Income Inequality and the Rise of the Working Rich

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（所得格差と高額所得労働者の出現）

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Chapter 1

Introduction

1.1 Stylized Facts

Kuznets [1955] proposed a famous hypothesis that income inequality follows an inverse-U curve. Simply put, the hypothesis states that it takes time for economic development to benefit a large mass of the population. Thus, income inequality rises in the early stage, but falls in the later stage as more and more individuals benefit from economic development. This hypothesis optimistically implies that there is no need to worry about the (short-term) high concentration of the national income in a few hands.

With more complete data, Piketty [2014] illustrates that the fall in income inequality in the first half of the 20th century, observed by Kuznets, was abrupt (see Figure 1.1). The abrupt reduction is not consistent with the gradual process of Kuznets' hypothesis. Piketty (p. 15) argues instead that the abrupt reduction was the result of the world wars, the violent economic shocks (e.g., the great depression), and political shocks. These shocks reduced capital by a substantial amount and hence affected top capital income disproportionally because wealth was very concentrated. More importantly, Piketty reports that, in the last several decades, many developed countries have witnessed substantial increases in income inequality.

The increases in income inequality in developed countries are not unprecedented. As shown in Figure 1.1, for example, the top 10% income share in the United States rose to a very high level in the early twentieth century. The recent increases, nevertheless, have one distinctive feature. That is, *the recent increases in income inequality are largely due to substantial increases in the top earned income share* [see Piketty and Saez [2003], Atkinson and Piketty [2007], and Atkinson et al. [2011]]. As one can tell from Figure 1.1, the top 10% earned income share has been rising significantly since the early 1980s (from 28.4% in 1980 to 43.5% in 2013), while the top 10% capital income share has remained roughly unchanged.

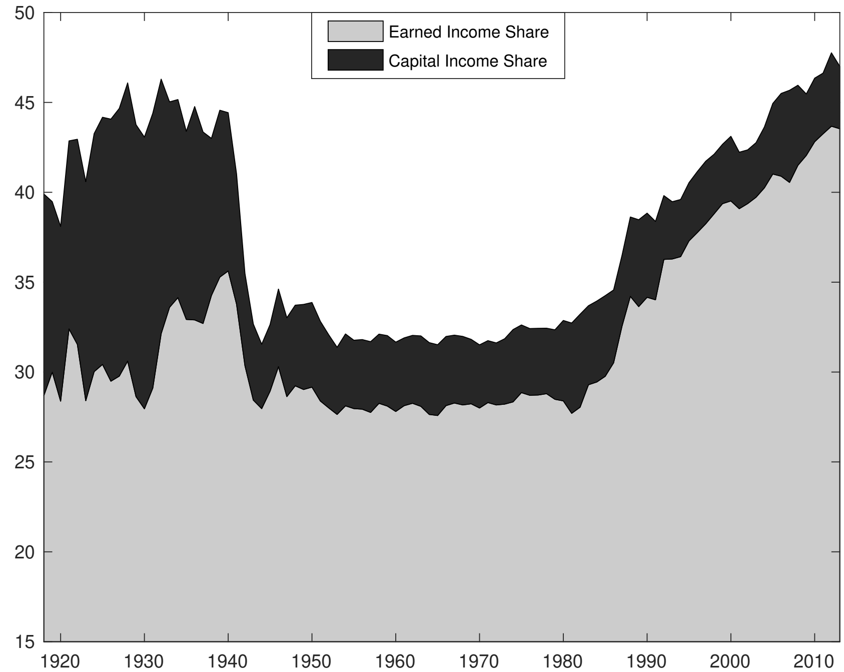

Figure 1.1: Top 10% Income Share and Composition in the United States, 1918-2013.

Note: Income here is the total income, consisting of earned income (including wages and entrepreneurial income) and capital income (including dividends, interest income, and rent).

Source: The World Wealth and Income Database.

Piketty and Saez [2003] refer to the top earned income group as "the working rich." Accordingly, we refer to the recent phenomenon of the increase in the top earned income share as "the rise of the working rich."

Throughout this dissertation, concerning the working rich, we focus only on entrepreneurs and managers/CEOs.[1] (Other working rich include lawyers, athletes, and celebrities.) We focus on two aspects of the rise of the working rich. First, *there are large increases in top*

[1]Throughout this dissertation, we use the terms "managers" and "CEOs" interchangeably. This group of individuals receives much attention in the literature. Piketty [2014] uses the term "the supermanagers," and refers to the recent phenomenon of the significant increase in managers' compensation as "the rise of the supermanagers." In this dissertation, we use the term "the working rich" because both wages and entrepreneurial income have comparable contributions to the recent increase in income inequality. In the United States, for example, the top 10% entrepreneurial income share (the top 10% wage share) rises by 5.4% from 2.7% in 1980 to 8.1% in 2013 (9.7% from 25.7% to 35.4%). The top 1% rises by 4.4% from 1.1% to 5.5% (4.9% from 4.9% to 9.8%).

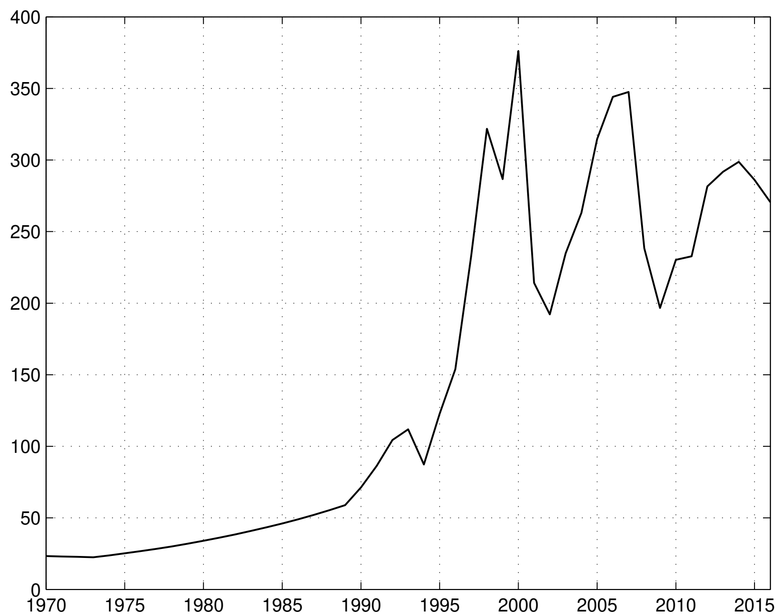

Figure 1.2: CEO-to-Worker Compensation Ratio in the United States, 1970-2015.

Source: The Economic Policy Institute.

earned income relative to top capital income and workers' wages. The increases in top earned income relative to top capital income are obvious from Figure 1.1. As already mentioned, the top earned income share has been rising significantly, while the top capital income share has remained roughly unchanged. Furthermore, we can tell from Figure 1.2 (which displays the CEO-to-Worker Compensation Ratio) that CEOs' earnings have also been rising significantly relative to workers' wages [see also Mishel and Davis [2014]]. The CEO-to-Worker Compensation Ratio is around 30 in the 1970s but is more than 200 in the 2010s. Second, the recent phenomenon also involves rising income inequality among the working rich. We know from Figure 1.1 that there is a large increase in the top 10% earned income share in the United States. Figure 1.3 breaks down the top 10% earned income share in Figure 1.1 into three groups, namely the top 10% to 1%, the top 1% to 0.1%, and the top 0.1%. As is evident, the increase in the top 10% earned income share from 1980 to 2013 is around 15% (from around 28% to 43%). The increase in the top 1% is around 9%, and the increase in the top 0.1% alone is around 5% (which is one-third of the increase in the top 10%). In short, *among the working rich, the richest enjoy the highest increases in income share.*

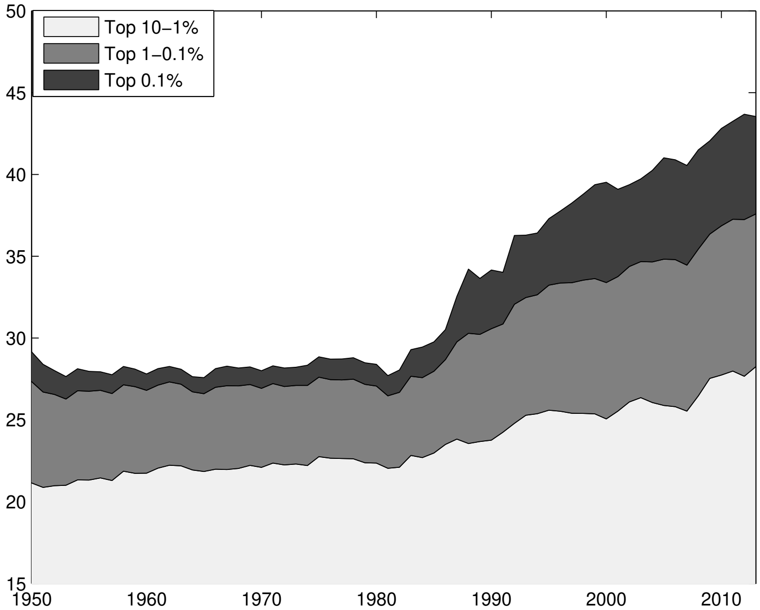

Figure 1.3: Top 10%-1%, Top 1%-0.1%, and Top 0.1% Earned Income Shares in the United States, 1950-2013.

Source: The World Wealth and Income Database.

1.2 A Selective Literature Review

There is a large body of literature on the recent phenomenon of the rise of the working rich.[2] It is not possible to cover the whole body of literature. Here, we only provide a brief and selective literature review. One should refer to, to name a few, Gabaix and Landier [2008], Frydman and Jenter [2010], and Kaplan and Rauh [2010] and the references therein for more reading.

We can say that entrepreneurs own firms and receive a hundred percent of profits. For the increase in entrepreneurial income, profits must increase. Similarly, we can say that managers receive a certain share of profits. For the increase in managers' wages, profits and/or the managers' share of profits must increase. To be sure, in general, the managers' share of profits is endogenous and depends on wage setting. An increase in, say, productivity can affect both profits and managers' share of profits. Nevertheless, for the sake of argument,

[2] As mentioned in footnote 1, managers receive much attention. Concerning works that study entrepreneurs, we are only aware of Gabaix et al. [2016] and Jones and Kim [2018].

it is convenient to think of profits and managers' share of profits separately. In this way, we can review the literature on the (relative) increase in top earned income in two groups. The first group concerns firm performance (i.e., the increase in profits), which applies to both entrepreneurial income and managers' pay. The second group concerns the increase in managers' share of profits.

Closely related to the first group, Gabaix and Landier [2008] and Terviö [2008] develop models, showing that the bigger the firm size, the higher the CEO compensation. Gabaix and Landier also provide evidence, showing that there is a one-to-one relationship between firm size and CEO compensation (see their Figure I). This one-to-one relationship implies that the (relative) increase in top CEO compensation can be explained by remarkable performances of top firms. In Gabaix's and Landier's words, when firm size increases by 500%, CEO compensation also increases by 500%.

One of the most (if not the most) significant causes of remarkable performance of top firms is technological progress. In the economics of superstars, Rosen [1981] argues that because of "imperfect substitution" among individuals (or firms), technological progress benefits those at the top (i.e., the superstars) disproportionately. In particular, Rosen argues that even among those with the same profession (say singing), from consumers' perspectives, they are not perfect substitutes. As Rosen (p. 846) puts it, "lesser talent often is a poor substitute for greater talent ... hearing a succession of mediocre singers does not add up to a single outstanding performance." Consequently, technological progress benefits the highly talented individuals (i.e., superstars) more. A new invention (say the invention of CDs), for instance, allows the superstars to enlarge their market sizes with disproportionately low costs.

Frydman and Saks [2010] provide evidence, supporting the strong correlation between firm size and CEO compensation in recent decades. However, they find a weak correlation prior to the mid-1970s (see their Figure 5). This evidence signifies the importance of the second group, described above. One explanation for the increase in managers' share of profits is the increase in demand for top managers. Khurana [2002] argues that investors in the 1980s suddenly started to look for leaders with charisma and good public image (i.e., superstar CEOs), rather than those with only firm-specific talents. Murphy and Zábojník [2004, 2007] show that when general managerial skills become more important than firm-specific skills, demand in the market for managers will increase (because firms in various industries have to compete for managers in the same market), and so will managers' compensation. Similarly, Cuñat and Guadalupe [2009] argue that globalization and foreign trade raise competition among firms, leading to higher demand for top managers.

Another explanation is the fat cat theory or the theory of managerial power [see, among others, Yermack [1997], Bertrand and Mullainathan [2001], and Bebchuk and Fried [2005]]. Bebchuk and Fried, for instance, argue that poor corporate governance allows managers to influence the pay-setting process, say, through managers' power to benefit directors, friendship and loyalty, and directors' incentive to be re-elected.[3] Similarly, Piketty [see chapter 9 in Piketty [2014] and also Piketty and Saez [2006]] argues that because it is impossible to estimate marginal productivity, it is inevitable that managers who set their own pay treat themselves generously. In addition, social norms have evolved in a way that tolerates extremely generous pay to top managers.

1.3 An Overview

This dissertation belongs to the first group described in the previous section. In the spirit of Gabaix and Landier [2008] as well as Frydman and Saks [2010], we assume throughout this dissertation that there is a one-to-one relationship between earned income and firm profit.[4] For the causes of the increase in profits, we consider the increase in either productivity or capital stock (where the importance of the latter is implied in the models). *The objective of this dissertation is to discuss when the increase in either productivity or capital stock leads to an increase in income inequality, which involves the rise of the working rich.*

This dissertation consists of five chapters. Chapter 2 deals with technical difficulty, concerning the change in income inequality. Chapter 3 deals with the first aspect of the rise of the working rich, i.e., the increase in top earned income relative to top capital income and workers' wages. Chapter 4 deals with the second aspect, i.e., inequality among the working rich. Finally, Chapter 5 presents concluding remarks. For the remainder of this chapter, we provide an overview of Chapters 2-4.

Chapter 2

The analysis of the change in inequality involves two issues. The first issue concerns robustness. There are many inequality measures (e.g., the Gini coefficient, coefficient of variation, Theil index, Atkinson index, and so on), and it is possible that one measure rises while an-

[3] Blanchard et al. [1994] and Bertrand and Mullainathan [2001] provide evidence, showing that managers' earnings are responsive to observable lucky dollars (e.g., windfall gains from a won or settled lawsuit), which have little to do with their performance.

[4] It is important to be aware that, in Gabaix and Landier [2008] and Frydman and Saks [2010], the one-to-one relationship concerns firm size (i.e., market capitalization), rather than profit. Considering profit, however, is convenient for exposition. As we shall see, there is no need to differentiate between managers and entrepreneurs.

other falls. The second issue, particularly in theoretical models, concerns technical difficulty. That is, calculations of changes in inequality measures can be laborious.

One solution to the issues above is to find a sufficient condition for the increase in inequality. In the literature, the first-degree Lorenz dominance (i.e., the non-intersecting Lorenz curves) has been shown to be a sufficient condition [see Fields and Fei [1978] and also Aaberge [2009]]. It is robust to many inequality measures, such as the Gini coefficient, coefficient of variation, and the Theil and Atkinson indices. Nevertheless, verifying the first-degree Lorenz dominance in theoretical models can still be burdensome, especially when one does not assume any specific form of the underlying distribution function.

In Chapter 2, we propose a new sufficient condition, which is stronger than the first-degree Lorenz dominance. The sufficient condition depends only on changes in (income) shares. To be specific, it states that *inequality rises, if there is a cutoff on the distribution, and changes in shares to the left (right) of the cutoff are negative (positive).* To illustrate this visually, in Figure 1.4 we depict distributions of ten-year changes in income share by deciles from 1970 to 2010 in the United States. All graphs in Figure 1.4 satisfy the sufficient condition mentioned above.

Similar to the first-degree Lorenz dominance, the sufficient condition applies to various inequality measures, including some of the most commonly used measures in theoretical models (e.g., the Gini coefficient and the coefficient of variation). Furthermore, as we shall see, it can easily be verified and hence will simplify the analyses in Chapters 3-4 considerably. Furthermore, as shown in Figure 1.4, it has an intuitive geometric interpretation. As one can see, it provides a clear picture of which part of the distribution contributes the most to the change in inequality. This geometric interpretation is conceivably useful in the analysis of the rise of the working rich, which involves a disproportionately large increase in the very top income share.

Chapter 3

To deal with the first aspect of the rise of the working rich, in chapter 3 we develop a model with three groups of individuals, namely the working rich (including entrepreneurs and managers), capitalists, and workers. Entrepreneurs run their own firms, while managers are hired by capitalists to run firms with a fixed fraction of profits as compensation.

In the spirit of Rosen [1981], we assume that firms are imperfect substitutes and are monopolistically competitive. In the literature of monopolistic competition, it is well-known that the predictions of the models depend largely on assumptions about preferences or, equivalently, demand functions. Concerning income inequality, Behrens et al. [2017] use the additive pref-

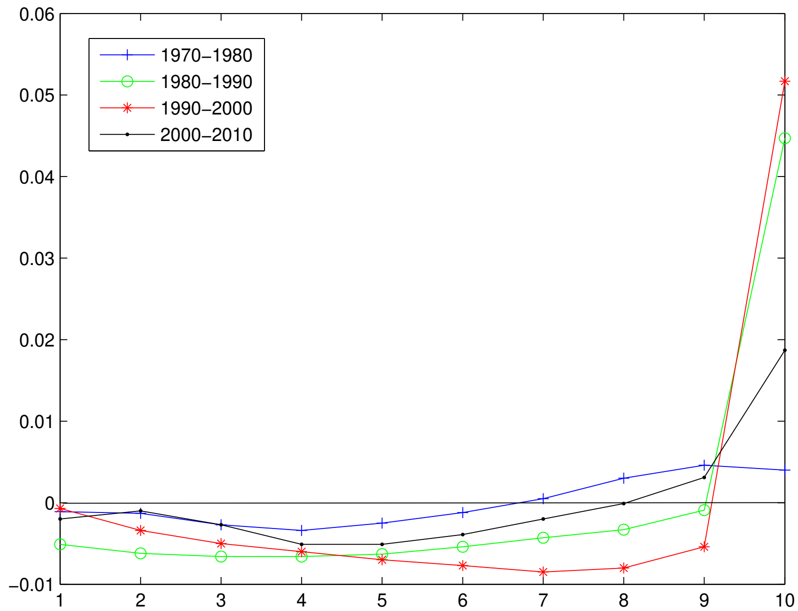

Figure 1.4: Distributions of Changes in Income Share in the United States.

Note: Each graph displays the distribution of ten-year changes in income shares by deciles. For instance, from 2000 to 2010, the income share of the last decile increases by 0.0187 (or, equivalently, 1.87% from 43.88% in 2000 to 45.75% in 2010), while that of the first decile falls by 0.002.

Source: The World Wealth and Income Database.

erences of Zhelobodko et al. [2012] and show that theoretical predictions are ambiguous and depend on whether preferences exhibit increasing relative love for variety (RLV), constant RLV, or decreasing RLV. In terms of demand functions, the increasing, constant, and decreasing RLV imply that the demand elasticity is respectively increasing in, independent of, and decreasing with demand. In Chapter 3, we emphasize the importance of the increasing demand elasticity—or, in the terminology of Mrázová and Neary [2017], "the strict subconvexity"—in the analysis of the rise of the working rich. Concretely, we show that an increase in either productivity or capital stock leads to an increase in top earned income relative to top capital income and workers' wages and an increase in income inequality when the elasticity of the average marginal cost (with respect to either productivity or capital stock) is sufficiently high. We show further that when the demand function is strictly subconvex, the elasticity of the average marginal cost tends to be strictly increasing in productivity and capital stock. If this is the case, the elasticity of the average marginal cost is sufficiently high, only when productivity and capital stock are sufficiently high. This result explains why the rise of the working rich is a phenomenon in the later stage of development.

Chapter 4

In Chapter 4, because we are only interested in the second aspect concerning inequality among the working rich, we consider an economy with entrepreneurs only.[5] Based on the results in Chapter 3, we consider a linear demand function, which is strictly subconvex. We will show that, concerning inequality among the working rich, the strict subconvexity is not the whole story. The change in inequality also depends on "the source of firm heterogeneity" and on whether there is "free entry."

The closest works to our model are Behrens and Robert-Nicoud [2014] and Behrens et al. [2017], which use Melitz-type models with heterogeneous entrepreneurs to discuss the links among market size, self-selection into entrepreneurship, and inequality. They consider the effect of market size (i.e., the population) on income inequality. In our model, as already mentioned, we focus instead on the effects of productivity and capital stock. Our model has one distinctive feature. That is, in conventional Melitz-type models, productivity is the source of firm heterogeneity. In our model, we also consider the case of demand heterogeneity, which has been found to be an important source of firm heterogeneity [see Hottman et al. [2016]].[6] With a simple linear demand function, we show that *the change in income inequality depends on the choice of the source of heterogeneity*. To be specific, when productivity and capital stock are sufficiently high, we obtain a fall in income inequality when considering productivity heterogeneity, but we obtain a rise in income inequality when considering demand heterogeneity.

The underlying mechanism of the model is as follows. An increase in either productivity or capital stock lowers the average marginal cost. The fall in the average marginal cost has three effects on income inequality. First, it raises the profits of all entrepreneurs and hence the average income. For the convenience of terminology, we call this effect "the average income

[5] We can simply exclude workers and keep managers, entrepreneurs, and capitalists. The underlying mechanism in such a setting, concerning inequality among the working rich, is not significantly different from the setting with entrepreneurs only.

[6] Concretely, Hottman et al. [2016] show that "appeal"—a parameter in the utility function, reflecting consumer tastes and characteristics of goods—is one of the main sources of heterogeneity. The difference in appeal can explain 50-70% of the variance in firm size.

effect" (henceforth AIE), which is conceivably inequality-reducing. Second, in the spirit of the economics of superstars of Rosen [1981], the higher the income, the higher the increase in income is. We call this effect "the superstar effect" (henceforth SE). The SE widens the spread of the income distribution and hence is inequality-enhancing. Third, the fall in the average marginal cost allows potential entrepreneurs to enter the market. We call this effect "the entry effect" (henceforth EE). New entrants raise the demand for and the price of capital and hence, ceteris paribus, raise the average marginal cost. Because the fall in the average marginal cost has the SE, the increase through the EE has the anti-SE.

To see the importance of free entry, we first consider a case without the EE. We show that income inequality rises when the SE dominates the AIE. The SE, nevertheless, does not guarantee that *the richest entrepreneurs enjoy the highest increases in income share* (as shown in Figure 1.3). This fact requires a significantly strong SE and, consistent with Chapter 3, is true only in the later stage of development when productivity and capital stock are sufficiently high. Next, we consider a case with the EE. In our model, the EE is not strong enough to offset the SE but still has an important implication for the rise of the working rich. Concretely, with the EE, it is possible that we can never obtain the fact mentioned above.

In Chapter 4, we also relax the monopolistic competition assumption and explore the implication of strategic interactions among (top) entrepreneurs. Our discussion points to the importance of the change in market shares—which does not play any apparent role in the discussion with monopolistic competition—in the analysis of the rise of the working rich.

Chapter 2

A Sufficient Condition for the Increase in Inequality

2.1 Introduction

Concerning the analysis of inequality (in say income), one area of interest is how to measure inequality. There are many inequality measures, which can be broadly summarized in two groups, namely rank-independent (or non-positional) and rank-dependent (or positional) measures. The former—including, among others, the coefficient of variation, Theil index, and Atkinson index [Atkinson [1970]]—depends only on (income) shares. The latter—including, among others, the Gini coefficient, the extended Gini coefficients of Yitzhaki [1983] and Chakravarty [1988], and the Lorenz family of Aaberge [2000]—depends on shares and positions on the distribution.

Another area of interest is the ranking of different distributions (i.e., whether one distribution is more equal than another). To rank distributions, we can pick some inequality measures and calculate the inequality corresponding to the distributions in question. But, among many inequality measures, which measures should we choose? One way to deal with this problem is to choose measures, which satisfy some desirable principles. One principle—which receives much attention in the literature—is the (Pigou-Dalton) principle of transfers which, loosely speaking, states that an income transfer from a rich individual to a poor individual reduces inequality [see, for example, Definition 2.1 in Aaberge [2009]].

Many inequality measures—including the Gini coefficient and the coefficient of variation—have been shown to satisfy the principle of transfers [see, for example, Fields and Fei [1978]]. Fields and Fei also show that inequality measures, which satisfy this principle, rank one distribution higher than another if the former *first-degree Lorenz-dominates* the latter, i.e., if the Lorenz curve of the former lies everywhere above that of the latter. In other words, the first-degree Lorenz dominance is a sufficient condition for the ranking of distributions or, in terms of the change in inequality, is a *sufficient condition* for the increase in inequality (measured by any measure which satisfies the principle of transfers).[1]

The first-degree Lorenz dominance, as a sufficient condition, is very useful for the ranking of distributions or the change in inequality. If it holds, there is no need to calculate changes in inequality measures. Also, the robustness is guaranteed, since it applies to many inequality measures, which satisfy the desirable principle of transfers. *The objective of this chapter is to propose a new sufficient condition,* which is stronger than the first-degree Lorenz dominance and is useful particularly in theoretical models. The motivation is that, in theoretical models, verifying the first-degree Lorenz dominance can be burdensome, especially when one does not assume any specific form of the underlying distribution function. The objective here is to propose a new sufficient condition, which can be easily verified.

The rest of this chapter is organized as follows. Section 2.2 provides the proof of the sufficient condition, Section 2.3 provides a simple example, and Section 2.4 provides a summary.

2.2 The Sufficient Condition

This section proposes a new sufficient condition for the increase in inequality. We will consider continuous probability distributions. The discrete version of the sufficient condition is provided in appendix.

To begin, suppose that there is a continuum of individuals with mass one. Let f be the density function of the population, and F be the cumulative distribution function with support on $[a, b]$, where $a < b$ and b can be infinitely large. We want to consider inequality in a variable $I_x(x, \alpha)$, which can be, say, the income of individual $x \in [a, b]$. The subscript x is used to indicate that the functional form of $I_x(x, \alpha)$ can vary across individuals. $\alpha \in \mathbb{R}$ is an exogenous variable. The goal of this chapter is to propose a sufficient condition for the increase (or fall) in inequality caused by an increase in α.

Assumption 2A

2A.1 $f(x) > 0$ for all $x \in [a, b]$.

2A.2 $I_x(x, \alpha) \geq I_y(y, \alpha) \geq 0$ for all $x, y \in [a, b]$ and $x > y$, and $I_u(u, \alpha) > I_v(v, \alpha)$ for some $u, v \in [a, b]$ and $u > v$.

[1]See Aaberge [2009] for the discussion of the second-degree Lorenz dominance, which applies to the case of intersecting Lorenz curves.

2A.3 $\partial I_x(x,\alpha)/\partial\alpha$ exists and is continuous for all $x \in [a,b]$ and $\alpha \in \mathbb{R}$.

Given that f is the density function of the population, 2A.1 holds trivially. 2A.2 states that $I_x(x,\alpha)$ is non-decreasing with respect to x and non-negative. This assumption allows us to determine the distribution of (changes in) shares and, in most cases, holds trivially in applications. 2A.3 is a natural assumption since this chapter is about the effect of the change in α.

The Lorenz curve is defined by

$$L(v,\alpha) \equiv \int_a^{F^{-1}(v)} \frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)} dF(x), \qquad \forall v \in [0,1], \tag{2.1}$$

where $\mathcal{I}(\alpha) \equiv \int_a^b I_x(x,\alpha)dF(x)$. Before we describe the sufficient condition, it is worthwhile to discuss the first-degree Lorenz dominance. In the following definition, we state the first-degree Lorenz dominance in terms of the change in the Lorenz curve, i.e., $\partial L(v,\alpha)/\partial\alpha$, which by Assumption 2A, exists and is continuous for all $v \in [0,1]$ and $\alpha \in \mathbb{R}$.

Definition 2.1 *We say the Lorenz curve satisfies the first-degree Lorenz dominance when*

$$\frac{\partial L(v,\alpha)}{\partial\alpha} = \int_a^{F^{-1}(v)} \frac{\partial}{\partial\alpha}\left[\frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)}\right] dF(x) \leq 0, \qquad \forall v \in [0,1], \tag{2.2}$$

and the inequality holds strictly for some $v \in (0,1)$.

The first-degree Lorenz dominance (i.e., non-intersecting Lorenz curves) states that when, say, α rises to α', the new Lorenz curve $L(v,\alpha')$ and the original Lorenz curve $L(v,\alpha)$ do not intersect (see Figure 2.1). It is then obvious from Figure 2.1 that the first-degree Lorenz dominance is a sufficient condition for the increase in the Gini coefficient, which is proportional to the area between the 45° line and the Lorenz curve. We can infer from the work of Fields and Fei [1978] that the first-degree Lorenz dominance is also a sufficient condition for the increase in other inequality measures, including the coefficient of variation, Theil index, and Atkinson index.

It is obvious from equation (2.2) that verifying the first-degree Lorenz dominance can be burdensome, especially with a general form of the distribution function F (see also the example in the next section). In this chapter, we propose a new sufficient condition, depending only on changes in shares which, for the convenience of notation, are defined by

$$s_x(x,\alpha) \equiv \frac{\partial}{\partial\alpha}\left[\frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)}\right], \qquad \forall x \in [a,b].$$

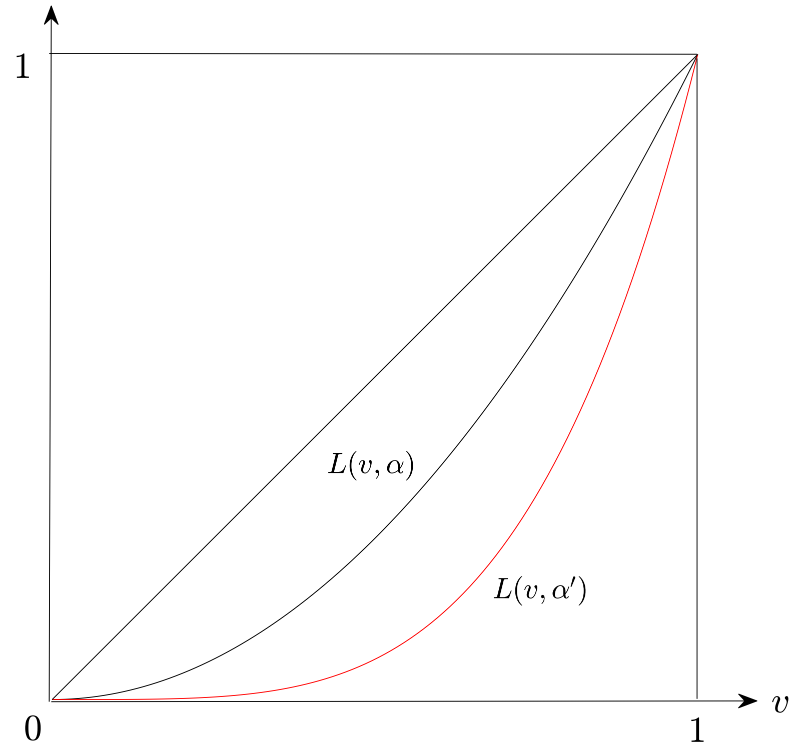

Figure 2.1: First-Degree Lorenz Dominance and Lorenz Curves.

By the definition of $\mathcal{I}(\alpha)$, $s_x(x,\alpha)$ must satisfy

$$\int_a^b s_x(x,\alpha)dF(x) = 0. \tag{2.3}$$

The sufficient condition, described in the following lemma, implies the first-degree Lorenz dominance.[2]

Lemma 2.1 *Assume Assumption 2A. If there exists* $c \in (a,b)$ *such that*

(i) $s_x(x,\alpha) \le 0$ *for all* $x \in [a,c]$,

(ii) $s_x(x,\alpha) \ge 0$ *for all* $x \in [c,b]$, *and*

(iii) $I_x(x,\alpha)s_x(x,\alpha) \neq I_c(c,\alpha)s_x(x,\alpha)$ *for some* $x \in [a,b]$,

the first-degree Lorenz dominance holds.[3]

[2] The fact that conditions (i)-(iii) in Lemma 2.1 form a sufficient condition for the increase in inequality will be stated formally in Theorem 2.1.

[3] By definition, we have

$$s_x(x,\alpha) = \frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)}\left[\frac{\partial I_x(x,\alpha)/\partial\alpha}{I_x(x,\alpha)} - \frac{\mathcal{I}'(\alpha)}{\mathcal{I}(\alpha)}\right].$$

Proof Condition (i) and equation (2.2) imply that

$$\frac{\partial L(v,\alpha)}{\partial \alpha} = \int_a^{F^{-1}(v)} s_x(x,\alpha)dF(x) \leq 0,$$

for all v such that $F^{-1}(v) \leq c$. For all v satisfying $F^{-1}(v) \geq c$, we can use equation (2.3) to rewrite equation (2.2) as

$$\frac{\partial L(v,\alpha)}{\partial \alpha} = -\int_{F^{-1}(v)}^{b} s_x(x,\alpha)dF(x) \leq 0,$$

where the inequality follows immediately from condition (ii).

It remains to show that $\partial L(v,\alpha)/\partial\alpha > 0$ for some $v \in (0,1)$. Condition (iii) implies that $s_d(d,\alpha) \neq 0$ for some $d \in [a,b]$. It follows that $s_d(d,\alpha) < 0$ if $d \in [a,c)$, and $s_d(d,\alpha) > 0$ if $d \in (c,d]$. By continuity (see 2A.3), if $d \in [a,c)$, we must have $s_x(x,\alpha) < 0$ for all $x \in [a,c)$ and in the neighborhood of d; if $d \in (c,d]$, we must have $s_x(x,\alpha) > 0$ for all $x \in (c,d]$ and in the neighborhood of d. It follows that $\partial L(v,\alpha)/\partial\alpha > 0$ for some $v \in (0,1)$ as desired. ■

Since the first-degree Lorenz dominance is a sufficient condition for the increase in inequality, from Lemma 2.1, we know that conditions (i)-(iii) form another sufficient condition.[4] To be formal, in the following, let us consider the sufficient condition with specific inequality measures.

Although there are many measures, many rank-independent and rank-dependent measures can be, respectively, written in some general forms:[5]

$$\begin{aligned} J_{ri}(\alpha) &= H\left(\int_a^b U\left(\frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)}\right)dF(x)\right), & (2.4)\\ J_{rd}(\alpha) &= 1-\int_0^1 P'(v)\frac{\partial L(v,\alpha)}{\partial v}dv = 1-P'(1)+\int_0^1 P''(v)L(v,\alpha)dv, & (2.5)\end{aligned}$$

where the second equality in equation (2.5) follows from integration by parts, and H, U, and P are at least-twice continuously differentiable and satisfy the following assumption.

Assumption 2B

2B.1 $H'U'' > 0$.

Hence, we can also write conditions (i)-(iii) in terms of growth rates.

[4]The first-degree Lorenz dominance is shown to be a sufficient condition with rank-dependent measures, i.e., equation (2.5) [see Theorem 2.1 in Aaberge [2009]]. As shown in Theorem 2.1, conditions (i)-(iii) also apply to rank-independent measures, i.e., equation (2.4).

[5]Note that we obtain the coefficient of variation when setting $U\left(I_x/\mathcal{I}\right) = [(I_x/\mathcal{I})-1]^2$ and $H\left(\int UdF\right) = \left(\int UdF\right)^{1/2}$ and the Gini coefficient when setting $P(v) = 2v - v^2$.

2B.2 $P''(v) < 0$ for all $v \in [0,1]$.

One can refer to Lambert and Lanza [2006] for the discussion of equation (2.4) and to Mehran [1976] and Yaari [1988] [see also Aaberge [2009]] for the discussion of equation (2.5). 2B.1-2B.2 are required for the principle of transfers [see footnote 4 in Lambert and Lanza [2006] and Proposition 1 in Yaari [1988]]. As stated in Theorem 2.1 below, these assumptions are also required for the sufficient condition.

Theorem 2.1 *Assume Assumption 2A-2B. If conditions (i)-(iii) in Lemma 2.1 hold, we have* $J'_{ri}(\alpha) > 0$ *and* $J'_{rd}(\alpha) > 0$.

Proof Changes in equations (2.4)-(2.5) are, respectively, given by

$$J'_{ri}(\alpha) = \int_a^b H'U'\left(\frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)}\right) s_x(x,\alpha)dF(x) \equiv \int_a^b M\left(I_x(x,\alpha)\right) s_x(x,\alpha)dF(x), \tag{2.4'}$$

$$J'_{rd}(\alpha) = \int_0^1 P''(v)\frac{\partial L(v,\alpha)}{\partial \alpha}dv. \tag{2.5'}$$

Let us begin with equation (2.5'). By Lemma 2.1 and 2B.2, we know that $P''(v)[\partial L(v,\alpha)/\partial\alpha] \geq 0$ for all $v \in [0,1]$. It follows that $J'_{rd}(\alpha) \geq 0$. Also, we have $P''(v)[\partial L(v,\alpha)/\partial\alpha] > 0$ for some $v \in (0,1)$. Then, with a similar discussion to that in the proof of Lemma 2.1, continuity ensures that we must have $J'_{rd}(\alpha) > 0$.

Now, for equation (2.4'), 2B.1 implies that $M' > 0$. Then, it follows from 2A.1-2A.2 and conditions (i)-(ii) that

$$M(I_x(x,\alpha))s_x(x,\alpha)f(x) \geq M(I_c(c,\alpha))s_x(x,\alpha)f(x), \qquad \forall x \in [a,b]. \tag{2.6}$$

Equations (2.3) and (2.4') and condition (2.6) imply that

$$J'_{ri}(\alpha) = \int_a^b M(I_x(x,\alpha))s_x(x,\alpha)dF(x) \geq M(I_c(c,\alpha))\int_a^b s_x(x,\alpha)dF(x) = 0.$$

Furthermore, with condition (iii), inequality in condition (2.6) must hold strictly for some $x \in [a,b]$. Then, as before, the continuity ensures that $J'_{ri}(\alpha) > 0$ as desired. ■

In words, conditions (i)-(iii) state that there is a cutoff (i.e., c) on the distribution and changes in shares to the left (right) of the cutoff are negative (positive). The fact that these conditions lead to an increase in inequality is very intuitive. Loosely speaking, these conditions state that the shares fall at the bottom but rise at the top. We can then infer that if the inequality signs in conditions (i)-(ii) reverse, we must have a fall in inequality, as stated in the following corollary.

Corollary 2.1 *Assume Assumption 2A.-2B. If there exists* $c \in (a,b)$ *such that* *(i)* $s_x(x,\alpha) \geq 0$ *for all* $x \in [a,c]$,

(ii) $s_x(x,\alpha) \leq 0$ *for all* $x \in [c,b]$, *and*

(iii) $I_x(x,\alpha)s_x(x,\alpha) \neq I_c(c,\alpha)s_x(x,\alpha)$ *for some* $x \in [a,b]$,

we have $J'_{ri}(\alpha) < 0$ *and* $J'_{rd}(\alpha) < 0$.

2.3 An Example

Consider a simple example, taken from Behrens and Robert-Nicoud [2014], which is about inequality in entrepreneurial incomes. The income of each entrepreneur x is given by

$$I_x(x,\alpha) = \left(\frac{1}{\alpha} - \frac{1}{x}\right)^2.$$

In Behrens and Robert-Nicoud [2014], α belongs to (a,b) so all entrepreneurs with $x < \alpha$ cannot survive the market, and $I_x(x,\alpha)$ is defined on $[\alpha,b]$. Here, for simplicity, we assume that $\alpha < a$, so $I_x(x,\alpha)$ is defined on $[a,b]$.

The Lorenz curve is given by equation (2.1), and its change is given by

$$\frac{\partial L(v,\alpha)}{\partial \alpha} = \int_a^{F^{-1}(v)} s_x(x,\alpha)dF(x), \qquad \forall v \in [0,1], \tag{2.7}$$

where the change in income share, $s_x(x,\alpha)$, is given by

$$s_x(x,\alpha) = \frac{2}{\alpha^2\mathcal{I}^2}\int_a^b \left(\frac{1}{\alpha} - \frac{1}{x}\right)\left(\frac{1}{\alpha} - \frac{1}{y}\right)\left(\frac{1}{y} - \frac{1}{x}\right)dF(y), \qquad \forall x \in [a,b]. \tag{2.8}$$

Behrens and Robert-Nicoud [2014] assume a Pareto distribution to calculate the Gini coefficient and to show that an increase in α raises the Gini coefficient. In fact, it is possible to show that this result holds regardless of the distribution function F. Although it is not impossible to consider the first-degree Lorenz dominance—by using equations (2.7)-(2.8)—to obtain this result, it is obviously laborious.

With the knowledge of the sufficient condition in Theorem 2.1, we can simply consider $s_x(x,\alpha)$ as a function of $x \in [a,b]$ and can rewrite $s_x(x,\alpha)$ as

$$s_x(x,\alpha) = \frac{2E(Y)}{\alpha^2\mathcal{I}^2}\left[X - \frac{E(Y^2)}{E(Y)}\right]X,$$

where $X \equiv \alpha^{-1} - x^{-1}$, $Y \equiv \alpha^{-1} - y^{-1}$, and $E(Z) \equiv \int_a^b Z dF(y)$ for any variable Z. It is obvious that $s_x(x,\alpha)$ is zero at c given by

$$c = \left[\frac{1}{\alpha} - \frac{E(Y^2)}{E(Y)}\right]^{-1}.$$

Since $Y > 0$ for all $y \in [a, b]$ and $Y_a < Y < Y_b$ for all $y \in (a, b)$ where $Y_a \equiv \alpha^{-1} - a^{-1}$ and $Y_b \equiv \alpha^{-1} - b^{-1}$, it is straightforward to show that $Y_a < E(Y^2)/E(Y) < Y_b$. Hence, c belongs to (a, b). Since X is strictly increasing in x, it follows that $s_x(x, \alpha) < 0$ for $x \in [a, c)$ and $s_x(x, \alpha) > 0$ for $x \in (c, b]$. Then, Theorem 2.1 implies that the Gini coefficient rises.

2.4 Summary

In this chapter, we propose a sufficient condition for the increase in inequality, which has three conceivable advantages. First, similar to the first-degree Lorenz dominance, it is robust to various inequality measures, including the Gini coefficient and the coefficient of variation. Second, it is more straightforward to verify the sufficient condition in this chapter, than the first-degree Lorenz dominance. As we shall see, it greatly simplifies the analyses in Chapter 3-4. Third, it has an intuitive geometric interpretation (recall Figure 1.4), which provides a clear picture of which part of the distribution contributes the most to the change in inequality. This third advantage is conceivably useful in the analysis of the rise of the working rich, which involves a disproportionally large increase at the top.

Appendix: Discrete Version of the Sufficient Condition

Suppose that there are $n > 1$ types of individuals, where n can be infinitely large. Let f_i be the population share of each type $i = 1, ..., n$, $I_i(\alpha)$ be a variable corresponding to $I_x(x, \alpha)$, $\mathcal{I}(\alpha) \equiv \sum_{i=1}^{n} f_i I_i(\alpha)$, and $s_i(\alpha)$ be the first-order derivative of $I_i(\alpha)/\mathcal{I}(\alpha)$.

Assumption 2A'

2A.1' $f_i > 0$ for all $i = 1, ..., n$.

2A.2' $I_i(\alpha) \geq I_j(\alpha) \geq 0$ for all $i > j$, and $I_k(\alpha) > I_l(\alpha)$ for some $k > l$.

2A.3' $I_i'(\alpha)$ exists for all $i = 1, ..., n$ and $\alpha \in \mathbb{R}$.

Assumption 2B remains the same.

Theorem 2.1' *Assume Assumption 2A' and 2B. If there exists some i^*, where $1 \leq i^* < n$, such that*

(i) $s_i(\alpha) \leq 0$ *for all* $i \leq i^*$,

(ii) $s_i(\alpha) \geq 0$ *for all* $i \geq i^* + 1$, *and*

(iii) $I_i(\alpha)s_i(\alpha) \neq I_{i^*}(\alpha)s_i(\alpha)$ *for some* $i < i^*$, *or* $I_j(\alpha)s_j(\alpha) \neq I_{i^*+1}(\alpha)s_j(\alpha)$ *for some* $j > i^* + 1$,

we have $J'_{ri}(\alpha) > 0$ *and* $J'_{rd}(\alpha) > 0$.

The proof is virtually the same to that of Theorem 2.1 and hence will not be provided here.

Chapter 3

Top Earned Income, Workers' Wages, and Top Capital Income

3.1 Introduction

In this chapter, we develop a model with three groups of individuals, namely the working rich (including entrepreneurs and managers), capitalists, and workers. Entrepreneurs run their own firms, while managers are hired by capitalists to run firms with a fixed fraction of profits as compensation. The objective is to discuss when an increase in either productivity or capital stock leads to the first aspect of the rise of the working rich—namely, the increases in top earned income (i.e., entrepreneurial income and managers' compensation) relative to workers' wages and top capital income—and the increase in income inequality.

We show that an increase in either productivity or capital stock raises profits and hence earned income, by lowering the average marginal cost of the economy. When the elasticity of the average marginal cost (with respect to either productivity or capital stock) is sufficiently high, an increase in either productivity or capital stock leads to rapid growth of profit and, under certain conditions, to the first aspect of the rise of the working rich and the increase in income inequality.

The elasticity of the average marginal cost depends on assumptions about demand functions. We show that when the demand elasticity is strictly increasing in demand or, in the terminology of Mrázová and Neary [2017], when the demand function is "strictly subconvex," the elasticity of the average marginal cost is likely to be strictly increasing in productivity and capital stock. If this is the case, we must obtain a sufficiently high elasticity of the average marginal cost only in the later stage of development when productivity and capital stock are sufficiently high.

The rest of this chapter is organized as follows. Section 3.2 describes the model. Section 3.3 provides the discussion of the main results. Section 3.4 provides a simple example of a linear demand function, which is strictly subconvex. Finally, Section 3.5 presents a summary.

3.2 The Model

Consider an economy with $L > 0$ individuals and $n > 0$ firms. The economy consists of managers, entrepreneurs, capitalists, and workers with the population of respectively L^m, L^e, L^c, and L^w, where $L^m + L^e + L^c + L^w = L$.

Our main focus is on managers and entrepreneurs, i.e., the working rich. We assume that all workers are identical, and each worker can supply one unit of labor inelastically with wage $w > 0$. All capitalists are also identical. They do not work and own all capital stock $\kappa > 0$ in the economy, which can be rented out to firms at the interest rate $r > 0$. They also own some firms and hire one manager to run each of their firms with a fixed fraction of profits as compensation. Each of the remaining firms is owned and run by one entrepreneur.[1] In our setting, there is no need to specify whether firms are owned by entrepreneurs or capitalists. We can simply state that the entrepreneur or manager, who runs firm i, receives a fixed fraction $\beta^i \in (0, 1]$ of profit, and capitalists receive the remaining $1 - \beta^i$.

In summary, the income of each worker is w; that of each manager/entrepreneur, running firm i, is $\beta^i \pi^i$—where π^i is the profit of firm i—which is what we call (top) earned income; and, that of each capitalist is $\left[r\kappa + \sum_{i=1}^{n}(1-\beta^i)\pi^i\right]/L^c$, consisting of interest income $r\kappa/L^c$ and dividend $\sum_{i=1}^{n}(1-\beta^i)\pi^i/L^c$. Because the capitalists receive all capital income, it is readily possible to refer to capital income of the capitalists as top capital income.

Each manager/entrepreneur runs one firm and produces one good with the inverse demand function

$$P^i = \lambda^{-1} p^i\left(y^i\right), \qquad \forall i = 1, ..., n, \tag{3.1}$$

where P^i is the price of good i, y^i is the demand, $\lambda > 0$ is a demand shifter (highlighting the marginal utility of income). The inverse demand function is downward sloping, i.e., $p_y^i < 0$ where, throughout this chapter, we use g_x to denote a (partial) differentiation of any function g with respect to any variable x, for notational convenience. Also, we use $\varepsilon_g(x) = |xg_x/g| \geq 0$ to denote the elasticity of the function g with respect to x.

[1] Because each firm is run by either one manager or one entrepreneur, we have to impose that $L^m + L^e = n$. Hence, there is no entry or exit in this chapter. We will consider the effect of free entry in the next chapter.

3.2.1 Profit Maximization

In a general equilibrium model, because the market demand function is a sum of individual demand functions, we must impose certain conditions (say, the Gorman form) on preferences to derive the inverse demand function (3.1).[2] In this chapter, we will not deal with this problem, proceed with the general form of the inverse demand function (3.1), and treat λ parametrically as in a partial equilibrium model.[3]

Each firm i rents k^i units of capital and hires ℓ^i workers to produce output y^i. The production function is

$$y^i = \frac{z}{c^i} f\left(k^i, \ell^i\right) - \phi^i,$$

where $z > 0$ highlights total factor productivity, $1/c^i > 0$ highlights productivity heterogeneity, and $\phi^i \geq 0$ as we shall see highlights fixed cost. The function f is at least twice continuously differentiable, satisfies $f_k > 0$, $f_\ell > 0$, $f_{kk} < 0$, and $f_{kk} f_{\ell\ell} > f_{k\ell}^2$, and is homogeneous of degree one.[4]

From cost minimization, we have

$$\frac{f_\ell\left(k^i, \ell^i\right)}{f_k\left(k^i, \ell^i\right)} = \frac{w}{r}, \qquad \forall i = 1, ..., n. \tag{3.2}$$

Because f is homogeneous of degree one, f_ℓ and f_k are homogeneous of degree zero, and f_ℓ / f_k depends only on k^i/ℓ^i. It is straightforward to show that k^i/ℓ^i, satisfying equation (3.2), is

[2] To keep the generality of preferences and hence the inverse demand function, Behrens et al. [2017] assume that consumption is the same for all individuals. Because individuals have different income levels, to ensure identical consumption, it is necessary to assume that κ is a consumption good and that the utility function is quasi-linear. Treating κ as a capital good not only simplifies the analysis but also allows us to incorporate interest income.

[3] In a general equilibrium model, λ is an endogenous variable. Treating λ parametrically is in the spirit of the monopolistic competition literature [see Zhelobodko et al. [2012] and Behrens et al. [2017]] and simplifies the analysis considerably. We will relax this assumption in the next chapter (Section 3.4, to be precise) to explore the implication of oligopolistic competition.

[4] Note that despite the homogeneity assumption, the production function still exhibits increasing returns to scale when $\phi^i > 0$.

strictly increasing in w/r.[5] Hence, we must have

$$\frac{k^i}{\ell^i} = \frac{\sum_{j=1}^n k^j}{\sum_{j=1}^n \ell^j} = \frac{\kappa}{L^w}, \qquad \forall i = 1, ..., n. \tag{3.3}$$

Furthermore, equation (3.2), along with Euler's theorem, implies that

$$rk^i + w\ell^i = ac^i y^i + ac^i \phi^i, \qquad \forall i = 1, ..., n.$$

where $a = r/zf_k = w/zf_\ell$ is independent of output y^i. Using the above equation and inverse demand function (3.1), we can write the profit π^i of each firm i as

$$\pi^i = P^i y^i - rk^i - r\ell^i = \lambda^{-1}\left(p^i - mc^i\right) y^i - ac^i\phi^i, \qquad \forall i = 1, ..., n,$$

where $m \equiv \lambda a > 0$. Because firms treat λ parametrically, we can consider mc^i as the marginal cost of firm i, where m is the common term. For the convenience of terminology, we call m "the average marginal cost"—because we can normalize the mean of the distribution of c^i to one without any loss of generality—which will play a central role throughout this dissertation.

The first-order and second-order conditions for profit maximization are respectively given by

$$p^i + y^i p_y^i = p^i\left[1 - \varepsilon_{p^i}(y^i)\right] = mc^i, \tag{3.4}$$

$$\rho^i \equiv -\frac{y^i p_{yy}^i}{p_y^i} < 2, \qquad \forall i = 1, ..., n, \tag{3.5}$$

where $\varepsilon_{p^i}(y^i) \equiv -y^i p_y^i / p^i > 0$ is the demand elasticity. Borrowing the terminology from Mrázová and Neary [2017], we call ρ^i the "convexity" of the demand function. Given condition (3.5), it is straightforward to show that output y^i is strictly decreasing with m, i.e., $y_m^i < 0$. This result is hardly surprising since m is the average marginal cost.

Because outputs depend only on the average marginal cost m, for the convenience of exposition, let us redefine profit in terms of a so that profits also depend only on the average marginal cost m:

$$e^i \equiv \frac{\pi^i}{a} = \left(\frac{p^i}{m} - c^i\right) y^i - c^i\phi^i = \frac{p^i y^i}{m}\left[\varepsilon_{p^i}(y^i) - \frac{mc^i\phi^i}{p^i y^i}\right], \qquad \forall i = 1, ..., n, \tag{3.6}$$

[5] Specifically, using Euler's theorem (i.e., $\ell f_\ell + k f_k = f$), we can rewrite equation (3.2) as

$$\frac{w}{r} = \frac{f\left(k^i/\ell^i, 1\right)}{f_k\left(k^i/\ell^i, 1\right)} - \frac{k^i}{\ell^i}.$$

It is, then, straightforward to show that

$$\frac{d(k^i/\ell^i)}{d\,(w/r)} = -\frac{f_k^2}{f f_{kk}} > 0,$$

where the inequality is immediately implied by assumption.

where we have made use of equation (3.4) to obtain the last equality. Similarly, we can also redefine income in terms of a:

$$I^w \equiv \frac{w}{a} = zf_\ell, \tag{3.7}$$

$$I^{e,i} \equiv \frac{\beta^i \pi^i}{a} = \beta^i e^i, \qquad \forall i = 1, ..., n, \tag{3.8}$$

$$L^c I^c \equiv \frac{r\kappa + \sum_{i=1}^n (1-\beta^i)\pi^i}{a} = I^r + \sum\nolimits_{i=1}^n (1-\beta^i)e^i, \tag{3.9}$$

where $I^r \equiv z\kappa f_k$ is interest income, and I^w, $I^{e,i}$, and I^c are respectively the income of each worker, that of each manager/entrepreneur running firm i, and that of each capitalist.

3.2.2 The Equilibrium

Note from equations (3.6)-(3.9) that income depends only on k^i/ℓ^i and m. Because k^i/ℓ^i is given by equation (3.3), to complete the model, it remains to derive the average marginal cost m from the market-clearing condition for the capital market or the labor market, respectively, given by

$$\begin{aligned} \sum\nolimits_{i=1}^n k^i &= \kappa, \\ \sum\nolimits_{i=1}^n \ell^i &= L^w. \end{aligned}$$

We can summarize the market-clearing conditions above in the following equation:

$$\sum\nolimits_{i=1}^n c^i \left(y^i + \phi^i\right) = z\left(\kappa f_k + L^w f_\ell\right) = zf(\kappa, L^w), \tag{3.10}$$

where we have made use of equation (3.2) and Euler's theorem.

Lemma 3.1 *The average marginal cost m is strictly decreasing with productivity z and capital stock κ, i.e., $m_x < 0$, where $x \in \{z, \kappa\}$.*

Proof Because $y^i_m < 0$ for all $i = 1, ..., n$, the desired result follows immediately from equation (3.10). ■

Loosely speaking, because m is proportional to the interest rate (i.e., $m = \lambda r / z f_k$), we can consider m as a market-clearing price in the capital market. An increase in total supply κ or a decrease in demand (because of an increase in productivity z) will lower the market-clearing price m.

Lemma 3.2 *For each firm i, we have*

$$\frac{x}{e^i}\frac{\partial e^i}{\partial x} = \varepsilon_m(x)\varepsilon_{e^i}(m) = \varepsilon_m(x)\frac{p^i y^i}{m e^i}, \tag{3.11}$$

where $x \in \{z, \kappa\}$.

Proof The first equality in equation (3.11) follows immediately from the fact that e^i, for all i, depends only on the average marginal cost m. For the second equality, using equation (3.6), we can show that

$$\varepsilon_{e^i}(m) = -\frac{me^i_m}{e^i} = \frac{p^i y^i}{me^i}.$$

The above equation immediately gives the desired result. ■

The increase in either productivity or capital stock (i.e., the increase in x) affects profits only through a fall in the average marginal cost m. The elasticity $\varepsilon_m(x)$ highlights the sensitivity of m to x. The higher the elasticity $\varepsilon_m(x)$, the more the firms benefit. At the same time, the elasticity $\varepsilon_{e^i}(m)$ highlights the sensitivity of profits to the fall in the average marginal cost m. The higher the elasticity $\varepsilon_{e^i}(m)$, the faster the profit grows.

Lemma 3.2 states that the higher the elasticity of the average marginal cost (with respect to either productivity or capital stock), the higher the growth rate of earned income. We show in the next section that, under certain conditions, when the elasticity $\varepsilon_m(x)$ is sufficiently high, we obtain the first aspect of the rise of the working rich and the increase in income inequality.

3.3 Main Results

We are now ready to discuss our main results, concerning the increases in top earned income relative to workers' wages (Subsection 3.3.1) and to top capital income (Subsection 3.3.2), and the increase in income inequality (Subsection 3.3.3). As already mentioned, we will show that these increases require a sufficiently high elasticity of the average marginal cost $\varepsilon_m(x)$. Then, we will show that the value of the elasticity $\varepsilon_m(x)$ depends on the assumption about the demand function (Subsection 3.3.4).

3.3.1 Top Earned Income and Workers' Wage

Let $\mathcal{T}$ be the set of firms whose entrepreneurs/managers have the highest incomes, i.e., $\beta^i\pi^i > \beta^j\pi^j$, $\forall i \in \mathcal{T}$ and $\forall j \notin \mathcal{T}$. In other words, $\beta^i\pi^i$ for all $i \in \mathcal{T}$ is the top earned income.

In this subsection, we discuss the increase in the ratio of the average earned income in the top group $\mathcal{T}$ to workers' wages, caused by an increase in either productivity z or capital stock κ. That is, we discuss the increase in the ratio R^{ew}, defined by

$$R^{ew} \equiv \frac{|\mathcal{T}|^{-1}\sum_{i\in\mathcal{T}} I^{e,i}}{I^w} = |\mathcal{T}|^{-1}\sum_{i\in\mathcal{T}}\beta^i R^{ew,i},$$

where $|\mathcal{T}|$ is the number of elements in the set $\mathcal{T}$, and $R^{ew,i}$ highlights the ratio of the income of each entrepreneur/manager i to workers' wages and is defined by

$$R^{ew,i}(z,\kappa) \equiv \frac{e^i}{I^w}, \qquad \forall i.$$

Because R^{ew} depends positively on $R^{ew,i}$ for all $i \in \mathcal{T}$, we can simply focus on $R^{ew,i}$.

Proposition 3.1 *When either productivity or capital stock increases, the earned income of the entrepreneur/manager i grows faster than the workers' wages do, i.e., $R_x^{ew,i} > 0$, if, and only if, the following condition holds*

$$\varepsilon_m(x) > \frac{\varepsilon_{I^w}(x)}{\varepsilon_{e^i}(m)}, \tag{3.12}$$

where $x \in \{z, \kappa\}$, and $\varepsilon_{I^w}(x)$ is given by

$$\varepsilon_{I^w}(x) = \begin{cases} 1 & \text{if } x = z, \\ \frac{\kappa}{f_\ell}\frac{\partial f_\ell}{\partial \kappa} & \text{if } x = \kappa. \end{cases} \tag{3.13}$$

Proof With equation (3.7), it is straightforward to show that

$$\frac{x}{I^w} I_x^w \quad = \quad \varepsilon_{I^w}(x). \tag{3.14}$$

Then, by the definition of $R^{ew,i}$ and equations (3.11) and (3.14), we must have

$$\frac{x}{R^{ew,i}} R_x^{ew,i} = \varepsilon_m(x)\varepsilon_{e^i}(m) - \varepsilon_{I^w}(x), \qquad \forall i. \tag{3.15}$$

Equation (3.15) immediately gives rise to condition (3.12). ■

Condition (3.12) does not always hold in general. It depends on productivity and capital stock (i.e., the value of x). We will discuss assumptions under which condition (3.12) is likely to hold in Subsection 3.3.4 and provide an example accordingly in Section 3.4. For now, what we can tell from Proposition 3.1 is that *top earned income grows faster than workers' wages when the elasticity of the average marginal cost is sufficiently high.* This is not surprising because we know from Lemma 3.2 that a high elasticity of the average marginal cost leads to high growth rates in profit and hence earned income.

3.3.2 Top Earned Income and Top Capital Income

In this subsection, we discuss the increase in the ratio of top earned income to top capital income, i.e., the increase in the ratio R^{ec}, given by

$$\frac{L^c}{|\mathcal{T}|} R^{ec}(z,\kappa) = \frac{|\mathcal{T}|^{-1}\sum_{i\in\mathcal{T}} I^{e,i}}{I^c} = \frac{L^c}{|\mathcal{T}|}\frac{\sum_{i\in\mathcal{T}}\beta^i R^{ec,i}}{1+\sum_{i=1}^n (1-\beta^i) R^{ec,i}},$$

where we add the term $L^c/|\mathcal{T}|$ to save notation, and $R^{ec,i} \equiv e^i/I^r$.

Proposition 3.2 *When either productivity or capital stock increases, top earned income grows faster than top capital income, i.e., $R_x^{ec} > 0$, if, and only if, the following condition holds*

$$\Gamma^{ec}\varepsilon_m(x) > mI^r R^{ec}\varepsilon_{I^r}(x), \tag{3.16}$$

where $x \in \{z, \kappa\}$, and Γ^{ec} and $\varepsilon_{I^r}(x)$ are given by

$$\Gamma^{ec} \equiv \sum\nolimits_{i\in\mathcal{T}} \beta^i p^i y^i - R^{ec}\sum\nolimits_{i=1}^{n}(1-\beta^i)p^i y^i, \tag{3.17}$$

$$\varepsilon_{I^r}(x) \equiv \frac{xI_x^r}{I^r} = \begin{cases} 1 & \text{if } x = z, \\ 1 + \frac{\kappa}{f_k}\frac{\partial f_k}{\partial \kappa} & \text{if } x = \kappa. \end{cases} \tag{3.18}$$

Proof Differentiating R^{ec} with respect to $x \in \{z, \kappa\}$ yields (see Appendix 3A)

$$xmL^c I^c R_x^{ec} = \Gamma^{ec}\varepsilon_m(x) - mI^r R^{ec}\varepsilon_{I^r}(x). \tag{3.19}$$

Equation (3.19) immediately gives rise to condition (3.16). ■

Recall from Proposition 3.1 that a high elasticity $\varepsilon_m(x)$ is required for the increase in top earned income relative to workers' wages. We know from Proposition 3.2 that the same can be said for the increase in top earned income relative to top capital income when $\Gamma^{ec} > 0$, i.e., when

$$\sum\nolimits_{i\in\mathcal{T}} \beta^i p^i y^i > R^{ec}\sum\nolimits_{i=1}^{n}(1-\beta^i)p^i y^i. \tag{3.20}$$

This condition is required because the capitalists earn dividends, which are proportional to profits. The share of profit, received by the capitalists as dividends, must not be too high; otherwise, when the elasticity $\varepsilon_m(x)$ (and hence the growth rate of profit) is sufficiently high, the increase in dividends dominates the increase in top earned income. When condition (3.20) holds, condition (3.16) simply requires a high growth rate of earned income and a low growth rate of the interest income highlighted respectively by $\varepsilon_m(x)$ and $\varepsilon_{I^r}(x)$.

Condition (3.20) holds trivially when $\beta^i = 1$ for all i (i.e., when all firms are owned by entrepreneurs), when all firms are identical, or when the following conditions hold:

$$\beta^i \to 1, \qquad \forall i \notin \mathcal{T},$$
$$\beta^i > \frac{R^{ec}}{1 + R^{ec}}, \qquad \forall i \in \mathcal{T}.$$

The first condition states that the capitalists do not have much stake in firms outside the top group; or, in other words, most firms outside the top group are owned by entrepreneurs. The second condition states that managers in the top group have sufficiently strong bargaining power; otherwise, the capitalists obtain most of the gains of the increase in profit.

3.3.3 Income Inequality

In this subsection, we consider the change in income inequality. With the sufficient condition in Chapter 2, it suffices to consider only changes in income share.

Define by S^w, $S^{e,i}$, and S^c, respectively, the income share of each worker, that of the manager/entrepreneur of firm i, and that of each capitalist:

$$\begin{aligned} S^w &\equiv \frac{I^w}{\mathcal{I}}, \\ S^{e,i} &\equiv \frac{I^{e,i}}{\mathcal{I}}, \qquad \forall i, \\ S^c &\equiv \frac{I^c}{\mathcal{I}}, \end{aligned}$$

where $\mathcal{I}$ is the aggregate income and is given by

$$\mathcal{I} \equiv L^w I^w + \sum\nolimits_{i=1}^{n} I^{e,i} + L^c I^c = \sum\nolimits_{i=1}^{n} \frac{p^i y^i}{m},$$

where we have made use of equations (3.6)-(3.10) to obtain the second equality.

Lemma 3.3 *The income share of each worker falls, that of each manager/entrepreneur rises, and that of each capitalist rises—i.e., $S_x^w < 0$, $S_x^{e,i} > 0$, and $S_x^c > 0$—if, and only if, the following respective conditions hold:*

$$\varepsilon_m(x) > \varepsilon_{I^w}(x) - \gamma(x), \tag{3.21}$$

$$\frac{c^i\left(y^i + \phi^i\right)}{e^i}\varepsilon_m(x) > \gamma(x), \qquad \forall i, \tag{3.22}$$

$$\Gamma^c \varepsilon_m(x) > L^c I^c \gamma(x) - I^r \varepsilon_{I^r}(x), \tag{3.23}$$

where $x \in \{z, \kappa\}$, Γ^c and $\gamma(x)$ are given by

$$\Gamma^c \equiv \sum\nolimits_{i=1}^{n} (1-\beta^i) c^i (y^i + \phi) - I^r, \tag{3.24}$$

$$\gamma(x) \equiv \begin{cases} z f(\kappa, L_w)/\mathcal{I} & \text{if } x = z, \\ z\kappa f_k/\mathcal{I} & \text{if } x = \kappa, \end{cases} \tag{3.25}$$

and $\varepsilon_{I^w}(x)$ and $\varepsilon_{I^r}(x)$ are respectively given by equations (3.13) and (3.18).

Proof Differentiating income share S^w, $S^{e,i}$, and S^c with respect to $x \in \{z, \kappa\}$ gives (see Appendix 3A)

$$\begin{aligned} \frac{x}{S^w} S_x^w &= \varepsilon_{I^w}(x) - \varepsilon_m(x) - \gamma(x), & (3.26) \\ \frac{x}{S^{e,i}} S_x^{e,i} &= \frac{c^i(y^i + \phi^i)}{e^i} \varepsilon_m(x) - \gamma(x), \qquad \forall i, & (3.27) \\ x\mathcal{I} L^c S_x^c &= \Gamma^c \varepsilon_m(x) - L^c I^c \gamma(x) + I^r \varepsilon_{I^r}(x). & (3.28) \end{aligned}$$

These equations immediately give rise to conditions (3.21)-(3.23). ■

The growth rate of the aggregate income $\mathcal{I}$ is $\varepsilon_m(x) + \gamma(x)$ [see equation (3A.5) in Appendix 3A]. Condition (3.22), simply put, states that when the growth rate of earned income—highlighted by $\varepsilon_m(x)$—is higher than that of the aggregate income, earned income share rises. We can also say the same for the workers' wage share as shown in condition (3.21). Alternatively, we can say that when the elasticity $\varepsilon_m(x)$ is sufficiently high, workers' wage share falls because the high increase in earned income leads to a high increase in the aggregate income (relative to workers' wage). Similarly, by raising the aggregate income, the increase in earned income also, ceteris paribus, lowers capital income share. However, the total change in capital income share is ambiguous, because of the increase in dividends. If the capitalists' stake in firms is non-trivial, capital income share will rise. To be precise, when the elasticity $\varepsilon_m(x)$ is sufficiently high, capital income share S^c rises when $\Gamma^c > 0$ or, equivalently, when

$$\sum\nolimits_{i=1}^{n} \left[\left(1 - \beta^i\right) - \frac{\kappa f_k}{f(\kappa, L^w)} \right] c^i \left(y^i + \phi^i\right) > 0, \tag{3.29}$$

where we have made use of equation (3.10).

Suppose for a moment that firms are identical. Then, condition (3.29) becomes

$$\frac{1}{n} \sum\nolimits_{i=1}^{n} (1 - \beta^i) > \frac{\kappa f_k}{f(\kappa, L^w)} = \varepsilon_f(\kappa), \tag{3.29'}$$

where the right hand side is the capital share of the total production or, equivalently, the elasticity of the total production with respect to capital stock. Condition (3.29'), as well as condition (3.23), tells us that when profits grow rapidly [i.e., when the elasticity $\varepsilon_m(x)$ is sufficiently high], for the increase in capital income share, what is important is the capitalists' stake in each firm (on average), rather than the capital share of the total production.

Now, let us turn to the discussion of income inequality. To apply Theorem 2.1', we have to impose restrictions on the rank (Assumption 2A'). Assume that

$$S^w < S^{e,i} < S^c, \qquad \forall i. \tag{3.30}$$

The first inequality states that workers' wage share is strictly lower than earned income share. The second inequality states that top capital income is important among the very top income earners [see Figure 8.9-8.10 in Piketty [2014]].

Proposition 3.3 *If conditions (3.21)-(3.23) and (3.30) hold, an increase in either productivity or capital stock raises income inequality.*

Proof The proof follows immediately from Theorem 2.1' and Lemma 3.3. ■

In Proposition 3.1-3.3, we provide sufficient conditions for the increase in top earned income relative to workers' wage and top capital income and the increase in income inequality. In general forms, it is not possible to guarantee that these conditions hold. What we can tell is that these conditions are likely to hold when the elasticity of the average marginal cost $\varepsilon_m(x)$ is sufficiently high.

Recall that we can derive the average marginal cost from equations (3.4) and (3.10). It is obvious from these equations that the elasticity of the average marginal cost depends on assumptions about demand functions. In the next subsection, we emphasize the importance of one specific property of demand functions, namely "the strict subconvexity" (to be described). We will show that if demand functions are strictly subconvex, the elasticity of the average marginal cost is likely to be strictly increasing in productivity and capital stock. If this is the case, we obtain a sufficiently high elasticity of the average marginal cost (and hence Proposition 3.1-3.3) when productivity and capital stock are sufficiently high. This explains why the rise of the working rich is a phenomenon in the later stage of development. To confirm this result, we will consider a simple example in Section 3.4.

3.3.4 Strictly Subconvex Demand and the Rise of the Working Rich

Suppose for a moment that firms are identical and $x = z$. In this case, interestingly, conditions (3.12), (3.16) and (3.21)-(3.23) in Proposition 3.1-3.3 are equivalent to condition (3.31) in the following corollary.

Corollary 3.1 *If firms are identical, provided that conditions (3.29)-(3.30) hold, an increase in productivity raises the ratio of earned income to workers' wages, the ratio of earned income to capital income, and income inequality if the following condition holds:*

$$\frac{y\varepsilon_p'(y)}{\varepsilon_p(y)} > -\frac{\phi}{y}\left[(2-\rho) + \frac{1-\varepsilon_p(y)}{\varepsilon_p(y)}\frac{mc}{p}\right], \tag{3.31}$$

where $\varepsilon_p'(y)$ is the differentiation of $\varepsilon_p(y)$ with respect to y.

Proof When firms are identical and $x = z$, condition (3.12) becomes

$$\varepsilon_m(z) > \frac{1}{\varepsilon_e(m)} = \frac{me}{py} = \varepsilon_p(y) - \frac{mc\phi}{py}, \tag{3.32}$$

where the first and second equalities follow, respectively, from equations (3.11) and (3.6). In this proof, we first show that conditions (3.16) and (3.21)-(3.23) are also equivalent to condition (3.32). Next, we show that condition (3.32) is equivalent to condition (3.31). The desired result, then, follows immediately from Propositions 3.1-3.3.

Since firms are identical and $x = z$, equations (3.17) and (3.25) become

$$\Gamma^{ec} = py\left[\sum\nolimits_{i\in\mathcal{T}}\beta^i - R^{ec}\sum\nolimits_{i=1}^{n}(1-\beta^i)\right] = \frac{py}{e}I^r R^{ec}, \tag{3.17'}$$

$$\gamma(z) = \frac{zf(\kappa, L^w)}{\mathcal{I}} = \frac{mc(y+\phi)}{py} = 1 - \frac{me}{py}. \tag{3.25'}$$

where the second equality of equation (3.17') follows from the definition of R^{ec}, and we have made use of equations (3.6) and (3.10) to obtain equation (3.25'). Substituting equation (3.17'), the third equality of equation (3.25'), and the second equality, respectively, into conditions (3.16), (3.21), and (3.22) immediately gives rise to condition (3.32).

For condition (3.23), use equations (3.24) and (3.25') to obtain

$$\begin{aligned} L^c I^c \gamma(x) - I^r \varepsilon_{I^r}(x) &= \left[I^r + e\sum\nolimits_{i=1}^{n}(1-\beta_i)\right]\gamma(x) - I^r \\ &= \gamma(x)e\sum\nolimits_{i=1}^{n}(1-\beta_i) - I^r\left[1-\gamma(x)\right] = \frac{me}{py}\Gamma^c. \end{aligned}$$

Because, by assumption, condition (3.29) holds, we have $\Gamma^c > 0$. Then, substituting the above equation into condition (3.23) gives condition (3.32) as desired.

It remains to show the equivalence between conditions (3.31) and (3.32). Using the two equalities in equation (3.4), we can show that

$$\rho = 1 + \varepsilon_p(y) - \frac{y\varepsilon_p'(y)}{\varepsilon_p(y)}, \tag{3.33}$$

$$\varepsilon_y(m) = \frac{1}{\varepsilon_p(y)}\frac{1-\varepsilon_p(y)}{2-\rho}. \tag{3.34}$$

Furthermore, since firms are identical, equation (3.10) becomes $nc(y+\phi) = zf(\kappa, L_w)$. It follows that

$$\varepsilon_y(m)\varepsilon_m(z) = 1 + \frac{\phi}{y}.$$

Substituting the above equation and equations (3.33)-(3.34) into condition (3.32) and rearranging terms will give condition (3.31) as desired. ■

It is noteworthy that if there is no fixed cost, i.e., if $\phi = 0$, condition (3.31) becomes

$$\varepsilon_p'(y) > 0. \tag{3.31'}$$

In other words, if the demand elasticity is strictly increasing in demand, condition (3.31) holds for all values of productivity and capital stock. Mrázová and Neary [2017] refer to the above condition as "the strict subconvexity" of the demand function. If $\phi > 0$, condition (3.31) holds trivially when the demand function is either strictly subconvex or isoelastic (or simply subconvex), i.e., when $\varepsilon_p'(y) \geq 0$. Although we cannot rule out the possibility that condition (3.31) hold when the demand function is strictly superconvex, i.e., when $\varepsilon_p'(y) < 0$, the possibility is obviously slim, compared with the case of subconvex demand.

Now, let us return to the general case with firm heterogeneity. In this case, it is not possible to obtain a nice condition as in condition (3.31) or (3.31'). In fact, with firm heterogeneity, it is possible that Propositions 3.1-3.3 do not hold for certain values of productivity and capital stock, even when the demand function is strictly subconvex (see the example in the next section). Corollary 3.1, nonetheless, suggests the importance of the strict subconvexity.

We can simply focus on the elasticity of the average marginal cost $\varepsilon_m(x)$ because, as we have already mentioned in previous subsections, Propositions 3.1-3.3 hold when this elasticity is sufficiently high. To begin, we can use equation (3.10) to show that [see also equation (3A.4) in Appendix 3A]

$$\varepsilon_m(x) = \delta(x) \frac{\sum_{i=1}^n c^i \left(y^i + \phi^i\right)}{\sum_{i=1}^n c^i y^i \varepsilon_{y^i}(m)} \equiv \delta(x) M(m), \tag{3.35}$$

where $\delta(x)$ is given by

$$\delta(x) = \frac{\gamma(x)\mathcal{I}}{z f(\kappa, L^w)} = \begin{cases} 1 & \text{if } x = z, \\ \kappa f_k / f & \text{if } x = \kappa. \end{cases} \tag{3.36}$$

Note that if $x = z$, $\delta(x)$ equals one and hence its differentiation with respect to x is zero, i.e., $\delta_x(x) = 0$. When $x = \kappa$, we also have $\delta_x(x) = 0$ if the function f is a Cobb-Douglas function. Suppose that this is the case. Then, we can ignore x and view the elasticity of the average marginal cost, given in equation (3.35), as a function of m. This means that we can simply focus on the function $M(m)$, the differentiation M_m of which satisfies the following (see Appendix 3A):

$$\begin{aligned} \left[\sum_{i=1}^n c^i y^i \varepsilon_{y^i}(m)\right] \frac{m M_m}{M} = & \left\{ \sum_{i=1}^n c^i y^i \varepsilon_{y^i}(m)^2 - \frac{\left[\sum_{i=1}^n c^i y^i \varepsilon_{y^i}(m)\right]^2}{\sum_{i=1}^n c^i \left(y^i + \phi^i\right)} \right\} \\ & - \sum_{i=1}^n c^i y^i \varepsilon_{y^i}(m)^2 \left[\frac{1}{1 - \varepsilon_{p^i}(y^i)} \frac{y^i \varepsilon_{p^i}'(y^i)}{\varepsilon_{p^i}(y^i)} - \frac{y^i \rho_y^i}{2 - \rho^i} \right]. \end{aligned} \tag{3.37}$$

The first term on the right-hand side is non-negative (Cauchy-Schwarz inequality). Then, if the demand function is superconvex, i.e., if $\varepsilon'_{p^i}(y^i) \leq 0$, for M_m is strictly negative, it requires that $\rho^i_y < 0$. Otherwise, i.e., if $\rho^i_y \geq 0$,[6] M_m is positive, and an increase in x (by lowering m) lowers the elasticity of the average marginal cost $\varepsilon_m(x)$. Then, when productivity and capital stock are sufficiently high, the elasticity $\varepsilon_m(x)$ is low, and it is unlikely that Propositions 3.1-3.3 are true.

M_m is likely to be strictly negative when the demand function is strictly subconvex and $\rho^i_y \leq 0$. In the next section, we consider an example of a linear demand function, which is strictly subconvex and has zero convexity, i.e., $\rho^i = 0$. In this example, we confirm that M_m is indeed strictly negative. That is, the elasticity $\varepsilon_m(x)$ is strictly increasing in productivity and capital stock. Hence, we obtain Proposition 3.1-3.3 particularly when productivity and capital stock are sufficiently high (see Figure 3.1-3.3).

Before proceeding to the next section, it is worthwhile to discuss why, with strictly subconvex demands, Propositions 3.1-3.3 are likely to hold when productivity and capital stock are sufficiently high. With strictly subconvex demands, there is a less-than 100 percent pass-through, i.e., $d \log p / d \log m < 1$ [see equation (5) in Mrázová and Neary [2017]]. In other words, a fall in the average marginal cost lowers prices by less than one hundred percent and, hence, must raise markups, given by $p/mc = 1/[1 - \varepsilon_p(y)]$. As a result, an increase in either productivity or capital stock, by lowering the average marginal cost, must lead to a high increase in profit (since firms can raise both outputs and markups).

Alternatively, if the inverse demand function is derived from utility maximization, $p(y)$ is the marginal utility u_y. Then, the demand elasticity satisfies

$$\varepsilon_p = -\frac{y u_{yy}}{u_y} \equiv r(y).$$

Zhelobodko et al. [2012] call $r(y)$ "the relative love for variety." Strict subconvexity is equivalent to "increasing relative love for variety," i.e., $r'(y) > 0$. This property implies that the higher the consumption, the lower the elasticity of substitution among goods [see Zhelobodko et al. [2012]]. In other words, when consumption levels rise, consumers perceive goods as being more

[6] Behrens et al. [2017] provide an example of a versatile utility function, the (inverse) demand function and the convexity derived from which are given by

$$\begin{aligned} p &= (a+y)^{-\sigma} + b, \\ \rho &= \frac{(1+\sigma)y}{a+y}. \end{aligned}$$

The above demand function is isoelastic, strictly superconvex, and strictly subconvex if (a, b) equals, respectively, $(0, 0)$, $(0, 1)$, and $(1, 0)$. It follows immediately that $\rho_y = 0$ ($\rho_y > 0$) when the demand function is either isoelastic or strictly superconvex (strictly subconvex).

differentiated and hence are willing to pay more for each good. As a result, there is a large increase in profits when productivity and capital stock (and hence outputs) are sufficiently high.

3.4 An Example

In this section, we consider a simple example with a linear demand function and a Cobb-Douglas production function:

$$\begin{aligned} p^i &= \bar{y}^i - y^i, \qquad \forall i, \\ f(k,\ell) &= k^\alpha \ell^{1-\alpha}, \end{aligned}$$

where $\bar{y}^i > 0$ and $\alpha \in (0,1)$. Using equations (3.4) and (3.10), we can show that

$$y^i = \left(\bar{y}^i - mc^i\right)/2, \qquad \forall i, \tag{3.38}$$

$$m = \left[\sum\nolimits_{i=1}^{n} c^i \left(\bar{y}^i + 2\phi^i\right) - 2z\kappa^\alpha (L^w)^{1-\alpha}\right] / \|c\|^2, \tag{3.39}$$

where $\|c\|^2 = \sum_{i=1}^n (c^i)^2$. With equations (3.38)-(3.39), it is straightforward to show that

$$\varepsilon_{p^i}(y^i) = \frac{y^i}{\bar{y}^i - y^i} = \frac{\bar{y}^i - mc^i}{\bar{y}^i + mc^i}, \qquad \forall i, \tag{3.40}$$

$$\varepsilon_m(x) = \delta(x) \frac{2z\kappa^\alpha (L^w)^{1-\alpha}}{m\|c\|^2}, \tag{3.41}$$

where $\delta(x)$ is given by equation (3.36). Note from the first equality of equation (4.40) that, for each good, the demand function is strictly subconvex. Note also that, as already mentioned in the previous section, the elasticity of the average marginal cost is indeed strictly increasing in $x \in \{z, \kappa\}$.

To simplify calculations, assume throughout this section that $c^i = 1$ and $\phi^i = 0$ for all i. Hence, equation (3.39) becomes

$$m = E_1 - \frac{2z\kappa^\alpha \left(L^w\right)^{1-\alpha}}{n} < \min_i \{\bar{y}^i\}, \tag{3.42}$$

where $E_1 = n^{-1} \sum_{i=1}^n \bar{y}^i$, and the inequality ensures that all firms have strictly positive outputs.

For the rest of this section, we reconsider the discussion in Section 3.3. That is, we reconsider the ratio of top earned income to workers' wages in subsection 3.4.1, the ratio of top earned incomes to top capital income in subsection 3.4.2, and income inequality in subsection 3.4.3.

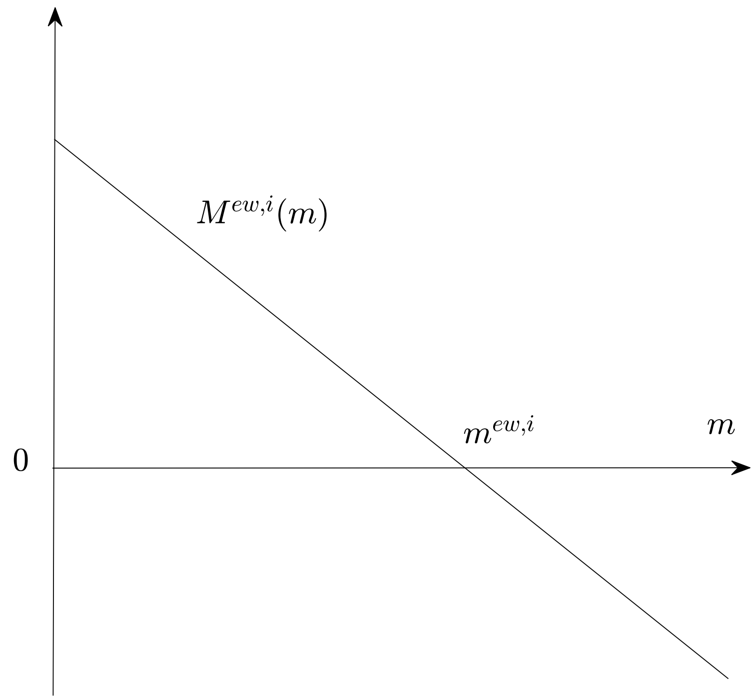

Figure 3.1: Change in the Ratio of Top Earned Income to Workers' Wages.

3.4.1 Top Earned Incomes and Workers' Wages

In our example, we have $\varepsilon_{I^w}(x) = \delta(x)$. Moreover, since $\phi = 0$, we know from equations (3.6) and (3.11) that $\varepsilon_{e^i}(m) = 1/\varepsilon_{p^i}(y^i)$ for all i. Substituting these equations, along with equations (3.40)-(3.41), into equation (3.15) yields

$$\frac{xm\left(\bar{y}^i - m\right)}{\delta(x)R^{ew,i}} R_x^{ew,i} = \bar{y}^i E_1 - m\left(2\bar{y}^i - E_1\right) \equiv M^{ew,i}(m), \qquad \forall i,$$

It is obvious from the above equation that when the average marginal cost is sufficiently low, top earned income will grow faster than workers' wages, i.e., $R_x^{ew,i} > 0$ for all i. In general, if firms in the top group $\mathcal{T}$ have the highest $\bar{y}^i$, we must have $2\bar{y}^i > E_1$ for $i \in \mathcal{T}$. Then, as depicted in Figure 3.1, the $M^{ew,i}(m)$ schedule is downward sloping. As the average marginal cost falls (i.e., as productivity and capital stock rise), the ratio R^{ew} first falls in the early stage when the average marginal cost m is high but then rises in the later stage when the average marginal cost is sufficiently low. In other words, top earned income grows faster than workers' wages only in the later stage of development when productivity and capital stock are sufficiently high.

Before proceeding, it is worth mentioning that the average marginal cost in our example has an upper limit, i.e., $m < \min_i\{\bar{y}_i\}$ [see equation (3.42)]. Hence, when discussing Figure 3.1, as well as Figure 3.2-3.3 in the following subsections, it is important to take this upper limit into account. We refrain from doing this, however. Here, we simply want to confirm the results shown in the previous section. That is, because the demand function is strictly subconvex (with zero convexity), when the average marginal cost m is sufficiently low or, equivalently, when productivity and capital stock are sufficiently high, we obtain the increase in top earned income relative to workers' wage and top capital income, and the increase in income inequality.

3.4.2 Top Earned Income and Top Capital Income

In our example, we can rewrite R_x^{ec}, given by equation (3.19), as follow (see Appendix 3B for the derivation)

$$\begin{aligned}\frac{8x(mL^cI^c)^2}{(E_1-m)\delta(x)}R_x^{ec} = &\left[\Gamma_0^c\sum\nolimits_{i\in\mathcal{T}}\beta^i - B\sum\nolimits_{i\in\mathcal{T}}\beta^i\bar{y}^i\right]m^2 \\ &-\sum\nolimits_{i=1}^n(1-\beta^i)(\bar{y}^i)^2\left[\sum\nolimits_{i\in\mathcal{T}}\beta^i - BR_0^{ec}\right]m + \Gamma_0^{ec}\sum\nolimits_{i=1}^n(1-\beta^i)(\bar{y}^i)^2 \\ \equiv &am^2 - bm + c \equiv M^{ec}(m), \end{aligned} \tag{3.43}$$

where B, $R_0^{ec} > 0$, Γ_0^c, and Γ_0^{ec} are given in Appendix 3B.[7] It is convenient to rewrite Γ_0^c and Γ_0^{ec} here:

$$\Gamma_0^c = \lim_{m\to 0} 2\Gamma^c = \sum\nolimits_{i=1}^n(1-\beta^i)\bar{y}^i - \alpha n E_1, \tag{3.44}$$

$$\Gamma_0^{ec} = \lim_{m\to 0}(2\Gamma^{ec}/m) = \sum\nolimits_{i\in\mathcal{T}}\beta^i\bar{y}^i - \Gamma_0^c R_0^{ec}. \tag{3.45}$$

Recall that, for the increase in the ratio R^{ec}, the share of profit, received by capitalists as dividends, must not be too high. In other words, condition (3.20), i.e., $\Gamma^{ec} > 0$, holds. Consistently, here, we assume that $\Gamma_0^{ec} > 0$ and hence $c > 0$. Then, it follows immediately from equation (3.43) that the ratio R^{ec} rises, when the average marginal cost is sufficiently low. See also Figure 3.2, which depicts the $M^{ec}(m)$ schedule when $a > 0$ in panel (a) and when $a < 0$ in panel (b).[8]

[7] Note that we recycle the notation here. a and c, given in equation (3.43), are different from those in Section 3.2.

[8] Since $R_0^{ec} > 0$, $\Gamma_0^{ec} > 0$ (by assumption) implies that

$$\Gamma_0^c < \frac{\sum_{i\in\mathcal{T}}\beta^i\bar{y}^i}{R_0^{ec}}.$$

It follows that

$$a = \Gamma_0^c\sum\nolimits_{i\in\mathcal{T}}\beta^i - B\sum\nolimits_{i\in\mathcal{T}}\beta^i\bar{y}^i < \frac{\sum_{i\in\mathcal{T}}\beta^i\bar{y}^i}{R_0^{ec}}\left[\sum\nolimits_{i\in\mathcal{T}}\beta^i - BR_0^{ec}\right].$$

(a) Case 1: $a > 0$ (b) Case 2: $a < 0$

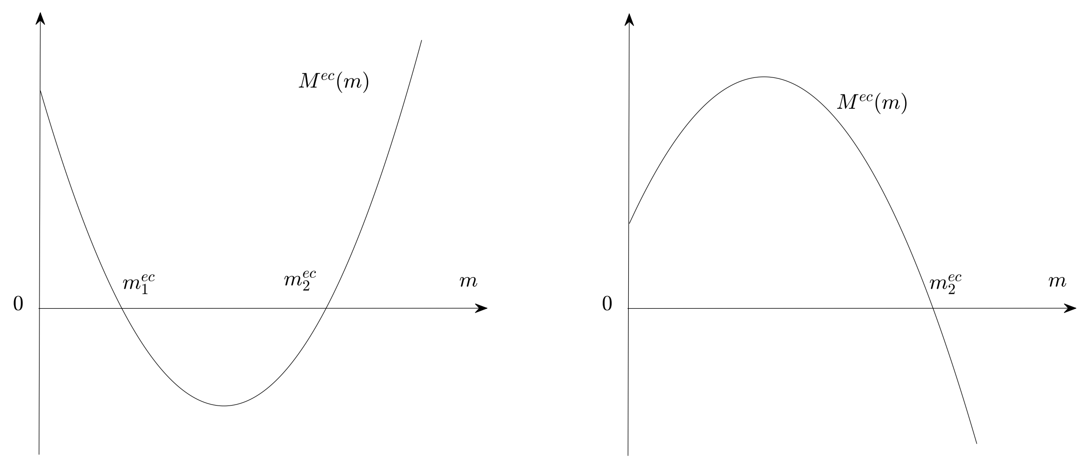

Figure 3.2: Change in the Ratio of Top Earned Income to Top Capital Income.

3.4.3 Income Inequality

In our example, we can rewrite equations (4.26)-(4.28) as (see Appendix 3B)

$$M^w(m) \equiv \frac{xm(E_2 - m^2)}{\delta(x)S^w} S_x^w = -E_1 m^2 + 2E_2 m - E_1 E_2, \tag{3.46}$$

$$M^{e,i}(m) \equiv \frac{x(\bar{y}^i - m)(E_2 - m^2)}{2\delta(x)(E_1 - m)S_x^{e,i}} S_x^{e,i} = E_2 - m\bar{y}^i, \qquad \forall i, \tag{3.47}$$

$$M^c(m) \equiv \frac{2xm\mathcal{I}L^c(E_2 - m^2)}{\delta(x)(E_1 - m)} S_x^c = \Gamma_0^c m^2 - \left[BE_2 + \sum\nolimits_{i=1}^{n} (1 - \beta_i)(\bar{y}^i)^2\right] m + \Gamma_0^c E_2, \tag{3.48}$$

where $E_2 \equiv n^{-1} \sum_{i=1}^{n} (\bar{y}^i)^2$.

Recall that, for the increase in capital income share, it requires that $\Gamma^c > 0$. Consistently, here, we assume that $\Gamma_0^c > 0$. Hence, it is obvious from equations (3.46)-(3.48) that, in the later stage of development when the average marginal cost is sufficiently low, the earned income and capital income shares rise, while workers' wage share falls. See Figure 3.3, which depicts the $M^w(m)$ schedule, the $M^{e,i}(m)$ schedule, and the $M^c(m)$ schedule, respectively, in panel (a), (b), and (c).

Now, for the increase in income inequality, recall from Proposition 3.3 that it requires

It is obvious that if $a \geq 0$, we must have $b > 0$. Hence, when $a = 0$, similar to Figure 3.1, the $M^{ec}(m)$ schedule is downward sloping. When $a > 0$, the two roots of $M^{ec} = 0$ must be strictly positive. When $a < 0$, since $c > 0$, one root is strictly positive, while the other is strictly negative.

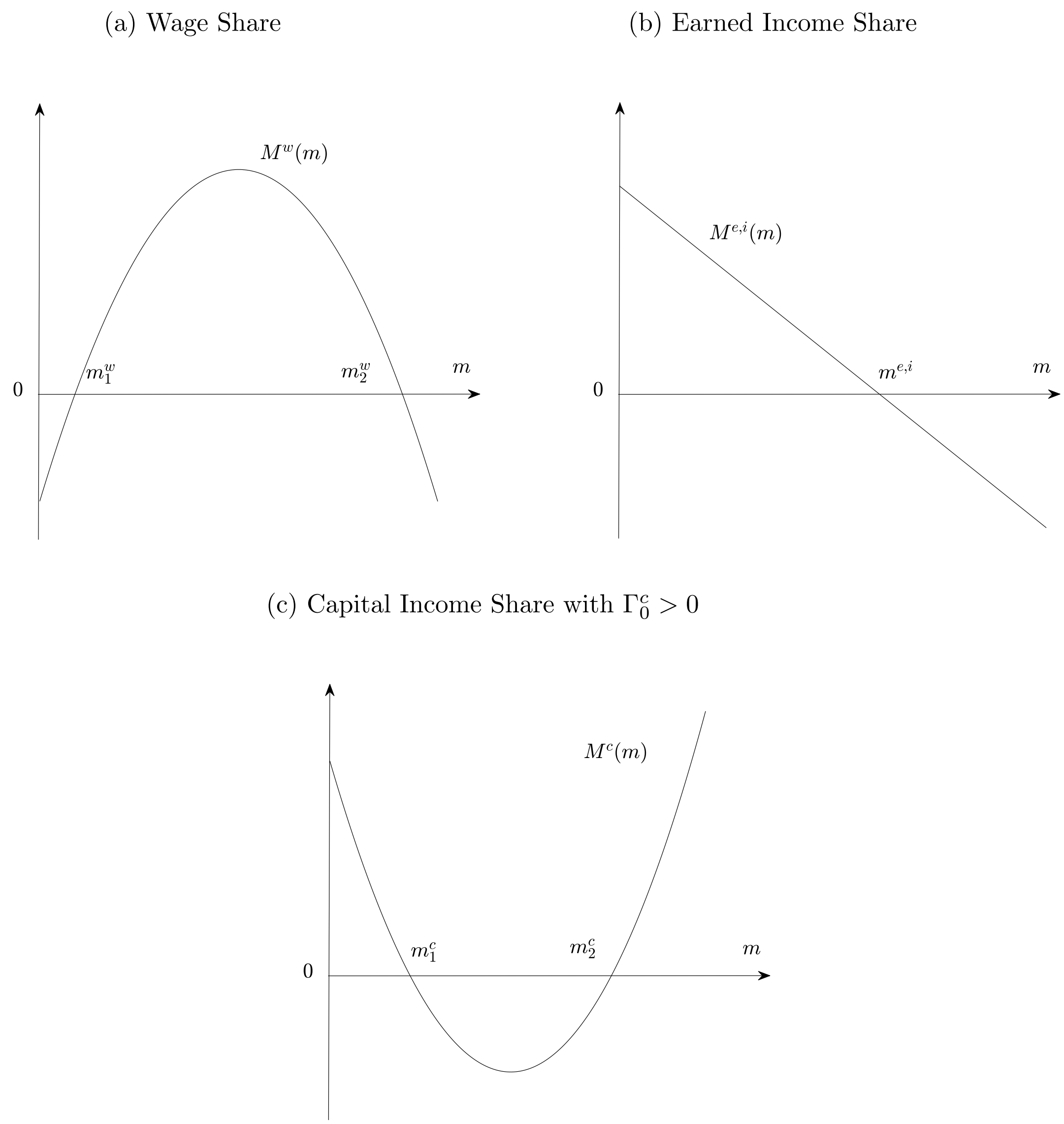

Figure 3.3: Changes in Income Shares

conditions (3.30), which in our example can be rewritten as (see Appendix 3B)

$$\left[1+\frac{2n(1-\alpha)}{\beta^i L^w}\right]m^2 - 2\left[\bar{y}^i + \frac{n(1-\alpha)E_1}{\beta^i L^w}\right]m + (\bar{y}^i)^2 > 0, \tag{3.49}$$

$$\left(B-\beta^i L^c\right)m^2 - 2\left(\Gamma_0^c - \beta^i L^c \bar{y}^i\right)m + \left[\sum\nolimits_{j=1}^n (1-\beta^j)(\bar{y}^j)^2 - \beta^i L_c(\bar{y}^i)^2\right] > 0, \qquad \forall i, \tag{3.50}$$

where the first condition ensures that $S^w < S^{e,i}$, while the second condition ensures that $S^{e,i} < S^c$. When the average marginal cost m is sufficiently low, the first condition holds trivially. The second condition also holds trivially if the following condition holds

$$\sum\nolimits_{j=1}^n (1-\beta^j)(\bar{y}^j)^2 > \beta^i L_c(\bar{y}^i)^2, \qquad \forall i. \tag{4.50'}$$

In short, when $\Gamma_0^c > 0$ and condition (4.50') hold, income inequality rises when the average marginal cost is sufficiently low or, equivalently, when productivity and capital stock are sufficiently high.

3.5 Summary

In this chapter, we develop a simple model to discuss the increase in income inequality, which involves the first aspect of the recent phenomenon of the rise of the working rich, i.e., the increase in top earned income (including entrepreneurial income and managers' compensation) relative to workers' wages and top capital income.

The model is based on one assumption. That is, there is a one-to-one relationship between earned income and profit. With this assumption, if profits of (top) firms grow rapidly, under certain conditions, we will obtain an increase in income inequality and the first aspect of the rise of the working rich. We show that firms grow rapidly when the elasticity of the average marginal cost (with respect to either productivity or capital stock) is sufficiently high. We show further that if the demand function is "strictly subconvex," i.e., when the demand elasticity is strictly increasing in demand, the elasticity of the average marginal cost is likely to be strictly increasing in productivity and capital stock. If this is the case,[9] the elasticity of the average marginal cost is sufficiently high only when productivity and capital stock are sufficiently high.

In short, the strict subconvexity of demand functions can explain the fact that the first aspect of the rise of the working rich is a phenomenon in the later stage of development.

[9] We confirm that this is indeed the case when the demand function is linear and hence is strictly subconvex.

Appendix 3A

This appendix provides calculations of equations (3.19), (3.26)-(3.28), and (3.37) in Section 3.3.

Equation (3.19): By the definition of R^{ec}, we have

$$R_x^{ec} = \frac{\sum_{i\in\mathcal{T}} \beta^i R_x^{ec,i}}{1+\sum_{i=1}^n (1-\beta^i) R^{ec,i}} - R^{ec} \frac{\sum_{i=1}^n (1-\beta^i) R_x^{ec,i}}{1+\sum_{i=1}^n (1-\beta^i) R^{ec,i}}. \tag{3A.1}$$

Using equation (3.11) and the definition of $R^{ec,i}$, we can show that

$$xR_x^{ec,i} = R^{ec,i}\left[\varepsilon_m(x)\varepsilon_{e^i}(x) - \varepsilon_{I^r}(x)\right] = \varepsilon_m(x)\frac{p^i y^i}{mI^r} - R^{ec,i}\varepsilon_{I^r}(x). \tag{3A.2}$$

Define A by

$$A \equiv xmI^r\left[1+\sum\nolimits_{i=1}^n (1-\beta^i)R^{ec,i}\right] = xmL^c I^c, \tag{3A.3}$$

where the second equality follows from equation (3.9).

Using (3A.2)-(3A.3), we can rewrite equation (3A.1) as

$$\begin{aligned}
AR_x^{ec} =& \sum_{i\in\mathcal{T}} \beta^i \left[\varepsilon_m(x)p^i y^i - mI^r R^{ec,i}\varepsilon_{I^r}(x)\right] - R^{ec}\sum_{i=1}^n (1-\beta^i)\left[\varepsilon_m(x)p^i y^i - mI^r R^{ec,i}\varepsilon_{I^r}(x)\right] \\
=& \left[\sum_{i\in\mathcal{T}} \beta^i p^i y^i - R^{ec}\sum_{i=1}^n (1-\beta^i)p^i y^i\right]\varepsilon_m(x) \\
& - mI^r\varepsilon_{I^r}(x)\left[\sum_{i\in\mathcal{T}} \beta^i R^{ec,i} - R^{ec}\sum_{i=1}^n (1-\beta^i)R^{ec,i}\right] \\
=& \Gamma^{ec}\varepsilon_m(x) - mI^r R^{ec}\varepsilon_{I^r}(x),
\end{aligned}$$

where the third line follows immediately from the definition of R^{ec}.

Equations (3.26)-(3.28): Using equation (3.10), we can show that

$$\varepsilon_m(x)\sum\nolimits_{i=1}^n c^i y^i \varepsilon_{y^i}(m) = \gamma(x)\mathcal{I}, \tag{3A.4}$$

where $\gamma(x)$ is given by equation (3.25). Also, by the definition of $\mathcal{I}$, we can show that

$$\frac{x}{m\mathcal{I}}\frac{\partial(m\mathcal{I})}{\partial x} = \frac{x}{m\mathcal{I}}\frac{\partial \sum_{i=1}^n p^i y^i}{\partial x} = \frac{\varepsilon_m(x)}{\mathcal{I}}\sum_{i=1}^n c^i y^i \varepsilon_{y^i}(m) = \gamma(x), \tag{3A.5}$$

where we have made use of equation (3.4) to obtain the third equality and (3A.4) to obtain the last equality.

By definition, we have $S^w = mI^w/(m\mathcal{I})$. Using this equation, along with (3A.5), immediately gives equation (3.26). Also, by definition, we have $S^{e,i} = \beta^i me^i/(m\mathcal{I})$. Using this equation, along with equations (3.11) and (3A.5), gives equation (3.27):

$$\begin{aligned}\frac{x}{S^{e,i}}S^{e,i}_x &= [\varepsilon_{e^i}(m) - 1]\varepsilon_m(x) - \gamma(x)\\ &= \frac{p^iy^i - me^i}{me^i}\varepsilon_m(x) - \gamma(x) = \frac{c^i(y^i+\phi^i)}{e^i}\varepsilon_m(x) - \gamma(x),\end{aligned}$$

where we have made use of equation (3.6) to obtain the last equality.

It remains to show equation (3.28). By definition, we have

$$L^cS^c = \frac{mI^r}{m\mathcal{I}} + \sum\nolimits_{i=1}^n \frac{1-\beta^i}{\beta^i}S^{e,i}$$

Differentiate the above equation with respect to x and use equation (3.27) to obtain

$$\begin{aligned}x\mathcal{I}L^cS^c_x &= I^r\left[\varepsilon_{I^r}(x) - \varepsilon_m(x) - \gamma(x)\right] + \sum\nolimits_{i=1}^n (1-\beta^i)e^i\frac{x}{S^{e,i}}S^{e,i_x}\\ &= I^r\left[\varepsilon_{I^r}(x) - \varepsilon_m(x) - \gamma(x)\right] + \sum\nolimits_{i=1}^n (1-\beta^i)e^i\left[\frac{c^i(y^i+\phi^i)}{e^i}\varepsilon_m(x) - \gamma(x)\right]\\ &= \left[\sum\nolimits_{i=1}^n (1-\beta^i)c^i(y^i+\phi^i) - I^r\right]\varepsilon_m(x) - \left[I^r + \sum\nolimits_{i=1}^n(1-\beta^i)e^i\right]\gamma(x) + I^r\varepsilon_{I^r}(x)\\ &= \Gamma^c\varepsilon_m(x) - L^cI^c\gamma(x) + I^r\varepsilon_{I^r}(x),\end{aligned}$$

where the last line follows from equations (3.9) and (3.24).

Equations (3.37): Equation (3.34) holds for all i. Differentiating this equation with respect to m yields

$$\varepsilon'_y(m) = \frac{\varepsilon_y(m)^2}{m}\left[\frac{1}{1-\varepsilon_p(y)}\frac{y\varepsilon'_p(y)}{\varepsilon_p(y)} - \frac{y\rho_y}{2-\rho}\right], \tag{3A.6}$$

where we have omitted good indices to save notation.

Differentiate $M(m)$, given in equation (3.35), with respect to m to obtain

$$\begin{aligned}\left[\sum\nolimits_{i=1}^n c^iy^i\varepsilon_{y^i}(m)\right]\frac{mM_m}{M} &= \frac{m}{M}\sum\nolimits_{i=1}^n c^iy^i_m - m\sum\nolimits_{i=1}^n c^i\left[y^i\varepsilon'_{y^i}(m) + y^i_m\varepsilon_{y^i}(m)\right]\\ &= -\frac{\left[\sum_{i=1}^n c^iy^i\varepsilon_{y^i}(m)\right]^2}{\sum_{i=1}^n c^i(y^i+\phi^i)} - \sum\nolimits_{i=1}^n c^iy^i\left[m\varepsilon'_{y^i}(m) - \varepsilon_{y^i}(m)^2\right]\\ &= \left\{\sum_{i=1}^n c^iy^i\varepsilon_{y^i}(m)^2 - \frac{\left[\sum_{i=1}^n c^iy^i\varepsilon_{y^i}(m)\right]^2}{\sum_{i=1}^n c^i(y^i+\phi^i)}\right\} - m\sum_{i=1}^n c^iy^i\varepsilon'_{y^i}(m).\end{aligned}$$

Substituting (3A.6) into the above equation immediately gives equation (3.37).

Appendix 3B

This appendix provides calculations of equations (3.43), (3.46)-(3.48), and (3.49)-(3.50) in Section 3.4. Before we begin, let us introduce some notations:

$$\begin{aligned}
B &\equiv \sum\nolimits_{i=1}^{n}(1-\beta^i) - 2\alpha n, \\
R_0^{ec} &\equiv \frac{\sum_{i\in\mathcal{T}}\beta^i\left(\bar{y}^i\right)^2}{\sum_{i=1}^{n}(1-\beta^i)\left(\bar{y}^i\right)^2} = \lim_{m\to 0} R^{ec}, \\
\Gamma_0^c &\equiv \sum\nolimits_{i=1}^{n}(1-\beta^i)\bar{y}^i - \alpha n E_1 = \lim_{m\to 0} 2\Gamma^c, \\
\Gamma_0^{ec} &\equiv \sum\nolimits_{i\in\mathcal{I}}\beta^i\bar{y}^i - \Gamma_0^c R_0^{ec} = \lim_{m\to 0}\frac{2\Gamma^c}{m},
\end{aligned}$$

where equalities concerning $\lim_{m\to 0}$ can be easily verified from (3B.4)-(3B.6) and (3B.8) below.

Equation (3.43): From equation (3.42), we have $z\kappa^\alpha(L^w)^{1-\alpha} = n(E_1-m)/2$. Substituting this equation into equation (3.41) as well as the definition of I^r gives

$$\varepsilon_m(x) = \delta(x)\frac{E_1-m}{m}, \tag{3B.1}$$

$$I^r = z\kappa f_k = \alpha z\kappa^\alpha(L^w)^{1-\alpha} = \frac{\alpha n(E_1-m)}{2}. \tag{3B.2}$$

Also, in our example, we have

$$4me^i = (\bar{y}^i - m)^2, \qquad \forall i. \tag{3B.3}$$

Using (3B.3), we can rewrite R^{ec} as

$$R^{ec} = \frac{\sum_{i\in\mathcal{T}}\beta^i(\bar{y}^i-m)^2}{4A/x}, \tag{3B.4}$$

where A is given by (3A.3) and now, with (3B.2)-(3B.3), must satisfy

$$\begin{aligned}
\frac{4A}{x} &= 2\alpha mn(E_1-m) + \sum\nolimits_{i=1}^{n}(1-\beta^i)(\bar{y}^i-m)^2 \\
&= Bm^2 - 2\Gamma_0^c m + \sum\nolimits_{i=1}^{n}(1-\beta^i)(\bar{y}^i)^2.
\end{aligned} \tag{3B.5}$$

To derive equation (3.43), we have to use equation (3.19). It is obvious that we have to calculate Γ^{ec}, which is given by equation (3.17) and now can be rewritten as

$$\begin{aligned}
\frac{4A}{x}4\Gamma^c =& \frac{4A}{x}\sum\nolimits_{i\in\mathcal{I}}\beta^i\left[(\bar{y}^i)^2 - m^2\right] - \sum\nolimits_{i=1}^{n}(1-\beta^i)\left[(\bar{y}^i)^2-m^2\right]\sum\nolimits_{i\in\mathcal{I}}\beta^i(\bar{y}^i-m)^2 \\
=& m\left\{\left[B+\sum\nolimits_{i=1}^{n}(1-\beta^i)\right]m - 2\Gamma_0^c\right\}\sum\nolimits_{i\in\mathcal{I}}\beta^i\bar{y}^i(\bar{y}^i-m) \\
&+ 2m\left\{-\alpha nm^2 - \Gamma_0^c m + \sum\nolimits_{i=1}^{n}(1-\beta^i)(\bar{y}^i)^2\right\}\sum\nolimits_{i\in\mathcal{T}}\beta^i(\bar{y}^i-m),
\end{aligned} \tag{3B.6}$$

where we use of $\bar{y}^2 - m^2 = \bar{y}(\bar{y}-m) + m(\bar{y}-m)$ and $(\bar{y}-m)^2 = \bar{y}(\bar{y}-m) - m(\bar{y}-m)$, along with (3B.5), to obtain the second equality.

In our example, we have $\varepsilon_{I^r}(x) = \delta(x)$. Then, we can substitute this equation, along with (3B.1)-(3B-2) into equation (3.19) to obtain equation (3.43):

$$\begin{aligned}
\frac{16A^2}{x\delta(x)(E_1-m)}R_x^{ec} =& \frac{4A}{x}\frac{4\Gamma^c}{m} - 2\alpha nm\frac{4A}{x}R^{ec} \\
=& \frac{4A}{x}\frac{4\Gamma^c}{m} - 2\alpha nm\sum\nolimits_{i\in\mathcal{T}}\beta^i\bar{y}^i(\bar{y}^i-m) + 2\alpha nm^2\sum\nolimits_{i\in\mathcal{T}}\beta^i(\bar{y}^i-m) \\
=& 2\left(Bm-\Gamma_0^c\right)\sum\nolimits_{i\in\mathcal{I}}\beta^i\bar{y}^i(\bar{y}^i-m) \\
&+ 2\left[-\Gamma_0^c m + \sum\nolimits_{i=1}^n(1-\beta^i)(\bar{y}^i)^2\right]\sum\nolimits_{i\in\mathcal{T}}\beta^i(\bar{y}^i-m) \\
=& 2\left[\Gamma_0^c\sum\nolimits_{i\in\mathcal{T}}\beta^i - B\sum\nolimits_{i\in\mathcal{T}}\beta^i\bar{y}^i\right]m^2 \\
&- 2\sum\nolimits_{i=1}^n(1-\beta^i)(\bar{y}^i)^2\left[\sum\nolimits_{i\in\mathcal{T}}\beta^i - BR_0^{ec}\right]m + 2\Gamma_0^{ec}\sum\nolimits_{i=1}^n(1-\beta^i)(\bar{y}^i)^2,
\end{aligned}$$

where we use (3B.4) to obtain the second equality, and (3B.5) to obtain the third equality. We can immediately derive equation (3.43) from the above equation, as well as (3A.3).

Equation (3.46)-(3.48): From equation (3.36), we have

$$\gamma(x) = \delta(x)\frac{zf(\kappa, L^w)}{\mathcal{I}} = \delta(x)\frac{n(E_1-m)/2}{n(E_2-m^2)/4m} = \delta(x)\frac{2m(E_1-m)}{E_2-m^2}, \tag{3B.7}$$

where the second equality follows from equation (3.42) and the definition of $\mathcal{I}$.

It is straightforward to derive equations (3.46)-(3.47) from equations (3.26)-(3.27), by using equations (3.38), (3B.1) and (3B.7). Hence, here, we only provide the derivation of equation (3.48). To be convenient, let us rewrite equation (3.48).

$$x\mathcal{I}L^cS_x^c = \Gamma^c\varepsilon_m(x) - L^cI^c\gamma(x) + I^r\varepsilon_{I^r}(x), \tag{3.48}$$

where $\varepsilon_{I^r}(x) = \delta(x)$, $L^cI^c = A/xm$ [see (3A.3)], and Γ^c is given by equation (3.24) and now satisfies

$$\begin{aligned}
2\Gamma^c = 2\sum\nolimits_{i=1}^n(1-\beta^i)y^i - 2I^r &= \sum\nolimits_{i=1}^n(1-\beta^i)(\bar{y}^i-m) - \alpha n(E_1-m) \\
&= \Gamma_0^c - (B+\alpha n)m.
\end{aligned} \tag{3B.8}$$

where we use equations (3.38) and (3B.2) to obtain the second equality.

Substituting (3B.1)-(3B.2) and (3B.7) into equation (3.48) yields

$$
\begin{aligned}
\frac{2mx\mathcal{I}L^c}{\delta(x)(E_1 - m)} S_x^c &= 2\Gamma^c - \frac{4A}{x}\frac{m}{E_2 - m^2} + \alpha n m \\
&= \Gamma_0^c - Bm - \frac{4A}{x}\frac{m}{E_2 - m^2} \\
&= \frac{\Gamma_0^c m^2 - \left[BE_2 + \sum_{i=1}^n (1-\beta^i)(\bar{y}^i)^2\right] m + \Gamma_0^c E_2}{E_2 - m^2},
\end{aligned}
$$

where we use (3B.8) to obtain the second line and (3B.5) to obtain the third line.

Equation (3.49)-(3.50): Condition (30) is equivalent to $I^w < I^{e,i} < I^c$ for all i, where $I^{e,i} = \beta^i e^i$ and $I^c = A/xmL^c$ can be derived from, respectively, (3B.3) and (3B.5). Substituting these equations into $I^{e,i} < I^c$ and rearranging terms, we immediately obtain condition (3.50).

L^w is given by equation (3.7) and is now given by

$$
I^w = zf_\ell = \frac{(1-\alpha)}{L^w} z\kappa^\alpha (L^w)^{1-\alpha} = \frac{n(1-\alpha)(E_1 - m)}{2L^w},
$$

where the third equality follows from equation (3.42). Substituting the above equation and $I^{e,i} = \beta^i e^i$ into $I^w < I^{e,i}$ and rearranging terms, we immediately obtain condition (3.49).

Chapter 4

Income Inequality among the Working Rich

4.1 Introduction

In this chapter, we deal with the second aspect of the rise of the working rich, concerning inequality among the working rich. As mentioned in Chapter 1, to keep the analysis as simple as possible, we consider an economy with only entrepreneurs, facing linear demand functions, which are strictly subconvex. We will show that inequality among the working rich depends on "the source of firm heterogeneity" and on whether there is "free entry."

As in Chapter 3, we show that an increase in either productivity or capital stock raises profits, by lowering the average marginal cost (Lemma 4.1). As already described in Chapter 1, we break down the effect of the fall in the average marginal cost on the income distribution into three effects (Lemma 4.2-4.4), namely the average-income effect (the AIE), the superstar effect (the SE), and the entry effect (the EE). The AIE is inequality-reducing, while the SE is inequality-enhancing. The EE, which is an anti-SE, has no first-order effect but has a second-order effect, which has an important implication for inequality among the working rich.

Recall from Chapter 3 that firms are differentiated in terms of productivity and demand (and fixed costs). The results do not depend on these different sources of firm heterogeneity. In this chapter, we show that the change in inequality among the working rich (entrepreneurs to be precise) depends on whether we consider productivity heterogeneity or demand heterogeneity. To be specific, we show that the SE is relatively weak under productivity heterogeneity. When productivity and capital stock are sufficiently high, income inequality falls under productivity heterogeneity but rises under demand heterogeneity.

Under demand heterogeneity, we show that inequality among the working rich also depends on the EE. Specifically, we show that income inequality rises when the SE is stronger than the AIE, but for the fact that the richest entrepreneurs enjoy the highest increases in income shares (as shown in Figure 1.3), the SE must be significantly strong. The introduction of the EE weakens the SE (through its second-order effect). With the EE, it is possible that we can never obtain the fact mentioned above. To put it differently, this fact requires a weak EE.

The discussion so far is based on monopolistic competition. We also extend the model to allow for strategic interactions among entrepreneurs. As in Shimomura and Thisse [2012], we assume that there are some large firms, which are oligopolistically competitive. The remaining firms are small and are monopolistically competitive. As in the literature of oligopolistic competition, market shares—which arguably highlight market power—become important among large firms. We show that the model with monopolistic competition underestimates the increases in profits of large (or top) firms, i.e., underestimates the SE, because large firms do not make use of their nontrivial market shares.

The organization of this chapter is as follows. Section 4.2 describes the model with monopolistic competition. Section 4.3 describes the main results. Section 4.4 extends the model to allow for oligopolistic competition. Finally, Section 4.5 gives a summary.

4.2 The Model

Consider an economy, in which each individual is a potential entrepreneur, who can produce one differentiated good. Because there are only entrepreneurs, without loss of generality, we can consider a continuous distribution and normalize the population to one. Also, assume that each individual is endowed with $\kappa > 0$ units of capital, which is not a consumption good and can be rented out with the interest rate $r \geq 0$.

Each entrepreneur faces a linear (inverse) demand function

$$P(i) = \lambda^{-1} \left[\bar{y}(i) - y(i) \right] = \lambda^{-1} p(i), \qquad \forall i \in \Omega, \tag{4.1}$$

where Ω is the set of all varieties, $P(i)$ is the price, $y(i)$ is the demand, $\lambda > 0$ highlights the Lagrange multiplier (see below), and $\bar{y}(i)$ highlights demand heterogeneity. Borrowing the terminology from Hottman et al. [2016], we call $\bar{y}(i)$ "appeal" (see footnote 6 in Chapter 1).

Unlike Chapter 3 where we want to keep the generality of the demand function, here, it is possible to explicitly derive the demand function (4.1) from utility maximization. Specifically, let $\mathcal{J}$ be the set of the population.[1] The utility function of each individual $j \in \mathcal{J}$ is given by

$$U^j = \int_{i\in\Omega} \bar{y}^j(i) y^j(i) di - \frac{1}{2}\int_{i\in\Omega} y^j(i)^2 di,$$

where $y^j(i)$ is individual j's consumption of variety $i \in \Omega$, and $\bar{y}^j(i)$ highlights appeal. With a standard budget constraint, given by $\int_{i\in\Omega} P(i)y^j(i)di = I^j$ where I^j is the total income, we can write the first-order condition as

$$\bar{y}^j(i) - y^j(i) = \lambda^j P(i), \qquad \forall i \in \Omega, \forall j \in \mathcal{J},$$

where $\lambda^j > 0$ is the Lagrange multiplier. Adding the above equation across all individuals and rearranging terms, we obtain the market inverse demand function (4.1), where $y(i) \equiv \int_{j\in\mathcal{J}} y^j(i)dj$, $\bar{y}(i) \equiv \int_{j\in\mathcal{J}} \bar{y}^j(i)dj$, and

$$\lambda \equiv \int_{j\in\mathcal{J}} \lambda^j dj = \frac{\int_{i\in\Omega} P(i)\bar{y}(i)di - \int_{j\in\mathcal{J}} I^j dj}{\int_{i\in\Omega} P(i)^2 di}.$$

Recall that, in Chapter 3, we treat λ as an exogenous variable. It is obvious from the above equation that λ depends on incomes (and hence profits) and prices. Thus, equation (4.1) is not the final form of the (inverse) demand function. Nevertheless, as in Chapter 3, we assume that entrepreneurs treat λ parametrically. We relax this assumption and consider, as an extension, the case of oligopolistic competition in Section 4.4.

4.2.1 Profit Maximization

Since each individual is a potential entrepreneur, throughout this chapter, we use capital (which is not a consumption good) as only one factor of production.[2]

Each entrepreneur $i \in \Omega$ rents capital $k(i)$ from a perfect capital market with the interest rate $r \geq 0$ and has a constant productivity $z/c(i) > 0$, where $z > 0$ is a scalar used to capture the increase in productivity, and $1/c(i) > 0$ highlights productivity heterogeneity. Profit $\pi(i)$ is given by

$$\pi(i) = P(i)y(i) - rk(i) = \lambda^{-1}\left[p\left(y(i), \bar{y}(i)\right) - mc(i)\right] y(i), \qquad \forall i \in \Omega,$$

where $m \equiv \lambda r/z \geq 0$, as in Chapter 3, is the average marginal cost.

Maximizing profit $\pi(i)$ subject to the linear demand function (4.1) yields

$$y(i) = \frac{\bar{y}(i) - mc(i)}{2}, \qquad \forall i \in \Omega. \tag{4.2}$$

[1] Note that since each good is produced by one individual, Ω is a subset of $\mathcal{J}$. Ω and $\mathcal{J}$ are equivalent when all individuals can survive the market.

[2] Using labor in our simple setting only complicates the analysis and does not improve the results. For a different setting with labor, see Behrens et al. [2017].

Similar to Chapter 3, let us redefine profits in terms of r/z:

$$\pi(\bar{y}(i), c(i), m) \equiv \frac{\pi(i)}{r/z} = \frac{[\bar{y}(i) - mc(i)]^2}{4m}, \qquad \forall i \in \Omega. \tag{4.3}$$

It is obvious that profit $\pi(\bar{y}(i), c(i), m)$ is strictly increasing in appeal $\bar{y}(i)$ and productivity $1/c(i)$ and is strictly decreasing with the average marginal cost m.

4.2.2 The Equilibrium

The market-clearing condition for the capital market is given by

$$\int_{i\in\Omega} c(i)y(i)di = \alpha. \tag{4.4}$$

where $\alpha \equiv \kappa z > 0$—which is strictly increasing in capital stock κ and productivity z—will be the main parameter in this chapter.[3] To complete the model, we can derive the average marginal cost m, using equations (4.2) and (4.4). Before doing this, it is worth noting from equation (4.2) that all entrepreneur i, with $m > \bar{y}(i)/c(i)$, will not survive the market. Hence, only i with $m \leq \bar{y}(i)/c(i)$ is included in equation (4.4).

For expositional purposes, define $x \equiv \bar{y}(i)/c(i)$, and assume that x follows a distribution F defined on an interval $[a, b]$, where $0 < a < b < \infty$. For the rest of this chapter, we use x as individual/entrepreneur index, and we can rewrite entrepreneur x's output and profit as

$$y(x, m) = c(x)\frac{x - m}{2}, \tag{4.5}$$

$$\pi(x, m) = \frac{c(x)^2(x - m)^2}{4m}, \qquad \forall x \geq m. \tag{4.6}$$

We know from the above discussion that all entrepreneurs with $x < m$ will not survive the market. A fall in the average marginal cost m will cause new entry, if and only if $m > a$. By assuming that $a > 0$, we can break down the analysis into three cases. In case 1, the average marginal cost m belongs to $(0, a]$, and there is no entry. In this case, we can derive the average marginal cost m from equations (4.4)-(4.5):

$$m = \left(\int_a^b xc(x)^2 dF(x) - 2\alpha \right) / \|c\|^2, \tag{4.7}$$

where $\|c\|^2 = \int_a^b c(x)^2 dF(x)$. This case occurs when α belongs to $[\alpha^*, \bar{\alpha})$, where

$$\alpha^* \equiv \frac{1}{2}\int_a^b (x - a)c(x)^2 dF(x) > 0,$$

$$\bar{\alpha} \equiv \frac{1}{2}\int_a^b xc(x)^2 dF(x) > 0.$$

[3] Note that we have reused the notation. α, here, is different from that in Chapter 3 but is the same as that in Chapter 2.

In case 2, α belongs to $(0, \alpha^*)$, and the average marginal cost m belongs to $(a, b]$ and satisfies

$$m = \left(\int_m^b x c(x)^2 dF(x) - 2\alpha \right) / \|c\|^2, \tag{4.8}$$

where $\|c\|^2$ in this case is $\int_m^b c(x)^2 dF(x)$. By comparing this case to case 1, we can see the role played by the EE.

Finally, case 3 occurs when α equals $\bar{\alpha}$ and the average marginal cost m is zero.

Lemma 4.1 *For $\alpha \in (0, \bar{\alpha})$, the average marginal cost m is strictly decreasing with α, i.e., $m'(\alpha) < 0$.*

Proof In case 1, the desired result follows immediately from equation (4.7). In case 2, we can rewrite equation (4.8) as

$$\int_m^b (x - m) c(x)^2 dF(x) = 2\alpha.$$

The desired result follows immediately from the above equation, since the left hand side is strictly decreasing with m. ■

As in Lemma 3.1, Lemma 4.1 states that an increase in either productivity or capital stock, i.e., an increase in α, lowers the average marginal cost.

4.2.3 The Underlying Mechanism

The total income $I_x(x, \alpha)$ of each individual $x \in [a, b]$ is given by

$$I_x(x, \alpha) = \begin{cases} \alpha & \text{if } x < m, \\ \alpha + \pi(x, m) & \text{if } x \geq m, \end{cases} \tag{4.9}$$

where $\alpha = r\kappa/(r/z)$ highlights capital income, and $\pi(x, m)$ highlights entrepreneurial income (or earned income).

Our goal in this chapter is to discuss how an increase in either productivity or capital stock (i.e., an increase in α) affects inequality in income $I_x(x, \alpha)$. Before doing this, it is worthwhile to discuss how the increase in α, through a fall in the average marginal cost m, affects the income distribution. Concretely, this subsection describes the AIE, the SE, and the EE.

To begin, recall that marginal costs of all entrepreneurs are proportional to the average marginal cost m. A fall in m causes a fall in marginal costs and an increase in profits of all entrepreneurs, leading to an increase in the aggregate/average income, $\mathcal{I}(\alpha)$, defined by

$$\mathcal{I}(\alpha) \equiv \int_a^b I_x(x,\alpha)dF(x) = \begin{cases} \int_a^b \pi(x,m)dF(x) + \alpha & \text{if } m \in [0,a], \\ \int_m^b \pi(x,m)dF(x) + \alpha & \text{if } m \in (a,b], \end{cases} \tag{4.10}$$

The result above is formally stated in the following lemma:

Lemma 4.2 (The average-income effect) *For $\alpha \in (0,\bar{\alpha})$, an increase in either productivity or capital stock raises the average income, i.e., $\mathcal{I}'(\alpha) > 0$.*

Proof The proof follows immediately from Lemma 4.1 and equations (4.6) and (4.10). ■

The AIE is arguably inequality-reducing. It, nevertheless, does not guarantee a fall in income inequality. To analyze the change in income inequality, it is also important to see the change in the spread of the distribution, i.e., the SE.

Lemma 4.3 (The Superstar Effect) *For $\alpha \in (0,\bar{\alpha})$, under either demand heterogeneity or productivity heterogeneity, we have*[4]

$$\frac{\partial}{\partial x}\left[\frac{\partial I_x(x,\alpha)}{\partial \alpha}\right] > 0, \qquad \forall x \in [a,b]. \tag{4.11}$$

That is, when either productivity or capital stock rises, the higher the income, the higher the increase in income.

Proof Differentiate equation (4.6) with respect to m to obtain

$$\frac{\partial \pi(x,m)}{\partial m} = -\frac{c(x)^2(x^2-m^2)}{4m^2}, \qquad \forall x \geq m. \tag{4.12}$$

We can then show that

$$\begin{aligned} \frac{\partial}{\partial \bar{y}(x)}\left|\frac{\partial \pi(x,m)}{\partial m}\right| &= \frac{\bar{y}(x)}{2m^2} > 0, \\ \frac{\partial}{\partial c(x)}\left|\frac{\partial \pi(x,m)}{\partial m}\right| &= -\frac{c(x)}{2} < 0, \qquad \forall x \geq m. \end{aligned}$$

These equations, along with equation (4.9), immediately give rise to the desired result. ■

[4] More precisely, we should write

$$\frac{\partial}{\partial \bar{y}(x)}\left[\frac{\partial I_x(x,\alpha)}{\partial \alpha}\right] > 0, \qquad \frac{\partial}{\partial c(x)}\left[\frac{\partial I_x(x,\alpha)}{\partial \alpha}\right] < 0.$$

We use condition (4.11) for notational convenience.

We call condition (4.11) the SE (i.e., the superstar effect) because it states, in the spirit of the economics of superstars of Rosen [1981], that the more appealing and more productive entrepreneurs enjoy higher increases in profits. The SE is arguably inequality-enhancing since it implies that the spread of the income distribution widens. In fact, it is important for the increase in income inequality. To be specific, as in Chapter 2, let $s_x(x, \alpha) \equiv \partial[I_x(x, \alpha)/\mathcal{I}(\alpha)]/\partial\alpha$ be the differentiation of income share $I_x(x, \alpha)/\mathcal{I}(\alpha)$ with respect to α. Then, we must have

$$\mathcal{I}(\alpha)\frac{\partial s_x(x, \alpha)}{\partial x} = \frac{\partial}{\partial x}\left[\frac{\partial I_x(x, \alpha)}{\partial \alpha}\right] - \frac{\partial I_x(x, \alpha)}{\partial x}\frac{\mathcal{I}'(\alpha)}{\mathcal{I}(\alpha)}.$$

Note that the first term on the right-hand side of the above equation highlights the SE, and the second term highlights the AIE. The second term is strictly positive. If the first term is non-positive, $s_x(x, \alpha)$ is strictly decreasing with x and income inequality must fall (Corollary 2.1).

Note finally that, for $\alpha \in (0, \alpha^*)$ or $m \in (a, b]$, a fall in the average marginal cost m causes new entry. This effect is stated formally in the following lemma.

Lemma 4.4 (The entry effect) *For $\alpha \in (0, \alpha^*)$, an increase in either productivity or capital stock raises the population share of surviving entrepreneurs.*

Proof The population share of surviving entrepreneurs is $\int_m^b dF(x)$. It is then effortless to show that this population share rises when m falls. ■

Intuitively, we can imagine that entrepreneurs have to compete with one another for the limited amount of capital stock. The tougher the competition, the fewer the surviving entrepreneurs. An increase in the total supply κ or a fall in the total demand (due to an increase in productivity z) implies that capital becomes more abundant. Consequently, the competition loosens and more entrepreneurs survive the market.

Unlike the AIE and SE, it is not obvious whether the EE is inequality-reducing or inequality-enhancing. To see the impact of the EE, note that the market-clearing condition for the capital market is given by

$$\int_{\hat{x}}^{b} c(x)y(x, m)dF(x) = \alpha,$$

where $\hat{x} = m$, and the left-hand side highlights the total demand for capital, while the right-hand side highlights the total supply. We can tell that if $y(\hat{x}, m) > 0$, ceteris paribus, a fall in $\hat{x}$ raises the total demand for capital (because of the demand of the new entrants) and hence the average marginal cost m. Since a fall in m causes the SE, the EE—through an increase in m—must cause the anti-SE.

In our model, as one can tell from equation (4.5), we have $y(\hat{x}, m) = 0$. Hence, a fall in $\hat{x}$ has no first-order effect, but it still has the second-order effect. To see this, given that $y(\hat{x}, m) = 0$, we can use the above equation to show that

$$m'(\alpha) = \left[\int_{\hat{x}}^{b} c(x) \frac{\partial y(x,m)}{\partial m} dF(x) \right]^{-1} < 0.$$

It is then straightforward to verify that a fall in $\hat{x}$ causes a fall in the average marginal cost to be smaller, i.e., $\partial |m'(\alpha)| / \partial \hat{x} > 0$, since $\partial y(\hat{x}, m)/\partial m < 0$. This second-order effect, as we shall see, has an important implication for the relative increase at the top.

4.3 Main Results

In this section, we discuss inequality among entrepreneurs caused by an increase in α. It is of interest to discuss both inequality in entrepreneurial income $\pi(x, m)$ and that in total income $I_x(x, \alpha)$. Let us first consider the former before moving on to the latter.

Proposition 4.1 *For $\alpha \in (0, \bar{\alpha})$, an increase in either productivity or capital stock lowers entrepreneurial income inequality under either productivity heterogeneity or demand heterogeneity.*

Proof Our goal is to show the fall in inequality in entrepreneurial income $\pi(x, m)$. Let Π be the aggregate profit:

$$\Pi(m) \equiv \int_{\hat{x}}^{b} \pi(x, m) dF(x),$$

where $\hat{x}$ equals m when $m > a$ and equals a when $m \leq a$. Given that $\pi(m, m) = 0$, we can write the differentiation of Π as

$$\Pi'(m) = \int_{\hat{x}}^{b} \frac{\partial \pi(x,m)}{\partial m} dF(x) = -\int_{\hat{x}}^{b} \frac{c(x)^2 (x^2 - m^2)}{4m^2} dF(x) = -\frac{\Pi + \alpha}{m}, \tag{4.13}$$

where we use equation (4.12) to obtain the second equality and equations (4.7)-(4.8) to obtain the third equality.

Let $s_\pi(x, \alpha)$ be the change in earned income share $\pi(x, m)/\Pi$:

$$s_\pi(x, \alpha) \equiv \frac{d}{d\alpha} \left[\frac{\pi(x,m)}{\Pi} \right] = -m'(\alpha) \frac{\alpha c(x)^2 (x - m)}{4m^2 \Pi^2} (\gamma - x), \qquad \forall x \in [\hat{x}, b]$$

where $\gamma \equiv m(2\Pi + \alpha)/\alpha$, and we have made use of equations (4.12)-(4.13). Provided that $-m'(\alpha) > 0$ and

$$\int_{\hat{x}}^{b} s_\pi(x, \alpha) dF(x) = 0,$$

we must have $\gamma \in (\hat{x}, b)$. It is then obvious that conditions (i)-(iii) in Corollary 2.1 hold. Hence, as desired, entrepreneurial income inequality must fall. $\blacksquare$

Recall that, whether there is free entry, the EE has no first-order impact on entrepreneurial income inequality. The change in inequality depends on the AIE and the SE. For Proposition 4.1 to hold, i.e., for a fall in entrepreneurial income inequality, the AIE must be stronger than the SE. To see this, define the entrepreneurial income share gap between the richest and the poorest by

$$G_\pi(m) \equiv \frac{\pi(b,m) - \pi(\hat{x},m)}{\Pi}.$$

The SE states that the numerator rises (Lemma 4.3), when the average marginal cost m falls. For the AIE to be stronger, the fall in m must lower the gap G_π. With straightforward calculations, we can show that

$$\begin{aligned} G'_\pi(m) &= \frac{\bar{c}^4(b-\hat{x})}{8m^2\Pi^2}\int_{\hat{x}}^b (x-m)\left[(b-x)+(\hat{x}-m)\right]dF(x) > 0, \\ G'_\pi(m) &= \frac{\bar{\bar{y}}\left[c(\hat{x})-c(b)\right]}{8m^2\Pi^2}\int_{\hat{x}}^b c(x)(x-m)\left\{c(\hat{x})(\hat{x}-m)+m\left[c(x)-c(b)\right]\right\}dF(x) > 0, \end{aligned}$$

where the first equation is under demand heterogeneity, i.e., $c(x) = \bar{c}$ for all $x \in [a,b]$. The second equation is under productivity heterogeneity, i.e., $\bar{y}(x) = \bar{\bar{y}}$ for all $x \in [a,b]$. It is obvious that, in either case, $G'_\pi(m) > 0$ as desired.

Now, let us turn to the discussion of total income inequality. As already mentioned, we break down the discussion into three cases.

Case 1: $\alpha \in [\alpha^*, \bar{\alpha})$

Assume throughout this subsection that $\alpha \in [\alpha^*, \bar{\alpha})$, and hence there is no entry.

Define the total income share gap between the richest and the poorest by

$$G_I(m(\alpha),\alpha) \equiv \frac{I_b(b,\alpha) - I_a(a,\alpha)}{\mathcal{I}(\alpha)} = \frac{m\pi(b,m) - m\pi(a,m)}{m\mathcal{I}(\alpha)},$$

where we know from equation (4.10) that $\mathcal{I}(\alpha) = \Pi + \alpha$. Using (4.6), we can show that the total effect of α on income share gap G_I is given by

$$\begin{aligned} \frac{dG_I}{d\alpha} &= m'(\alpha)\frac{c(a)^2(a-m) - c(b)^2(b-m)}{2m\mathcal{I}(\alpha)} - \frac{G_I}{m\mathcal{I}(\alpha)}\frac{d[m\mathcal{I}(\alpha)]}{d\alpha} \\ &= -m'(\alpha)\frac{bc(b)^2 - ac(a)^2}{2m\mathcal{I}(\alpha)} - m\left[m'(\alpha)\frac{c(a)^2 - c(b)^2}{2m\mathcal{I}(\alpha)} + \frac{G_I}{m\mathcal{I}(\alpha)}\right], \end{aligned} \tag{4.14}$$

where, to obtain the second equality, we use equation (4.13), which implies that $d[m\mathcal{I}(\alpha)]/d\alpha = m$.

Unlike the case of entrepreneurial income share gap G_π, an increase in α now does not necessarily lower the income share gap G_I. Since the direct effect of an increase in α is negative, the ambiguity of the change in G_I must arise from the indirect effect through a fall in m. To see this, note that by definition we have

$$\begin{aligned}\frac{\partial G_\pi/\partial m}{G_\pi} &= \frac{\partial\left[\pi(b,m)-\pi(a,m)\right]/\partial m}{\pi(b,m)-\pi(a,m)}-\frac{\partial\Pi/\partial m}{\Pi},\\ \frac{\partial G_I/\partial m}{G_I} &= \frac{\partial\left[\pi(b,m)-\pi(a,m)\right]/\partial m}{\pi(b,m)-\pi(a,m)}-\frac{\partial\Pi/\partial m}{\Pi+\alpha}.\end{aligned}$$

Compared with the first equation above, we see that the introduction of capital income lowers the absolute value of the second term on the right-hand side of the second equation (i.e., weakens the AIE). As we shall see shortly, the SE can now be stronger than the AIE, and hence there can be an increase in income inequality.

It is noteworthy from equation (4.14) that when the average marginal cost is sufficiently low (i.e., when $m\to 0$), the second term on the right-hand side can be neglected, and the income share gap G_I rises under demand heterogeneity but falls under productivity heterogeneity. As stated in the following proposition, this is also true for income inequality.

Proposition 4.2 *When productivity and capital stock are sufficiently high, and hence the average marginal cost is sufficiently low, an increase in either productivity or capital raises (lowers) income inequality under demand (productivity) heterogeneity.*

Proof Let $s_I(x,\alpha)$ be the change in the income share of each individual x:

$$\begin{aligned}s_I(x,\alpha)\equiv\frac{d}{d\alpha}\left[\frac{I_x(x,\alpha)}{\mathcal{I}(\alpha)}\right] &= \frac{d}{d\alpha}\left[\frac{mI_x(x,\alpha)}{m\mathcal{I}(\alpha)}\right]\\ &= -m'(\alpha)\frac{c(x)^2(x-m)-2\alpha}{2m\mathcal{I}(\alpha)}+m\frac{\Pi-\pi(x,m)}{m\mathcal{I}(\alpha)^2},\end{aligned}\qquad(4.15)$$

where, similar to equation (4.14), we have made use of equation (4.6) and $d[m\mathcal{I}(\alpha)]/d\alpha=m$.

When $m\to 0$, the second term of the second line of equation (4.15) disappears. Under demand heterogeneity, it is obvious that $s_I(x,\alpha)$ is strictly increasing in x. Then, since $\int_a^b s_I(x,\alpha)dF(x)=0$, there must exist $\gamma\in(a,b)$ such that conditions (i)-(iii) in Theorem 2.1 hold. Thus, income inequality must rise.

Under productivity heterogeneity, we have $c(x)=\bar{y}(x)/x$ where $\bar{y}(x)=\bar{\bar{y}}$ is identical for all $x\in[a,b]$. Then, $s_I(x,\alpha)$ is strictly decreasing with x since $c(x)^2x=\bar{\bar{y}}^2/x$. Again, since $\int_a^b s_I(x,\alpha)dF(x)=0$, we must have $\gamma\in(a,b)$ such that conditions (i)-(iii) in Corollary 2.1 hold. Thus, income inequality must fall. ■

Proposition 4.2 states a crucial result. That is, the choice about the source of heterogeneity can have serious consequences. To understand why, recall that income inequality rises when the SE is stronger than the AIE. In our setting, the SE is relatively weak under productivity heterogeneity. To be more specific, recall that the marginal cost of each entrepreneur i is $mc(i)$. Since $\partial[mc(i)]/\partial m = c(i)$, the fall in the average marginal cost m lowers the marginal costs of the least productive entrepreneurs disproportionally. This effect is not strong enough to cause the anti-SE, but it leads to a weak SE and, hence, to a fall in income inequality.

Given Proposition 4.2, since we want to discuss the increase in income inequality, for the rest of this chapter, we only consider demand heterogeneity and assume, without loss of generality, that $c(i) = 1$ for all i.

Proposition 4.3 *For $\alpha \in [\alpha^*, \bar{\alpha})$, under demand heterogeneity, an increase in either productivity or capital stock raises income inequality if the following condition holds:*

$$x^* \equiv \frac{E_2 + m^2}{2m} \geq \frac{b^2 - E_2}{2\,(b - E_1)}, \tag{4.16}$$

where $E_k \equiv \int_a^b x^k dF(x)$.

Proof Since $c(i) = 1$ for all i, we must have

$$\begin{aligned} 2\alpha &= E_1 - m, \\ \pi(x, m) &= \left(x^2 - 2mx + m^2\right)/4m, \\ \Pi &= \left(E_2 - 2mE_1 + m^2\right)/4m, \\ \mathcal{I} &= \left(E_2 - m^2\right)/4m. \end{aligned}$$

Using these equations, we can rewrite equation (4.15) as

$$s_I(x, \alpha) = \frac{-mx^2 + \left(E_2 + m^2\right)x - \left[\left(E_2 + m^2\right)E_1 - mE_2\right]}{4m^2\mathcal{I}(\alpha)^2} \equiv \frac{s(x, m)}{4m^2\mathcal{I}(\alpha)^2}. \tag{4.17}$$

The $s(x, m)$ schedule is depicts in Figure 4.1 (the solid line).

It is straightforward to verify that condition (4.16) is equivalent to $s(b, m) \geq 0$. Then, it is obvious from Figure 4.1 that conditions (i)-(iii) in Theorem 2.1 hold. Hence, as desired, income inequality must rise. ■

Figure 4.1 displays the $s(x, m)$ schedule and its shift—i.e., the $s(x, m')$ schedule—when the average marginal cost falls from m to a sufficiently low level m' at which $x^* \geq b$. Since capital income is equally distributed, Figure 4.1 also displays the distribution of changes in entrepreneurial income shares. As one can see, the increase in income inequality needs

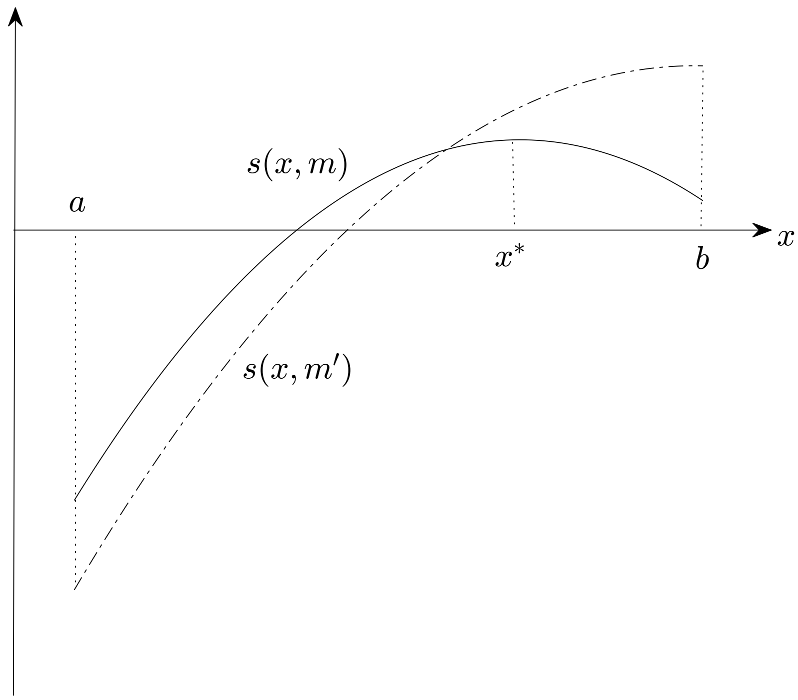

Figure 4.1: Distributions of Changes in income shares.

not involve the second aspect of the rise of the working rich, i.e., the fact that the richest entrepreneurs enjoy the highest increases in income shares. For this fact to be the case, it requires that

$$x^* \geq b, \tag{4.18}$$

which is a stronger condition than condition (4.16).[5]

Proposition 4.4 *For $\alpha \in [\alpha^*, \bar{\alpha})$, under demand heterogeneity, an increase in either productivity or capital stock leads to an increase in income inequality and the second aspect of the rise of the working rich only when condition (4.18) holds or, equivalently, only when*

$$m \leq b - \sqrt{b^2 - E_2}. \tag{4.19}$$

Proof We can immediately obtain condition (4.19) by substituting x^*, given in condition (4.16), into condition (4.18). ■

[5]It is straightforward to verify that

$$b \geq \frac{b^2 - E_2}{2(b - E_1)}.$$

Hence, if condition (4.18) holds, condition (4.16) will hold trivially.

Since the average marginal cost m is strictly decreasing with productivity and capital stock (Lemma 3.1), consistent with Chapter 3, Proposition 4.4 states that the increase in income inequality involves the second aspect of the rise of the working rich only when productivity and capital stock are sufficiently high.[6]

Using our discussion described in subsection 4.2.3, we can say that the increase in income inequality requires that the SE be stronger than the AIE, but the second aspect of the rise of the working rich requires that the SE be significantly stronger than the AIE. To see this, recall that by definition we have

$$s_I(x,\alpha) = \frac{1}{\mathcal{I}(\alpha)}\left[\frac{\partial I_x(x,\alpha)}{\partial \alpha} - I_x(x,\alpha)\frac{\mathcal{I}'(\alpha)}{\mathcal{I}(\alpha)}\right] \equiv \frac{SE(x) - I_x(x,\alpha)AIE(x)}{\mathcal{I}(x)}.$$

The increase in income inequality requires that $SE(b) - I_b(b,\alpha)AIE(b) \geq 0$ (Proposition 4.3), while the second aspect of the rise of the working rich requires that $SE(x) - I_x(x,\alpha)AIE(x)$ be the highest at $x = b$.

Case 2: $\alpha \in (0, \alpha^*)$

Suppose that $\alpha \in (0,\alpha^*)$ and hence $m > a$, where m satisfies equation (4.8). In this case, entrepreneur $x \in [a, m)$ will not survive the market. Define by $\nu \equiv \int_m^b dF$ the population share of surviving entrepreneurs. Obviously, a fall in m raises ν; that is, there is the EE (Lemma 4.4).

Proposition 4.5 *For $\alpha \in (0,\alpha^*)$, under demand heterogeneity, an increase in either productivity or capital stock raises income inequality if the following condition holds:*

$$x_m^* \equiv \frac{E_{2m} + \nu m^2}{2m\nu} \geq \frac{\nu b^2 - E_{2m}}{2\left(\nu b - E_{1m}\right)}, \tag{4.20}$$

where $E_{km} \equiv \int_m^b x^k dF(x)$.

Proof It is straightforward to show that

$$\begin{aligned} 2\alpha &= E_{1m} - \nu m, \\ \Pi &= \left(E_{2m} - 2mE_{1m} + \nu m^2\right)/4m, \\ \mathcal{I} &= \left(E_{2m} - \nu m^2\right)/4\nu m. \end{aligned}$$

[6] Graphically, it is straightforward to verify that x^* is strictly decreasing with m. Thus, as productivity and capital stock rise and hence the average marginal cost falls, x^* rises toward b.

Similar to the proof of Proposition 4.3, making use of these equations, we can show that

$$\begin{aligned}
A &\equiv \frac{d}{d\alpha}\left[\frac{\alpha}{\mathcal{I}(\alpha)}\right] = -\frac{\left(E_{2m}+\nu m^2\right)E_{1m}-2\nu m E_{2m}}{4\nu m^2\mathcal{I}(\alpha)^2}, && (4.21)\\
B(x) &\equiv \frac{d}{d\alpha}\left[\frac{\pi(x,m)}{\mathcal{I}(\alpha)}\right] = \frac{-\nu m x^2+\left(E_{2m}+\nu m^2\right)x-mE_{2m}}{4\nu m^2\mathcal{I}(\alpha)^2}, \quad \forall x\in[m,b], && (4.22)\\
s_m(x,m) &\equiv B(x)+\nu^{-1}A\\
&= \frac{-\nu m x^2+\left(E_{2m}+\nu m^2\right)x-\left[\left(E_{2m}+\nu m^2\right)\nu^{-1}E_{1m}-mE_{2m}\right]}{4\nu m^2\mathcal{I}(\alpha)^2}. && (4.23)
\end{aligned}$$

It is straightforward to verify that condition (4.20) is equivalent to $s_m(b,m)\geq 0$.

With the EE, we cannot immediately apply Theorem 2.1 to the increase in income inequality. As one might already be able to tell from equation (4.23), to apply Theorem 2.1, we have to modify changes in income shares slightly. To see this, let us explicitly consider the Lorenz curve, $L(v,\alpha)$, $\forall v\in[0,1]$, which is given by

$$L(v,\alpha)=\begin{cases} v\frac{\alpha}{\mathcal{I}(\alpha)} & \text{if } v<1-\nu,\\ (1-\nu)\frac{\alpha}{\mathcal{I}(\alpha)} & \text{if } v=1-\nu,\\ v\frac{\alpha}{\mathcal{I}(\alpha)}+\int_m^{F^{-1}(v)}\frac{\pi(x,m)}{\mathcal{I}(\alpha)}dF(x) & \text{if } v>1-\nu.\end{cases}$$

The change in the Lorenz curve is then given by

$$\frac{\partial L(v,\alpha)}{\partial\alpha}=\begin{cases} vA & \text{if } v<1-\nu,\\ (1-\nu)A+f(m)m'(\alpha)\frac{\alpha}{\mathcal{I}(\alpha)} & \text{if } v=1-\nu,\\ (1-v)(\nu^{-1}-1)A+L_m(v) & \text{if } v>1-\nu,\end{cases}$$

where $L_m(v)$ is defined by

$$L_m(v)\equiv\int_m^{F^{-1}(v)}s_m(x,m)dF(x),\qquad \forall v\in(1-\nu,1].$$

Recall from the proof of Theorem 2.1 that, for the increase in income inequality, it requires that $\partial L(v,\alpha)/\partial\alpha<0$, for all $v\in[0,1]$. Then, it suffices to show that if condition (4.20) holds, i.e., if $s_m(b,m)\geq 0$, we must have have $A<0$ and $L_m(v)<0$ for all $v\in(1-\nu,1]$.

Let us start with $L_m(v)<0$. It is straightforward to show that

$$\int_m^{F^{-1}(1)}s_m(x,m)dF(x)=\int_m^b s_m(x,m)dF(x)=0.$$

Then, we can use Lemma 2.1 to obtain the desired result. That is, it suffices to ensure that $s_m(x,m)$, where $x\in[m,b]$, satisfies conditions (i)-(iii) in Lemma 2.1. Similar to $s(x,m)$ in Proposition 4.3, $s_m(x,m)$ is strictly concave with respect to x. Then, since $s_m(b,m)\geq 0$, the graph of $s_m(x,m)$ is similar to that of $s(x,m)$ in Figure 4.1. Hence, as desired, conditions (i)-(iii) in Lemma 2.1—concerning $s_m(x,m)$ for all $x \in [m,b]$—must hold.

We can tell from the above discussion that $s_m(m,m) < 0$. Then, since $B(m) = 0$ [see equation (4.22)], it follows that $s_m(m,m) = \nu^{-1}A < 0$ as desired. ■

It is obvious from equation (4.22) that the change in entrepreneurial income share $B(x)$ is strictly concave with respect to x, and $B(x)$ is maximized at x_m^* given in condition (4.20). Then, as in Case 1, the second aspect of the rise of the working rich requires that $x_m^* \geq b$, which is a stronger condition than condition (4.20).

Proposition 4.5 *For $\alpha \in (0, \alpha^*)$, under demand heterogeneity, an increase in either productivity or capital stock leads to an increase in income inequality and the second aspect of the rise of the working rich only when $x_m^* \geq b$.*

Recall that Proposition 4.4 in Case 1 requires that the SE be significantly stronger than the AIE. Now, with free entry, there is another force at play, i.e., the EE. To see this, recall that x^* given in condition (4.16) is strictly decreasing with m. Hence, as m falls, x^* rises toward b. A fall in m now does not necessarily raise x_m^*:

$$\begin{aligned}\frac{dx_m^*}{dm} &= -\frac{\nu^{-1}E_{2m} - m^2}{2m^2} + \frac{1}{2m}\frac{d(\nu^{-1}E_{2m})}{dm} \\ &= -\frac{\nu^{-1}E_{2m} - m^2}{2m^2} + \frac{1}{2m}\frac{\nu^{-1}E_{2m} - m^2}{\nu}f(m).\end{aligned}$$

The second term in the first equality (which is zero when there is no entry) highlights the EE. From the second equality, we know that this term is strictly positive. Hence, ceteris paribus, when the average marginal cost m falls, the EE lowers x_m^*, and the condition that $x_m^* \geq b$ becomes less likely. In fact, it is possible that this condition does not hold for all $m \in (a,b)$. For example, with a simple Pareto distribution, i.e.,

$$F(x) = \frac{b}{b-a}\left(1 - \frac{a}{x}\right),$$

we have $x_m^* = (b+m)/2$. It is obvious that $x_m^* < b$ for all $m < b$.[7]

In short, the second aspect of the rise of the working rich requires a weak EE. The reason is simply that the EE, as we have already discussed (see Lemma 4.4), weakens the SE. In our model, we have a weak EE when productivity and capital stock are sufficiently high (i.e.,

[7]If F is a uniform distribution, i.e., if $F(x) = (x-a)/(b-a)$, we have $x_m^* = (b^2 + mb + 4m^2)/6m$. It is, then, straightforward to show that $x_m^* \geq b$ holds for $m < b$ when $m \leq b/4$.

when the average marginal cost is sufficiently low), since the mass of potential entrepreneurs is limited by a fixed mass of the population.

Case 3: $\alpha = \bar{\alpha}$

In Case 1-2, we discuss when an increase in either productivity or capital stock, through a fall in the average marginal cost m, leads to an increase in income inequality and the rise of the working rich. When the average marginal cost m is zero, we can no longer carry out such discussions; but, we can still make one interesting inference, concerning cross-country differences.

Before we begin, it is worth mentioning that although income inequality has been increasing in many rich countries in the last several decades, there are large differences among these countries. Figure 4.2 displays top 1% income shares in some rich countries [see Figure 8-9 in Atkinson et al. [2011] for more countries]. As one can see, since the early 1980s, top 1% income shares in the United States and Canada have increased significantly, while those in France, Japan, and Australia (although also on the rise) have remained at relatively low levels.

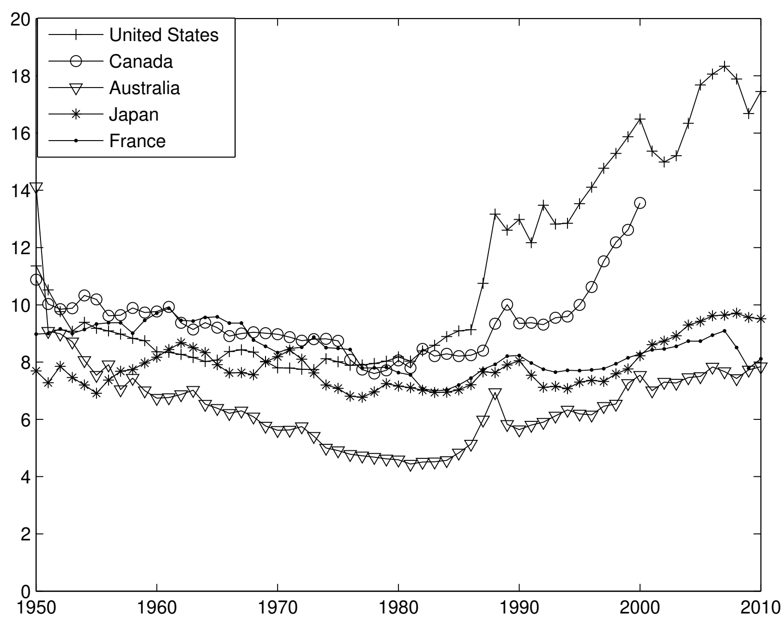

Figure 4.2: Top 1% Income Shares in the Rich Countries, 1950-2010.

Source: The World Wealth and Income Database.

Large cross-country differences cause one problem when one considers technological progress (and the increase in capital stock) as the cause of the increase in income inequality. Concretely, as Piketty [2014] points out, rich countries arguably have experienced similar economic development. Technological progress, which has widespread effects, does not seem to be the cause of the large differences. In other words, technological progress is not the whole story. Country specific factors arguably also play important roles. Piketty [2014] points to differences in social norms. Our model suggests another factor, namely differences in variance in demand/appeal.

We know from Case 1 that, in the later stage of development when productivity and capital stock are sufficiently high, income inequality rises under demand heterogeneity as productivity and capital stock rise. By assumption, demands are bounded from above, and so are productivity and capital stock (recall that α must be lower than $\bar{\alpha}$) and income inequality. At the limit (i.e., when $\alpha = \bar{\alpha}$ and $m = 0$), we have

$$\lim_{\alpha \to \bar{\alpha}} \frac{I_x(x, \alpha)}{\mathcal{I}(\alpha)} = \frac{\bar{y}(x)^2}{\int_a^b \bar{y}(z)^2 dF(z)}, \qquad \forall x \in [a, b].$$

Hence, income inequality is independent of productivity and capital stock and depends only on inequality in demand/appeal.

The result above allows us to make one remark about large differences among developed countries. Concretely, under demand heterogeneity, income inequality converges to an upper limit, which depends only on the variance in demand. As a result, countries with low limits will not experience high increases in income inequality, even when experiencing similar economic development to other countries. Intuitively, if individuals value different goods relatively equally, holding everything else constant, they will allocate their incomes to different goods relatively equally. Then, income inequality will be relatively low. In contrast, in say materialistic countries where luxurious goods are highly appealing to the population, when the countries are richer (due to the increase in either productivity or capital stock), they will witness high increases in income inequality because individuals spend more on highly appealing luxurious goods, belonging to those at the top.

It is worth stressing that the limit of income inequality, as already mentioned, arises from the assumption about preferences and hence needs not exist in general. Also, given the emphasis on the importance of the distribution of appeal/demand in a cross-country comparison, it is important to take into account international trade (which is ignored in our model). Therefore, the discussion here is by no means a complete discussion of the cross-country comparison. The objective here is simply to provide some insight, which can be drawn immediately from our model. With the above discussion, we can expect that, in say a Melitz-type model with international trade, since only top firms can export, international trade leads to higher limits to income inequality. Thus, it is conceivable that trade barriers and trade liberalization have important implications for cross-country differences (not to mention within-country income inequality).

4.4 An Extension: Oligopolistic Competition

The model in Section 4.2 is based on one assumption. That is, all entrepreneurs/firms are monopolistically competitive. In the literature, this assumption is often justified by the statement that the number of firms is significantly large. Hence, each firm is (infinitesimally) small in scale, and the effect of each firm on market variables (λ in this case) can be neglected. In such a case, firms are not involved in strategic interactions. As Neary [2010] indicates, allowing for strategic interactions in a general equilibrium model generates technical difficulty. The merit of monopolistic competition is the tractability of the analyses of economic problems [Matsuyama [1995]], although at the expense of the generality of firm behaviors.

Some industries, such as restaurants, cereal, and shoes are often raised as examples of monopolistically competitive markets. Monopolistic competition, however, is not a good representation of firm behaviors in many real markets. There is evidence [e.g., Hottman et al. [2016] and see also Table 1 in Neary [2010]] that, in many industries, there are indeed many small firms, but there are a few large firms with nontrivial market shares.[8] Accordingly, in this section, we extend the model in Section 4.2 to allow for strategic interactions.

One question, concerning oligopolistic competition, is whether firms are involved in Cournot competition or Bertrand competition. Unlike monopolistic competition, under oligopolistic competition, Cournot competition and Bertrand competition lead to different market outcomes [see, for example, Parenti et al. [2017]]. In this section, we only consider the former, which involves relatively simplified calculations. Also, for simplicity, we assume that there is no entry and only consider the case of demand heterogeneity.

Another question is whether all firms are involved in strategic interactions. Since, as mentioned above, there are many small firms and a few large firms in many industries, we follow the work of Shimomura and Thisse [2012] and assume that only a few large firms

[8] Also, Neary [2003] argues that oligopolistic competition with strategic interactions provides a better explanation for globalization. Neary proposes the so-called "general oligopolistic equilibrium" (or GOLE), to promote oligopolistic competition in trade theories [see Colacicco [2015] for the survey of GOLE]. To allow for strategic interactions, Neary uses the large-in-small-but-small-in-large assumption. That is, each firm can be large in its own industry but is small in the whole economy. Hence, we can neglect the effect of each firm on the whole economy, but not the effect on the industry.

are involved in strategic interactions, while the rest are monopolistically competitive. The discussion here readily encompasses the case, in which all firms are involved in strategic interactions.

Shimomura and Thisse [2012] use a CES preference. Here, we continue to use the quadratic utility function, which leads to the linear demand function (4.1). Profit maximization of small firms remains the same, and outputs are still given by equation (4.2). For large firms, since λ must be taken into account, to solve profit maximization, we have to use the aggregate budget constraint, given by

$$\sum\nolimits_{i\in\Omega^L} P^i y^i + \sum\nolimits_{i\in\Omega^S} P^i y^i = \sum\nolimits_{j\in\mathcal{J}} I^j \equiv I, \tag{4.24}$$

where $\Omega^L \subset \Omega$ is the set of large firms, and $\Omega^S \subset \Omega$ is the set of small firms. Note that, unlike the case of monopolistic competition, firm size is now not negligible. Hence, by convention, it is more appropriate to consider a discrete model, rather than a continuous one.[9]

For notational convenience, define

$$R^i \equiv \lambda P^i y^i = \left(\bar{y}^i - y^i\right) y^i, \qquad \forall i \in \Omega, \tag{4.25}$$

which highlights revenues. It follows immediately from equations (4.24)-(4.25) that

$$\lambda = I^{-1} \sum\nolimits_{i\in\Omega} R^i \equiv I^{-1} R. \tag{4.26}$$

λ, given by equation (4.26), depends only on outputs and total income I. As Parenti et al. [2017] point out, a major difficulty with oligopolistic competition is the income effect (or, equivalently, the Ford effect). There is no income effect in monopolistic competition or GOLE (see footnote 8). In the former, recall that firms treat λ and hence income parametrically. In the latter, the large-in-small-but-small-in-large assumption (see footnote 8) excludes the possibility that firms take the market variable (total income I in this case) into account.

To keep the analysis as simple as possible, we also assume that there is no income effect. Then, using equations (4.25)-(4.26), we can carry out profit maximization under Cournot competition. From profit maximization, we can write markup μ^i and output y^i as

$$\mu^i = \left(1 - \frac{y^i}{P^i}\frac{\partial P^i}{\partial y^i}\right)^{-1} = \theta^i \frac{\bar{y}^i - 2y^i}{\bar{y}^i - y^i}, \tag{4.27}$$

$$y^i = \frac{\bar{y}^i - \theta^i m}{2}, \qquad \forall i \in \Omega, \tag{4.28}$$

where m is the same as in previous sections, and θ^i is given by

$$\theta^i = \begin{cases} 1 & \text{if } i \in \Omega^S \\ R/\left(R - R^i\right) & \text{if } i \in \Omega^L. \end{cases} \tag{4.29}$$

[9] In Shimomura and Thisse [2012], variables of small firms are written in continuous forms. This difference in notation does not cause any significant change in the results.

For large firms, θ^i highlights the market share R^i/R. We can tell from equations (4.27)-(4.29) that the key distinctive feature of strategic interactions is the involvement of market shares, highlighted by θ^i (which equals one under monopolistic competition). It is obvious from equations (4.27)-(4.28) that the market share highlights market power. The higher the market share, the lower the output and the higher the markup.

As in Section 4.2, to complete the model, we can substitute equation (4.28) into the market-clearing condition (4.4). Although θ^i depends on y^i and hence it is not possible to obtain the closed form of the solution, the following lemma states that Lemma 4.1 still holds.

Lemma 4.1' *Outputs are strictly decreasing with the average marginal cost, which in turn is strictly decreasing with productivity and capital stock.*

Proof Total differentiation of R^i and θ^i are given by

$$dR^i = \left(\bar{y}^i - 2y^i\right) dy^i = m\theta^i dy^i, \qquad \forall i \in \Omega,$$
$$\frac{d\theta^i}{\theta^i} = \theta^i \frac{dR^i}{R} - \left(\theta^i - 1\right) \frac{dR}{R}, \qquad \forall i \in \Omega^L,$$

where we have made use of equation (4.28). Substituting these equations into the total differentiation of equation (4.28) yields

$$\frac{dR^i}{R} = -a^i \frac{dm}{m} + a^i \left(\theta^i - 1\right) \frac{dR}{R}, \qquad \forall i \in \Omega, \tag{4.30}$$

where a^i satisfies

$$\frac{1}{a^i} \equiv \begin{cases} \frac{2R}{m^2}, & \text{if } i \in \Omega^S \\ \theta^i + \frac{2R}{(m\theta^i)^2}, & \text{if } i \in \Omega^L. \end{cases}$$

Add both sides of equation (4.30) across all $i \in \Omega$ and rearrange terms to obtain

$$\frac{dR}{R} = -\frac{A}{1 - A_\theta} \frac{dm}{m} < 0,$$

where $A = \sum_{i \in \Omega} a^i > 0$ and $A_\theta = \sum_{i \in \Omega} a^i \left(\theta^i - 1\right) < 1$.[10] Substituting the above equation back into equation (4.30) yields

$$\frac{dR^i}{R} = -a^i \left[1 + \frac{A\left(\theta^i - 1\right)}{1 - A_\theta}\right] \frac{dm}{m} < 0, \qquad \forall i \in \Omega. \tag{4.31}$$

[10] $A_\theta < 1$ because, for all $i \in \Omega^L$, we have

$$a_i \left(\theta^i - 1\right) = \frac{\theta^i - 1}{\theta^i + \frac{2R}{\left(\theta^i mc^i\right)^2}} < \frac{\theta^i - 1}{\theta^i} = \frac{R^i}{R},$$

where we have made use of the definition of θ^i to obtain the last equality.

Since dy^i is proportional to dR^i, it follows that output y^i is strictly decreasing with the average marginal cost m, as desired. This result, along with the market-clearing condition (4.4), implies that the average marginal cost must be strictly decreasing with productivity and capital stock. ■

As in Section 4.2, given Lemma 4.1', to analyze the effect of an increase in either productivity or capital stock on income inequality, we can simply discuss the effect of the fall in m. Technical difficulty prevents us from carrying out a fully fledged discussion. In this section, we simply want to show that monopolistic competition underestimates the increase in profits of the large firms, i.e., underestimates the SE. To do this, as in Section 4.2, let us redefine profits in terms of r/z:

$$\pi\left(\bar{y}^i, m\right) = \frac{\pi^i}{r/z} = \left(\frac{p^i}{m} - 1\right) y^i, \qquad \forall i \in \Omega.$$

Differentiating profits with respect the average marginal cost m yields

$$\frac{\partial \pi\left(\bar{y}^i, m\right)}{\partial m} = -\frac{R^i}{m^2} + \frac{\left(\theta^i - 1\right)}{m\theta^i}\frac{\partial R^i}{\partial m}, \qquad \forall i \in \Omega. \tag{4.32}$$

The second term on the right hand side of equation (4.32) arises from the change in the market share. We can tell from equation (4.31) that this term is strictly negative for large firms, but is zero under monopolistic competition (since $\theta^i = 1$). It follows that, as already mentioned, monopolistic competition underestimates the increase in profits of large firms. This result is hardly surprising, since monopolistically competitive firms, by ignoring strategic interactions, do not take full advantages of their market power.

The above discussion suggests that monopolistic competition underestimates the SE between large firms and small firms. It is not immediately obvious whether this result is also true among large firms. The terms $\partial R^i/\partial m$ can be smaller for larger firms because, with strong market power, they restrict the increase in production to ensure a high increase in prices.[11] Nevertheless, we can show that, under certain conditions, monopolistic competition also underestimates the SE among large firms.

4.5 Summary

In Chapter 3, we show that income inequality and the first aspect of the rise of the working rich depend on assumptions about demand functions. That is, when the demand function is

[11] This fact is captured by the term θ^i in the definition of a^i. This term arises from the change in the market share $d\theta^i$.

strictly subconvex, we obtain the increase in income inequality and the increase in top earned incomes relative to workers' wages and top capital incomes, when productivity and capital stock are sufficiently high.

In this chapter, we deal with the second aspect of the rise of the working rich. We show that, concerning inequality among the working rich, the assumption about demand functions is not the whole story. Concretely, we show that even when the demand function is strictly subconvex and when productivity and capital stock are sufficiently high, income inequality falls if we consider productivity (rather than demand) as the source of firm heterogeneity. We also show that even when the demand function is strictly subconvex, it is possible that we can never obtain the second aspect of the rise of the working rich, when we allow for free entry. With these results, we can expect that there is a large difference among firms in terms of demand/appeal, and that there is low entry, in countries experiencing the second aspect of the rise of the working rich.

In this chapter, we also show that the relative increase in top earned incomes is significantly high if top firms are large firms, which are involved in strategic interactions. This result suggests the importance of the introduction of large firms into the models. Unfortunately, with technical difficulty, we are not able to carry out a fully fledged discussion, which is left to future work.

Chapter 5

Concluding Remarks

This dissertation develops simple models to discuss the recent phenomenon of the rise of the working rich (including entrepreneurs and managers), which involves (i) the increases in top earned income (including entrepreneurial income and managers' compensation) relative to workers' wages and top capital income, and (ii) rising inequality among the working rich, i.e., the fact that, among the working rich, the richest enjoy the highest increases in earned income share.

5.1 Summary and Discussion

In the past several decades, we have witnessed the rise of many "superstar firms." In 2018, for example, Apple made headlines, by becoming the first one-trillion-dollar company. Other large companies such as Facebook, Amazon, and Google are also not far behind.

Song et al. [2015] and Barth et al. [2016] provide evidence, suggesting that different firms pay different wages, and the increase in earning inequality is largely due to the increase in dispersion of wages across (rather than within) firms. This evidence points to the importance of the rise of the superstar firms—which (can afford to) pay substantially higher wages than the average wage of the economy—in the analysis of the rise of the working rich. As already mentioned in Chapter 1, there is indeed evidence, reporting a close link between the size of top firms and CEO compensation. Gabaix and Landier [2008] and Frydman and Saks [2010], for instance, report a one-to-one relationship. Gabaix and Landier report that the average market value of the largest 500 firms in the United States has increased by 500% from 1980 to 2003 [see their Figure I and see also Figure 5 in Frydman and Saks [2010]]. CEO compensation has also increased by 500% in the same period.

Throughout this dissertation, we assume that there is a one-to-one relationship between the income of the working rich (i.e., top earned income) and firm profits. In other words, consistent with the evidence described above, the analysis throughout this dissertation is based on the importance of the rise of the superstar firms.

Globalization and technological progress are often raised as the causes of the increase in income inequality. Jaumotte et al. [2013], using newly compiled panel data of 51 countries from 1981 to 2003, find that technological progress has a great effect on inequality (greater than globalization does). Accordingly, in this dissertation, we consider the increase in productivity (because of technological progress), as well as the increase in capital stock, as the main cause of the rise of the superstar firms and hence the increase in income inequality. We show that the increase either in productivity or capital stock leads to significant increases in profits of superstar firms if they are facing "strictly subconvex demand functions," i.e., if the demand elasticity is strictly increasing demand/output [Mrázová and Neary [2017]].[1] Intuitively, the strict subconvexity—which is equivalent to the increasing relative love for variety [Zhelobodko et al. [2012]]—implies that consumers view goods as being more differentiated when consumption levels rise and, hence, are willing to pay more for each good. As a result, the increase in either productivity or capital stock leads to a high increase in profits because firms can raise not only outputs but also markups. *The increase in profits is significantly high, in the later stage of development when productivity and capital stock (and hence outputs) are sufficiently high.*

In relative terms, we show that when the demand function is strictly subconvex, profits (and hence earned income) grow faster than workers' wages, particularly when productivity and capital stock are sufficiently high. The more rapid growth of profits implies that workers' wages grow more slowly than the aggregate income. Consequently, workers' wage shares of GDP fall. This result provides interesting insight into the fall in the labor share of GDP in recent decades, which is well documented. The closest works to our model in this literature are Autor et al. [2017a,b], which also point to the rise of superstar firms. The difference is that Autor et al. [2017a,b] focus on firm heterogeneity. To be specific, they argue that labor shares of production in larger firms are lower. Then, if, in their own words, 'industries are increasingly characterized by a "winner take most" feature where one firm (or a small number of firms) can gain a very large share of the market,' the labor share of GDP must fall [Autor et al. (2017b, p. 180)]. In our model in Chapter 3, we argue instead that, with strictly subconvex demands, when productivity and capital stock are sufficiently high, profits grow rapidly, and the growth of workers' wages is less than one hundred percent of that of profits.

[1] Compared with strict superconvexity, strict subconvexity is said to be more plausible and consistent with much available empirical evidence [see footnote 10 in Mrázová and Neary [2017]].

The remaining of growth in profits is received in other forms of income (e.g., dividends paid to capitalists). As a result, workers' wages grow more slowly than the aggregate income, and hence the workers' wage share of GDP must fall.

The same argument also applies to the fall in (top) capital income share, if the capitalists do not receive a high fraction of profits as dividends. Otherwise, the high increase in dividends will dominate the increase in earned income, and capital income will grow faster than the aggregate income. Interestingly, in our model, we show that the change in the capital income share depends on the difference between the fraction of profits received as dividends and the capital share of the total production (or, equivalently, the elasticity of total production with respect to capital stock). This result provides one testable prediction. That is, given that the top capital income share has remained stable in the past several decades (see Figure 1.1), we can expect that the average dividend share of profits roughly equals the capital share of the total production.

In short, in this dissertation, we show that the strict subconvexity of demand functions can explain why top earned income increases relative to top capital income and workers' wages, particularly when productivity and capital stock are sufficiently high. Concerning inequality among the working rich, however, the strict subconvexity is not the whole story. The change in inequality among the working rich depends on the sources of firm heterogeneity. Concretely, we show that even when the demand function is strictly subconvex, income inequality falls when productivity and capital stock are sufficiently high, if productivity is the source of firm heterogeneity (but rises if demand is the source). The reason is simply that, under productivity heterogeneity, an increase in either productivity or capital stock lowers marginal costs of the least productivity firms disproportionally (provided that all firms have full access to new technology).

We show further that rising inequality among the working rich requires a low new entry. This is because, by raising the demand for and hence the price of capital, new entry, ceteris paribus, leads to a higher average marginal cost, which disproportionately affects top firms. This result is consistent with the fact that incumbent firms in the United States, for example, are aging while young firms—which are very productive (say because they adopt new technologies) and play an important role in creative destruction—are less prevalent and more likely to fail [see, among others, Hathaway and Litan [2014] and Millar and Sutherland [2016]].

5.2 Future Research

Although our models present many interesting results, they have some shortcomings that should be addressed in future work. First, our main results are based mainly on monopolistic competition. As shown in Chapter 4, monopolistic competition underestimates the increase in top earned income. This result raises the question of whether our model in Chapter 3 overestimates the importance of the strict subconvexity of demand functions. Answering this question, unfortunately, involves technical difficulty (recall the discussion in Section 4.4) and hence is left to future work.

Second, the increase in income inequality in both Chapters 3 and 4 involves a large fall in bottom income share (see Figure 4.1). As shown in Figure 1.4, in contrast, the largest fall is in the middle. One conceivable way to overcome this shortcoming is to allow for spending on, say, R&D and advertisement to raise productivity and appeal, respectively. If top firms increase their spending on advertisement, for example, they will benefit at the cost of bottom firms (i.e., those arguably in the middle of the income distribution), because the increase in demand for their goods may raise the price of inputs and hence the average marginal cost. At the same time, the bottom income earners might also benefit because of increases in demand for inputs.

Finally, although we mention economic development, our models are static models. We simply consider the effect of an exogenous increase in capital stock, rather than endogenous capital accumulation. Hence, it is not possible to discuss the implication of say the strict subconvexity on wealth inequality.

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