehat{\varphi}_{r_{t-1}, r_{t+1}}(s_t) = \widehat{\varphi}_{r_{t-1}, r_{t+1}}(t)$. Obviously, an increase of d^R has the same effect of an increase of $d^{M,R}$. Moreover, if $G(s)/s$ is increasing, this is the same for the case that $d^{M,R}$ increases in each period. Since the effect of increase in $d^{M,R}$ depends on the size of $d^{M,R}$ and distribution function, it is not easy to determine the shape of $\widehat{\varphi}$. In this case, it is not clear the relation between the problem with increasing scales and that with decreasing scales.

To simplify the analysis, we consider the case that $G(s) = \alpha s$, $\alpha \in \mathbb{R}_{++}$. Thus, if $\alpha > 1$, the scales are increasing. Then, $\widehat{\varphi}$, $\widehat{D}$, and $\widehat{I}$ are constant across the periods since $G(s)/s = G^2(s)/G(s) = \alpha$. Thus, we write these variable as a function of α.

[7] For the detail definitions, see appendix.
[8] For the detail, see the proof of the lemma.

We also have that the possible equilibria with pure strategy are all conceal and all reports since $\widehat{\varphi}$ is constant. Thus, we consider $\widehat{\varphi}_{0,0}$ and $\widehat{\varphi}_{1,1}$. For simplicity, we consider the case that $\widehat{D}^U < 0$ (and thus $\widehat{D}^R < 0$ for each α). Consider the effect of α. Recall Remark 4.2. We have shown that if f is nondecreasing, $\widehat{\varphi}$ is a convex function of punishment. Since an increasing in scale has the similar effect of an increasing in punishment, $\widehat{\varphi}$ would also be high when α is large enough and small enough. In fact, we show

Proposition 4.7. *Suppose that* $\widehat{D}^R < 0$. *Then, there exists* $\bar{\alpha}$ *such that for each* $\alpha > \bar{\alpha}$, $\widehat{\varphi}_{r_{t-1},r_{t+1}} > 0$ *and* $\widehat{\varphi}_{r_{t-1},r_{t+1}}$ *is increasing in* α.

Proposition 4.8. *Suppose that* $\widehat{D}^R < 0$ *and* F *is a uniform distribution on* $(0, \bar{c})$. *Suppose also that* p, b^M *and* $d^{M,U}$ *are sufficiently small,* $q^S < 1$ *and* $\bar{c}$ *is sufficiently large. Then, there exists* $\bar{\alpha}$ *such that for each* $\alpha < \bar{\alpha}$, $\widehat{\varphi}_{r_{t-1},r_{t+1}}$ *is decreasing in* α.

4.7 Discussion and Conclusion

This paper studies the subordinates' motivation of concealment of problems. We show that the main cause of concealment is incentive to reduce the future responsibility. Increasing in punishment for manager for a reported problem is considered as an easy way to decrease negligences of the managers. However, in our model, heavier punishment makes the responsibility heavier and in turn, it may strengthen the incentive to conceal the problem.

Comparative statics also shows that concealment is likely to occur when the scale is increasing. If scale is increasing, the harm would also be increasing. Therefore, such problem should be solved in their early stage, but in equilibrium, they will not be reported and thus, resolution of a problem tend to delay or a problem remains unsolved until an accident occurs. This is the trouble. We need to overcome this trouble by controlling punishments and rewards considering social welfare.

This paper assumes that there is only one subordinate in each period and he is sure to be a manager. One may wonder that if there are many subordinates, since a few of them are promoted managers, the incentive to reduce the responsibility may not work.

However, in such case, since there are many subordinates, the responsibility of reporting a problem to a manager is also small for each subordinate. Thus, it is natural to assume that the responsibility of subordinates decreasing in the number of subordinates. Since the present matter is such responsibility, even when the possibility of being a manager is small, the incentive to reduce the responsibility still may work.

We leave some questions for future researches. The first question is optimizing punishment and reward to maximize social welfare. In this paper, we omit the discussion about social welfare, but it is important to determine public policy. The second question is generalizing our model to include many players. In our model, only two players in the firm, but it is not realistic. The last question is generalizing our model to allow nonlinear relation between costs, punishment, reward and scales. In each generalization, the incentive to reduce responsibility also works, but it is a question that what properties hold in equilibria.

Appendix

4.A Omitted Proofs

4.A.1 Proof in Section 4.3

Proof of Fact 4.1. Differentiating $G(s)/s$ gives

$$\left(\frac{G(s)}{s}\right)' = \frac{G'(s)s - G(s)}{s^2} = \frac{\frac{s^2}{2}G''(\lambda s) - G(0)}{s^2}.$$

The second equality is by the Taylor expansion of G, that is, $G(0) = G(s) - sG'(s) + \frac{s^2}{2}G''(\lambda s)$ for some $\lambda \in (0, 1)$. Since $G(0) = 0$, then $G(s)/s$ is increasing if G is convex and is decreasing if G is concave. □

4.A.2 Proofs in Section 4.4

Proof of Lemma 4.2. Note that $I(r_{t-1})$ is decreasing in r_{t-1}. Since $D^R < 0$, $\varphi_{r_{t-1},r_{t+1}}(t) > \varphi_{r'_{t-1},r_{t+1}}(t)$ for each $r_{t-1} > r'_{t-1}$. Since $D^R < D^U$, $\varphi_{r_{t-1},r_{t+1}}(t) > \varphi_{r_{t-1},r'_{t+1}}(t)$ for each $r_{t+1} > r'_{t+1}$. Therefore, for each $r_{t-1}, r_{t+1} \in [0, 1]$, $\varphi_{1,1}(s) \geqslant \varphi_{r_{t-1},r_{t+1}}(t) \geqslant \varphi_{0,0}(t)$. □

Proof of Lemma 4.3. Suppose that for each t, $\varphi_{1,1}(t) \geqslant 0$. Consider the behavior of subordinate at period t. Suppose that the other player takes the action such that he reports the problem when he detects it. Then, $r_{t-1} = r_{t+1} = 1$ and thus, reporting the problem is one of his best response.

Suppose that $\varphi_{1,1}(t^*) < 0$ for some $t^* \in \mathbb{N}$. Then, since $\varphi_{r,r'}(t^*)$ is increasing in r, r', $\varphi_{r,r'}(t^*) \leqslant \varphi_{1,1}(t^*) < 0$ for each $r, r' \in [0, 1]$. Therefore, for the subordinate t^*, concealing the problem is a strict dominant strategy. Therefore, reporting for each period is not an equilibrium. □

Proof of Lemma 4.4. Suppose that for each t, $\varphi_{0,0}(t) \leqslant 0$ and $\varphi_{1,0}(0) \leqslant 0$. Then, for each subordinate

$t > 0$, as in the proof of Lemma 4.3, we can show that concealing is the best response. Then, since subordinate 1 conceals, $r_{-1} = 1$, and $\varphi_{1,0}(0) \leqslant 0$, for subordinate 0, concealing is the best response.

To prove contraposition, suppose that $\varphi_{0,0}(t) > 0$ for some t or $\varphi_{1,0}(0) > 0$. If $\varphi_{0,0}(t) > 0$ for some t, since $\varphi_{0,0}(t) < \varphi_{r,r'}(t)$ for each $r, r' \in [0, 1]$, for subordinate t, reporting is a strict dominant strategy.

Consider the latter case, $\varphi_{1,0}(0) > 0$. Then, since subordinate 1 conceals and $r_{-1} = 1$, reporting is the best response. In each case, the strategy profile where each subordinate reports is not an equilibrium. □

Proof of Lemma 4.5. Suppose that $t^{1,0}$ exists. Therefore, for some t, $\varphi_{1,0}(t) < 0$. Then, since $b^S + pd^S > 0$ and $s_t > 0$, $\delta(1-p)[I(1)D^R - D^U] < 0$. Since s_{t+1}/s_t is increasing over time t, $\varphi_{1,0}(t)$ is decreasing in t.

Consider the behavior of subordinate $t^* \in \mathbb{N}$. Suppose that subordinate in each $t \neq t^*$ follows the strategy of the statement. Suppose also that $t^{1,0} > 0$.

Consider the case that $t^* > t^{1,0}$. We show that subordinate t^* conceals. Note that each subordinate $t > t^*$ conceals. Thus, $r_{t^*+1} = 0$. Since s_{t+1}/s_t is increasing in t, $\varphi_{1,0}(t^*) < 0$. Then, since $\varphi_{r_{t-1},r_{t+1}}$ is increasing in r_{t-1}, $\varphi_{r_{t-1},0}(t^*) < 0$. Therefore, for subordinate t^*, concealing is the best response.

Consider the case that $t^* \leqslant t^{1,0}$. We show that subordinate t^* reports. Note that each subordinate $t < t^*$ reports. Thus, $r_{t^*-1} = 1$. Since s_{t+1}/s_t is increasing in t, $\varphi_{1,0}(t^*) > 0$. Then, $\varphi_{r_{t-1},r_{t+1}}$ is increasing in r_{t+1}, since and $\varphi_{1,r_{t+1}}(t^*) > 0$. Therefore, for subordinate t^*, reporting is the best response.

Suppose that $t^* = 0$. Then, since $r_{-1} = 1$ and $0 \leqslant t^{1,0}$, as in shown in above, reporting is the best response. □

Proof of Theorem 4.1. By Lemma 4.3, 4.4 and 4.5, each of the strategies in the statement is a PBE under some condition.

Consider a pure strategy equilibrium except for strategy profile 1 and 2. We show that this is equivalent to strategy profile 3. Then, there exists $t^R \in \mathbb{N}$ such that the subordinate in period t^R reports the problem if he detects it and there also exists t^C such that the subordinate in period t^C does not report the problem even if he detects it.

To show the proposition, we prove

Claim 4.1. *Suppose that the hypothesis of Proposition 4.1 holds. Let t^R be a period at which the subordinate reports and t^C a period at which the subordinate conceals. Then,* *(i) In each PBE, for each $t \leqslant t^R$, the subordinate in period $t \geqslant 0$ reports the problem.*

(ii) In each PBE, for each $t \geqslant t^C$, the subordinate in period $t \geqslant 0$ does not report the problem.

Proof of Claim 4.1. We first show the first part of the claim. Let $t \leqslant t^R$.

Suppose by contradiction that t does not report the problem. Then, there is $t^* \in \{t, t+1, \ldots, t^R - 1\}$ such that subordinate t^* does not report and subordinate $t^* + 1$ reports.

Then, since subordinate t^* does not report at an equilibrium and $r_{t^*+1} = 1$, $\varphi_{r_{t^*-1},1}(t^*) < 0$. Since $\varphi_{r_{t^*-1},1}$ is increasing in r_{t^*-1} and there is no t such that $\varphi_{0,1}(t^*) = 0$, $\varphi_{0,1}(t^*) < 0$. Thus, in $\varphi_{0,1}$, the coefficient of s_{t+1}/s_t is negative. Note that s_{t+1}/s_t is nondecreasing in t and thus, $\varphi_{0,1}(t^* + 1) < 0$. Since $\varphi_{0,r_{t^*+1}}(t^* + 1)$ is increasing in r_{t^*+1}, $\varphi_{0,r_{t^*+1}}(t^* + 1) < 0$. Therefore, for subordinate $t^* + 1$, not reporting is the unique best response, which is a contradiction the fact that subordinate $t^* + 1$ report at an equilibrium.

As in the same way, we can show that in equilibrium, for each $t \geqslant t^C$, the subordinate in period t does not report the problem even if he detects it. □

By Claim 4.1, there exists $\widehat{t}$ such that for each $t \geqslant \widehat{t}$, $r_t = 0$ and for each $t < \widehat{t}$, $r_t = 1$.

Suppose that $\varphi_{1,0}(\widehat{t}) > 0$. Then, since $r_{\widehat{t}+1} = 0$ and $r_{\widehat{t}-1} = 1$, reporting is the best response, is a contradiction. Therefore, $\varphi_{1,0}(\widehat{t}) < 0$. Suppose that $\varphi_{1,0}(\widehat{t} - 1) < 0$. Then, since $r_{\widehat{t}} = 0$ and $r_{\widehat{t}-2} = 1$, not reporting is the best response, is a contradiction. Therefore, $\varphi_{1,0}(\widehat{t} - 1) > 0 > \varphi_{1,0}(\widehat{t})$. Thus, $\widehat{t} - 1 = t^{1,0}$. □

Proof of Theorem 4.2. (1) Suppose that there is no equilibrium such that no one reports in problem P. Then, by Proposition 4.4, $\varphi_{0,0}(t) > 0$ for some $t \in \mathbb{N}$ or $\varphi_{1,0}(0) > 0$.

Case 1. Suppose that $\varphi_{0,0}(t) > 0$ for some $t \in \mathbb{N}$. Then, we first show that $\varphi'_{0,0}(t) > 0$ for each $t \in \mathbb{N}$. Consider the case that $I(0)D^R - D^U \geqslant 0$. Then $\varphi'_{0,0}(t) > 0$ for each $t \in \mathbb{N}$.

Consider the case that $I(0)D^R - D^U < 0$. Then, we have $\widetilde{\varphi}_{0,0}(t) > \varphi_{0,0}(t)$. This implies that $\widetilde{\varphi}_{0,0}(t') > 0$ for each $t' \in \mathbb{N}$.

Since $\varphi'_{r_{t-1},r_{t+1}}(t') > \varphi'_{0,0}(t') > 0$ for each $t' \in \mathbb{N}$. Then, reporting is the dominant strategy for each subordinate t. Therefore, the unique PBE is the strategy that each player reports the problem P'.

Case 2. Suppose that $\varphi_{1,0}(0) > 0$. Then, as in the previous case, we have that $\varphi'_{1,0}(t) > 0$ for each $t \in \mathbb{N}$. Then, for subordinate 0, reporting is strict dominant strategy. Since $\varphi'_{1,0}(s'_1) > 0$ and $\varphi'_{1,0}(t) < \varphi'_{1,r}(t)$ for each r, for subordinate 1 reporting is the unique best response. Continuing this process, reporting is the best response for each subordinate t.

(2) Suppose that there is no equilibrium such that no one conceals in problem P'. Then, by Proposition 4.3, $\varphi'_{1,1}(t) < 0$ for some t. This implies that $-(1 - I(1))D^R - (1 - q)(D^U - D^R) < 0$. Therefore, we have $\varphi'_{1,1} > \varphi_{1,1}$, which implies that $\varphi_{1,1}(t) < 0$ for each $t \in \mathbb{N}$. Then, $\varphi_{r_{t-1},r_{t+1}}(t') < 0$ for each $t' \in \mathbb{N}, r_{t-1}, r_{t+1} \in [0, 1]$. This implies that not reporting is the dominant strategy, and thus, unique equilibrium is the strategy such that each player does not report the problem P. □

Proof of Proposition 4.1. Under Assumption 4.1, first note that for each t, $B_t > A_t$ and $B_t \geqslant (1 - q^S)\widetilde{B_t} + q^S A_t$. This is because $B_\tau - A_\tau$ is calculated as

$$B_\tau - A_\tau = (1 - p)(\bar{c}^U q^* - \bar{c}^R) + p(F^R - q^* F^U)SD(s_t).$$

When $q^* = 1$,

$$B_\tau - A_\tau = (1 - p)(\bar{c}^U - \bar{c}^R)s_t + p(F^R - F^U)SD(s_t) \geqslant 0,$$

since $SD(s_t) \geqslant d^{M,R}s_t$ and $\bar{c}^R - \bar{c}^U = \int_{\frac{pd^{M,U}}{1-p}}^{\frac{pd^{M,R}}{1-p}} c dF(c)$. When $q^* = 0$, as in the same way,

$$B_\tau - A_\tau = -(1 - p)(\bar{c}^R) + pF^R SD(s_t) \geqslant 0.$$

Since $B_\tau - A_\tau$ is a linear function of q^*, $B_\tau - A_\tau \geqslant 0$.

As in the same way,

$$B_t - q^S A_t - (1 - q^S)\widetilde{B_t} = (1 - p)q^S[\widetilde{c}^U - \bar{c}^R]s_t + pSD(s_t)q^S(F^R - F^U) \geqslant 0.$$

Then, since $1 - q^* F^U > 1 - F^R$, $SW_{RCt^*+1} - SW_{RCt^*} > 0$. □

4.A.3 Proofs in Section 4.6.1

Proof of Lemma 4.6. Let $(r_t)_{t \in \mathbb{N}} \in \{0, 1\}^{\mathbb{N}}$ be the profile of probability of report in an equilibrium. By the assumption, there exists t such that $\varphi_{1,1}(t) < 0$. Suppose that $r_t = 1$. Then, since $\varphi_{1,1}(t + 1) < 0$ and $\varphi_{1,0}(t + 1) < 0$, we have $r_{t+1} = 0$. On the other hand, since $\varphi_{1,0}(t) < 0$ and $\varphi_{0,0}(t) < 0$, $r_t = 0$, a contradiction. Therefore, $r_t = 0$. Let $t^* = \min\{t : \varphi_{1,1}(t) < 0\}$. Since $\varphi_{1,1}(t)$ is decreasing in t, for each $t \geqslant t^*$, $r_t = 0$.

Let $T = \{t < t^* : \varphi_{0,0}(t) < 0\}$. If $T = \varnothing$, we are done. Suppose that $T \neq \varnothing$. Let $t^{**} = \max T$. Then, since $\varphi_{0,0} < \varphi_{1,1}$, $t^{**} = t^* - 1$. Since $\varphi_{1,0}(t^{**}) < 0$, $\varphi_{0,0}(t^{**}) < 0$ and $r_{t^*} = 0$, $r_{t^{**}} = 0$. Note that $\varphi_{1,0}(t) < 0$, $\varphi_{0,0}(t) < 0$ for each $t \in T$. Therefore, $r_{t^{**}-1} = 0$ if $t^{**} - 1 \in T$. Continuing this process, $r_t = 0$ for each $t \in T$. □

Proof of Proposition 4.2. Note that if $\varphi_{1,0}(t) > 0$, reporting is strict dominant strategy. Thus, in equilibrium, $r_t = 1$. By lemma 4.6, for each t such that $\varphi_{0,0}(t) < 0$, subordinate t does not report. For each t, such that $\varphi_{0,0}(t) < 0$, since $\varphi_{1,0}(t) < 0$, no one has an incentive to deviate.

If $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| = 0$, we are done. Consider the case that $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| = 1$. Let t be the element. Then, $r_{t-1} = 1$ and $r_{t+1} = 0$. Since $\varphi_{1,0}(t) < 0$, not reporting is the best response, which concludes the proof. □

Proof of Proposition 4.3. Let $t^* := \max\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}$. By lemma 4.6, $r_{t^*+1} = 0$.

Suppose that $r_{t^*} = 0$. Then, since $\varphi_{0,0}(t^*) > 0$, $r_{t^*-1} = 1$. Since $|\{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}| \geqslant 2$, $t^* - 1 \in \{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}$. Then, it must be $r_{t^*-2} = 0$ since $\varphi_{1,0}(t^* - 1) < 0$. However, since $\varphi_{0,1}(t^* - 2) > 0$ and $\varphi_{1,1}(t^* - 2) > 0$, the best response for subordinate $t^* - 2$ is reporting, a contradiction. Suppose that $r_{t^*} = 1$. Then, since $\varphi_{1,0}(t^*) < 0$, $r_{t^*-1} = 0$. However, since $\varphi_{1,1}(t^* - 1) > 0$ and $\varphi_{0,1}(t^* - 1)$, reporting is the best response for subordinate $t^* - 1$, a contradiction. □

Proof of Lemma 4.7. Let $(r_t)_{t\in\mathbb{N}}$ be the probability of report in an equilibrium. Suppose that $\varphi_{0,0}(t) \leqslant 0$ for some $t \in \mathbb{N}$. Let $t_* := \min\{t : \varphi_{0,0}(t) < 0\}$. Suppose that $r_{t_*} = 1$. Then, since for each $t > t_*$, $\varphi_{1,1}(t) < 0$ and $\varphi_{1,0}(t) < 0$, $r_{t_*+1} = 0$. Suppose that $r_{t_*+2} = 1$. then, in turn, $r_{t_*+3} = 0$. However, since $\varphi_{0,0}(t_* + 2) < 0$ this implies that $r_{t_*+2} = 0$, a contradiction. Therefore, $r_{t_*+2} = 0$. As in the same way, we can show that $r_{t_*+j} = 0$ for each $j > 0$.

Then, suppose that $r_{t_*-1} = 1$. Since $\varphi_{1,0}(t_*) < 0$, it is a contradiction. Then, suppose that $r_{t_*-1} = 0$. Then, since $\varphi_{0,0}(t_*) < 0$, it is also a contradiction. Therefore, $r_{t_*} = 0$. As in the same fashion, we can show that $r_{t_*+j} = 0$ for each $j > 0$.

Let $T = \{t : \varphi_{1,1}(t) < 0 < \varphi_{0,0}(t)\}$. Suppose that $|T| \geqslant 2$. Let $t^* = \max T$. We now show that if $r_{t^*-j} = 1$, $r_{t^*-j-1} = 0$ and if $r_{t^*-j} = 0$, $r_{t^*-j-1} = 1$. for each $j = 0, 1, \ldots, |T| - 1$.

We consider the case for $j = 0$. Suppose by contradiction that $r_{t^*} = 1$ and $r_{t^*-1} = 1$. Then, since $\varphi_{1,0}(t^*) < 0$ and $r_{t^*+1} = 0$, $r_{t^*} = 0$ is the best response, a contradiction.

Suppose by contradiction that $r_{t^*} = 0$ and $r_{t^*-1} = 0$. Then, since $\varphi_{0,0}(t^*) > 0$ and $r_{t^*+1} = 0$, $r_{t^*} = 1$ is the best response, a contradiction.

Suppose that when $j = k$, the statement is true and consider the case for $j = k + 1$. Suppose by contradiction that $r_{t^*-j} = 1$, $r_{t^*-j-1} = 1$. By induction assumption, since $r_{t^*-j+1} = 0$ and $\varphi_{1,0}(t^* - j - 1) < 0$, $r_{t^*-j} = 0$ is the best response, a contradiction.

Suppose by contradiction that $r_{t^*-j} = 0$, $r_{t^*-j-1} = 0$. By induction assumption, since $r_{t^*-j+1} = 1$ and $\varphi_{0,1}(t^* - j - 1) > 0$, $r_{t^*-j} = 1$ is the best response, a contradiction. □

Proof of Theorem 4.4. Consider the behavior of subordinate t such that $\varphi_{0,0}(t) < 0$. Since $\varphi_{1,0}(t) < 0$, if players after period t take the strategy that not reporting the problem, for subordinate t, not reporting is the best response. Since s_t is increasing in t and thus, $\varphi_{0,0}(t)$ is decreasing in t, for each $t > t'$, $\varphi_{1,0}(t') \leqslant \varphi_{0,0}(t') < 0$. Therefore, for each subordinate t', not reporting is the best response.

Consider the following strategy profile:

1. subordinate $t \leqslant t_*$ reports

2. subordinate $t_* + 2k + 1$ reports and subordinate $t_* + 2k$ does not report for each $0 \leqslant k \leqslant [t^* - t_*]/2$.

3. subordinate $t > t^*$ does not report the problem.

As in the proof of lemma 4.7, for each t except for t^*, t_*, this strategy profile is the best response of itself. Consider subordinate t_*. Since $\varphi_{1,1}(t_*) > 0$ and subordinate $t_* - 1$ and $t_* + 1$ reports, reporting is the best response.

For subordinate t^*, we have the following two cases:

Case 1. Suppose that $|\{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}|$ is an even number. Note that $\varphi_{1,0}(t^*) < 0$. Since $|\{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}|$ is an even number , subordinate $t^* - 1$ reports and subordinate $t^* + 1$ does not report. Then, not reporting is the best response for subordinate t^*.

Case 2. Suppose that $|\{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}|$ is an odd number. Note that $\varphi_{0,0}(t^*) > 0$. Since $|\{t : \varphi_{0,0}(t) > 0 > \varphi_{1,1}(t)\}|$ is an odd number, subordinate $t^* - 1$ does not reports and subordinate $t^* + 1$ does not report. Then, reporting is the best response for subordinate t^* .

Therefore, this strategy profile is a PBE. □

Proof of Proposition 4.4. Note that

$$\frac{\partial I(0)}{\partial d^{M,R}} = \frac{-\frac{p}{1-p} f\left(w^R\right)}{(q^S + (1-q^S)q^M)\left[1 - F\left(w^U\right)\right] + (1-q^S)(1-q^M)}$$
$$\frac{\partial D^R}{\partial d^{M,R}} = -p \int_{w^R} dF(c) < 0,$$

where $w^R = (b^M + pd^{M,R})/(1-p)$ and $w^U = (b^M + pd^{M,U})/(1-p)$. In this case, since $D^R > 0$, $\frac{\partial \varphi_{0,0}(s)}{\partial d^{M,R}} < 0$. □

Proof of Proposition 4.5. Differentiating $\varphi_{0,0}$ by b^M yields

$$\frac{\partial \varphi_{0,0}(t)}{\partial b^M} = \delta(1-p)\left(\frac{\partial I(0)}{\partial b^M} D^R + \frac{\partial D^R}{\partial B^M} I(0) - \frac{\partial D^U}{\partial b^M}\right)\frac{s_{t+1}}{s_t}.$$

Note that

$$\frac{\partial I(0)}{\partial b^M} = -\frac{1}{1-p}\frac{f\left(w^R\right)\left[Q\left[1-F\left(w^U\right)\right] + Q'\right] - Qf\left(w^U\right)\left[1-F\left(w^R\right)\right]}{Q\left[1-F\left(w^U\right)\right] + Q'}$$
$$\frac{\partial D^R}{\partial b^M} = F\left(w^R\right), \quad \frac{\partial D^U}{\partial b^M} = F\left(w^U\right),$$

where $Q = (q^S + (1-q^S)q^M)$ and $Q' = (1-q^S)(1-q^M)$. Note that the probability $I(0)$ is maximized when $q^S = 1$ and minimized when $q^S = q^M = 0$. Then, we have

$$\frac{\partial I(0)}{\partial b^M} D^R + \frac{\partial D^R}{\partial B^M} I(0) - \frac{\partial D^U}{\partial b^M} < \frac{\partial I(0)}{\partial b^M} D^R + F(w^R)\frac{1-F(w^R)}{1-F(w^U)} - F(w^U), \quad \text{and}$$
$$\frac{\partial I(0)}{\partial b^M} D^R + \frac{\partial D^R}{\partial B^M} I(0) - \frac{\partial D^U}{\partial b^M} > \frac{\partial I(0)}{\partial b^M} D^R + F(w^R)(1-F(w^R)) - F(w^U).$$

Since $f(x)/(1-F(x))$ is increasing in x and $w^R > w^U$, $\frac{\partial I(0)}{\partial b^M}$ is negative. Then, if $D^R > 0$ and $F(w^R)(1-F(w^R)) < F(w^U)(1-F(w^U))$, $\frac{\partial \varphi_{0,0}(s)}{\partial b^M} < 0$. Conversely, if $D^R < 0$ and $F(w^R)(1-F(w^R)) > F(w^U)$, $\frac{\partial \varphi_{0,0}(s)}{\partial b^M} > 0$. □

Proof of Theorem 4.5. Suppose that there is no PBE such that no one reports in problem P. Thus, some subordinate reports. If the subordinate is subordinate 0, $\varphi_{1,0}(0) \geqslant 0$. Consider the case that $\varphi_{1,0}(0) < 0$. We now show that for some $t \in \mathbb{N}$, $\varphi_{0,0}(t) \geqslant 0$. Suppose by contradiction that for each t, $\varphi_{0,0}(t) < 0$. Then, since subordinate 0 conceals, there is a PBE such that each subordinate t conceals, a contradiction.

Therefore, for some $t \in \mathbb{N}$, $\varphi_{0,0}(t) > 0$ or $\varphi_{1,0}(0) > 0$.

Case 1. Suppose that for some $t \in \mathbb{N}$, $\varphi_{0,0}(t) \geqslant 0$. Then, as in the proof of Theorem 4.2, for each t, $\varphi'_{0,0}(t) > 0$. Then, we have $\varphi'_{0,1}(t) > 0$ for each t since $\varphi'_{0,1} > \varphi'_{0,0}$. Since $\varphi_{0,0} \leqslant \varphi_{1,1}$, $\varphi'_{1,1}(t) > 0$. Since $\varphi'_{1,1}(t) > 0$, the strategy profile that each player reports the problem is a PBE.

Suppose by contradiction that there is a pure strategy PBE such that someone conceals the problem. Let such player be subordinate t. Then, it must be $\varphi'_{1,0}(t) < 0$. Let the equilibrium strategy profile of subordinates denote by r. Suppose that $r_{t+1} = 1$. Then, since $\varphi'_{0,1}(t) > 0$ and $\varphi'_{1,1}(t) > 0$, the best response is $r_t = 1$, a contradiction. Suppose that $r_{t+1} = 0$. Consider the case that $r_{t+2} = 1$. Then, as in the same way, we have a contradiction. Therefore, we consider the case that $r_{t+2} = 0$. However, in this case, since $\varphi'_{0,0}(t) > 0$ for each s, $\varphi'_{0,0}(t+1) > 0$. Thus, the best response for subordinate $t+1$ is $r_{t+1} = 1$, a contradiction.

Case 2. Suppose that $\varphi_{1,0}(0) \geqslant 0$. Then, since $\varphi'_{1,0}(t) < \varphi'_{1,r}(t)$ for each $r > 0$, as in the proof of Theorem 4.2, each subordinate reports the problem in PBE.

The latter case is shown in the same way. □

Equilibrium with mixed strategy

In this subsection, we consider equilibria with mixed strategy. We first prove Theorem 4.3.

Proof of Theorem 4.3. Let $T_{10,00} = \{t : \varphi_{1,0}(t) < 0 < \varphi_{0,0}(t)\}$. We show the case for $|T_{10,00}| \leqslant 1$ in the main text. We first consider the case that $|T_{10,00}| = 2$. Let $T_{10,00} = \{t_*, t_* + 1\}$. Then, let r_{t_*}, r_{t_*+1} be the numbers that satisfy $\varphi_{1,r_{t_*}}(t_*) = 0$, $\varphi_{r_{t_*+1},0}(t_* + 1) = 0$. Let $r_t = 1$ for each $t < t_*$ and $r_t = 0$ for each $t > t_* + 1$. Then, r is an equilibrium.

As an induction assumption, we suppose that there is an equilibrium when $|T_{10,00}| = k$. Consider the case that $|T_{10,00}| = k + 1$.

Suppose by contradiction that there is no equilibria when $|T_{10,00}| = k+1$. Let $t_* = \min T_{10,00}$. Since $|T_{10,00} \setminus \{t_*\}| = k$, if $r_{t_*} = 1$, an equilibrium exists. Let M be the set of equilibrium strategy when $r_{t_*} = 1$ is fixed. Therefore, $r_{t_*} = 1$ is not the best response for subordinate t_*, which implies that $\varphi_{1,r_{t_*+1}}(t_*) < 0$ for each $r \in M$. Since $\varphi_{1,1}(t_*) > 0$, $r_{t_*+1} < 1$. Suppose that $r_{t_*+1} = 0$. Then, since $\varphi_{0,0}(t) > 0$ and $\varphi_{0,1}(t) > 0$ for each $t \in T_{10,00}$, $r_{t_*+2} = 1$ is the best response for subordinate $t_* + 1$'s action.[9] On the other hand, since $\varphi_{1,1}(t) > 0$ (and $\varphi_{0,1}(t) > 0$), $r_{t_*+1} = 1$ is the best response for subordinate

[9] Since $k \geqslant 2$, $t_* + 2 \in T_{10,00}$.

$t_* + 2$'s action, a contradiction. Therefore, $r_{t_*+1} \in (0,1)$. This implies that $\varphi_{1,r_{t_*+2}}(t_*+1) = 0$. Since $\varphi_{1,0}(t_*+1) < 0 < \varphi_{1,1}(t_*+1)$, $r_{t_*+2} \in (0,1)$. This also implies that $\varphi_{r_{t_*+1},r_{t_*+3}}(t_*+2) = 0$.

Note that $t_* + k - 1 \in T_{10,00}$ but $t_* + k \notin T_{10,00}$. Note also that $r_{t_*+k} = 0$.

Case 1. Suppose that $r_{t_*+k-1} = 1$. Then, since $\varphi_{r',1}(t) > 0$ for each $t \in T_{10,00}$, $r_{t_*+k-2} = 1$. However, since $\varphi_{1,0}(t_*+k-1) < 0$, the best response is $r_{t_*+k-1} = 0$, a contradiction.

Case 2. Suppose that $r_{t_*+k-1} = 0$. Suppose also that $r_{t_*+k-2} = 0$, then, since $\varphi_{0,0}(t_*+k-1) > 0$, the best response is $r_{t_*+k-1} = 1$, a contradiction. Therefore, we have $r_{t_*+k-2} \in (0,1)$, that is, $\varphi_{r_{t_*+k-3},r_{t_*+k-1}}(t_*+k-2) = 0$. Therefore, $r_{t_*+k-3} \in (0,1)$ since there is no $t \in T_{10,00}$ such that $\varphi_{1,0}(t) = 0$, $\varphi_{0,0}(t) = 0$, $\varphi_{1,1}(t) = 0$, or $\varphi_{0,1}(t) = 0$.

Case 2-1. Suppose that $r_{t_*+k-4} = 1$, then, for each $j < k-3$, if $r_{t_*+j} = 1$, r is an equilibrium, a contradiction.

Case 2-2. Suppose that $r_{t_*+k-4} = 0$, then, since $\varphi_{0,r'}(t) > 0$ for each $r' \in [0,1]$ and $t \in T_{10,00}$, $r_{t_*+k-3} = 1$ is the best response, a contradiction.

Therefore, we have $r_{t_*+k-4} \in (0,1)$. Continuing this process, $r_{t_*+j} \in (0,1)$ for each $j = 1, \ldots, k-2$.

Case 3. Suppose that $r_{t_*+k-1} \in (0,1)$, then, since $\varphi_{r_{t_*+k-2},0}(t_*+k-1) = 0$, $r_{t_*+k-2} \in (0,1)$ and as in the case 2, we have $r_{t_*+j} \in (0,1)$ for each $j = 1, \ldots, k-1$.

By cases 1,2 and 3, we have, $\varphi_{r_{t_*+j-1},r_{t_*+j+1}}(t_*+j) = 0$ for each $j = 1, \ldots, k-2$. We also have that $\varphi_{r_{t_*+k-2},r_{t_*+k}}(s_{t_*+k-1}) = 0$.

Now let $\widehat{r}_{t_*+1}$ be the number that satisfies $\varphi_{1,\widehat{r}_{t_*+1}}(t_*) = 0$. Then, $\widehat{r}_{t_*+1} > r_{t_*+1}$, $\varphi_{\widehat{r}_{t_*+1},r_{t_*+3}}(t_*+2) < 0$. Then, there is $\widehat{r}_{t_*+3} > r_{t_*+3}$ such that $\varphi_{\widehat{r}_{t_*+1},\widehat{r}_{t_*+3}}(t_*) = 0$. In turn, $\varphi_{\widehat{r}_{t_*+3},r_{t_*+5}}(t_*+4) < 0$. Let k^* be the largest even number less than $k-1$. Continuing this process, for each $j = 2, 4, \ldots, k^*$ there exists $\widehat{r}_{t_*+1}, \widehat{r}_{t_*+3}, \ldots, \widehat{r}_{t_*+k^*+1}$ such that $\varphi_{\widehat{r}_{t_*+j-1},\widehat{r}_{t_*+j+1}}(t_*+j) = 0$.

Consider the case that $k-1$ is an even number. Then, $k^* = k-3$. Then, let $\widehat{r}_{k-1} = r_{k-1}$. Note that $\varphi_{r_{t_*+k-2},r_{t_*+k}}(t_*+k-1)r_{t_*+k-1} = 0$. Suppose that $r_{t_*+k-1} > 0$. Then, $\varphi_{r_{t_*+k-2},0}(t_*+k-1) = 0$, and thus, $\varphi_{\widehat{r}_{t_*+k-2},0}(t_*+k-1) < 0$. Then, let $\widehat{r}_{t_*+k-1} = 0$. Since $\varphi_{r_{t_*+k-3},r_{t_*+k-1}}(t_*+k-2) = 0$, $\varphi_{r_{t_*+k-3},0}(t_*+k-2) < 0$. Then there exists $\widehat{r}_{t_*+k-3} < r_{t_*+k-3}$ such that $\varphi_{\widehat{r}_{t_*+k-3},0}(t_*+k-2) = 0$. Then, in turn, $\varphi_{r_{t_*+k-5},\widehat{r}_{t_*+k-3}}(t_*+k-4) < 0$. Continuing this process, there exists $r_{t_*}, r_{t_*+2}, \ldots, r_{t_*+k-1}$ for each $j = 2, 4, \ldots, k-1$, $\varphi_{\widehat{r}_{t_*+j-2},\widehat{r}_{t_*+j}}(t_*+j+1) = 0$. Then, $\widehat{r}$ satisfies equilibrium condition.

Suppose that $r_{t_*+k-1} = 0$. Then, since $\varphi_{r_{t_*+k-2},0}(t_*+k-1) < 0$, $\varphi_{\widehat{r}_{t_*+k-2},0}(t_*+k-1) < 0$. For each $j = 2, 4, \ldots, k-1$, let $\widehat{r}_{t_*+j} = r_{t_*+j}$. Then, since $\varphi_{r_{t_*+j-1},r_{t_*+j+1}}(t_*+j) = 0$, we also have $\varphi_{\widehat{r}_{t_*+j-1},\widehat{r}_{t_*+j+1}}(t_*+j) = 0$. Then, $\widehat{r}$ satisfies equilibrium condition.

Note that in either case, $\widehat{r}_{t_*+k-1} = 0$ when k is odd number. Thus, there is an equilibrium such that $\widehat{r}_{t_*+k-1} = 0$ when k is odd.

Consider the case that $k-1$ is odd. Then, $k^* = k-2$. As we shown above, $r_{t_*+k-1} = 0$ for some $r \in M$. Then, since $\varphi_{r_{t_*+k-2},0}(t_* + k - 1) < 0$, there is $\widehat{r}_{t_*+k-2} < r_{t_*+k-2}$ such that $\varphi_{\widehat{r}_{t_*+k-2},0}(t_* + k - 1) = 0$. Then, in turn, $\varphi_{r_{t_*+k-4},\widehat{r}_{t_*+k-2}}(t_* + k - 3) < 0$. Continuing this process, there exists $r_{t_*}, r_{t_*+2}, \ldots, r_{t_*+k-2}$ for each $j = 2, 4, \ldots, k - 1$, $\varphi_{\widehat{r}_{t_*+j-2},\widehat{r}_{t_*+j}}(t_* + j + 1) = 0$. Then, $\widehat{r}$ satisfies equilibrium condition.

Thus, in each case, we can construct an equilibrium strategy, a contradiction. □

The following propositions show the properties of equilibria with mixed strategy. Combining Theorem 4.3 and the following proposition, if $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$,[10] we can characterize the equilibrium with mixed strategy.

Proposition 4.9. *Suppose that $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$. Then, in each equilibrium, for each t such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.*

Proof of Proposition 4.9. Suppose that $\varphi_{0,1}(t) = 0$ for some $t \in \mathbb{N}$. Then, there exists t such that $\varphi_{0,1}(t) < 0$. Then, for each r, r', $\varphi_{r,r'}(t) < 0$. Thus, for the subordinate in period t, not reporting is strict dominant strategy. Since $\varphi_{0,1}(t)$ is decreasing in t, for each $t' > t$, subordinate in period t' conceals in equilibrium.

Let $T := \{t' : \varphi_{0,1}(t') \geqslant 0 \text{ and } \varphi_{0,0}(t') < 0\}$. If $T = \emptyset$, we are done. Suppose that $T \neq \emptyset$. Consider $t^* := \max T$. Then, we have $\varphi_{0,1}(t^* + 1) < 0$. Thus, $r_{t^*+1} = 0$. Since $\varphi_{0,0}(t) \geqslant \varphi_{1,0}(t)$, $\varphi_{1,0}(t^*) < 0$. Therefore, not reporting is best response for the subordinate in period t^*. Thus, we have $r_{t^*} = 0$. Continuing this process, we have that for each $t \in T$, subordinate t does not report in equilibrium. □

The above proposition needs the assumption that $\varphi_{0,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$. If it fails to hold, the equilibrium has the following properties.

Proposition 4.10. *Suppose that $\varphi_{1,1}(t) \leqslant 0$ for some $t \in \mathbb{N}$ and $\varphi_{0,1}(t) \geqslant 0$ for each $t \in \mathbb{N}$. Let $t^* := \min\{t : \varphi_{1,1}(t) < 0\}$. Then, in each equilibrium, the following statements hold. (1) Suppose that $r_{t^*} = 0$. Then, for each $t < t^*$ such that $\varphi_{0,0}(t) < 0$, subordinate t conceals the problem.*

(2) Suppose that $r_{t^*} \neq 0$. Then, for each t such that $\varphi_{1,1}(t) < 0$, subordinate t completely mixes reporting and concealing.

[10] Note that $\varphi_{0,1}(t) \leqslant 0$ implies that $\varphi_{1,1}(t) \leqslant 0$.

Proof of Proposition 4.10. (1) This is same for the Proposition 4.9.

(2) Let $t^* := \min\{t : \varphi_{1,1}(t) < 0\}$. Suppose that the subordinate in period t^* reports. Then, since $\varphi_{1,1}(t^*) < 0$, for each r and $t' > t^*$, $\varphi_{1,r}(t') < 0$. Therefore, $r_{t^*+1} = 0$. On the other hand, since $\varphi_{0,0}(t^*) < 0$, $\varphi_{r,0}(t^*) < 0$ for each $r \in [0, 1]$. Therefore, not reporting is the best response, a contradiction.

Suppose that the subordinate in period t^* reports with probability $r_{t^*} < 1$. This implies that $\varphi_{r_{t^*-1},r_{t^*+1}}(t^*) = 0$. If $r_{t^*+1} = 0$, $\varphi_{r_{t^*-1},r_{t^*+1}}(t^*) < 0$, a contradiction. If $r_{t^*-1} = 1$, it also contradicts. Therefore, $r_{t^*+1} > 0$ and $r_{t^*-1} < 1$. We consider the following three cases:

1. $r_{t^*+1} = 1$ and $r_{t^*-1} = 0$,

2. $r_{t^*+1} = 1$ and $r_{t^*-1} > 0$,

3. $r_{t^*+1} < 1$.

Case 1. Suppose that $r_{t^*+1} = 1$ and $r_{t^*-1} = 0$, then, since $\varphi_{0,1}(t) > 0$ for each $t \in \mathbb{N}$, a contradiction.

Case 2. Suppose that $r_{t^*+1} = 1$ and $r_{t^*-1} > 0$. Then, $\varphi_{r_{t^*},r_{t^*+2}}(t^* + 1) > 0$. Since $r_{t^*+1} = 1$ and $\varphi_{1,r}(t^* + 2) < 0$, $r_{t^*+2} = 0$. Thus, since $\varphi_{0,0}(t^* + 1) < 0$ and $\varphi_{1,0}(t^* + 1) < 0$, not reporting is the best response for subordinate $t^* + 1$, a contradiction.

Case 3. Suppose that $r_{t^*+1} < 1$. Since $r_{t^*+1} > 0$, $\varphi_{r_{t^*},r_{t^*+2}}(t^* + 1) = 0$. Then, $r_{t^*+2} > 0$ since $\varphi_{r_{t^*},0}(t^* + 1) < 0$. If $r_{t^*+2} = 1$, as in case 2, we have a contradiction. Continuing this process, for each $t' > t^*$, $\varphi_{r_{t'-1},r_{t'+1}}(t') = 0$. □

4.A.4 Proofs in Section 4.6.2

Proof of Lemma 4.8. When the problem is reported, the manager in period t's objective is

$$\max_{\rho} \rho b^M s_t + (1 - \rho)(-pd^{M,R})s_t - c\chi(\rho)s_t,$$

equivalently

$$\max_{\rho} \rho b^M + (1 - \rho)(-pd^{M,R}) - c\chi(\rho). \tag{4.2}$$

By the Karush-Kuhn-Tucker condition, the optimal probability $\rho^*(c)$ satisfies

$$b^M + pd^{M,R} - \chi'(\rho^*(c))c + \lambda = 0, \lambda\rho^*(c) = 0,$$

for some nonnegative real number λ. Since χ'^{-1} is increasing, if $b^M + pd^{M,R} < \chi'(0)c$, it must be $\lambda > 0$. Therefore, $\rho^*(c) = 0$. On the other hand, if $b^M + pd^{M,R} \geqslant \chi'(0)c$, $\lambda = 0$. Thus, problem (4.2) has the interior solution. Therefore,

$$\rho^*(c) = \begin{cases} 0 & \text{if} \quad b^M + pd^{M,R} < \chi'(0)c, \\ \chi'^{-1}((b^M + pd^{M,R})/c) & \text{if} \quad b^M + pd^{M,R} \geqslant \chi'(0)c. \end{cases}$$

Since χ'^{-1} is increasing, $\rho^*(c)$ is decreasing in c. Then, the expected utility of facing the reported problem as a manager is

$$\widetilde{D}^R = \int [\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - c\chi(\rho^*(c))]\, dF(c).$$

Consider the manager's problem when the manager knows the problem before the subordinate's report. If he ignores the problem and the problem is unreported, the expected utility is $-pd^{M,U}$. If he ignores the problem and the problem is reported, the expected utility is $\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c$. Let r be the probability that the subordinate reports the problem. Then, the expected utility of ignoring the problem is

$$r[\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - c\chi(\rho^*(c))] + (1 - r)[-pd^{M,U}].$$

On the other hand, if he does not ignore the problem, that is, $\rho > 0$ the expected utility is $\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c$. Thus, the manager ignores the problem if and only if $\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c < -pd^{M,U}$.

As in the previous section, let $\widetilde{D}^U$ be the expected utility of facing the unreported problem, that is,

$$\widetilde{D}^U = \int [\max\{\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c, -pd^{M,U}\}]\, dF(c).$$

It is easy to show that $\widetilde{D}^U \geqslant \widetilde{D}^R$. Let c^* be the number that solves $\rho^*(c^*)b^M + (1 - \rho^*(c^*))(-pd^{M,R}) - \chi(\rho^*(c^*))c^* = -pd^{M,U}$. Note that $\rho^*(c)b^M + (1 - \rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c$ is decreasing in

c. This is because, by the envelope theorem, differentiating the above function by c yields that $-\chi(\rho^*(c)) < 0$. Then, $\widetilde{D}^U$ is written as

$$\widetilde{D}^U = \int^{c^*} [\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c]\, dF(c)$$

$$-pd^{M,U}(1 - F(c^*)).$$

Therefore, the probability that the reported problem remains unsolved in the next period, which is denoted by $\widetilde{I}(r_{t-1})$ is

$$\widetilde{I}(r_{t-1}) := \Bigg((q^S(1-r_{t-1}) + (1-q^S)q^M) \int_{c^*} (1-\rho^*(c))\, dF(c)$$

$$+ (1-q^S)(1-q^M)\left(\int (1-\rho^*(c))\, dF(c)\right)\Bigg)$$

$$\times \left((q^S(1-r_{t-1}) + (1-q^S)q^M)(1-F(c^*)) + (1-q^S)(1-q^M)\right)^{-1}$$

Consider the subordinate's behavior. The expected utility of reporting is

$$\delta \widetilde{I}(r_{t-1})\widetilde{D}^R s_{t+1} + b^S s_t.$$

The expected utility of not reporting is

$$-pd^S s_t + \delta\left[p\widetilde{D}^R s_{t+1} + (1-p)(q^S r_{t+1}\widetilde{D}^R s_{t+1} + (1-q^S r_{t+1})\widetilde{D}^U s_{t+1})\right].$$

Then, calculating the difference of these equation yields $\widetilde{\varphi}_{r_{t-1}, r_{t+1}}$. □

Proof of Theorem 4.6. Recall that

$$\widetilde{\varphi}_{1,0}(t) := \delta(1-p)\left[(\widetilde{I}(r_{t-1}) - 1)\widetilde{D}^R - (1-p)(\widetilde{D}^U - \widetilde{D}^R)\right]\frac{s_{t+1}}{s_t} + b^S + d^S.$$

Then,

$$\frac{\partial\widetilde{\varphi}_{1,0}(t)}{\partial d^{M,R}} = \delta(1-p)\left[\frac{\partial\widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R + (\widetilde{I}(1)-p)\frac{\partial\widetilde{D}^R}{\partial d^{M,R}} - \frac{\partial\widetilde{D}^U}{\partial d^{M,R}}\right]\frac{s_{t+1}}{s_t}.$$

Consider the case that $d^{M,R} \to \infty$. Then, for sufficiently large $d^{M,R}$, for each $c \in (0, \bar{c})$, $[b^M + pd^{M,R}]/c > \chi'(0)$. Therefore, the maximization problem (4.2) has an interior solution. Therefore by the first order condition, we have $\chi'(\rho^*(c)) = [b^M + pd^{M,R}]/c$. Then, since $\lim_{\rho\to\bar{\rho}}\chi'(\rho) = \infty$, as $d^{M,R} \to \infty$, $\rho^*(c) \to \bar{\rho}$. This implies that $\chi(\rho^*(c)) \to \infty$. Then, we have that for each $c > 0$, as $d^{M,R} \to \infty$, $\rho^*(c)b^M + (1-\rho^*(c))(-pd^{M,R}) - \chi(\rho^*(c))c \to \infty$. Then, for sufficiently large $d^{M,R}$, $c^* < 0$. This implies that for sufficiently large $d^{M,R}$, since $c^* < 0$ is not in the support of F, $f(c^*) = 0$. Therefore, for sufficiently large $d^{M,R}$,

$$\frac{\partial\widetilde{I}(r)}{\partial d^{M,R}} = -\int \frac{\partial\rho^*(c)}{\partial d^{M,R}}\, dF(c).$$

This also implies that $\partial\widetilde{D}^U/\partial d^{M,U} = 0$ for sufficiently large $d^{M,R}$. Therefore, to determine the sign of $\frac{\partial\widetilde{\varphi}_{1,0}(s)}{\partial d^{M,R}}$, we consider only

$$\frac{\partial\widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R + (I(1)-p)\frac{\partial\widetilde{D}^R}{\partial d^{M,R}}.$$

(1) We consider the case that $\bar{\rho} < 1$. We write $\widetilde{D}^R$ as a function of $d^{M,R}$ explicitly, that is $\widetilde{D}^R(d^{M,R})$. Since $\frac{\partial\widetilde{D}^R(d^{M,R})}{\partial d^{M,R}} = -p\int(1-\rho^*(c))\, dF(c) < -p$, $|\widetilde{D}^R(d^{M,R}) - \widetilde{D}^R(0)| < pd^{M,R}$. Therefore, $|\widetilde{D}^R(d^{M,R})| < pd^{M,R} + |\widetilde{D}^R(0)|$.

To verify $\lim_{d^{M,R}\to\infty}\frac{\partial\widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R(d^{M,R})$, we consider $\partial\rho^*(c)/\partial d^{M,R}$. Note that $\frac{\partial\rho^*(c)}{\partial d^{M,R}} > 0$. To show this, recall that $\rho^*(c) = \chi'^{-1}([b^M + pd^{M,R}]/c)$. Since χ'^{-1} is increasing, $\frac{\partial\rho^*(c)}{\partial d^{M,R}} > 0$. We also write $\rho^*(c)$ as a function of $d^{M,R}$, $\rho^*(c, d^{M,R})$. Note that $\rho^*(c, d^{M,R}) - \rho^*(c, d^{M,R}/2) = \int_{d^{M,R}/2}^{d^{M,R}}(\partial\rho^*(c,d')/\partial d')\, dd' \geqslant \min_{d\in[d^{M,R}/2, d^{M,R}]} d^{M,R}(\partial\rho^*(c,d)/\partial d^{M,R})/2$. Since $\rho^*(c, d^{M,R}) \to \bar{\rho}$ for each c as $d^{M,R} \to \infty$, $\rho^*(c, d^{M,R}) - \rho^*(c, d^{M,R}/2) \to 0$ as $d^{M,R} \to \infty$. Therefore,

$$\lim_{d^{M,R}\to\infty}\min_{d\in[d^{M,R}/2, d^{M,R}]}\frac{d^{M,R}}{2}\frac{\partial\rho^*(c,d)}{\partial d^{M,R}} = 0.$$

Since

$$\left|\frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R(d^{M,R})\right| < \max_c \frac{\partial \rho^*(c,d)}{\partial d^{M,R}}(pd^{M,R} + |\widetilde{D}^R(0)|),$$

the RHS converges to 0 as $d^{M,R} \to \infty$.

Consider $\widetilde{I}(1)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}}$. Since $c^* < 0$ for sufficiently large $d^{M,R}$, $\widetilde{I}(1) = \int (1-\rho^*(c))\, dF(c)$. Then,

$$(\widetilde{I}(1) - p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} = -p\left[\int (1-\rho^*(c,d^{M,R}))\, dF(c) - p\right]\left[\int (1-\rho^*(c,d^{M,R}))\, dF(c)\right].$$

Therefore, if $\bar{\rho} < 1-p$, since for each c, $\rho^*(c) \to \bar{\rho}$, $(\widetilde{I}(1)-p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} < 0$ and thus, $\frac{\partial \widetilde{\varphi}_{1,0}}{\partial d^{M,R}} < 0$. On the other hand, if $\bar{\rho} > 1-p$, $(\widetilde{I}(1)-p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} > 0$ and thus, $\frac{\partial \widetilde{\varphi}_{1,0}}{\partial d^{M,R}} > 0$. □

Proof of Proposition 4.6. Under the assumption, $\widetilde{D}^U = 0$. Note that if $q^M = 0$, $\widetilde{I}(1) = \int (1 - \rho^*(c))\, dF(c)$. Then,

$$\widetilde{\varphi}_{1,0}(t) = \delta(\widetilde{I}(1) - p)\widetilde{D}^R\frac{s_{t+1}}{s_t} + b^S + pd^S.$$

Note that since $\chi(\rho) = \rho/(\bar{\rho} - \rho)$, by the Karush-Kuhn-Tucker condition,

$$\rho^*(c) = \max\left\{\bar{\rho} - \sqrt{\bar{\rho}\frac{c}{b^M + pd^{M,R}}}, 0\right\}.$$

The first order derivative is

$$\delta\left[(\widetilde{I}(1)-p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} + \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R\right]\frac{s_{t+1}}{s_t}.$$

Consider $(\widetilde{I}(1)-p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} + \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R$. This is calculated as

$$\begin{aligned}
&(\widetilde{I}(1)-p)\frac{\partial \widetilde{D}^R}{\partial d^{M,R}} + \frac{\partial \widetilde{I}(1)}{\partial d^{M,R}}\widetilde{D}^R \\
&= -p(1-\bar{\rho})(1-\bar{\rho}-p) - p\sqrt{\bar{\rho}}\tau A\left(\frac{\bar{\rho}}{2} + \frac{\mu A^2}{2} + 1 - 2\bar{\rho} - p + 1 - \frac{pd^{M,R}}{2(b^M + pd^{M,R})}\right),
\end{aligned}$$

where $A = 1/(b^M + pd^{M,R})^{1/2}$, $\tau = \int^{\bar{\rho}/A^2} \sqrt{c}\, dF(c)$ and $\mu = \int^{\bar{\rho}/A^2} c\, dF(c)$. Then, if $1-p > \bar{\rho}$, the first order derivative is negative. □

4.A.5 Proof in Section 4.6.3

Proof of Lemma 4.9. Consider a manager in period t who is reported a problem. Let $\xi(s)$ be the probability that the manager $t+1$ ignores the problem. Then, the manager's expected utility of ignoring the problem is

$$-p(d^{M,R}s_t + cs_t) - (1-p)p\delta\xi(s_t)(1-\mu)d^R G(s_t).$$

The manager in period t solves the problem if and only if

$$\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(s_t)(1-\mu)d^R[G(s_t)/s_t]}{1-p} \geqslant c$$

Therefore, the probability that a manager ignores, ξ satisfies

$$\xi(s) = H(\xi)(s) := \left[1 - F\left(\frac{b^M + pd^{M,R} + \delta(1-p)\xi(G(s))(1-\mu)d^R \frac{G^2(s)}{G(s)}}{1-p}\right)\right]. \tag{4.3}$$

Then, if there is a ξ that satisfies the above condition, the managers' behavior is determined.

Claim 4.2. *Suppose that $G(s)/s$ and f are well defined on $\mathbb{R}_+$ and $\sup_{s\in\mathbb{R}_+} G(s)/s < \infty$. Then, there exists a function $\xi : \mathbb{R}_+ \to [0,1]$ that satisfies* (4.3).

Proof of Claim 4.2. To show the existence of ξ that satisfies (4.3), we show the existence of a fixed point of H.

Since each $[0,1]$ is a nonempty convex compact set, $[0,1]^{\mathbb{R}_+}$ is a convex set and by the Tychonoff theorem, $[0,1]^{\mathbb{R}_+}$ is a compact set under the product topology. Let O be the product topology of $\mathbb{R}^{\mathbb{R}_+}$. To show the existence of a fixed point of H, we use

Fact 4.2 (Aliprantis and Border 2006, p.206)**.** *$(\mathbb{R}^{\mathbb{R}_+}, O)$ is locally convex Hausdorff space.*

Fact 4.3 (Brouwer-Schauder-Tychonoff's fixed point theorem, Aliprantis and Border 2006, p.583)**.** *Let C be a nonempty compact convex subset of locally convex Hausdorff space, and let $f : C \to C$ be a continuous function. Then f has a fixed point.*

Therefore, we only to show that H is continuous on $[0,1]^{\mathbb{R}_+}$. To show this, let $\xi, \xi' \in [0,1]^{\mathbb{R}_+}$. Note that the product topology is generated by the family of seminorm $(|h(s)|)_{s\in\mathbb{R}_+}$ for each $h \in [0,1]^{\mathbb{R}_+}$.

Note also that by the mean value theorem, there exists $\widetilde{\xi} \in [0,1]$ such that

$$|H(\xi)(s) - H(\xi')(s)|$$
$$= \delta p(1-\mu)d^R \frac{G^2(s)}{G(s)} f\left(\frac{b^M + pd^{M,R} + \delta(1-p)p\widetilde{\xi}(1-\mu)d^R \frac{G^2(s)}{G(s)}}{1-p}\right)$$
$$\times |\xi(G(s)) - \xi'(G(s))|.$$

Then, since $G(s)/s$ is bounded above, when $\xi'(s) \to \xi(s)$ for each s, $H(\xi')(s) \to H(\xi)(s)$ for each s. Therefore, H is continuous. Then H has a fixed point, ξ. □

If the problem is unreported, the expected utility of ignoring the problem is

$$(1-r)(-pd^{M,U}s_t + cs_t) + r\max\Big\{b^M s_t - d^{M,R}s_t,$$
$$-p(d^{M,R}s_t + cs_t) - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)\Big\},$$

where r is the probability that the problem is reported in the next period. Therefore, the manager solves if and only if

$$b^M s_t - cs_t \geqslant -(1-r)p(d^{M,U} + c)s_t + r\max\Big\{b^M s_t - cs_t,$$
$$-pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)\Big\}$$

If $b^M s_t - cs_t \geqslant -pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)$, the condition is

$$b^M s_t - cs_t \geqslant -p(d^{M,U} + c)s_t.$$

If $b^M s_t - cs_t < -p(d^{M,R} + c)s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)$, the condition is

$$b^M s_t - cs_t \geqslant -(1-r)p(d^{M,U} + c)s_t - r(pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t)).$$

However, since $d^{M,R} > d^{M,U}$, $-pd^{M,R}s_t - (1-p)\delta p\xi(s_t)(1-\mu)d^R G(s_t) < -pd^{M,U}$. Therefore, if $b^M s_t - cs_t < -p(d^{M,R} + c)s_t - (1-p)\delta p \xi(s_t)(1-\mu)d^R G(s_t)$, the above condition must not be satisfied. Therefore, the manager solves if and only if $\frac{b^M + pd^{M,U}}{1-p} \geqslant c$. Thus, the probability that the manager ignores problem is

$$\widehat{I}(r_{t-1}, s) = \frac{(q^S(1-r_{t-1}) + q^M(1-q^S))\xi(s)}{[q^S(1-r_{t-1}) + q^M(1-q^S)]\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1-q^S)(1-q^M)}.$$

Let $\widehat{D}^U := \mathbb{E}\left[\max\left\{b^M - c, -p(d^{M,U} + c)\right\}\right]$, and

$$\widehat{D}^R(s) := \mathbb{E}\left[\max\left\{b^M - c, -p(d^{M,R} + c) - (1-p)\delta p \xi(G(s))(1-\mu)d^R \frac{G^2(s)}{G(s)}\right\}\right].$$

Then as in the basic model, we can define $\widehat{\varphi}_{r,r'}$. □

Proof of Proposition 4.7. Note that when $s_{t+1} = \alpha s_t$, $\widehat{D}^R$, ξ and $\widehat{I}$ are no longer depend on s but on α. Therefore, we write these variables as functions of α.

Note also that since ξ is independent of s, ξ is uniquely determined. This is because, (4.3) is written as

$$\xi(\alpha) = H_\alpha(\xi(\alpha)) := \left[1 - F\left(\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(\alpha)(1-\mu)d^R\alpha}{1-p}\right)\right]. \tag{4.4}$$

Then, since $H_\alpha(\xi)$ is decreasing in ξ, for each α, $\xi(\alpha)$ is uniquely determined.

Let $E(\alpha) := [-\{1 - \widehat{I}(r_{t-1}, \alpha)\}\widehat{D}^R(\alpha) - (1 - q^S r_{t+1})(\widehat{D}^U - \widehat{D}^R(\alpha))]$. The proof consists of the following six steps.

Step 1: *$\xi(\alpha)$ is decreasing in α.* First note the derivative of ξ. By the implicit function theorem, $\xi(G)$ is differentiable and by (4.4), the derivative is given by

$$\xi'(\alpha) = -\frac{A}{1 + A\alpha}\xi(\alpha) < 0,$$

where

$$A = p\delta(1-\mu)d^R f\left(\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(\alpha)(1-\mu)d^R\alpha}{1-p}\right).$$

Therefore, $\xi(\alpha)$ is decreasing in α. ■

Step 2: *$\xi(\alpha)\alpha$ is increasing in α.* By step 1, $\xi(\alpha)$ is decreasing in α. On the other hand, the RHS of (4.4) is a decreasing function of $\xi(\alpha)\alpha$. Therefore, $\xi(\alpha)\alpha$ is increasing in α. ■

Step 3: *$\xi(\alpha)\alpha$ converges to a real number as $\alpha \to \infty$.* Suppose by contradiction that $\xi(\alpha)\alpha$ does not converge. By step 2, since $\xi(\alpha)\alpha$ is increasing, $\xi(\alpha)\alpha \to \infty$. Then, there exists $\bar{\alpha}$ such that for each $\alpha > \bar{\alpha}$,

$$\frac{b^M + pd^{M,R} + \delta(1-p)p\xi(\alpha)(1-\mu)d^R\alpha}{1-p} > \bar{c}.$$

Then, since the support of f is $(0, \bar{c})$, $\xi(\alpha) = 0$, which implies that $\xi(\alpha)\alpha = 0$, a contradiction. ■

Step 4: $\lim_{\alpha\to\infty} E(\alpha) > 0$. By step 3, since $\xi(\alpha)\alpha$ converges as $\alpha \to \infty$, $\xi(\alpha) \to 0$. This implies that $\widehat{I}(r,\alpha) \to 0$. Then, since $\widehat{D}^R(\alpha) < 0$ and $\widehat{D}^U < 0$,

$$\lim_{\alpha\to\infty} E(\alpha) = -q^S r_{t+1}\widehat{D}^R(\alpha) - (1-q^S r_{t+1})\widehat{D}^U > 0.$$

■

Step 5: $\lim_{G\to\infty} \frac{\partial E(\alpha)}{\partial \alpha}\alpha \geqslant 0$. The derivative of $E(\alpha)$ is given by

$$\frac{\partial E(\alpha)}{\partial \alpha} = \frac{\partial \widehat{I}(r_{t-1},\alpha)}{\partial \alpha}\widehat{D}^R(\alpha) + (\widehat{I}(r_{t-1},\alpha) - q^S r_{t+1})\frac{\partial \widehat{D}^R(\alpha)}{\partial \alpha},$$

where

$$\frac{\partial \widehat{I}(r_{t-1},\alpha)}{\partial \alpha} = \frac{(1-q^S r_{t-1})\xi'(\alpha)}{[q^S(1-r_{t-1}) + q^M(1-q^S)]\left[1 - F\left(\frac{b^M + pd^{M,U}}{1-p}\right)\right] + (1-q^S)(1-q^M)} < 0,$$

$$\frac{\partial \widehat{D}^R(\alpha)}{\partial \alpha} = -(1-p)p\delta(1-\mu)d^R(\xi(\alpha)\alpha)'\xi(\alpha) < 0.$$

Note that since $\xi(\alpha) \to 0$, $\xi'(\alpha) \to 0$. Therefore, $\frac{\partial \widehat{I}(r_{t-1},\alpha)}{\partial \alpha} \to 0$ and $\frac{\partial \widehat{D}^R(\alpha)}{\partial \alpha} \to 0$. If $r_{t+1} > 0$, since $\widehat{I}(r_{t-1},\alpha) \to 0$ as $\alpha \to \infty$, $\frac{\partial E(\alpha)}{\partial \alpha} > 0$ for sufficiently large α. ■

Step 6: *Completing the proof.* Consider the case that $r_{t+1} = 0$. Then, since $\xi(\alpha)\alpha$ is bounded above, $\widehat{I}(r_{t-1},\alpha)\alpha$ is bounded above. Since $\frac{\partial \widehat{D}^R(\alpha)}{\partial \alpha} \to 0$, $(\widehat{I}(r_{t-1},\alpha) - q^S r_{t+1})\frac{\partial \widehat{D}^R(\alpha)}{\partial \alpha} \to 0$. Thus, since

$\widehat{D}^R(\alpha) < 0$ and $\frac{\partial \widehat{I}(r_{t-1},\alpha)}{\partial\alpha} < 0$, $\lim_{\alpha\to\infty}\frac{\partial E(\alpha)}{\partial\alpha}\alpha \geqslant 0$.

Note that $\widehat{\varphi}_{r_{t-1},r_{t+1}}(\alpha) = \delta(1-p)E(\alpha)\alpha + b^S + pd^S$. Then, if $E(\alpha) > 0$, $\widehat{\varphi}_{r_{t-1},r_{t+1}}(\alpha) > 0$. Thus, by step 5, $\lim_{\alpha\to\infty}\widehat{\varphi}_{r_{t-1},r_{t+1}}(\alpha) > 0$.

Note also that $\partial\widehat{\varphi}_{r_{t-1},r_{t+1}}(\alpha)/\partial\alpha = \delta(1-p)\frac{\partial E(\alpha)}{\partial\alpha}\alpha + \delta(1-p)E(\alpha)$. Then by steps 4 and 5, $\partial\widehat{\varphi}_{r_{t-1},r_{t+1}}(\alpha)/\partial\alpha > 0$. □

Proof of Proposition 4.8. As in the proof of Proposition 4.7, let

$$E(\alpha) = [-\{1-\widehat{I}(r_{t-1},\alpha)\}\widehat{D}^R(\alpha) - (1-q^S r_{t+1})(\widehat{D}^U - \widehat{D}^R(\alpha))]$$

Since $\partial\widehat{\varphi}_{r_{t-1},r_{t+1}}(\alpha)/\partial\alpha = \delta(1-p)\frac{\partial E(\alpha)}{\partial\alpha}\alpha + \delta(1-p)E(\alpha)$, it is sufficiently to show that $\frac{\partial E(0)}{\partial\alpha} < 0$ and $E(0) < 0$ for sufficiently large $\bar{c}$. Note that

$$\xi(0) = 1 - F\left(\frac{b^M + pd^{M,R}}{1-p}\right) = 1 - \frac{b^M + pd^{M,R}}{(1-p)\bar{c}}.$$

Then, we can write $\widehat{I}(r_{t-1},0) = A(\bar{c})\xi(0)$. Note that $A(\bar{c}) \geqslant 1$ and $\lim_{\bar{c}\to\infty} A(\bar{c}) = 1$. On the other hand,

$$\begin{aligned}
\widehat{D}^R(0) = {} & b^M\frac{b^M + pd^{M,R}}{(1-p)\bar{c}} - \left(\frac{b^M + pd^{M,R}}{(1-p)\bar{c}}\right)^2\frac{1}{\bar{c}} - pd^{M,R}\left(1 - \frac{b^M + pd^{M,R}}{(1-p)\bar{c}}\right) \\
& + \frac{p}{\bar{c}}\left(\frac{b^M + pd^{M,R}}{(1-p)\bar{c}}\right)^2 - p\bar{c} \\
\widehat{D}^U = {} & b^M\frac{b^M + pd^{M,U}}{(1-p)\bar{c}} - \left(\frac{b^M + pd^{M,U}}{(1-p)\bar{c}}\right)^2\frac{1}{\bar{c}} - pd^{M,U}\left(1 - \frac{b^M + pd^{M,U}}{(1-p)\bar{c}}\right) \\
& + \frac{p}{\bar{c}}\left(\frac{b^M + pd^{M,U}}{(1-p)\bar{c}}\right)^2 - p\bar{c}
\end{aligned}$$

Then, since $\widehat{D}^R(0) < 0$,

$$\begin{aligned}
E(0) &= [-\{1 - I(r_{t-1},0)\}\widehat{D}^R(0) - (1-q^S r_{t+1})(\widehat{D}^U - \widehat{D}^R(0))] \\
&< -\widehat{D}^R(0)\frac{b^M + pd^{M,R}}{1-p} - (1-q^S r_{t+1})(\widehat{D}^U - \widehat{D}^R(0)).
\end{aligned}$$

Letting $\bar{c} \to \infty$ yields that

$$\begin{aligned}
&\lim_{\bar{c}\to\infty} -\widehat{D}^R(0)\frac{b^M + pd^{M,R}}{1-p} - (1 - q^S r_{t+1})(\widehat{D}^U - \widehat{D}^R(0)) \\
&= p\frac{b^M + pd^{M,R}}{1-p} - (1 - q^S r_{t+1})p(d^{M,R} - d^{M,U}).
\end{aligned}$$

Therefore, if the following condition holds, $\lim_{\bar{c}\to\infty} E(0) < 0$.

$$p < \frac{(1 - q^S r_{t+1})(d^{M,R} - d^{M,U}) - b^M}{d^{M,R} - (1 - q^S r_{t+1})(d^{M,R} - d^{M,U})}.$$

The above condition holds when p, b^M and $d^{M,U}$ are sufficiently small.

Next, we consider $\frac{\partial E(\alpha)}{\partial \alpha}$, which is written as

$$\frac{\partial E(\alpha)}{\partial \alpha} = \frac{\partial I(r_{t-1}, \alpha)}{\partial \alpha}\widehat{D}^R(\alpha) + (\widehat{I}(r_{t-1}, \alpha) - q^S r_{t+1})\frac{\partial \widehat{D}^R(\alpha)}{\partial \alpha}.$$

Note that

$$\xi'(0) = -A\xi(0) = -\frac{p\delta(1-\mu)d^R}{\bar{c}}\xi(0), \quad \frac{\partial \widehat{D}^R(0)}{\partial \alpha} = -(1-p)\delta(1-\mu)d^R(\xi(0))^2.$$

Therefore,

$$\begin{aligned}
\lim_{\bar{c}\to\infty} \frac{\partial E(0)}{\partial \alpha} &= p^2\delta(1-\mu)d^R - (1 - q^S r_{t+1})(1-p)\delta(1-\mu)d^R \\
&= \delta(1-\mu)d^R(p^2 - (1 - q^S r_{t+1})(1-p)).
\end{aligned}$$

Thus, if $q^S < 1$ and p is sufficiently small, $\lim_{\bar{c}\to\infty} \frac{\partial E(0)}{\partial \alpha} < 0$. □

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