An Empirical Analysis of Relationship between Energy market and BRICS’s Currencies

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神戸大学大学院経済学研究科

経済学　専攻

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An Empirical Analysis of Relationship between Energy market and BRICS's Currencies
（エネルギー市場と BRICS の為替市場の関係に関する実証分析）

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Introduction

In this paper, we employ multiple methods (copula model, connectedness method and vine-copula model) to investigate the relationship between energy market (oil price and natural gas price) and BRICS's exchange rates.

With the development of emerging economies, developing economies have also become major players under intense international competition. In the meantime, BRICS, which is the association of five emerging economies (Brazil, Russia, India, China, and South Africa) is growing rapidly and these economies are likely to be a new center of gravity in the global economic system. On the other hand, energy is a key factor for development. Based on the BP Statistical Review of World Energy (2019), consumption of primary energy (commercially-traded fuels, including modern renewables used to generate electricity) in BRICS countries increased from 3750.2 million tons oil equivalent (Mtoe) in 2008 to 5222.5 Mtoe in 2018, an increase of 39.2% over the past decade, accounting for 37.7% of total global primary energy consumption. As the trade of energy resource is usually settled in US dollars, it is reasonable to assume that there is a relationship between the exchange rates of BRICS countries and energy markets.

To this end, we choose crude oil and natural gas as representatives of energy markets. Compared with other energy resources, crude oil is plentiful and widely used, it can be refined into many other forms as a power source. On the other hand, as a more environmentally friendly energy resource than other fuels such as coal, cross-regional trade of natural gas became viable thanks to the expansion of transmission pipeline networks and development of liquefied natural gas (LNG) storage. Besides, crude oil and natural gas are also two important energy sources in BRICS countries.

In Chapter 1, We study the dependence structure between West Texas Intermediate (WTI) oil prices and the exchange rates of BRICS countries from 4 October 2010 to 11 December 2018. We use the Normal, Plackett, rotated-Gumbel, and Student's t copulas to measure the constant dependence, and we capture the dynamic dependence using the Generalized Autoregressive Score (GAS) with the Student's t copula. We find a significant dependence existed in all exchange rate-oil price pairs, and the dependence is negative, which means that oil serves as a hedge against rising inflation in BRICS countries. The Russian Ruble (RUB)–WTI pair has the strongest dependence in both the constant and time-varying copula models, possibly because Russia is a large oil producer and has a high share of global oil exports, making Russia itself more dependent on oil. Moreover, we treat five exchange rate–oil pairs as portfolios and evaluate the Value at Risk and Expected Shortfall from the time-varying copula models. We find that both reach low values when the oil price falls sharply.

In Chapter 2, we investigate the connectedness between Henry Hub natural gas and BRICS's exchange rate in terms of time and frequency. This empirical work is based on the approach of connectedness proposed by Diebold and Yilmaz, who provided an effective way of valuing how much variation in one variable is responsible for the value of other variables, and the method proposed by Baruník and Křehlík, who decomposed the results from Diebold and Yilmaz into different frequencies. We collect data from 23 August 2010 to 20 June 2019 and test both return series and volatilities from GARCH models. The empirical results show that the total connectedness was modest, which means that most variation was due to the variation in the variables themselves. By taking the frequency decomposition of connectedness, we find that, in the return series, the short term contributes to the total connectedness the most, whereas the long term contributes most in relation to volatility. From the results of net pairwise connectedness between the natural gas price and exchange rates, we obtain a value of almost zero in each natural gas and exchange rate pair, which means that natural gas does not play an important role in explaining movements in the exchange rates. We also use the rolling-window method to conduct time-varying analysis. The results are similar to those of the constant analysis and we cannot say for certain that the natural gas price has a great influence on exchange rate movement.

In Chapter 3, we examine the role of BRICS's currency in energy market by using vine copula method. To conduct our analysis, we choose BRICS's exchange rates for the currency market, and WTI crude oil and Henry Hub natural gas future prices for the energy market. BRENT crude oil and NBP natural gas future prices are used to check the robustness of our research. The data period is from August 24, 2010 to November 29, 2019. The results of vine copula show that a negative dependence structure exists in most crude oil-exchange rate pairs while the results are the opposite in natural gas-exchange rate pairs. In each pair, the strength of dependence is weak except pairs with the ruble, which is considered to be caused by the Russian economy's high reliance on the energy market. Tail dependence based on vine copula suggests that there are almost no co-movements between energy price and exchange rate in extreme events. The empirical results provide evidence that BRICS's currencies can provide a hedge to crude oil movement but not to natural gas movement. BRICS's currencies can serve as a safe haven asset for both crude oil and natural gas and reduce the investment risk when faced with a market crash. In addition, two commonly used risk measurements, Value at Risk (VaR) and Expected Shortfall (ES) of two portfolios, are calculated. One consists of only crude oil and natural gas prices, which is the benchmark portfolio, the other is composed of two energy prices and BRICS's exchange rates. The results show adding BRICS's exchange rates into the benchmark portfolio is highly effective in terms of risk reduction, which confirms the conclusion from vine copula estimation.

Chapter 1

Conditional Dependence between Oil Prices and Exchange Rates in BRICS Countries: An Application of the Copula-GARCH Model

1.1 Introduction

Crude oil, as a non-renewable resource and a crucial commodity, plays a significant role in the world's economy. Over the past few decades, the crude oil price has experienced great fluctuations compared to the period from the Second World War to the early 1970s. For example, in June 2014, the West Texas Intermediate (WTI) crude oil price reached a peak of 105 US dollars per barrel, and then fell sharply to 59 US dollars per barrel in December 2014. In the early 1980s, many studies pointed out that oil price dynamics influenced economic activity (Hamilton 1983; Gisser and Goodwin 1986; Mork 1989). On the other hand, currency markets have also suffered multiple crises in recent years, such as the Latin American currency crisis of 1994, the Asian financial crisis of 1997, and most recently the fluctuation in the exchange rate between the US dollar and Chinese Yuan after a trade war, as each country continued to dispute tariffs in 2018. With these notable events, every abnormal foreign exchange movement had a huge impact on the economy. Analyzing the relationship between crude oil prices and exchange rates yields considerable information about the increasing importance of oil and currency markets in economics for market operators, investors, and economists.

On the theoretical side, the extant literature (Bénassy-Quéré et al. 2007; Bodenstein et al. 2011; Habib et al. 2016; Beckmann et al. 2017) has pointed out that there are three channels between oil price shocks and exchange rates: the terms of trade channel, wealth effects channel, and portfolio reallocation channel. The terms of trade channel considers a simple two-country open economy static model, which has two goods sectors—a traded goods sector and a non-traded goods sector. When there is a positive term of trade shock, the price of a basket of traded goods or non-traded goods in the domestic economy is relatively higher than it is in a foreign economy, leading to the appreciation of domestic currency. Hence, when oil prices rise, the currency of an energy-dependent country will appreciate, and vice versa. The wealth effects channel reflects the short-term effects while the portfolio reallocation channel reflects the medium- and long-term effects. When there is a positive oil price shock, the wealth transfers from the oil-importing country to the oil-exporting country, leading to large shifts in current account balances and portfolio reallocation. For this purpose, in order to adjust to clear the trade balance and asset markets, the currency of the oil-exporting country appreciates, and the currency of oil-importing country depreciates. Chen and Chen (2007) provide a simple model to illustrate the above theory:

$$\boldsymbol{p = \alpha p^{T} + (1 + \alpha) p^{N}} \tag{1}$$

$$\boldsymbol{p^{*} = \alpha^{*} p^{T*} + (1 + \alpha^{*}) p^{N*}} \tag{2}$$

where p (p^*) is the log-linear approximation of the domestic (foreign) consumer price index, and p^T (p^{T*}) and p^N (p^{N*}) are the prices of the traded and non-traded sectors in the home country (foreign country), respectively. The α (α*) weights correspond to expenditure shares on traded goods near the point of approximation for the domestic (foreign) country. The log of the real exchange rate q is defined as

$$\boldsymbol{q = s + p - p^{*}} \tag{3}$$

where s is the log of the nominal exchange rate. Therefore, the real exchange rate could be written as

$$\boldsymbol{q = \left(s + p^{T} - p^{T*}\right) + (1 - \alpha)\left(p^{N} - p^{T}\right) - (1 - \alpha^{*})(p^{N*} - p^{T})} \tag{4}$$

If $\alpha \simeq \alpha^*$, when the oil price rises, the price of the traded sector in the country which is more dependent on oil importation will increase, and thereby cause a depreciation (Chen and Chen 2007).

On the empirical side, there are also many studies that investigate the relationship between oil prices and exchange rates. Some of this literature finds a negative relationship. Akram (2009) uses quarterly data from 1990Q1 to 2007Q4, and a structural value at risk (VaR) model to analyze whether a decline in the US dollar contributes to higher commodity prices. Their results suggest that a weaker dollar leads to higher commodity prices, including oil prices. Basher et al. (2012) study the dynamic relationship between oil prices, exchange rates, and emerging market stock prices by modeling daily data from 1988 to 2008 using a structural VaR model. They find that positive shocks to oil prices lead to an immediate drop in the US dollar exchange rate, which has a statistically significant impact in the short-run. Reboredo and Rivera-Castro (2013) use a wavelet decomposition approach to investigate the relationship between oil prices (WTI) and US dollar exchange rates for a large set of currencies, including developed and emerging economies, net oil-exporting and oil-importing economies, and inflation-targeting countries. They find that oil price changes had no effect on exchange rates or vice versa in the pre-crisis period, but after the 2008 global financial crisis, they find a negative interdependence. By contrast, other research finds a positive relationship between oil prices and exchange rates. Bénassy-Quéré et al. (2007) study co-integration and causality between the real price of oil and the real value of the dollar from 1974 to 2005 using monthly data. Their results suggest that a 10% rise in the oil price coincides with a 4.3% appreciation of the dollar in the long run, and the causality runs from oil to the dollar. Ding and Vo (2012) use the multivariate stochastic volatility and the multivariate generalized autoregressive conditional heteroscedasticity (GARCH) models to investigate the volatility interactions between the oil market and the foreign exchange market. They divided the data into two parts to capture the structural breaks in the economic crisis, and they show that before the 2008 crisis, the two markets responded to shocks simultaneously and no interaction is found. However, during the financial crisis, they find a positive bidirectional volatility interaction between the two variables.

In addition to research in developed markets, other studies focus on this relationship in emerging markets. For example, Narayan et al. (2008) estimate the impact of oil prices on the nominal exchange rate on Fiji Island with GARCH and Exponential GARCH (EGARCH) models. They find that a rise in oil prices leads to an appreciation of the Fijian dollar. Narayan (2013) uses exchange rate data from 14 Asian countries and demonstrates that a higher oil price leads to a future depreciation of the Vietnamese Dong, but a future appreciation of the local currencies of Bangladesh, Cambodia, and Hong Kong.

Analysis of the correlation between the currency and oil markets also provides abundant information about whether the oil price could act as a hedge or as a safe haven against exchange rate risk. Much literature investigates the relationship between different assets to examine whether these variables act as hedges against each other. The methods used in these papers are varied, such as linear regression, quantile regression, etc. Considering that the characteristics of skew and leptokurtic kurtosis often have asymmetric behaviors, and the oil price and exchange rates seem unsuitable for modeling with linear models, in this study, we propose a copula model to investigate the dependency between oil prices and exchange rates. Compared to traditional methods, a copula model has several advantages. First, it can provide a valid joint distribution in combination with any univariate distribution, which makes it more flexible in modeling multivariate distributions. Second, it can capture a wide range of dependence structures, such as asymmetric, nonlinear, and tail dependence in extreme events. Prior studies apply copula models to examine the relationship between oil prices and exchanges rates (Patton 2006; Wu et al. 2012; Aloui et al. 2013; Ji et al. 2019). However, most of them focus on developed markets.

In this study, we investigate the dependence structure between the oil and currency markets. Our contribution to the literature is twofold. First, we propose both constant and time-varying copula-GARCH-based models to measure the dependence between the two markets elastically. By using a copula model, we can study the co-movement between oil prices and exchange rates during bearish and bullish markets, and we can also examine whether oil acts as a hedge against exchange rate risk in BRICS countries. Second, we focus on exchange rate data from the BRICS countries, which is a group of five developing countries (Brazil, Russia, India, China, and South Africa), to determine whether the dependence structure in emerging markets is the same as that in developed countries.

The rest of this paper proceeds as follows. In Section 2, we introduce the empirical methodology and estimation strategy. Section 3 presents the data and discusses the empirical results. In the final section, we present our conclusions. The results of ARMA-GARCH model and the robustness analysis are presented in Appendix A and B, respectively.

1.2 Empirical Methodology

Sklar (1959) introduced the notion of the copula, in which an n-dimensional distribution function which can be decomposed into two parts, namely, the marginal distribution and the copula.

We denote $X \equiv [X_1,\ldots,X_n]'$ as the variable set of interest, and X_i follows the marginal distribution F_i for each $i \in [0,n]$.

$$F_i(x_i) = P(X_i \leq x_i) \tag{5}$$

Following Sklar's theorem, there exists a copula $C:[0,1]^n \rightarrow [0,1]$ with uniform marginals that can map the univariate marginal distribution F_i to the multivariate distribution function F.

$$\boldsymbol{F}(x_1, \dots x_n) = C\big(F_1(x_1), \dots, F_n(x_n)\big) \tag{6}$$

The density of the multivariate distribution function **F** is

$$f(x_1, \dots x_n) = c\big(F_1(x_1), \dots, F_n(x_n)\big) \times f_1(x_1) \times \dots \times f_n(x_n) \tag{7}$$

We first obtain standardized residuals using an autoregressive moving-average GARCH (ARMA-GARCH) model for the daily returns of crude oil prices and exchange rates. Then, we transform the standardized residuals into a uniform distribution. Finally, we input the transformed uniform variates into the copula function to model the dependence structure.

1.2.1 ARMA-GARCH Model

To model the daily returns of crude oil prices and exchange rates, we use autoregressive-moving-average (ARMA) models with two kinds of GARCH models––the standard GARCH (sGARCH) model and the Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) model. In both GARCH models, we assume that the standard errors from the GARCH models follow a skewed t distribution.

We denote r_t as the daily returns, which follow

$$r_t = \phi_0 + \sum\nolimits_{i=1}^{p} \phi_{i,1} r_{t-i} + \sum\nolimits_{j=1}^{q} \phi_{j,2} \varepsilon_{t-j} + \varepsilon_t \tag{8}$$

where ϕ_0 is a constant and ε_t is a white noise error item. Table 1 presents the functions for the two GARCH models.

1.2.2 Marginal Distribution

Following Patton (2013), we consider both parametric and non-parametric models in modeling the marginal distribution.

For the parametric estimation, we choose the skewed t distribution as in Hansen (1994). The density function is

$$f(\varepsilon) = \begin{cases} bc(1 + \dfrac{1}{\nu - 2}(\dfrac{b\varepsilon + a}{1 - \lambda})^2)^{-(\nu+1)/2} & \varepsilon < -a/b \\ bc(1 + \dfrac{1}{\nu - 2}(\dfrac{b\varepsilon + a}{1 + \lambda})^2)^{-(\nu+1)/2} & \varepsilon \geq -a/b \end{cases} \tag{9}$$

$$a = 4\lambda c \left(\frac{\nu - 2}{\nu - 1}\right)$$

$$b^2 = 1 + 3\lambda^2 - a^2$$

$$c = \frac{\Gamma(\frac{\nu+1}{2})}{\sqrt{\pi(\nu-2)}\Gamma(\frac{\nu}{2})}$$

The skewed t distribution has two parameters to control its shape: a skewness parameter $\lambda \in (-1,1)$, which determines whether the mode of the density is skewed to the left or to the right, and a degrees of freedom parameter $v \in (2,\infty)$, which controls the kurtosis of the distribution. When $\lambda=0$, we obtain the Student's t distribution; when $v \to \infty$, we obtain the skewed normal distribution; and when $\lambda=0$ with $v \to \infty$, we obtain the standard normal distribution. For this property, the skewed t distribution has more flexibility in fitting the data than do the normal distribution or the Student's t distribution.

For the non-parametric estimation, we use the empirical distribution function (EDF):

$$\hat{F}_i(\varepsilon) \equiv \frac{1}{T+1}\sum_{t=1}^{T} 1\{\hat{\varepsilon}_{it} \leq \varepsilon\} \tag{10}$$

where T is the length of the data and $\hat{\varepsilon}_{it}$ indicates the standardized residuals from the GARCH model.

1.2.3 Copula Model

1.2.3.1 Constant Copula Model

There are three families of copula model: elliptical copulas (Normal and Student's t); Archimedean copulas (Gumbel, Clayton, and Frank); and quadratic copulas (Plackett). We used Normal, Clayton, rotated-Clayton, Plackett, Frank, Gumbel, rotated-Gumbel, SJC and Student's t copula models in our empirical analysis at first, but the log-likelihoods in the Clayton, Rotated-Clayton, Frank, Gumbel, rotated-Gumbel and SJC copulas are negative, which means that the dependence coefficients in these copula models are quiet low, so we only present the results of the Normal, Plackett, and Student's t copula models, which have a positive log-likelihood. The rotated-Gumbel copula has the lowest log-likelihood in most currency–oil price pairs, so we also present it in the paper for comparison.

The normal and Student's t copulas are based on an elliptical distribution, such as the normal or Student's t distribution. The normal copula is symmetric and has no tail dependence. $\theta>0$ and $\theta<0$ lead to positive and negative dependence, respectively. The Student's t copula is also symmetric, but can capture tail dependence in extreme events. When $\theta_1=0$ with $\theta_2\to\infty$, then it becomes independent between the two variables. Like the Normal copula, the Plackett copula is also symmetric and cannot capture either lower or upper tail dependence. $0<\theta<1$ implies negative dependence and $\theta>1$ implies positive dependence. The Gumbel copula can only capture extreme lower dependence, $\theta=1$ and $\theta\to\infty$ indicate independence and perfectly negative dependence, respectively.

Table 2 summarizes the properties of the abovementioned bivariate copula models.

1.2.3.2 Time-Varying Copula Model

In this study, we examined the time-varying Student's t copula using a Generalized Autoregressive Score (GAS) model. Creal et al. (2013) proposed the GAS model, which provides a general framework for modeling time variation. We consider δ_t as the parameter vector of the time-varying copula models, which we can express as the following function:

$$\delta_t = h(f_t) \tag{11}$$

where f_t is the time-varying parameter vector in the GAS model. Following Creal et al. (2013), we can assume that f_t is given by the familiar autoregressive updating equation:

$$\begin{gathered} f_{t+1} = \omega + \alpha S_{t-1} \cdot \nabla_t + \beta f_t \\ \nabla_t = \frac{\partial lnc(\boldsymbol{u_t}; f_t)}{\partial f_{t-1}} \\ f_t = h^{-1}(\delta_t) \end{gathered} \tag{12}$$

where ω is a constant, S_{t-1} is a time-dependent scaling matrix, $c(\cdot)$ is the density function of the copula model, and $\mathbf{u}_t$ is the vector of the probability integral transforms using the univariate marginals. According to Creal et al. (2013), we can set the scaling matrix S_{t-1} to be equal to the pseudo-inverse information matrix:

$$S_{t-1} = I_{t-1}^{-1} = E_{t-1}[\nabla_t \nabla_t']^{-1} = -E_{t-1}\left[\frac{\partial^2 lnc(\boldsymbol{u_t}; f_t)}{\partial f_{t-1} \partial f_{t-1}'}\right]^{-1} \tag{13}$$

For the time-varying Student's t copula, we use the function $\delta_t=[1-\exp(-f_t)]/[1+\exp(-f_t)]$ to ensure that the parameter lies within $(-1,1)$.

1.3 Empirical Analysis

1.3.1 Data and Summary Statistics

Our dataset contained the WTI oil prices in dollars and the nominal dollar-denominated exchange rates for the Brazilian Real (BRL), Russian Ruble (RUB), Indian Rupee (INR), Chinese Yuan (CNY), and South African Rand (ZAR). After 2000, China reformed its exchange rate regime twice. The first time was in 2005, when the authorities adopted a managed floating exchange rate system with reference to a basket of currencies, and the second time was in 2010, when the CNY shifted from a bilateral to a multilateral reference, with greater emphasis on the currency basket. Before each reform, CNY fluctuated with no change or within a narrow range. We thus chose the offshore exchange rate for CNY for our analysis. Our sample period was from 4 October 2010 to 11 December 2018, to match the data availability for the offshore CNY exchange rate. We downloaded all data from the DataStream database.

We obtained the stationary return series using the following function:

$$r_{i,t} = 100 \times ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right) \tag{14}$$

Since many events occurred during our sample period, such as China's central bank devaluation of its currency in 2015, the United Kingdom's vote to leave the European Union in 2016, etc., which may be directly or indirectly relevant to oil prices and exchange rates, we applied a methodology from Bai and Perron (1998, 2003) for testing the structural breaks, to avert unexpected changes in our dataset. The p-values of structural break tests on the return series are summarized in Table 3. The results showed an acceptance of the null hypothesis that there are no breaks, therefore we were able to use this sample period for further analysis.

Figure 1 shows the plot of the time-path of the return series, and Table 4 reports the descriptive statistics for daily returns. The mean of CNY is close to zero, and its standard deviation is the lowest of all return series. Thus, in some way, CNY remained stable under the government regulations. RUB has the most fluctuations among the five exchange rates, especially at the end of 2014, when the price of crude oil declined by nearly 50%, which was a major cause of the financial crisis in Russia. All return series are skewed and have high kurtosis, indicating that the distributions of the return series show obvious non-normality characteristics. The Jarque-Bera test performed on the return series also verifies our analysis.

Before beginning the copula analysis, we provide the results of the rank correlation and linear correlation in Table 5 for comparison. All coefficients are significant, indicating negative relationships between all currency–oil pairs. The RUB-WTI pair has the highest coefficient for both the rank and linear correlations, possibly because Russia is the second largest oil-exporting country in the world, and Russia exports more oil than the other countries in our sample.

1.3.2 Constant Copula Result

Tables 6-10 report the estimations of the four constant copulas (Normal, Plackett, rotated-Gumbel, and Student's t) for five currency–oil price pairs (BRL-WTI, RUB-WTI, INR-WTI, CNY-WTI, and ZAR-WTI), respectively. The parameters from the copula model are estimated based on the maximum likelihood method. In all pairs, the Student's t copula has the highest value of log-likelihood, while the rotated-Gumbel copula has the lowest. Almost all estimated copula parameters are significant at the 1% level. The results of the copula models reveal a significant dependence between the oil price and exchange rates. All copula models suggest a negative dependence among all considered pairs, for both the parametric and semi-parametric cases, which could be because oil is considered as a hedge against inflation in BRICS countries. For oil-importing countries, like China, India, and South Africa, the negative dependence is in line with the theoretical method, which illustrates that the currency of an oil-importing country will depreciate when oil price goes up. This result is also in accordance with other studies, such as that of Ghosh (2011), who assesses the relationship between the Indian Rupee and oil price and finds the existence of a negative correlation. Kin and Courage (2014) find that an increase in oil prices leads to a depreciation of the South African rank exchange rate. However, for oil-exporting countries such as Russia and Brazil, the result is the opposite with the theoretical model. We still found some studies which have the same results as those found in our analysis, such as Blokhina et al. (2016), who find a negative relationship by applying a linear regression to Russia. In each copula model, the RUB-WTI pair has the highest dependence coefficient, followed by ZAR-WTI, BRL-WTI, CNY-WTI, and INR-WTI, which is consistent with the rank and linear correlation results.

According to Genest et al. (2009), the Cramer–von Mises (CvM) test is the most powerful test to check the goodness-of-fit of copula models. Here, we used both the

CvM test and the Kolmogorov-Smirnov (KS) test to check the quality of the data fit. We defined the CvM and KS tests for the bivariate case as:

$$\widehat{\boldsymbol{C}}(\boldsymbol{u}) \equiv \frac{1}{T}\sum_{t=1}^{T}\prod_{j=1}^{n}\mathbf{1}\{\widehat{U}_{it} \leq u_j\}\ \forall i = 1,2$$

$$CvM = \sum_{t=1}^{T}\{C(U_{it};\theta) - \hat{C}(U_{it})\}^2\ \forall i = 1,2 \tag{15}$$

$$KS = \max_t |C(U_{it};\theta) - \hat{C}(U_{it})|\ \forall i = 1,2 \tag{16}$$

Table 11 presents the p-values of the goodness-of-fit test. For all pairs, the rotated-Gumbel copula is rejected at the 1% level of significance, except for in the parametric case of INR-WTI. The Student's t copula generates the highest p-value in most cases, indicating that it provides the best fit for the data.

Tail dependence describes co-movements between two variables during extreme events. In our case, lower tail dependence measures the probability of currency depreciation when the oil price falls. In contrast, upper tail dependence measures the probability of currency appreciation when the oil price rises. In Table 12, we present the tail dependence coefficients from the constant Student's t copula model, which can capture symmetrical tail dependence. The results show significant extreme market co-movements between the exchange rate and oil price. INR-WTI has the highest tail dependence, while RUB-WTI has the lowest for both the parametric and semi-parametric cases.

1.3.3 Time-Varying Copula Result

Table 13 presents the estimated time-varying Student's t copula model using the GAS model. For all pairs, the $\hat{\beta}$ estimate is highly significant and has a large value, except for the INR-WTI pair, indicating that the time-varying copula model parameters in this period have a great impact on the copula parameters of the following period.

Figure 2 presents the plot of the dynamic estimated parameters from time-varying Student's t copula in the semi-parametric case. The red line indicates the average of the estimated time-varying parameters. As in the results from the constant copula models, all pairs reveal negative dependence structures in the following order: RUB-WTI, ZAR-WTI, BRL-WTI, CNY-WTI, and INR-WTI. This result is in line with our results from constant copula models.

Figure 3 reports the plot of the conditional tail dependence with time deviation in the semi-parametric case. For clarity, we used different scales for the y axis for the different pairs. In general, for all pairs, the tail dependence from the time-varying copula, which is less than 0.0025, is much lower than that of the constant copula models (about 0.4).

Following Patton (2013), we tested the goodness-of-fit of the time-varying copula model by transforming the data through the Rosenblatt method, which is a multivariate version of the probability integral transformation proposed in Diebold et al. (1999) and developed further by Rémillard (2017). The transform uses the function:

$$\begin{aligned} V_{1t} &= U_{1t} \ \forall t \\ V_{2t} &= C_{2|1,t}(U_{2t}|U_{1t};\theta) \end{aligned} \tag{17}$$

Table 14 reports the p-values of the goodness-of-fit test. In the parametric case, the time-varying Student's t copula model is accepted with high p-values, except for INR-WTI pair with the KS test. In the semi-parametric case, the time-varying Student's t copula model is rejected for the BRL-WTI, INR-WTI, and ZAR-WTI pairs with the CvM test. Based on the results from the goodness-of-fit test, we consider that the constant copula model provides a better fit than the time-varying copula model for the INR-WTI pair.

1.3.4 Value at Risk and Excepted Shortfall

VaR and expected shortfall (ES) are statistics that quantify the risk of a portfolio. VaR measures the maximum expected loss with a confidence level, and ES measures the expected loss when the portfolio return is greater than the value of the VaR calculated with that confidence level. In this section, we treat the five exchange rate-oil price pairs (BRL-WTI, RUB-WTI, INR-WTI, CNY-WTI, and ZAR-WTI) as equally weighted portfolios, and then we calculate dynamic VaR and ES using a Monte Carlo simulation. The procedure has the following steps. Step 1: at each time t (t is from 1 to T, where T is the length of the return series), generate 5000 random trials of uniform variables using the parameters estimated from the time-varying Student's t copula model. Step 2: transform the random uniform variables into standardized residuals, then conduct a series of new asset returns using the estimated parameters from the GARCH models. Step 3: use the portfolio weights (1% in this study) and the new asset returns to calculate the portfolio returns and then calculate the VaR and ES. Step 4: repeat steps 1-3 T times.

Figure 4 plots the results. We can see that the VaR begins at 3%, decreases to 6% at the beginning of 2015 (9% for RUB–WTI), and from the middle of 2015 it decreases to reach approximately 8% by the beginning of 2016 for all pairs, due to the falling crude oil prices. Similarly, the ES also reaches a low value at that period.

1.4 Conclusions

This study examined both the constant and dynamic dependencies between crude oil prices (WTI) and exchange rates for the BRICS countries, which includes oil-exporting and oil-importing economies, from 4 October 2010 to 11 December 2018. We employed constant and time-varying copula-GARCH-based models to capture the nonlinear, asymmetric, and tail dependence between the two variables.

The empirical results showed, first, that a significant dependence existed in all exchange rate-oil price pairs, and the dependence is negative, which means that oil serves as a hedge against rising inflation in BRICS countries. For example, when one currency appreciates, consumers from that country will find the oil price less expensive and then increase their demand for oil, which will lead to a rise in the oil price. Second, in both the constant and time-varying copula models, the RUB-WTI pair has the strongest dependence, possibly because Russia is a large oil producer and has a high share of global oil exports, making Russia itself more dependent on oil. Third, from the goodness-of-fit test results, the Student's t copula model has the best performance among all the constant copula models considered, while for the time-varying copula models, it was rejected for the BRL-WTI, INR-WTI, and ZAR-WTI pairs. We also measured risk management by treating the five exchange rate-oil price pairs as portfolios and calculating the dynamic values of VaR and ES. Both reached very low values for all pairs when the oil price experienced a sharp decrease.

The results of this paper offer us several suggestions. First, the significant correlation between oil price and exchange rate indicates that enterprises and governments in BRICS countries, when the oil price fluctuates, should pay attention to the exchange rate risk. Second, the negative relationship between oil price and exchange rate shows that the oil could be a hedge against inflation in BRICS countries, which is useful for foreign investors to manage exchange rate risk.

However, in this paper, we only considered bivariate copula models. For the future extension of this research, we will focus on a high-dimensional copula model, which could provide a more flexible analysis.

Appendix A ARMA-GARCH Results

Table A1 summarizes the results of ARMA-GARCH models. We first use ARMA models to estimate the return series, and then use GARCH model to estimate the residuals from the ARMA models to obtain the standard residuals. We chose the ARMA model lags using the AIC criterion. All returns series are modeled by standard GARCH model at first, but the standard residuals in RUB and ZAR returns, could not pass the Ljung-Box test and Lagrange Multiplier test, which means the GARCH model is not fitted well in these returns. Hence, we chose GJR-GARCH model in fitting RUB and ZAR return data. As mentioned in the main text, each GARCH model tests the hypothesis that the error item follows a skewed t distribution. Almost all estimated parameters from the GARCH models are significant at the 1% level. Persistence coefficients ($\alpha+\beta$ in GARCH model, $\alpha+\beta+\gamma/2$ in GJR-GARCH model) in all models are very close to 1, which reveal the existence of volatility clustering. This appears show that when the volatility is high, it is likely to remain high and vice versa when the volatility is low. This consequence is also in line with previous research.

Table A2 provides the results of the Ljung-Box test of the standardized residuals and squared standard residuals (up to 20 lags), as well as the Lagrange Multiplier test of the standardized residuals. All values are insignificant, meaning that the standardized residuals obtained from the GARCH model have no serial correlation and autoregressive conditional heteroscedasticity.

Appendix B Robustness Analysis

To examine the robustness of our results, we used the Brent crude oil price to analyze five currency–oil price pairs: BRL-BRENT, RUB-BRENT, INR-BRENT, CNY-BRENT, and ZAR-BRENT. We modeled the dependence structure for exchange rates and oil prices.

We summarize the rank correlation and linear correlation results, and the tail dependence from the constant Student’s t copula model in Tables A3 and A4, respectively. We plot the estimated dynamic parameters and tail dependences in

Figures A1 and A2, respectively, and illustrate the results for the dynamic VaR and ES Figure A3.

All results are consistent with those for the exchange rate-WTI pairs, which confirm the suitability of our proposed approach to capture the dependence structure between exchange rates and crude oil prices.

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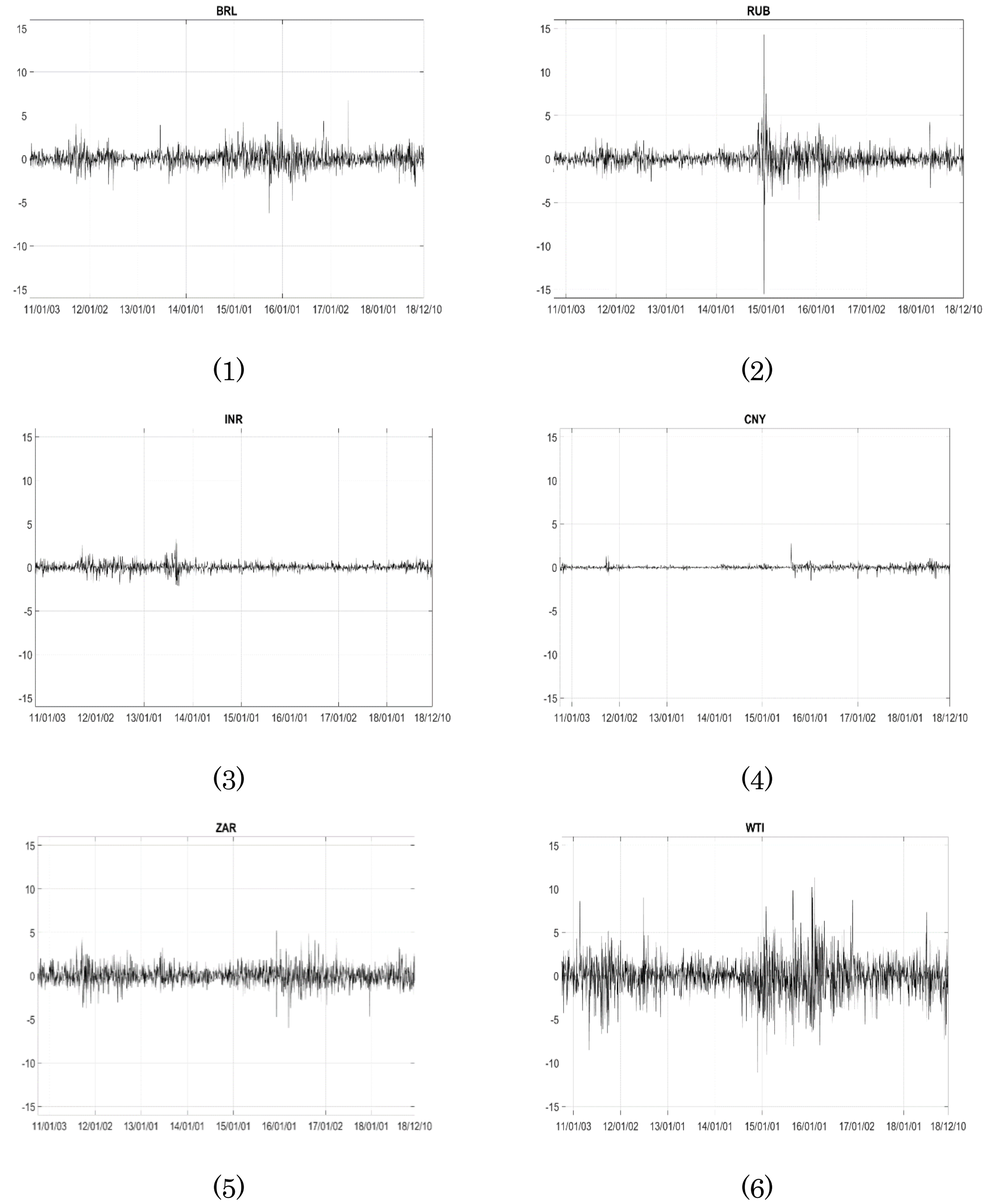
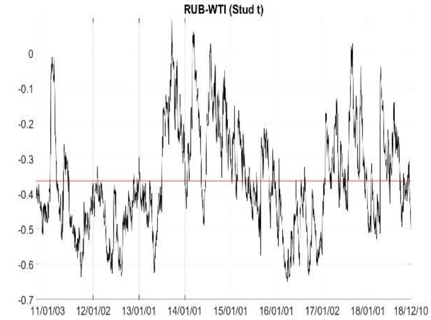


Figure 1. Daily returns on the exchange rates and crude oil prices. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. (1)-(6) refer to return series of BRL, RUB, INR, CNY, INR and WTI, respectively.



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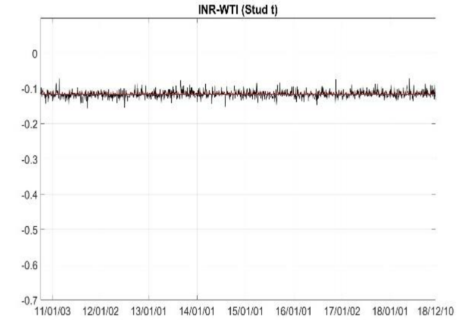

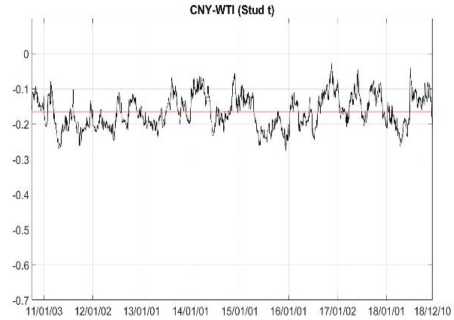

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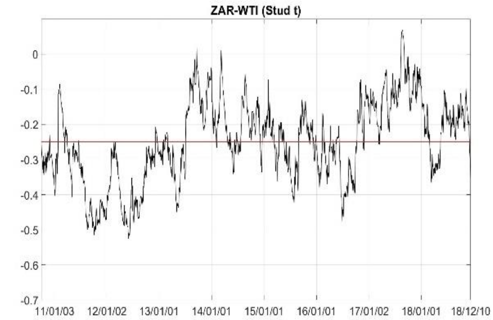

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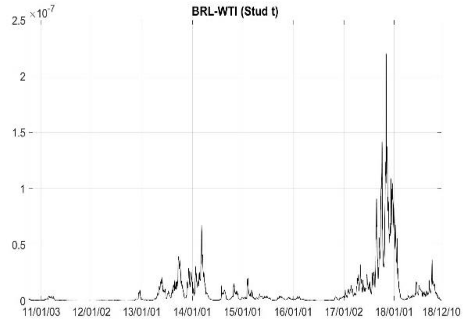
Figure 2. Dynamic parameters from the time-varying copula models. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. (1)-(5) refer to BRL-WTI, RUB-WTI, INR-WTI, CNY-WTI and ZAR-WTI pairs, respectively.



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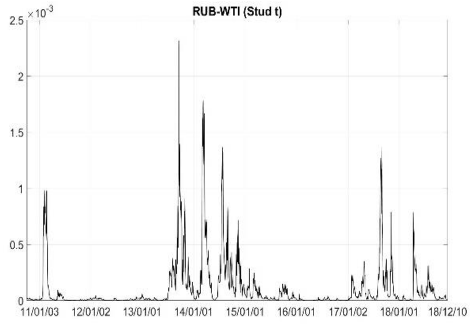

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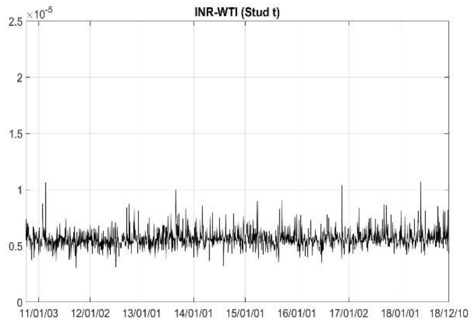

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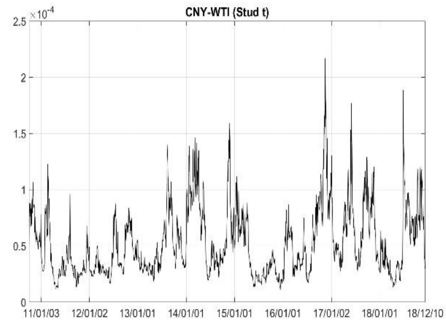

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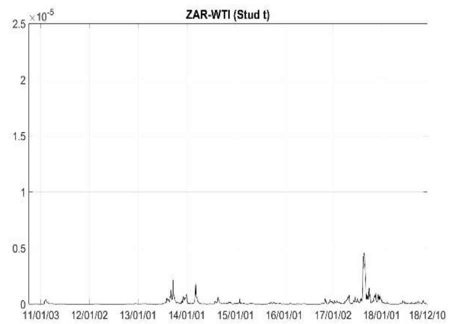

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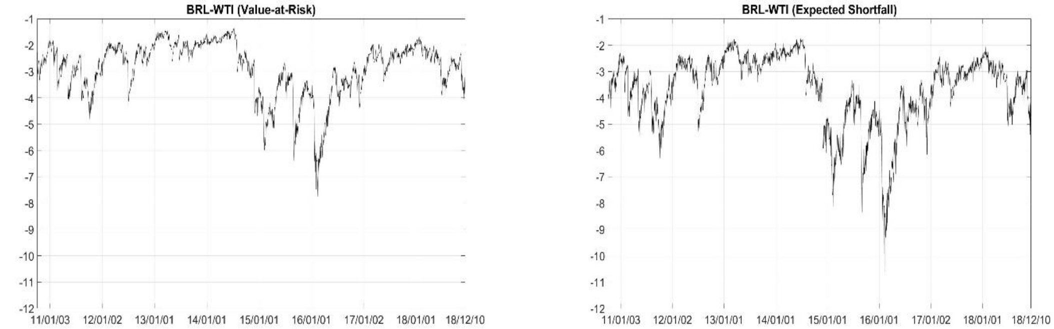
Figure 3. Conditional tail dependence from the time-varying copula models. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. (1)-(5) refer to BRL-WTI, RUB-WTI, INR-WTI, CNY-WTI and ZAR-WTI pairs, respectively.


BRL-WTI (Value-at-Risk)
BRL-WTI (Expected Shortfall)
-1 -2 -3 -4 -5 -6 -7 -8 -9 -10 -11 -12
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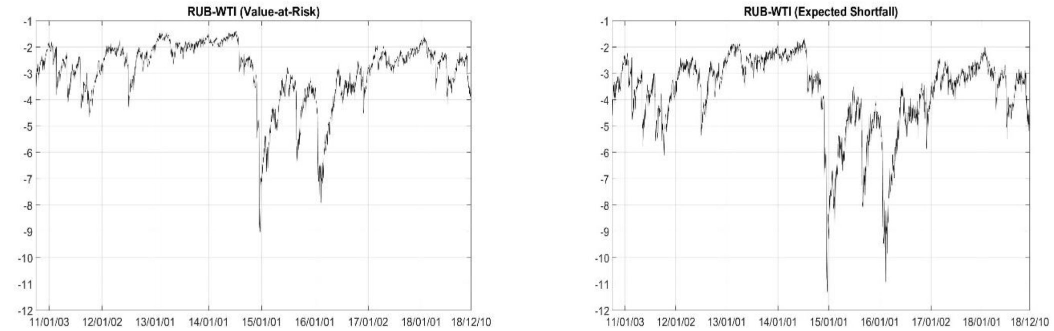
RUB-WTI (Value-at-Risk)
RUB-WTI (Expected Shortfall)
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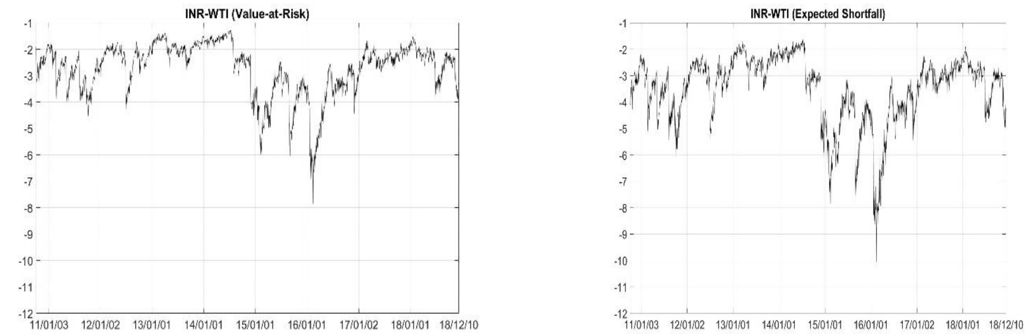
INR-WTI (Value-at-Risk)
INR-WTI (Expected Shortfall)
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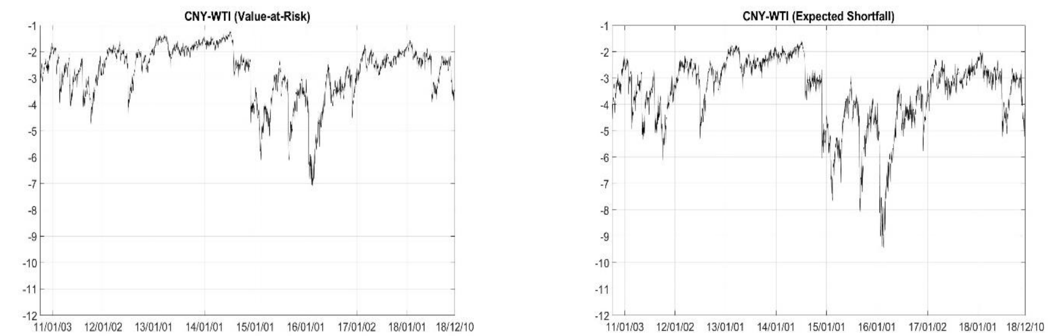
CNY-WTI (Value-at-Risk)
CNY-WTI (Expected Shortfall)
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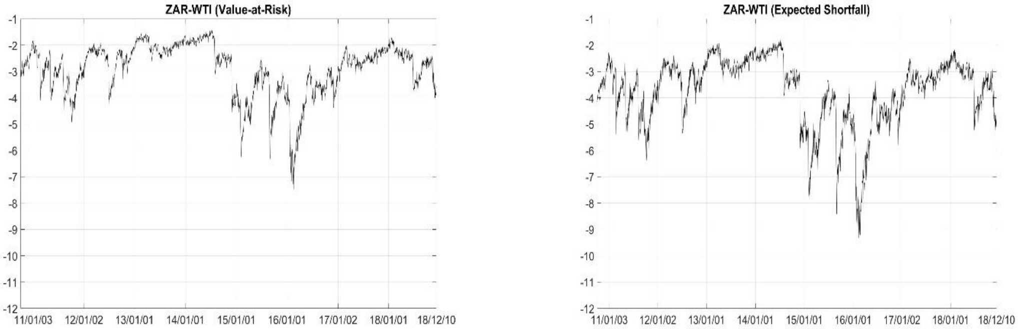

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Figure 4. Conditional 1% Value at Risk (left panel) and Expected Shortfall (right panel) for an equal-weighted portfolio based on the Student's t copula model. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. (1)-(5) refer to BRL-WTI, RUB-WTI, INR-WTI, CNY-WTI and ZAR-WTI pairs, respectively.

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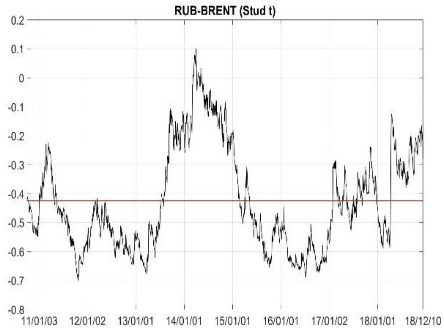

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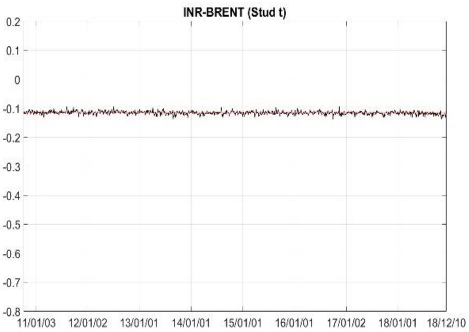

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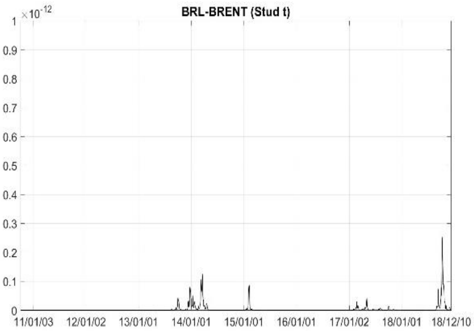
Figure A1. Dynamic parameters from the time-varying copula models. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; BRENT: Brent crude oil price. (1)-(5) refer to BRL-BRENT, RUB-BRENT, INR-BRENT, CNY-BRENT and ZAR-BRENT pairs, respectively.



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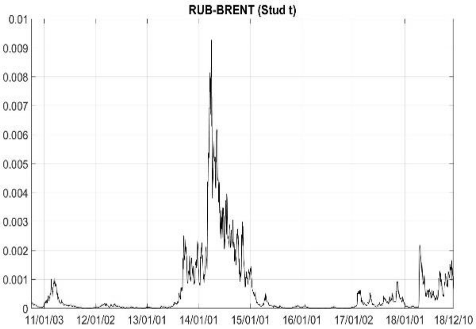

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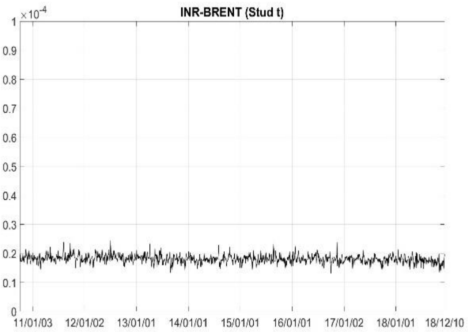

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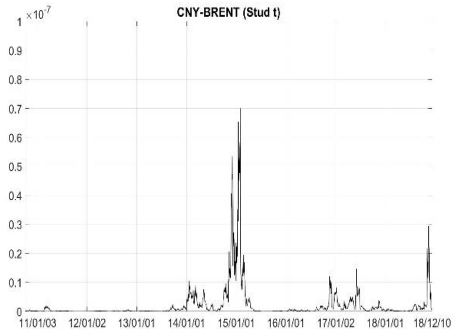

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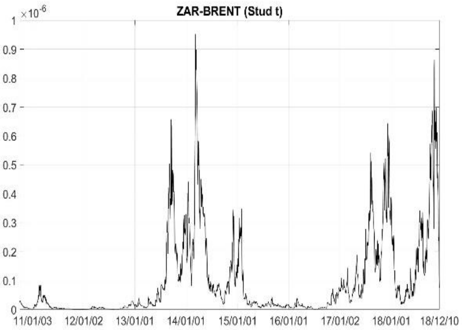

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Figure A2. Conditional tail dependence from the time-varying copula models. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; BRENT: Brent crude oil price. (1)-(5) refer to pair of BRL–BRENT, RUB–BRENT, INR–BRENT, CNY–BRENT and ZAR–BRENT pairs, respectively.

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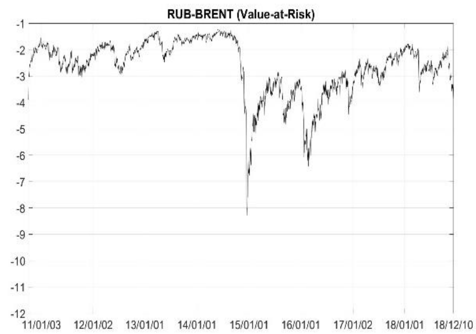

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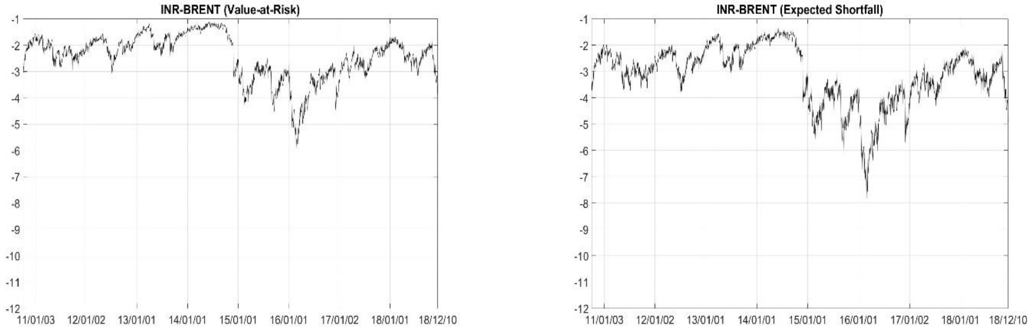

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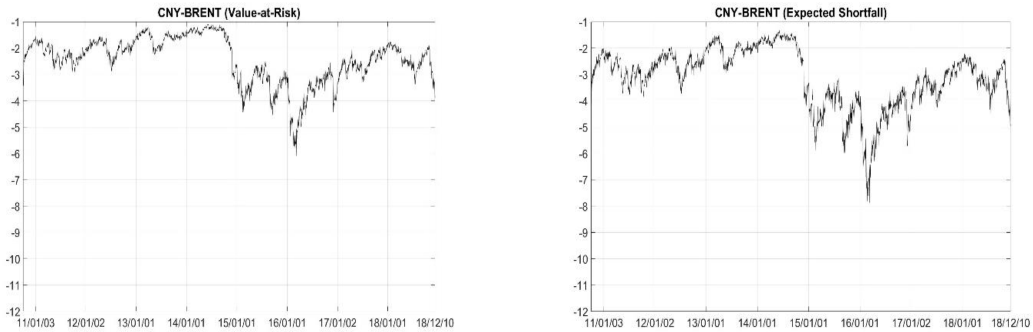

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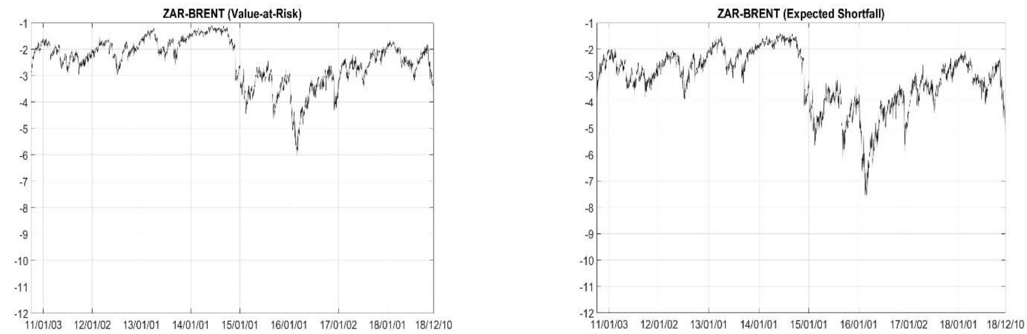

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Figure A3. Conditional 1% Value-at-Risk (left panel) and Expected Shortfall (right panel) for an equal-weighted portfolio based on the Student's t copula model. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; BRENT: Brent crude oil price. (1)-(5) refer to pair of BRL–BRENT, RUB–BRENT, INR–BRENT, CNY–BRENT and ZAR–BRENT pairs, respectively.

Table 1. Generalized autoregressive conditional heteroscedasticity (GARCH) model functions.

Type	Function
sGARCH	$\varepsilon_t = \sigma_t z_t$ $\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$
GJR-GARCH	$\varepsilon_t = \sigma_t z_t$ $\sigma_t^2 = \omega + (\alpha + \gamma I_{t-1})\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2$ $I_{t-1} = \begin{cases} 0 & \text{if } r_{t-1} \geq \mu \\ 1 & \text{if} r_{t-1} < \mu \end{cases}$

Notes: ε_t: a zero-mean white noise item, which is assumed to follow a skewed t distribution; μ: the expected return; z_t: an i.i.d. random variable with zero mean and a variance of one.

Table 3. P-value of Structural break tests.

Variable	p-value
BRL	0.2970
RUB	0.3960
INR	0.2790
CNY	0.1969
ZAR	0.4307
WTI	0.4737

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand.

Table 2. Bivariate copula models.

Type	Parameter(s)	Function	Lower Tail Dependence	Upper Tail Dependence
Normal	$\theta \in (-1,1)$	$\int_{-\infty}^{\Phi^{-1}(u_1)}\int_{-\infty}^{\Phi^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\theta^2}} \exp\left(-\frac{s^2-2\theta st+t^2}{2(1-\theta^2)}\right) dsdt$	0	0
Plackett	$\theta \in (0,\infty)$	$\frac{1+(\theta-1)(u_1+u_2)-\{[1+(\theta-1)(u_1+u_2)]^2-4u_1u_2\theta(\theta-1)\}^{1/2}}{2(\theta-1)}$	0	0
Rotated-Gumbel	$\theta \in (1,\infty)$	$\exp\{-[(-\ln u_1')^\theta + (-\ln u_2')^\theta]^{1/\theta}\}$	$2-2^{1/\theta}$	0
Student's t	$\rho(\theta_1) \in (-1,1)$ $\nu(\theta_2) \in (2,\infty)$	$\int_{-\infty}^{t^{-1}(u_1)}\int_{-\infty}^{t^{-1}(u_2)} \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left(-\frac{s^2-2\rho st+t^2}{2(1-u_2{}^2)}\right)^{-\frac{u_2+2}{2}} dsdt$	$2 \times F_t\left(-\sqrt{(u_2+1)\frac{\rho-1}{\rho+1}}, u_2\right)$	$2 \times F_t\left(-\sqrt{(u_2+1)\frac{\rho-1}{\rho+1}}, u_2\right)$

Notes: u_1, u_2: uniform variates. $u_1' = 1 - u_1$, $u_2' = 1 - u_2$; $\Phi^{-1}(\cdot)$: the inverse cumulative distribution function of the univariate standard normal distribution; $t^{-1}(\cdot)$: the inverse cumulative distribution function of the univariate Student's t distribution; $F_t(\cdot)$: the probability density function of the standard Student's t distribution. This table is based on Aloui et al. (2013), Patton (2013) and Nelsen (2006).

Table 4. Summary statistics.

Max	Min	Mean	Std Dev	Skewness	Kurtosis	JB-test
			CNY			
2.7613	−1.6089	0.0020	0.2306	0.4162	19.3577	23,875.7349 ***
			BRL			
6.7177	−6.2365	0.0394	0.9086	0.1419	7.5098	1817.2875 ***
			RUB			
14.2683	−15.5230	0.0365	1.0691	0.0078	43.8985	148,868.9913 ***
			INR			
3.2513	−2.1247	0.0223	0.4308	0.3526	8.5421	2777.8609 ***
			ZAR			
5.2017	−5.9939	0.0334	0.9905	0.2045	5.7404	683.2793 ***
			WTI			
11.2892	−11.1258	−0.0213	2.0619	0.0411	6.2304	929.3830 ***

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand. The sample period is from 4 October 2010 to 11 December 2018. JB-test: the Jarque–Bera test for normality. *** indicates a rejection of the null hypothesis, which states that the data is normally distributed at the 1% level of significance.

Table 5. Rank correlation and linear correlation.

Pair	Kendall's τ	Spearman's ρ	Pearson
CHY–WTI	−0.1033 ***	−0.1536 ***	−0.1382 ***
BRL–WTI	−0.1543 ***	−0.2284 ***	−0.2383 ***
RUB–WTI	−0.2460 ***	−0.3576 ***	−0.3483 ***
INR–WTI	−0.0710 ***	−0.1064 ***	−0.1243 ***
ZAR–WTI	−0.1626 ***	−0.2396 ***	−0.2607 ***

Notes: Kendall's τ: the Kendall's τ correlation coefficient, which measures the strength of dependence between two variables; Spearman's ρ: the Spearman's ρ correlation coefficient, which measures the degree of association between two variables; Pearson: the Pearson correlation coefficient, which measures the degree of the relationship between linearly related variables. *** indicates significance at 1% level.

Table 6. Constant copula parameter estimation (BRL-WTI).

Type	Parametric			Semi-Parametric		
	$\hat{\theta}$		logζ	$\hat{\theta}$		logζ
Normal	−0.2396 ***		63.3962	−0.2384 ***		62.4599
	(0.0201)			(0.0221)		
Plackett	0.4930 ***		58.7756	0.4948 ***		58.3472
	(0.0367)			(0.0320)		
Rotated-Gumbel	1.1000 ***		−94.1647	1.1000 ***		−94.4560
	(0.0044)			(0.0103)		
Student's t (ρ, v^{-1})	−0.2403 ***	0.0309 **	**64.4623**	−0.2398 ***	0.0301 *	**63.4614**
	(0.0218)	(0.0156)		(0.0202)	(0.0205)	

Notes: BRL: Brazilian Real; WTI: West Texas Intermediate crude oil price. $\hat{\theta}$: the estimated coefficient; Logζ: the log-likelihood of each constant copula model. The values in parentheses are the standard error of the parameter. The values in bold indicate the highest log-likelihood values. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 7. Constant copula parameter estimation (RUB-WTI).

Type	Parametric			Semi-Parametric		
	$\hat{\theta}$		logζ	$\hat{\theta}$		logζ
Normal	−0.3607 ***		148.9589	−0.3615 ***		149.5790
	(0.0239)			(0.0184)		
Plackett	0.3210 ***		152.4024	0.3211 ***		152.4935
	(0.0321)			(0.0159)		
Rotated-Gumbel	1.1000 ***		−121.9177	1.1000 ***		−120.9702
	(0.0037)			(0.0085)		
Student's t (ρ, v^{-1})	−0.3697 ***	0.0910 ***	**160.9720**	−0.3708 ***	0.0940 ***	**161.4015**
	(0.0231)	(0.0336)		(0.0199)	(0.0275)	

Notes: RUB: Russian Ruble; WTI: West Texas Intermediate crude oil price. $\hat{\theta}$: the estimated coefficient; Logζ: the log-likelihood of each constant copula model. The values in parentheses are the standard error of the parameter. The values in bold indicate the highest log-likelihood values. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 8. Constant copula parameter estimation (INR-WTI).

Type	Parametric			Semi-Parametric		
	$\hat{\theta}$		logζ	$\hat{\theta}$		logζ
Normal	−0.1166 ***		14.8911	−0.1157 ***		14.3927
	(0.0196)			(0.0197)		
Plackett	0.7166 ***		12.7553	0.7228 ***		12.3592
	(0.0514)			(0.0452)		
Rotated-Gumbel	1.1000 ***		−54.1551	1.1000 ***		−56.5984
	(0.0021)			(0.0099)		
Student's t (ρ, v^{-1})	−0.1169 ***	0.0505 **	**17.4666**	−0.1154 ***	0.0405 **	**16.0603**
	(0.0238)	(0.0230)		(0.0248)	(0.0230)	

Notes: INR: Indian Rupee; WTI: West Texas Intermediate crude oil price. $\hat{\theta}$: the estimated coefficient; Logζ: the log-likelihood of each constant copula model. The values in parentheses are the standard error of the parameter. The values in bold indicate the highest log-likelihood values. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 9. Constant copula parameter estimation (CNY-WTI).

Type	Parametric		Semi-Parametric	
	$\hat{\theta}$	logζ	$\hat{\theta}$	logζ

Normal	−0.1560 ***		26.4584	−0.1553 ***		26.0857
	(0.0222)			(0.0198)		
Plackett	0.6227 ***		26.2658	0.6214 ***		26.2913
	(0.0420)			(0.0368)		
Rotated-Gumbel	1.1000 ***		−67.8413	1.1000 ***		−65.9822
	(0.0030)			(0.0105)		
Student's t (ρ, v^{-1})	−0.1562 ***	0.0493 **	**29.7568**	−0.1591 ***	0.0515 **	**28.9640**
	(0.0209)	(0.0216)		(0.0216)	(0.0228)	

Notes: CNY: offshore Chinese Yuan; WTI: West Texas Intermediate crude oil price. $\hat{\theta}$: the estimated coefficient; Logζ: the log-likelihood of each constant copula model. The values in parentheses are the standard error of the parameter. The values in bold indicate the highest log-likelihood values. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 10. Constant copula parameter estimation (ZAR-WTI).

Type	Parametric			Semi-Parametric		
	$\hat{\theta}$		logζ	$\hat{\theta}$		logζ
Normal	−0.2587 ***		74.2223	−0.2575 ***		73.2669
	(0.0197)			(0.0199)		
Plackett	0.4699 ***		66.3718	0.4739 ***		65.4050
	(0.0391)			(0.0244)		
Rotated-Gumbel	1.1000 ***		−93.5894	1.1000 ***		−94.3130
	(0.0041)			(0.0092)		
Student's t (ρ, v^{-1})	−0.2605 ***	0.0416 **	**75.8064**	−0.2576 ***	0.0421 **	**74.8820**
	(0.0220)	(0.0205)		(0.0202)	(0.0224)	

Notes: ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. $\hat{\theta}$: the estimated coefficient; Logζ: the log-likelihood of each constant copula model. The values in parentheses are the standard error of the parameter. The values in bold indicate the highest log-likelihood values. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 11. P-value of Goodness-of-fit test for constant copula models.

Type	Parametric		Semi-Parametric	
	KS	**CvM**	**KS**	**CvM**
		BRL		
Normal	0.63	0.38	0.18	0.03
Plackett	0.67	0.31	**0.00**	**0.00**
Rotated-Gumbel	**0.00**	**0.00**	**0.00**	**0.00**
Student's t	0.70	0.36	0.16	0.03
		RUB		

Normal	0.50	1.00	0.13	0.13
Plackett	0.62	1.00	0.10	0.10
Rotated-Gumbel	**0.00**	**0.00**	**0.00**	**0.00**
Student's t	0.55	1.00	0.29	0.28
		INR		
Normal	**0.00**	0.55	0.13	0.08
Plackett	**0.00**	0.41	0.07	0.05
Rotated-Gumbel	0.03	0.37	**0.00**	**0.00**
Student's t	**0.00**	0.54	0.17	0.08
		CNY		
Normal	0.40	0.77	0.76	0.49
Plackett	0.31	0.73	0.40	0.28
Rotated-Gumbel	**0.00**	**0.00**	**0.00**	**0.00**
Student's t	0.34	0.69	0.71	0.70
		ZAR		
Normal	0.16	0.57	0.30	0.29
Plackett	0.39	0.49	0.05	0.03
Rotated-Gumbel	**0.00**	**0.00**	**0.00**	**0.00**
Student's t	0.43	0.64	0.40	0.26

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand. Bold numbers indicate a rejection of the null hypothesis, which stares that the copula model provides the best fit to the data at the 1% level of significance. KS: *p*-values from the Kolmogorov–Smirnov test; CvM: the p-value from Cramer–von Mises test.

Table 12. Tail dependence coefficients of the Student's t copula model.

Pair	Parametric	Semi-Parametric
	$\hat{\lambda}^L$ (= $\hat{\lambda}^U$)	$\hat{\lambda}^L$ (= $\hat{\lambda}^U$)
CNY–WTI	0.4353	0.4337
BRL–WTI	0.4132	0.4136
RUB–WTI	0.3516	0.3504
INR–WTI	0.4477	0.4511
ZAR–WTI	0.4033	0.4041

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price.

Table 13. Time-varying copula parameter estimation.

Parametric					Semi-Parametric				
$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{v}$	$\log\zeta$	$\hat{\omega}$	$\hat{\alpha}$	$\hat{\beta}$	$\hat{v}$	$\log\zeta$
				BRL					
−0.0034	0.0230 *	0.9931 ***	0.0267*	73.0383	−0.0034	0.0230 ***	0.9931 ***	0.0267 *	72.2516
(0.0489)	(0.0148)	(0.1442)	(0.0196)	-	(0.0000)	(0.0073)	(0.0010)	(0.0174)	-
				RUB					
−0.0180 **	0.0756 ***	0.9766 ***	0.0668 ***	193.0128	−0.0173 ***	0.0690 ***	0.9774 ***	0.0672 ***	193.5189
(0.0095)	(0.0158)	(0.0137)	(0.0245)	-	(0.0000)	(0.0166)	(0.0016)	(0.0263)	-
				INR					
−0.1491 ***	0.0176	0.3688	0.0504 ***	17.5825	−0.1513 ***	0.0156	0.3519	0.0405 **	16.1506
(0.0603)	(0.0525)	(0.2973)	(0.0191)	-	(0.0000)	(0.0379)	(0.4285)	(0.0239)	-
				CNY					
−0.0106	0.0219	0.9680 ***	0.0512 ***	31.9006	−0.0106 ***	0.0247	0.9687 ***	0.0541 ***	31.2668
(0.0507)	(0.0580)	(0.1725)	(0.0191)	-	(0.0000)	(0.0426)	(0.1028)	(0.0230)	-
				INR					
−0.0073	0.0415 **	0.9865 ***	0.0250*	91.4405	−0.0067 ***	0.0418 ***	0.9874 ***	0.0268 **	90.7733
(0.0346)	(0.0210)	(0.0578)	(0.0171)	-	(0.0000)	(0.0124)	(0.0017)	(0.0195)	-

Note: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand. $\hat{\omega}$, $\hat{\alpha}$, $\hat{\beta}$, and $\hat{v}$: the estimated coefficients; Logζ : the log-likelihood of each time-varying copula model. The values in parentheses are the standard error of the parameter. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table 14. P-value of goodness-of-fit test for the time-varying copula models.

Parametric		Semi-Parametric	
KS	**CvM**	**KS**	**CvM**
		BRL	
0.9	0.83	**0.00**	**0.00**
		RUB	
0.86	1.00	0.21	0.09
		INR	
0.00	0.55	**0.00**	**0.00**
		CNY	
0.64	0.62	0.79	0.87
		ZAR	
0.79	0.73	0.07	**0.01**

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand. Bold numbers indicate a rejection of the null hypothesis, which states that the copula model provides the best fit to the data at the 1% level of significance. KS: Kolmogorov–Smirnov test; CvM: Cramer–von Mises test.

Table A1. Marginal distribution parameter estimation.

Variable	φ	AR (1)	AR (2)	AR (3)	MA (1)	MA (2)	ω	α	β	γ	λ	ν	Persistence
BRL	0.0138	-	-	-	-	-	0.0068 **	0.0882 ***	0.9105 ***	-	1.0000 ***	5.5195 ***	0.9988
	(0.0153)						(0.0031)	(0.0164)	(0.0151)		(0.0291)	(0.6392)	
RUB	0.0239	0.0574 ***	-	-	-	-	0.0080 ***	0.1194 ***	0.9163 ***	−0.0891 ***	1.0000 ***	6.6418 ***	0.9911
	(0.0153)	(0.0219)					(0.0026)	(0.0222)	(0.0166)	(0.0190)	(0.0304)	(0.9238)	
INR	−0.0122	0.5749 ***	−0.9716 ***	0.0127	−0.5719 ***	0.9567 ***	0.0032 **	0.1096 ***	0.8891 ***	-	0.9000 ***	4.1632 ***	0.9987
	(0.0074)	(0.0390)	(0.0137)	(0.0271)	(0.0251)	(0.0049)	(0.0014)	(0.0236)	(0.0236)		(0.0229)	(0.4281)	
CNY	−0.0069 ***	0.4704 ***	−0.0623 ***	-	−0.4603 **	-	0.0004 ***	0.1077 ***	0.8913 ***	-	0.9982 ***	4.0425 ***	0.9990
	(0.0026)	(0.1816)	(0.0212)		(0.1806)		(0.0001)	(0.0179)	(0.0164)		(0.0281)	(0.3212)	
ZAR	0.0136	0.0173	-	-	-	-	0.0068 ***	0.0496 ***	0.9636 ***	−0.0423 ***	0.9000 ***	8.3036 ***	0.9921
	(0.0203)	(0.0218)					(0.0024)	(0.0068)	(0.0028)	(0.0106)	(0.0267)	(1.3302)	
WTI	−0.0049	-	-	-	−0.0390 *	-	0.0253 **	0.0551 ***	0.9408 ***	-	0.9121 ***	5.7172 ***	0.9959
	(0.0339)				(0.0213)		(0.0119)	(0.0109)	(0.0115)		(0.0262)	(0.7056)	

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. φ, AR(p), MA(q): the estimated parameters from the ARMA (p, q) model. ω, α, β (,γ): the estimated parameters from the GARCH (GJR-GARCH) model. λ, ν: the estimated parameters from skewed t model for the distribution of the error term. The persistence is calculated by α+β in GARCH model and by α+β+γ/2 in GJR-GARCH model. The values in parenthesis represent the standard errors of each parameter. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A2. Ljung-Box test and Lagrange Multiplier test.

Variable	Q (20)	Q^2 (20)	ARCH (20)
CNY	15.3600	0.1734	0.1692
p-value	0.7554	1.0000	1.0000
BRL	15.9640	6.3681	6.4523
p-value	0.7189	0.9983	0.9981
RUB	32.7250	26.9020	26.7650
p-value	0.0362	0.1380	0.1420
INR	23.5130	23.5590	23.3340
p-value	0.2643	0.2622	0.2727
ZAR	20.7420	24.8260	23.7330
p-value	0.4125	0.2082	0.2543
WTI	13.4890	10.3330	10.2270
p-value	0.8554	0.9617	0.9639

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; WTI: West Texas Intermediate crude oil price. Q (20): the Ljung-Box test statistics for serial correlation of order 20 for the standardized residuals from the GARCH model; Q^2(20): the Ljung-Box test statistics for serial correlation of order 20 for the squared standardized residuals from the GARCH model; ARCH (20): the Lagrange Multiplier test statistics for autoregressive conditional heteroscedasticity.

Table A3. Rank correlation and linear correlation.

Pair	Kendall's τ	Spearman's ρ	Pearson
CHY-BRENT	−0.0884 ***	−0.1318 ***	−0.1124 ***
BRL-BRENT	−0.1573 ***	−0.2329 ***	−0.2219 ***
RUB-BRENT	−0.2853 ***	−0.4104 ***	−0.3914 ***
INR-BRENT	−0.0718 ***	−0.1075 ***	−0.1181 ***
ZAR-BRENT	−0.1547 ***	−0.2282 ***	−0.2343 ***

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; BRENT: Brent crude oil price. Kendall's τ: the Kendall's τ correlation coefficient; Spearman's ρ: the Spearman's ρ correlation coefficient; Pearson: the Pearson correlation coefficient. *** indicates significance at 1% level.

Table A4. Tail dependence coefficients of the Student's t copula model.

Pair	Parametric	Semi-Parametric
	$\hat{\lambda}^L$ (= $\hat{\lambda}^U$)	$\hat{\lambda}^L$ (= $\hat{\lambda}^U$)
CNY-BRENT	0.4064	0.4064
BRL-BRENT	0.4194	0.4194
RUB-BRENT	0.3214	0.3214
INR-BRENT	0.4494	0.4494
ZAR-BRENT	0.4151	0.4151

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNY: offshore Chinese Yuan; ZAR: South African Rand; BRENT: Brent crude oil price.

Chapter 2

Connectedness Between Natural Gas Price and BRICS Exchange Rates: Evidence from Time and Frequency Domains

2.1 Introduction

In light of the increasing attention being paid to environmental sustainability, energy systems are gradually transitioning from a dependence on non-renewable resources to the use of environment-friendly resources. This will have a great impact on day-to-day life, economies, businesses, manufacturers, and governments. Compared to coal or petroleum, natural gas has many qualities that makes it burn more efficiently. It also generates fewer emissions of most types of air pollutants, including carbon dioxide. With the expansion of gas pipelines, the increasing number of gas liquefaction plants, and the exploitation of natural gas fields, it is reasonable to consider that the natural gas trade will become more globalized. Natural gas has become a major part of the world's energy consumption, demand, and supply in recent years. In 2018, for example, natural gas consumption rose by 5.3%, one of the fastest rates of growth since 1984. With the continuing rapid expansion in liquefied natural gas (LNG), the inter-regional natural gas trade grew by 4.3%, which was more than double the 10-year average (BP Statistical Review of World Energy 2019). As reported in the Global Energy Perspective 2019: Reference Case (Global Energy Perspective 2019), natural gas will be the only fossil fuel whose share of total energy demand continues to increase until 2035, and China will represent nearly half of the global demand growth. Other developing countries are also expected to increase their demand for natural gas.

Brazil, Russia, India, China, and South Africa (BRICS), a group of five fast-growing developing countries, play an important and expanding role in the world economy. In recent years, BRICS have represented an increasing share of global economic growth. According to the International Monetary Fund (IMF), as of 2018, the combined gross domestic product (GDP) of these five nations accounted for 23.2% of the gross world product (GWP). Given the growth of BRICS and the fact that energy is a crucial ingredient for economic development, these countries' relationship with natural gas will only become closer. According to the BP Statistical Review of World Energy 2019, in 2018, the total consumption of natural gas in BRICS was 835.8 billion cubic meters (BCM), which accounted for 21.7% of the total global consumption. In terms of imports, China became the second largest importer of LNG, with imports increasing from 4.6 BCM in 2008 to 73.5 BCM in 2018. India was the fourth largest importer, with imports increasing from 11.3 BCM to 30.6 BCM over the same period. In terms of exports, Russia was the largest exporter of pipeline gas. It also accounted for nearly 6% of total LNG exports. As the trade of natural gas is usually settled in US dollars, it is meaningful to study the relationship between the natural gas price and the BRICS's exchange rates.

Against this backdrop, this paper investigates the interdependence between the natural gas price and the BRICS's exchange rate. In doing so, this study is expected to offer valuable insights for market operators, investors, and economists. We use the Henry Hub natural gas futures as the data for the natural gas price. There are two reasons behind this choice of dataset: First, the shale gas revolution in America has dramatically increased US production of shale gas since 2007. World Energy Outlook 2018, produced by the International Energy Agency (IEA), has predicted that natural gas production in America will increase from 976 BCM in 2017 to 1328 BCM in 2040 and that this increase will be mainly due to the growth in shale gas production. Therefore, the Henry Hub natural gas price, which usually represents pricing for the North American natural gas market, has a great influence on the global energy market. We assume that this influence will become stronger over time. The second reason is that there are multiple natural gas price indexes in the world, such as the Japan Korea Marker and the UK National Balancing Point (NBP); however, we cannot predict which price index has strong connectedness with the BRICS's exchange rates. Therefore, we select the Henry Hub price given its characteristics of high liquidity and large trading volume.

Our contribution to the literature is twofold. First, we apply the connectedness methodology from Diebold and Yilmaz (2009, 2012 and 2015), which allows us to know how pervasive the risk is throughout the entire market by quantifying the contribution of each variable to the system. We also apply the time–frequency version of connectedness proposed by Baruník and Křehlík (2018) to find the connectedness between different variables in the short, medium, and long term.

Second, to the best of our knowledge, there is not much research on the relationship between the natural gas price and exchange rates. Nevertheless, there are many studies that analyzed the relationship between crude oil prices and foreign exchange rates, and almost all of them show that exchange rates are highly connected to the oil price. For example, in our previous research on the relationship between the West Texas Intermediate (WTI) crude oil price and BRICS's exchange rates using the copula method, we found a significant negative dependence between the two variables. Considering the globalization of natural gas trade, high demand growth (1.6% per year), and the expansive market share in the global energy market (World Energy Outlook 2018 has predicted that, by 2030, natural gas will overtake coal and become the second largest source of energy after oil.), it is reasonable to compare the relationship between the crude oil price and exchange rates with that between the natural gas price and exchange rates. Therefore, in this study, we also aim to determine whether BRICS's exchange rates are closely linked to the natural gas price, as they are to the oil price.

The rest of this paper is organized as follows: A brief review of relevant literature is provided in Section 2. Section 3 introduces the empirical methodology used in this study. Section 4 reports empirical results. Section 5 gives the conclusion. Finally, a robustness analysis is presented in the Appendix.

2.2 Literature Review

As we have mentioned above, there is not much literature that has analyzed the relationship between the natural gas price and exchange rates, as far as we know. However, there are many studies on the relationship between the exchange rate and other variables, such as the oil price and the stock market. Chen and Chen (2007) investigated the long-term relationship between different crude oil price indexes and G7 countries' exchange rates using the monthly panel data between January 1972 and December 2005. They found that oil prices may account for the movements of the real exchange rate and there is a link between oil prices and real exchange rates. Additionally, from the results of panel predictive regression, they found that the crude oil price has the ability to forecast the future exchange rate. Andrieș et al. (2014) identified the patterns of co-movement of the interest rate, stock price, and exchange rate in India using wavelet analysis. They used the data span from July 1997 to December 2010. The empirical results showed that exchange rates, interest rates, and stock prices are linked to each other and that the stock price fluctuations lag behind both the exchange rates and interest rates. Brahm et al. (2014) used monthly data to investigate the relationship between the crude oil price and exchange rates in the long term and short term, respectively. The data span was from January 1997 to December 2009. Empirical results indicated exchange rates Granger-caused crude oil prices in the short term, whereas crude oil Granger-caused exchange rates in the long term. Furthermore, based on impulse response analysis, exchange rate shock had a significant negative effect on crude oil prices. Jain and Pratap (2016) explored the relationship between global prices of crude oil and gold, the stock market in India, and the USD-INR exchange rate using the DCC-GARCH (dynamic conditional correlation-generalized autoregressive conditional heteroscedasticity) model. They also examined the lead lag linkages among these variables using symmetric and asymmetric non-linear causality tests. They used daily data from the period of 2006 to 2016, finding that a fall in the value of the Indian Rupee and the benchmark stock index was caused by a fall in gold and crude oil prices.

On the empirical side, the methodology used in this paper has already been applied in many fields. Maghyereh et al. (2016) used implied volatility indices (VIX) of the daily close price of crude oil in 11 countries. They found that the connectedness between oil and equity was dominated by the transmissions from the oil market to equity markets and most of the linkages between these two markets were established from mid-2009 to mid-2012, a period that witnessed the start of the global recovery. Lundgren et al. (2018) studied the renewable energy stock returns and their relation to the uncertainty of currency, oil price, stocks, and US treasury bonds. They used data covering the period from 2004 to 2016, and found that the European stock market depends on renewable energy stock prices. Singh et al. (2018) employed a dynamic and directional network connectedness between the implied volatility index (VIX) of the exchange rates of nine major currency pairs and the crude oil using the data between May 2017 and December 2017. They found that crude oil affected currencies more than currencies affected crude oil, but the reverse was true during the crude oil crisis period. Furthermore, their results revealed that

EUR–USD is more sensitive to crude oil price fluctuation than others. Ji et al. (2018) combined empirical mode decomposition with a connectedness methodology, and examined the dynamic connectedness among crude oil, natural gas, and refinery products using daily data between 3 January 2000 and 15 September 2017. Employing a constant analysis, they found that crude oil and its refinery product tend to be a net transmitter, while the natural gas tends to be a net receiver. In time-varying analysis, they found that the total connectedness generally increased until the 2014 crude oil crash, and then decreased sharply. Lovch and Perez-Laborda (2019) used the connectedness method and frequency decomposition method to investigate the relationship between the natural gas and crude oil price during the period from 1994 to 2018. They found that the volatility connectedness varied over time; the connectedness became weak after the financial crisis; and the volatility had long-run effects, except during some specific periods, when volatility shocks transmitted faster but dissipated in the short-run.

2.3 Empirical Methodology

In this paper, we employ two methods to establish the nature of the relationship between exchange rates and natural gas price. The first method is provided by Diebold and Yilmaz (DY) (2009; 2012; 2015), whose approach calculates the connectedness between different objects by introducing variance decomposition into vector autoregression (VAR) models. The second method is based on Baruník and Křehlík (BK) (2018), who proposed a new framework to estimate connectedness by using a spectral representation of variance decomposition. In conclusion, the DY framework describes the connectedness as "when shocks are arising in one variable, how would other variables be changing?", whereas the BK framework estimates the connectedness in short-, medium-, and long-term financial cycles.

2.3.1 Connectedness Table

Based on Diebold and Yilmaz (2015), a simplified connectedness table is presented in Table 1, which gives a clear picture of aggregated and disaggregated connectedness.

In table 1, x_i is the interested variable, whereas d_{ij} is the pairwise directional connectedness from x_j to x_i, which shows what percentage of the h-step-ahead forecast error variance in x_i is due to the shocks in x_j (Equation (1)). We can simply understand d_{ij} as how much future uncertainty of x_i is due to the shocks in x_j:

$$C_{i\leftarrow j} = d_{ij} \tag{1}$$

The column "From" is the total directional connectedness from x_j to others (Equation (2)), and the row "To" means the total directional connectedness from others to x_i (Equation (3)):

$$C_{\cdot\leftarrow j} = \sum_{\substack{i=1 \\ i\neq j}}^{N} d_{ij} \tag{2}$$

$$C_{i\leftarrow \cdot} = \sum_{\substack{j=1 \\ j\neq i}}^{N} d_{ij} \tag{3}$$

We were also interested in net pairwise directional connectedness (Equation (4)) and net total directional connectedness (Equation (5)), which are expressed as a negative value to indicate a net recipient and a positive value to indicate a net transmitter:

$$C_{ij} = C_{j\leftarrow i} - C_{i\leftarrow j} \tag{4}$$

$$C_i = C_{\cdot\leftarrow i} - C_{i\leftarrow \cdot} \tag{5}$$

Finally, the total connectedness (Equation (6)), calculated by the grand total of the off-diagonal entries of d_{ij}, is given in the lower-right cell of the connectedness table:

$$C = \frac{1}{N}\sum_{\substack{i,j=1 \\ i\neq j}}^{N} d_{ij} \tag{6}$$

2.3.2 Generalized Forecast Error Variance Decomposition (GFEVD)

Diebold and Yilmaz (2009) measured connectedness based on forecast error variance decompositions from VAR models, which were introduced by Sims (1980) and Koop et al. (1996). However, the calculation of variance decomposition requires orthogonalized shocks and depends on ordering the variables, so Diebold and Yilmaz (2012) exploited the generalized forecast error variance decomposition (GFEVD) of Pesaran and Shin (1998) to solve those problems. In this paper, we employ the method of GFEVD to calculate the connectedness.

We will give a brief introduction to GFEVD, followed by an explanation of Lütkepohl (2005) and Diebold and Yilmaz (2015).

For easy understanding, we first consider a VAR (1) process with N-variable:

$$\begin{gathered} y_t = v + A_1 y_{t-1} + u_t, \ \ t = 0, \pm 1, \pm 2 \ldots \\ E(u_t) = 0 \\ E(u_t u_t') = \Sigma_u \\ E(u_t u_s') = 0, \ \ t \neq s \end{gathered} \tag{7}$$

If the generation mechanism starts at time t = 1, we get:

$$\begin{gathered} y_1 = v + A_1 y_0 + u_1 \\ y_2 = v + A_1 y_1 + u_2 = v + A_1(v + A_1 y_0 + u_1) + u_2 \\ = (I_N + A_1)v + A_1^2 y_0 + A_1 u_1 + u_2 \\ \ldots \\ y_t = (I_N + A_1 + \cdots + A_1^{t-1})v + A_1^t y_0 + \sum_{m=0}^{t-1} A_1^m u_{t-m} \\ \ldots \end{gathered} \tag{8}$$

If all eigenvalues of A_1 have modulus less than 1 (VAR process is stable), we have:

$$\begin{gathered} (I_N + A_1 + \cdots + A_1^{t-1})v \ \rightarrow \ (I_N - A_1)^{-1} v \ \text{ as } t \rightarrow \infty \\ A_1^t y_0 \rightarrow 0 \ \ as\, t \rightarrow \infty \end{gathered} \tag{9}$$

Then, we can rewrite Equation (7) as:

$$\begin{gathered} y_t = \mu + \sum_{m=0}^{\infty} A_1^m u_{t-m}, \ \ t = 0, \pm 1, \pm 2 \ldots \\ \text{where } \ \mu \equiv (I_N - A_1)^{-1} v \end{gathered} \tag{10}$$

Secondly, let us consider a VAR (p) process:

$$y_t = v + A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, t = 0, \pm 1, \pm 2, \ldots \tag{11}$$

By using matrices, we can rewrite the VAR (p) process as a VAR (1) process:

$$\begin{gathered} Y_t = \boldsymbol{v} + \boldsymbol{A} Y_{t-1} + U_t \\ \underset{(Np * 1)}{Y_t} \equiv \begin{bmatrix} y_1 \\ y_2 \\ \cdots \\ y_{t-p+1} \end{bmatrix}, \ \underset{(Np \times 1)}{\boldsymbol{v}} \equiv \begin{bmatrix} v \\ 0 \\ \vdots \\ 0 \end{bmatrix} \\ \underset{(Np \times Np)}{\boldsymbol{A}} \equiv \begin{bmatrix} A_1 & A_2 & \ldots & A_{p-1} & A_p \\ I_N & 0 & \ldots & 0 & 0 \\ 0 & I_N & \ldots & 0 & 0 \\ \ldots & \ldots & \ldots & \ldots & 0 \\ 0 & 0 & \ldots & I_N & 0 \end{bmatrix}, \ \underset{(Np \times 1)}{U_t} = \begin{bmatrix} u_t \\ 0 \\ \cdots \\ 0 \end{bmatrix} \end{gathered} \tag{12}$$

Similar to Equation (10), Equation (12) can be rewritten as:

$$Y_t = \boldsymbol{\mu} + \sum_{m=0}^{\infty} \boldsymbol{A}^m U_{t-m}, \quad t = 0, \pm 1, \pm 2 \ldots \tag{13}$$

By pre-multiplying a N×Np matrix J≡[I_N:0:…:0], we get:

$$y_t = JY_t = J\boldsymbol{\mu} + \sum_{m=0}^{\infty} J\boldsymbol{A}^m U_{t-m} = J\boldsymbol{\mu} + \sum_{m=0}^{\infty} J\boldsymbol{A}^m J' J U_{t-m}$$

$$= \mu + \sum_{m=0}^{\infty} \Phi_m u_{t-m} \tag{14}$$

$$\underset{(N\times 1)}{\mu} = J\boldsymbol{\mu}, \quad \underset{(N\times N)}{\Phi_m} \equiv J\boldsymbol{A}^m J', \quad \underset{(N\times 1)}{u_t} = JU_t$$

Finally, we get a moving average (MA) representation of the VAR(p) process:

$$\begin{gathered} y_t = \mu + \sum_{m=0}^{\infty} \Phi_m u_{t-m} \\ E(u_t) = 0 \\ E(u_t u_t') = \Sigma_u \\ E(u_t u_s') = 0, \;\; t \neq s \end{gathered} \tag{15}$$

The h-step GFEVD can be expressed as:

$$\omega_{ij,h}^{g} = \frac{\sigma_{jj}^{-1} \sum_{m=0}^{h-1} (e_i' \Phi_m \Sigma_u e_j)^2}{\sum_{m=0}^{h-1} \left(e_i' \Phi_m \Sigma_u \Phi_m' e_j\right)} \tag{16}$$

where e_i is the i-th column of I_N and σ_{jj} is the j-th diagonal element of Σ_u.

Because the sums of the forecast error variance contribution are not necessarily in agreement, we contribute our generalized connectedness indexes as:

$$d_{ij} = \widetilde{\omega_{ij}^{g}} = \frac{\omega_{ij,h}^{g}}{\sum_{j=1}^{N} \omega_{ij,h}^{g}} \tag{17}$$

2.3.3 Spectral Representation of GFEVD

Based on the DY framework, the BK framework defines the general spectral representation of GFEVD and uses it to define the frequency-dependent connectedness measure, which is inspired by the previous research of Geweke (1982; 1984; 1986) and Stiassny (1996).

We still consider the MA representation of the VAR(p) process (Equation (15)). The BK framework provides a frequency response function (Equation (18)), which can be obtained as a Fourier transform of the coefficient Φ_m:

$$\Psi\left(e^{-i\lambda}\right) = \sum_{m} e^{-i\lambda m} \Phi_m, \;\; i = \sqrt{-1} \tag{18}$$

The generalized causation spectrum over frequencies $\lambda \in (-\pi, \pi)$ is defined as:

$$(f(\lambda))_{j,k} = \frac{\sigma_{kk}^{-1}\left|(\Psi(e^{-i\lambda})\Sigma_u)_{j,k}\right|^2}{(\Psi(e^{-i\lambda})\Sigma_u \Psi'(e^{+i\lambda}))_{j,j}} \tag{19}$$

where $(f(\lambda))_{j,k}$ represents the portion of the spectrum of x_j at a given frequency λ due to shocks in x_k. In order to obtain a natural decomposition of variance decomposition to frequencies, a weighting function is defined as:

$$\Gamma_j(\lambda) = \frac{(\Psi(e^{-i\lambda})\Sigma_u \Psi'(e^{-i\lambda}))_{j,j}}{\frac{1}{2\pi}\int_{-\pi}^{\pi}(\Psi(e^{-i\lambda})\Sigma_u \Psi'(e^{-i\lambda}))_{j,j}d\lambda} \tag{20}$$

where $\Gamma_j(\lambda)$ represents the power of the j-th variable at a given frequency.

The entire range of frequencies' influence of GFEVD from x_j to x_k is expressed as:

$$\omega_{jk}^{\infty} = \frac{1}{2\pi}\int_{-\pi}^{\pi}\Gamma_j(\lambda)(f(\lambda))_{j,k}d\lambda \tag{21}$$

Additionally, the GFEVD on specified frequency band $d=(a,b)$, $a,b \in (-\pi,\pi)$, $a<b$, is defined as:

$$\omega_{jk}^{d} = \frac{1}{2\pi}\int_{d}\Gamma_j(\lambda)(f(\lambda))_{j,k}d\lambda \tag{22}$$

As in Section 2.3.2, we contribute our scaled GFEVD on frequency band d as below, to make sure that the sums of variance contribution are in agreement:

$$d_{ij} = \widetilde{\omega_{jk}^{d}} = \frac{\omega_{jk}^{d}}{\sum_k \omega_{jk}^{\infty}} \tag{23}$$

2.4 Empirical Results

2.4.1 Data

For this study, we collected daily data from Bloomberg, including the Henry Hub natural gas futures price (GASF), and the nominal dollar-denominated exchange rates for the Brazilian Real (BRL), Russian Ruble (RUB), Indian Rupee (INR), offshore Chinese Yuan (CNH), and South African Rand (ZAR). We used the offshore Chines Yuan instead of the onshore Chinese Yuan (CNY) for the reason that China has reformed its exchange rate regime twice, once in 2005 and the other in 2010. Before and after each reform, CNY kept its exchange rate steady for a long time, with almost no fluctuation or only change in a narrow range. Therefore, we chose CNH, which has more fluctuations, to conduct our analysis. In order to match the data availability for CNH, we used the data sample period from 23 August 2010 to 20 June 2019.

The stationary return series were obtained from Equation (18), and are in percentage points:

$$r_{i,t} = 100 \times ln\left(\frac{p_{i,t}}{p_{i,t-1}}\right) \tag{24}$$

The return series for the natural gas price and exchange rate over time are plotted in Figure 1. Figure 1 shows that the GASF return had the highest volatility compared to the others. We consider that the natural gas price was largely affected by temperature, so most fluctuations occurred concentratedly during the winter season. A small number of fluctuations were recorded in the middle of the year, such as in 2012, when hot weather forecasts and elevated cooling demands created a great demand for natural gas. The RUB return fluctuated drastically at the end of 2014, when the crude oil crash happened, and caused the financial crisis in Russia.

Table 2 provides summary statistics for all return series. CNH has the lowest mean of all return series, as well as standard deviation. Therefore, in some way, CNH remained stable under government regulations. The GASF had the highest standard deviation, as shown in Figure 1. The distribution of all return series significantly deviated from normal, as demonstrated by the Jarque-Bera test.

We were interested in not only the connectedness of the return series, but also the volatility connectedness, because volatility can provide a measure of risk and is particularly crisis-sensitive (Diebold and Yilmaz 2011). As volatility is unobserved and must be estimated, we used generalized autoregressive conditional heteroscedasticity (GARCH) models to obtain the volatilities of BRL, INR, CNH, and GASF return series, and Glosten-Jagannathan-Runkle GARCH (GJR-GARCH) models to obtain the volatility of RUB and ZAR return series (for the sake of brevity, the results of the GARCH model and GJR-GARCH model are omitted).

The plots of volatility are presented in Figure 2. For simplicity's sake, we used a different scale for the y-axis in RUB and GASF. The GASF fluctuated violently and most fluctuations accumulated during the winter season, which is consistent with the return series. The volatilities of the five exchange rates reached a high level at the end of 2011, compared to the period before and after, when the eurozone debt crisis reached its peak. The BRL's volatility was turbulent after 2010, especially between 2015 and 2017, when Brazil experienced a severe economic crisis and faced a dramatic economic recession. The volatility of INR reached its peak at the end of 2013, as the Indian rupee had depreciated greatly. The description statistics for volatility are reported in Table 3. Similar to the return series, GASF has the highest standard deviation, whereas CNH has the lowest. All volatilities were skewed and had high kurtosis, indicating that the distributions showed obvious non-normality characteristics. The Jarque–Bera test also verifies our opinion.

2.4.2 Connectedness and Frequency Decomposition

As the calculation of the connectedness index is based on the VAR model, we conducted an augmented Dickey-Fuller (ADF) test for the unit root before applying the data to the VAR model.

However, it is well-known that the unit root hypothesis can be rejected if the data series contain structural break(s) (1989; Papell and Lumsdaine 1997; Andrews and Zivot 2002). As our sample period is long, from 2010 to 2019, which is almost 9 years, and several big events happened during the sample period, such as the 2014 crude oil crush, which may have had an impact on the economies and caused structural breaks, it is well-founded to consider that structural breaks may exist. Therefore, a Bai-Perron test for structural breaks was conducted. The p-values of the Bai-Perron test for the return series are presented in Table 4. All numbers indicate the acceptance of null hypothesis that no break exists. These results confirm the reliability of the ADF test.

The results of the ADF test are presented in Table 5. All results show that no unit root exists. The p lags of the VAR model were chosen by the Akaike information criterion (AIC). Return series used the VAR (1) model, whereas volatilities used the VAR (2) model (for the sake of brevity, the results of the VAR model are omitted).

The connectedness index based on DY and its spectral representation based on BK of short-, medium-, long-term are reported in Tables 6–9, respectively. The frequency band of short term, medium term, and long term in Table 6 roughly corresponds to 1 day to 5 days, 5 days to 21 days, and more than 21 days, respectively. The diagonal elements in both tables represent the own-market connectedness and are not important in our paper. We have more interest in the off-diagonal elements, which indicate pairwise connectedness between two variables: the values of the To row, which show when one variable receives a shock; how much influence would be exerted on other variables; the values of the “From” column, which measure the composition of one variable’s change; and the values of the “Net” row, which reveal whether a variable is a net recipient or a net transmitter. The “GAS-FX” column, which exhibits the net pairwise directional connectedness between GAS and the five exchange rates, is the most critical for our study.

As shown in Table 6, the total connectedness of the return series is 22.484%, which is almost twice as much as the connectedness between volatilities (11.697%), but both of them are modest. In this system, no matter what the connectedness from the return series or volatilities was, the shocks transmitted from ZAR to BRL contributed the largest value. The BRL was a net transmitter in return connectedness, but a net recipient in volatility connectedness. The RUB was opposite to BRL in that it was a net recipient in the return case, but a net transmitter in the volatility case. Furthermore, INR, CNH, and GASF were net receivers in both cases, whereas ZAR was a net transmitter. By obtaining the absolute value of the “Net” row, we found that INR had the strongest influence (11.517%) in all return series and ZAR was the most powerful variable (16.677%) in volatilities.

In Tables 7–9, we note that the sum of total connectedness in the short term, medium term, and long term is equal to the total connectedness shown in Table 6, which is in agreement with the definition of frequency decomposition for connectedness. It is interesting to find that the total connectedness from the return series is highest in the short term (17.756%), followed by the medium term (3.480%) and long term (1.248%). By contrast, from volatilities, the value is highest in the long term (11.232%), followed by the medium term (0.370%) and short term (0.095%), which means that the uncertainty transmitted by the shock has a long-term impact on the market, rather than the shock itself.

From the values of the GAS-FX column, we found that the net pairwise connectedness between GAS and the exchange rate was higher in volatilities than return series, but both were very weak. All values were almost zero. The possible reason for this is that the GASF data we have chosen are for Henry Hub natural gas, which could be seen as representative of the North American natural gas market. However, as the natural gas pipeline in North America can hardly reach any BRICS countries, and the distance between North America and BRICS countries makes the transportation cost of LNG expensive, whether as an import or export, North

American natural gas is not the primary selection for BRICS countries. Our opinion is also supported by statistics from the BP Statistical Review of World Energy 2019. Whether natural gas is traded by a pipeline or LNG, the quantity being directed from the US, Mexico, and Canada to BRICS countries is very low. Consequently, we could say that the natural gas price is unrelated to the exchange rate in BRICS countries.

2.4.3 Rolling-Window Analysis

We also conducted a rolling-window analysis to investigate the time-varying connectedness between GAS and exchange rates. The window size was 300 (we also obtained the dynamic connectedness from a window size of 400 and obtained similar results to the result produced from the 300 window size). Figure 3 plots the dynamic total connectedness. From Figure 3, we can see that the total connectedness from the return series begins with a high level (around 40%) in the first few windows, and then falls after 2011 (around 25%), when South Africa joined the BRICS group and the period in which the European debt crisis was at its peak. After a temporary rise in late 2012, the connectedness falls again at the beginning of 2013 (around 20%). From 2013 to mid-2015, the connectedness fluctuates between 20% and 25%, and then rises again to over 30% after mid-2015. The connectedness drops dramatically between 2017 and 2018, from almost 35% to around 20%, and then recovers slowly. The trend of dynamic connectedness from volatilities is similar to that of return series. There are several unusual peaks and troughs in the plot, which we think are related to big events, like the Russian financial crisis (2014), the Brazilian economic recession (2015), and the US–China trade war (2018). Figure 4 presents the frequency decomposition of dynamic connectedness. We find that, whether in the short, medium, or long term, the trend of return connectedness is similar to the dynamic total connectedness. However, for volatilities, in the short and medium term, the connectedness exhibits almost no change (except for some abrupt rises and falls), and the long term has a similar trend to total connectedness. We think that long-term connectedness exerts the most influence in the case of volatility.

The time-varying net pairwise connectedness between GAS and exchange rates return series is plotted in Figure 5. Like the result in Section 4.2, all values were low and almost all of them were below 2.5% in terms of the absolute value, so were negligible. The results from volatilities are presented in Figure 6. There are also several sudden rises and falls, which is consistent with the plot of total connectedness. In the net pairwise connectedness of GAS-BRL, GAS-INR, and GAS-ZAR pairs, except for the abnormal value at some points, the values were almost insignificant; thus, we can hardly say that the GAS has an influence on the exchange rate or vice versa. However, in GAS-RUB and GAS-CNH pairs, there are some significant positive or negative periods during our data span. Before 2014, GAS was a net transmitter to RUB, and then turned into a net recipient after 2014, when Russia was undergoing an economic crisis caused by the oil price crash. After 2016, GAS was a net transmitter to CNH, when Australia became the largest supplier of LNG to China instead of Qatar, and the trade kept increasing after that.

2.5 Conclusions

This paper examined the connectedness between the Henry Hub natural gas price and the BRICS's exchange rates. To that end, the connectedness methodology from Diebold and Yilmaz (2009, 2012, 2015) as well as frequency decomposition of connectedness proposed by Baruník and Křehlík (2018) were used. We collected data from 23 August 2010 to 20 June 2019 and tested both return series and volatilities from GARCH models.

Our empirical results show that the total connectedness was 22.5% in the return series and 11.7% in volatilities. Compared to results from previous studies—such as Lundgren et al. (2018) who found that the total volatility connectedness among renewable energy stock returns, investment assets, and several sources of uncertainty is 67.4%—our results are modest, which means that most variation was due to the variation in the variables themselves. By taking the frequency decomposition of connectedness, we found that, in the return series, the short term contributes to the total connectedness the most, whereas the long term contributes most in relation to volatility. From the results of net pairwise connectedness between the natural gas price and exchange rates, we obtained a value of almost zero in each natural gas and exchange rate pair, which means that natural gas does not play an important role in explaining movements in the exchange rates. We also applied a rolling-window approach to conduct the time-varying analysis. In short, the results are similar to those of the constant analysis and we cannot say for certain that the natural gas price had a great influence on exchange rate movement. Only in the plot of volatility connectedness were there several dramatic fluctuations, which we consider to be connected to some notable events, such as economic crises and trade frictions.

Our results are obviously different from the results of the studies on the relationship between the oil price and exchange rates, such as that conducted by Singh et al. (2018), who found that the total volatility connectedness between the oil price and nine exchange rates reached 72.96%. The shocks transmitted from crude oil to each exchange rate are also significant. We consider some possible reasons for the difference. First, crude oil can be used more widely across different fields than natural gas. For example, it can fuel our cars and make plastics, rubbers, and the like, which are uses that cannot be replaced by natural gas. As indicated in the BP Statistical Review of World Energy 2019, crude oil has the highest share in global energy consumption, and its consumption is almost double that of natural gas. Second, the production of crude oil far exceeds that of natural gas. Therefore, whether for energy import countries or energy export countries, crude oil is more easily traded. Third, compared to developed countries, awareness of the environment in developing countries is at a lower level. As the BRICS are the focus of our study, although their consumption of natural gas has increased in recent years, natural gas is still not the primary energy source for these countries (with the exception of Russia). India, China, and South Africa consumed coal the most in 2019, while Brazil consumed oil the most (BP Statistical Review of World Energy 2019).

Although crude oil plays an irreplaceable role in the energy market now, with increasing environmental awareness, we believe that natural gas will become more important and the connectedness between the natural gas price and exchange rates will become stronger in the future.

The empirical evidence in this study may have important implications for policymakers, especially those in oil-dependent countries. As much of the literature shows that exchange rates are highly dependent on the oil price, turbulence in the crude oil market could have a great impact on the foreign currency market, thus causing exchange rate pressure and even economic instability. In order to solve the foreign exchange fluctuation, monetary authorities need to accumulate or reduce foreign exchange reserves, which is not considered desirable in the real world. Changing the dependence structure in relation to energy—from depending on energy that is closely connected with the currency market, such as crude oil, to depending on energy that is hardly connected to the currency market, such as natural gas—could provide an efficient way of maintaining economic stability and reducing exchange rate pressure. By contrast, because of the low connectedness between the natural gas price and exchange rates, foreign exchange fluctuation may barely be affected by the natural gas price. Therefore, for investors, it is less risky to invest in gas-related financial products than oil-related financial products, which are highly connected with currency.

Although this paper conducted thorough research, there were several limitations in the empirical work. First, although we found that natural gas did not have a significant impact on the exchange rate, this result could be influenced by the data selection. We used Henry Hub as our natural gas price data, which represents the North American natural gas market. However, given the restriction of pipelines and high transportation cost, North American countries that produce natural gas may not be the primary selection for BRICS. Second, with technological improvements in exploiting natural gas and the increasing number of gas liquefaction plants, we assumed that the LNG price would have more influence on the exchange rate than the pipeline natural gas price did. However, owing to data limitations, we could only focus on the whole natural gas market, which may be the reason why the connectedness between the natural gas price and exchange rates was modest. Therefore, for further extension of this research, first, we want to collect different natural gas price data, such as the Netherlands Title Transfer Facility (TTF) index and Japan Korea Marker, to exclude the impact of data selection on the results. Second, we want to analyze the relationship between the crude oil price and exchange rates and the relationship between the crude oil price and natural gas price. This would allow us to compare the connectedness between the crude oil price and exchange rates with that between the natural gas price and exchange rates more rationally. Finally, if the data permit, we want to use the data on only LNG to find the connectedness between the natural gas price and foreign exchange rates more precisely.

Appendix A Robustness Analysis

We used the Henry Hub natural gas spot price (GASS) as the natural gas price data to examine the robustness of our results. (The natural gas futures index in the United Kingdom, which is known as the UK National Balancing Point (NBP), was also used to conduct the robustness check, but the results were quite similar to those from GASS, so we only present the connectedness table of NBP and exchange rates (Table A2) in Appendix).

The plot of GASS's return and volatility series are reported in Figure A1. We found that some values of volatility were extremely large (the maximum is over 800). We think that the reason for this is that the natural gas spot price was more easily affected by the change of demand and supply than the future price, even though the change was small.

We summarize the results of connectedness and the frequency decomposition of short, medium, and long term in Tables A1, A3-A5, respectively. The result is quite similar to that of GASF and exchange rates. We also used a 300 rolling-window to conduct the time-varying analysis. The dynamic connectedness and its spectral representation are plotted in Figures A2 and A3, respectively. The net pairwise connectedness of return series and volatilities are illustrated in Figures A4 and A5, respectively. All results are consistent with those from the analysis using the natural gas future price, except for the net pairwise connectedness of return series (Figure A4). Some values are opposite to the result above, but all of them are low, even the maximum value, which is less than 5% and negligible.

The results of robustness confirm the suitability of our proposed approach, which aimed to capture the relationship between the natural gas price and exchange rates.

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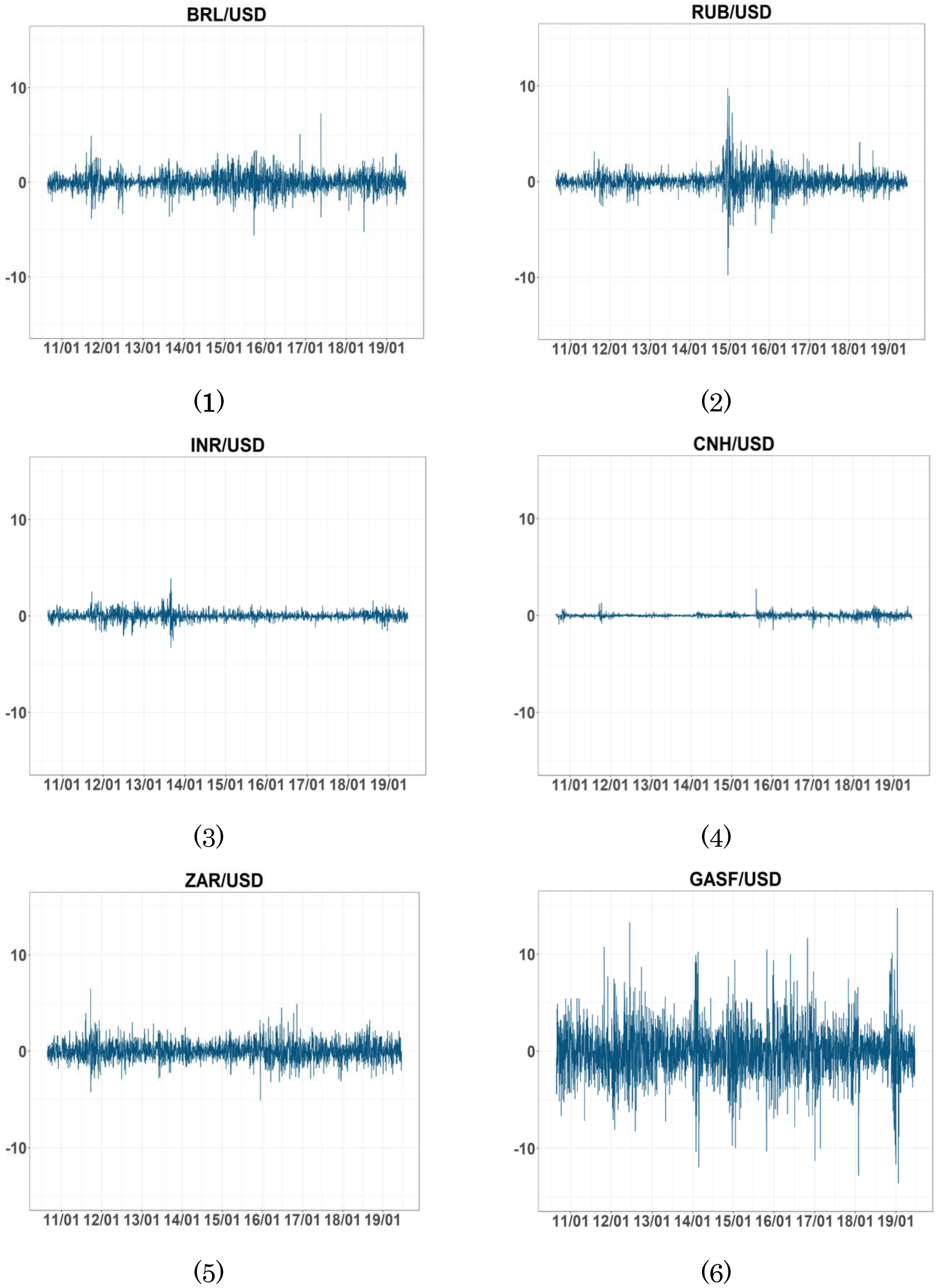
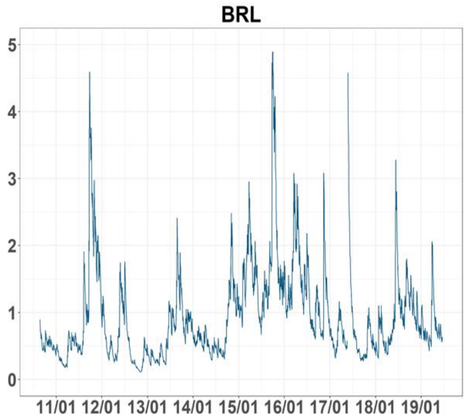


Figure 1. Daily return. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. (1-6) refer to BRL, RUB, INR, CNH, ZAR, and GASF return series, respectively.



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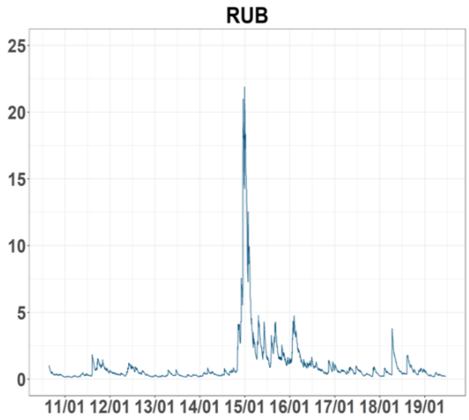

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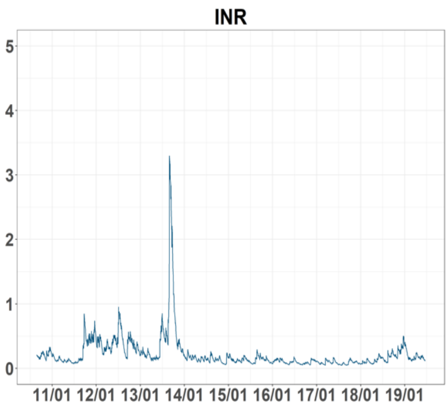

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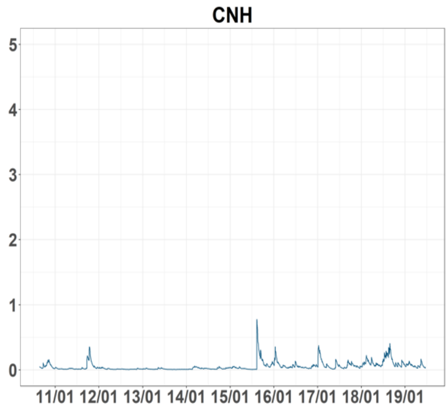

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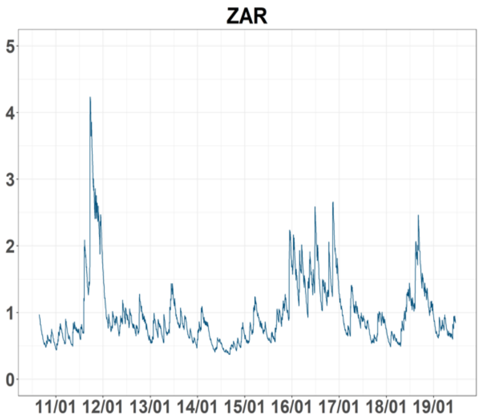

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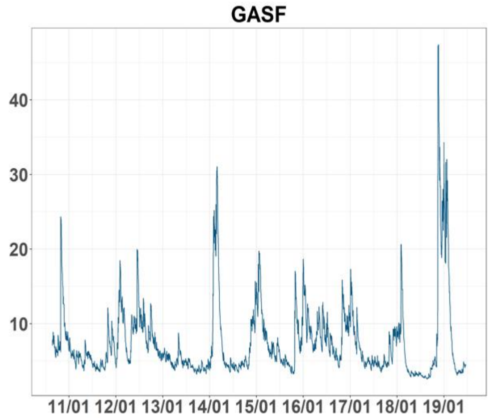

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Figure 2. Volatility. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. (1-6) refer to the volatility of BRL, RUB, INR, CNH, ZAR, and GASF return series, respectively.

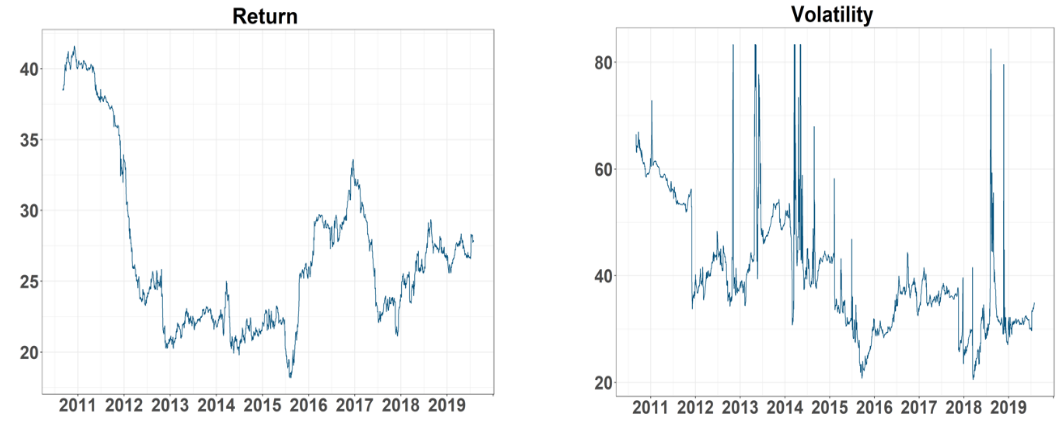

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Figure 3. Dynamic connectedness. Notes: (1): total connectedness of return series; (2): total connectedness of volatilities.

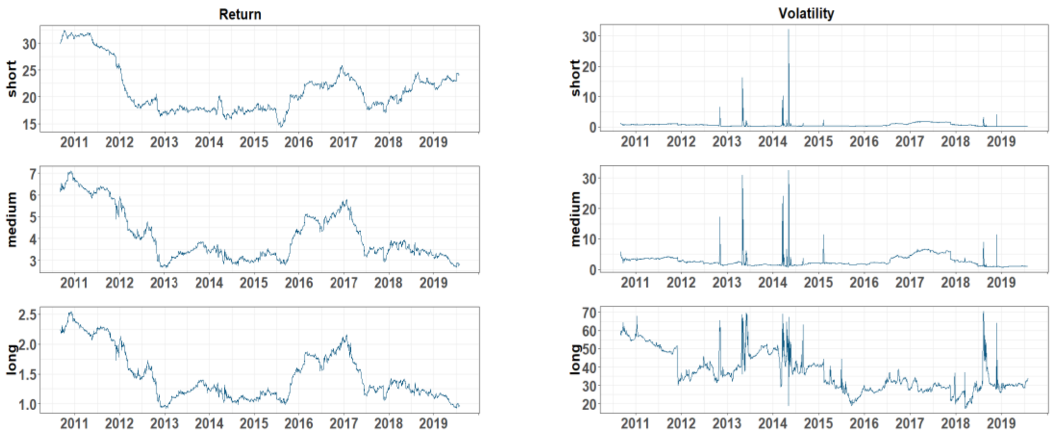

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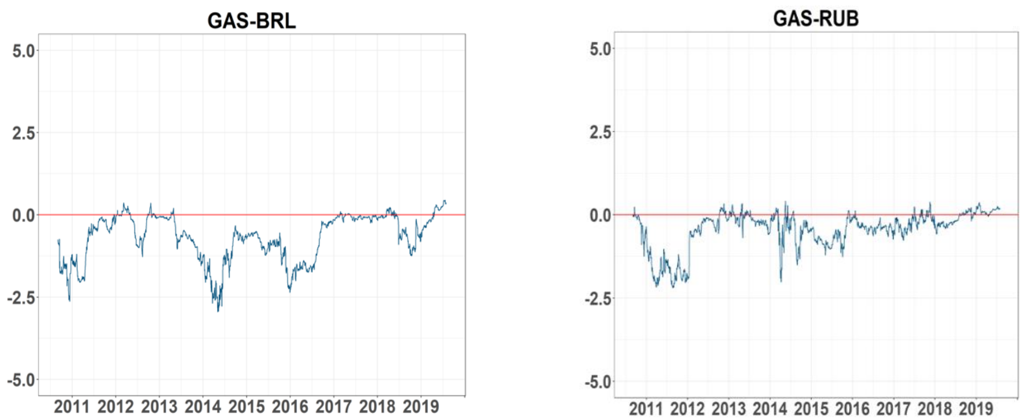
Figure 4. Frequency decomposition of dynamic connectedness. Notes: (1): frequency decomposition of total connectedness for return series; (2) frequency decomposition of total connectedness for volatilities.



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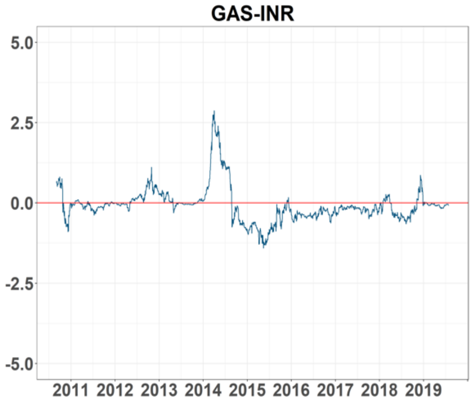

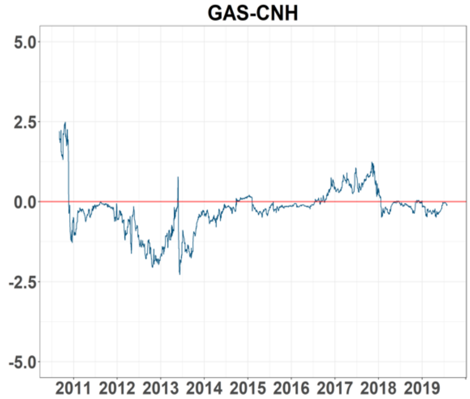

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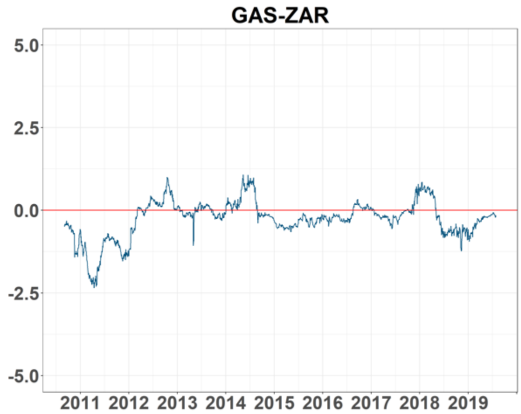

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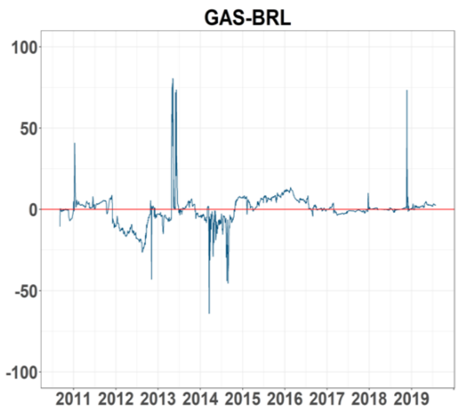
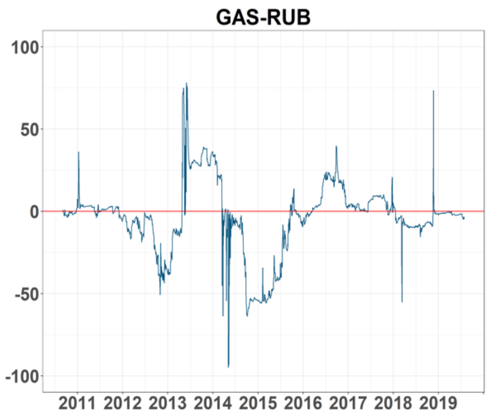
Figure 5. Net pairwise connectedness of return series. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. (1-5) refer to the net pairwise connectedness between GASF and BRL, RUB, INR, CNH, and ZAR return series, respectively.





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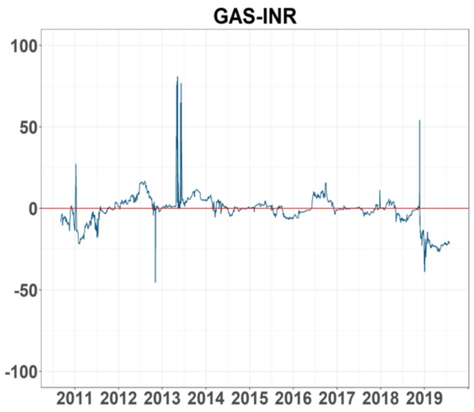

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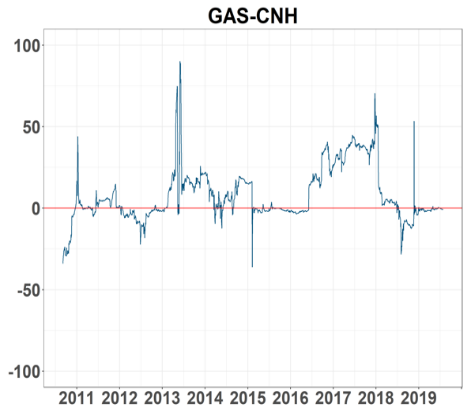

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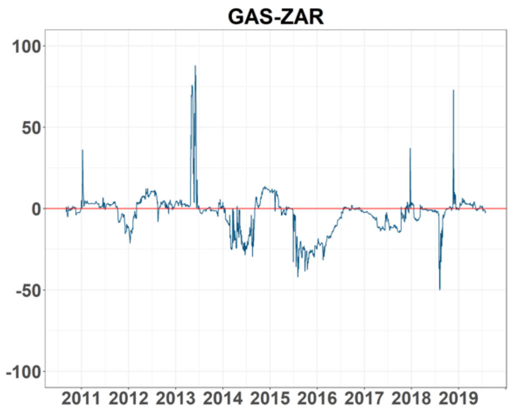

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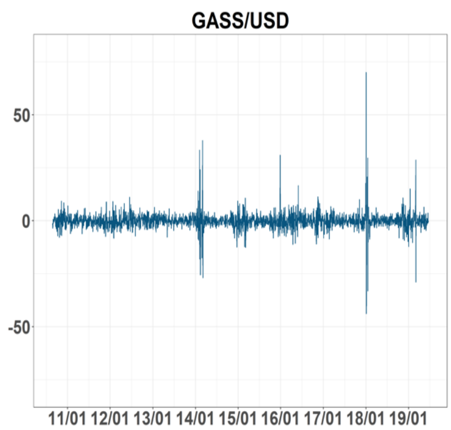
Figure 6. Net pairwise connectedness of volatility. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. (1-5) refer to net pairwise connectedness between the volatility of GASF and BRL, RUB, INR, CNH, and ZAR, respectively.



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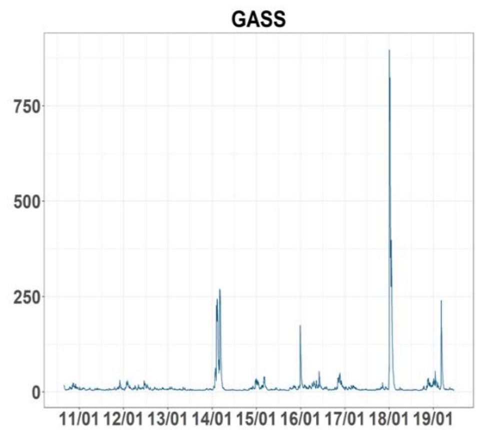

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Figure A1. Daily return and volatility of Henry Hub natural gas spot price. Notes: GASS: Henry Hub natural gas spot price. (1): return series; (2): volatility.

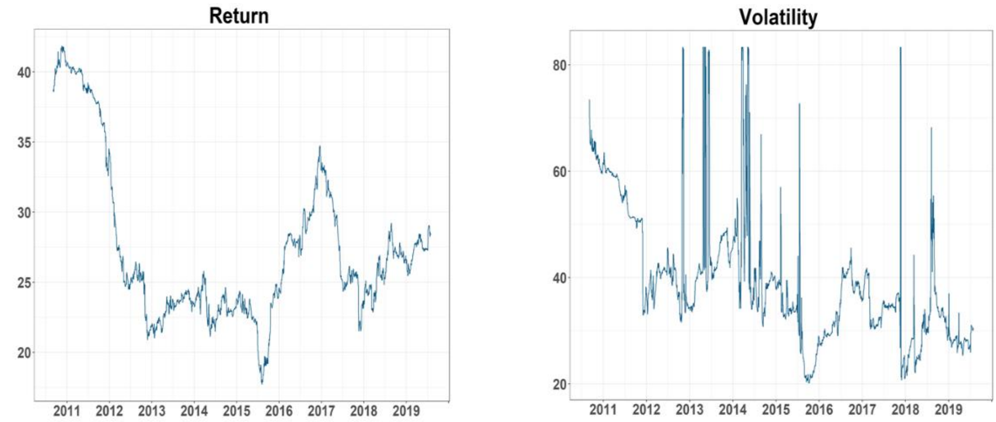

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Figure A2. Dynamic connectedness. Notes: (1): total connectedness of return series; (2): total connectedness of volatilities.

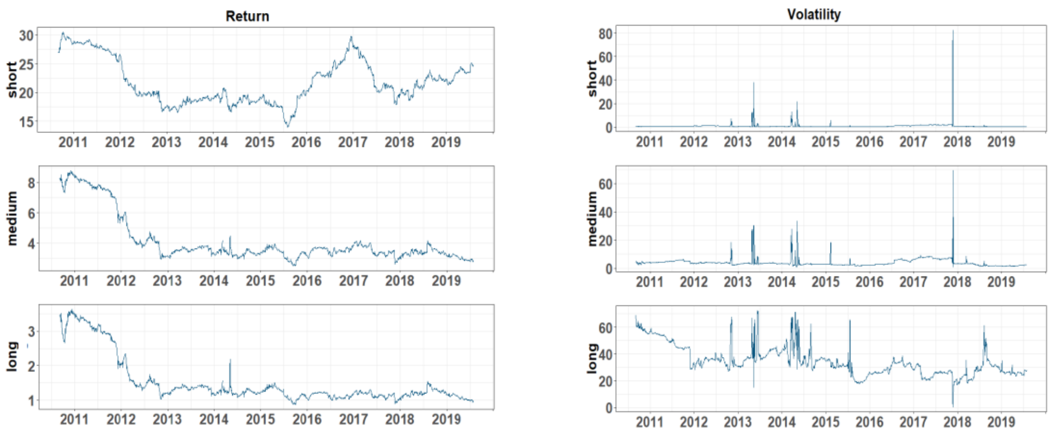

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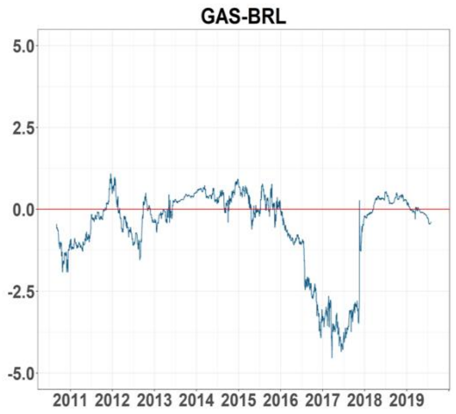
Figure A3. Frequency decomposition of dynamic connectedness. Notes: (1): frequency decomposition of total connectedness for return series; (2): frequency decomposition of total connectedness for volatilities.



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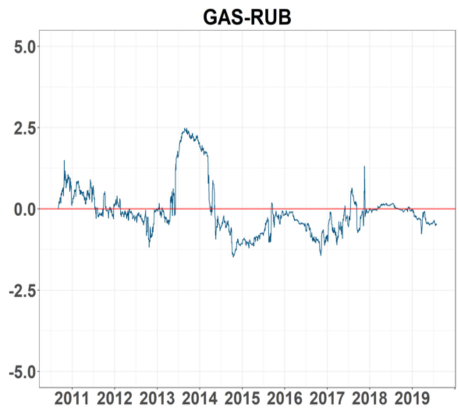

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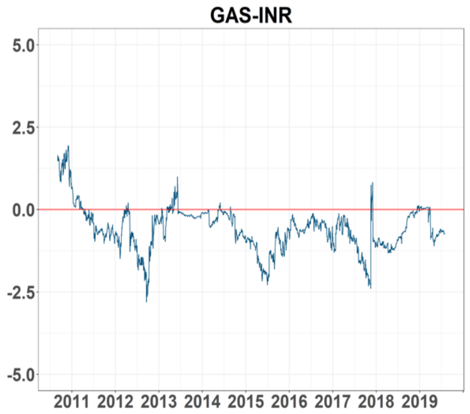

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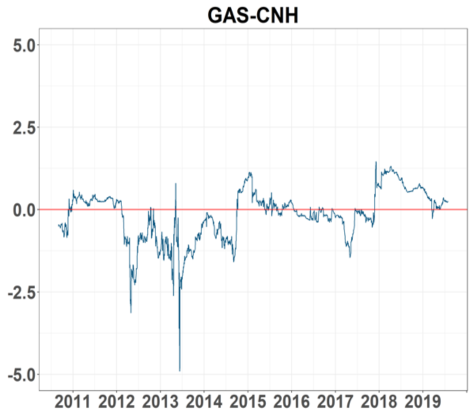

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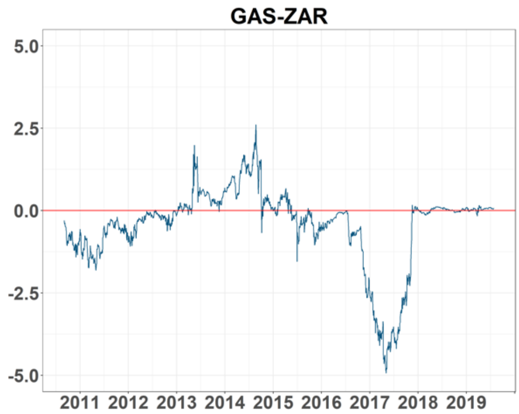

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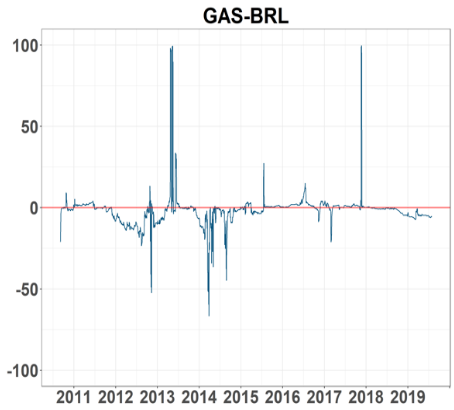
Figure A4. Net pairwise connectedness of return series. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASS: Henry Hub natural gas spot price. (1-5) refer to the net pairwise connectedness between GASS and BRL, RUB, INR, CNH, and ZAR return series, respectively.



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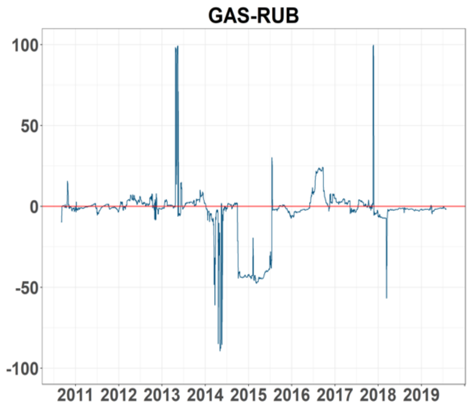

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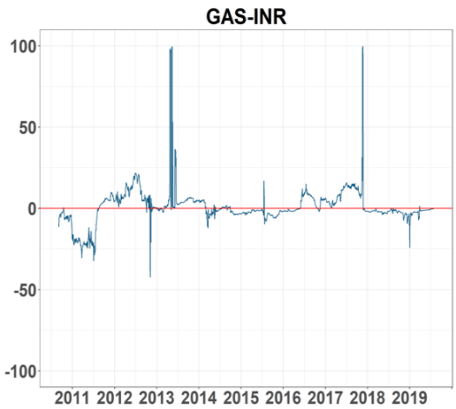

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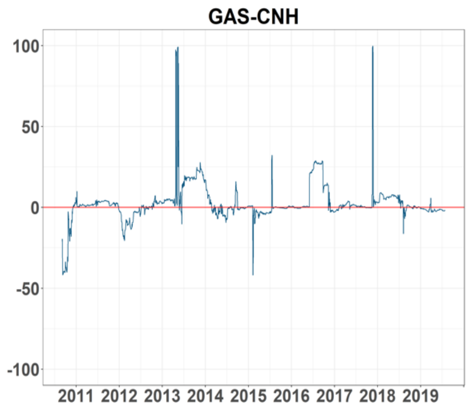

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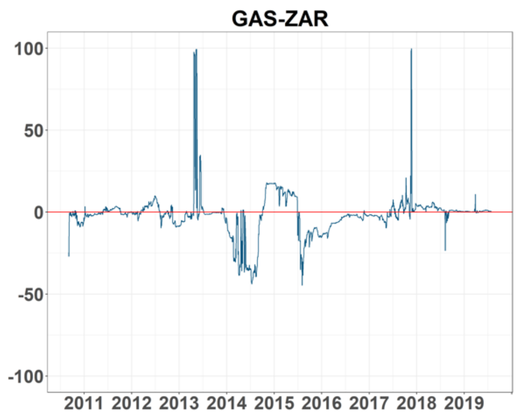

(5)

Figure A5. Net pairwise connectedness of volatility. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASS: Henry Hub natural gas spot price. (1-5) refer to the net pairwise connectedness between the volatility of GASS and BRL, RUB, INR, CNH, and ZAR, respectively.

Table 1. Connectedness table.

	x_1	x_2	…	x_N	From
x_1	d_{11}	d_{12}	…	d_{1N}	$\sum_{j=1}^{N} d_{1j}\ j \neq 1$
x_2	d_{21}	d_{22}	…	d_{2N}	$\sum_{j=1}^{N} d_{2j}\ j \neq 2$
⋮	⋮	⋮	…	⋮	⋮
x_N	d_{N1}	d_{N2}	…	d_{NN}	$\sum_{j=1}^{N} d_{Nj}\ j \neq N$
To	$\sum_{i=1}^{N} d_{i1}$ $i \neq 1$	$\sum_{i=1}^{N} d_{i2}$ $i \neq 2$	…	$\sum_{i=1}^{N} d_{iN}$ $i \neq N$	$\frac{1}{N}\sum_{i,j=1}^{N} d_{ij}\ i \neq j$

Source: Diebold and Yilmaz (2015).

Table 2. Summary statistics for daily returns.

	Min	Max	Mean	Std Dev	Skewness	Kurtosis	JB-Test
BRL	−5.601	7.270	0.034	0.949	0.140	3.784	1386.046 ***
RUB	−9.771	9.731	0.032	1.024	0.440	13.933	18,741.359 ***
INR	−3.294	3.904	0.017	0.451	0.286	7.546	5509.334 ***
CNH	−1.471	2.747	0.001	0.227	0.473	14.522	20,365.878 ***
ZAR	−5.081	6.444	0.029	0.986	0.271	1.994	411.515 ***
GASF	−18.055	16.691	−0.025	2.759	0.116	4.080	1607.127 ***

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. The sample period is from 23 August 2010 to 20 June 2019. JB-Test: the Jarque-Bera test for normality. *** indicates rejection of the null hypothesis that the data are normally distributed at the 1% level of significance.

Table 3. Summary statistics for volatilities of daily returns.

	Min	Max	Mean	Std Dev	Skewness	Kurtosis	JB-Test
BRL	0.111	5.466	0.957	0.707	2.127	6.536	5847.958 ***

RUB	0.135	21.907	1.061	2.044	6.108	44.688	206,321.592 ***
INR	0.048	3.297	0.223	0.277	6.275	52.188	276,967.370 ***
CNH	0.005	0.775	0.055	0.068	3.675	21.737	50,619.087 ***
ZAR	0.372	4.234	0.978	0.506	2.205	6.796	6312.786 ***
GASF	2.596	47.407	7.547	5.102	2.837	11.266	15,299.202 ***

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. The volatilities of BRL, INR, CNY, and GASF return series were calculated by the generalized autoregressive conditional heteroscedasticity (GARCH) (1,1) model, and the volatilities of RUB and ZAR return series were calculated by the Glosten–Jagannathan–Runkle (GJR)-GARCH (1,1) model. The sample period is from 23 August 2010 to 20 June 2019. *** indicates rejection of the null hypothesis that the data are normally distributed at the 1% level of significance.

Table 4. Bai–Perron breakpoint test on return series.

	BRL	RUB	INR	CNH	ZAR	GASF
p-value	0.769	0.448	0.316	0.190	0.773	0.536

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. Each number indicates the p-value of the Bai-Perron breakpoint test.

Table 5. Augmented Dickey–Fuller (ADF) test on return series and volatilities.

	BRL	RUB	INR	CNH	ZAR	GASF
			Return			
Dickey–Fuller	−12.050 ***	−11.581 ***	−11.718 ***	−11.470 ***	−14.284 ***	−13.837 ***
			Volatility			
Dickey–Fuller	−5.493 ***	−4.983 ***	−5.471 ***	−6.763 ***	−4.013 ***	−5.422 ***

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. Each number indicates that Dickey–Fuller is the ADF test statistic. *** indicates rejection of the null hypothesis that a unit root is present in the time series at the 1% level of significance.

Table 6. Connectedness among the natural gas future price and BRICS's exchange rates.

	BRL	RUB	INR	CNH	ZAR	GASF	From	GAS-FX
				Return				
BRL	69.719	7.790	1.360	3.129	17.692	0.310	30.281	−0.124
RUB	8.335	73.334	2.658	3.303	12.248	0.122	26.666	−0.113
INR	5.502	3.814	77.817	3.686	9.165	0.017	22.183	0.016
CNH	3.609	3.693	3.391	81.305	7.998	0.004	18.695	−0.024
ZAR	16.241	10.551	3.255	6.237	63.673	0.043	36.327	−0.012
GASF	0.433	0.235	0.001	0.028	0.055	99.248	0.752	
To	34.119	26.083	10.665	16.383	47.158	0.496	**22.484**	
Net	3.838	−0.583	−11.517	−2.312	10.831	−0.257		
				Volatility				
BRL	75.206	3.630	0.723	0.092	19.230	1.118	24.794	0.989
RUB	1.040	97.472	0.783	0.060	0.393	0.252	2.528	−3.667
INR	4.205	0.174	91.702	0.446	3.466	0.006	8.298	−0.026
CNH	3.768	0.197	2.853	86.083	6.943	0.156	13.917	−0.107
ZAR	12.265	0.244	1.080	0.449	85.171	0.791	14.829	−0.682
GASF	0.129	3.920	0.032	0.263	1.473	94.183	5.817	
To	21.407	8.165	5.471	1.310	31.506	2.324	**11.697**	
Net	−3.387	5.637	−2.826	−12.607	16.677	−3.493		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry

Hub natural gas futures price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASF and exchange rates, which is calculated by the GASF to others minus the others to GASF. The number in bold means the total connectedness. All results are expressed as a percentage.

Table 7. Connectedness among the natural gas future price and exchange rates in the frequency domain (short term).

	BRL	**RUB**	**INR**	**CNH**	**ZAR**	**GASF**	**From**	**GAS-FX**
				Return				
BRL	58.271	6.229	1.130	2.701	14.487	0.282	24.829	−0.111
RUB	6.470	58.659	2.207	2.684	9.513	0.096	20.969	−0.134
INR	3.796	2.569	62.837	2.626	5.975	0.016	14.982	0.015
CNH	2.838	3.122	2.748	65.387	6.316	0.004	15.029	−0.015
ZAR	13.332	8.809	2.767	5.097	51.584	0.042	30.048	0.004
GASF	0.393	0.229	0.001	0.020	0.039	82.464	0.682	
To	26.829	20.958	8.853	13.128	36.330	0.440	**17.756**	
Net	1.999	−0.010	−6.129	−1.901	6.283	−0.242		
				Volatility				
BRL	2.783	0.001	0.027	0.000	0.193	0.001	0.222	−0.001
RUB	0.002	1.226	0.011	0.000	0.022	0.000	0.036	−0.010
INR	0.012	0.006	1.107	0.006	0.039	0.001	0.063	−0.002
CNH	0.012	0.000	0.029	5.085	0.057	0.005	0.103	0.003
ZAR	0.085	0.012	0.019	0.004	1.181	0.000	0.120	−0.007
GASF	0.001	0.010	0.004	0.002	0.008	3.005	0.025	
To	0.112	0.030	0.090	0.012	0.319	0.008	**0.095**	
Net	−0.110	−0.006	0.026	−0.091	0.199	−0.017		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASF and exchange rates, which is calculated by the GASF to others minus the others to GASF. The number in bold means the total connectedness. The frequency band of short term roughly corresponds to 1 day to 5 days. All results are expressed as a percentage.

Table 8. Connectedness among the natural gas future price and exchange rates in the frequency domain (medium term).

	BRL	RUB	INR	CNH	ZAR	GASF	From	GAS-FX
				Return				
BRL	8.454	1.149	0.170	0.318	2.363	0.020	4.020	−0.009
RUB	1.372	10.802	0.334	0.457	2.011	0.019	4.193	0.015
INR	1.250	0.912	11.041	0.777	2.332	0.001	5.272	0.001
CNH	0.567	0.423	0.474	11.721	1.237	0.000	2.701	−0.006
ZAR	2.148	1.289	0.361	0.840	8.913	0.001	4.639	−0.011
GASF	0.030	0.005	0.000	0.006	0.012	12.391	0.053	
To	5.367	3.777	1.339	2.398	7.956	0.042	**3.480**	
Net	1.347	−0.416	−3.933	−0.303	3.316	−0.012		
				Volatility				
BRL	8.452	0.018	0.091	0.002	0.712	0.006	0.829	−0.002
RUB	0.012	4.530	0.035	0.000	0.082	0.002	0.130	−0.059
INR	0.072	0.034	4.791	0.020	0.193	0.001	0.320	−0.003
CNH	0.075	0.001	0.093	15.124	0.275	0.009	0.452	0.004
ZAR	0.273	0.036	0.062	0.004	3.877	0.001	0.375	−0.032
GASF	0.008	0.061	0.005	0.005	0.033	9.896	0.111	

To	0.440	0.150	0.285	0.029	1.296	0.019	**0.370**
Net	−0.389	0.020	−0.035	−0.423	0.920	−0.092	

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASF and exchange rates, which is calculated by the GASF to others minus the others to GASF. The number in bold means the total connectedness. The frequency band of medium term roughly corresponds to 5 days to 21 days. All results are expressed as a percentage.

Table 9. Connectedness among the natural gas future price and exchange rates in the frequency domain (long term).

	BRL	RUB	INR	CNH	ZAR	GASF	From	GAS-FX
				Return				
BRL	2.994	0.412	0.060	0.111	0.842	0.007	1.432	−0.003
RUB	0.493	3.873	0.118	0.162	0.724	0.007	1.504	0.006
INR	0.456	0.333	3.940	0.282	0.857	0.000	1.929	0.000
CNH	0.204	0.148	0.169	4.198	0.444	0.000	0.965	−0.002
ZAR	0.761	0.453	0.126	0.299	3.176	0.000	1.640	−0.004
GASF	0.010	0.001	0.000	0.002	0.004	4.392	0.017	
To	1.923	1.347	0.473	0.857	2.872	0.014	**1.248**	
Net	0.491	−0.157	−1.455	−0.108	1.232	−0.003		
				Volatility				
BRL	63.970	3.611	0.606	0.090	18.324	1.111	23.742	0.991
RUB	1.026	91.716	0.737	0.059	0.289	0.250	2.361	−3.598
INR	4.121	0.134	85.804	0.421	3.235	0.003	7.914	−0.020
CNH	3.681	0.196	2.731	65.874	6.611	0.142	13.361	−0.114

ZAR	11.906	0.196	1.000	0.442	80.113	0.790	14.334	−0.642
GASF	0.120	3.848	0.023	0.257	1.432	81.282	5.680	
To	20.855	7.985	5.097	1.269	29.891	2.297	**11.232**	
Net	−2.888	5.624	−2.817	−12.093	15.558	−3.384		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASF: Henry Hub natural gas futures price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASF and exchange rates, which is calculated by the GASF to others minus the others to GASF. The number in bold means the total connectedness. The frequency band of long term roughly corresponds to more than 21 days. All results are expressed as percentages.

Table A1. Connectedness between the natural gas spot price and BRICS's exchange rates.

	BRL	RUB	INR	CNH	ZAR	GASS	From	GAS-FX
				Return				
BRL	69.839	7.714	1.456	3.124	17.688	0.179	30.161	−0.249
RUB	8.323	73.405	2.681	3.276	12.230	0.085	26.595	−0.058
INR	5.571	3.833	77.727	3.652	9.066	0.152	22.273	0.040
CNH	3.588	3.648	3.378	81.296	7.953	0.137	18.704	0.136
ZAR	16.233	10.559	3.252	6.206	63.643	0.108	36.357	0.094
GASS	0.429	0.143	0.113	0.001	0.014	99.301	0.699	
To	34.143	25.896	10.879	16.259	46.951	0.660	**22.465**	
Net	3.983	−0.699	−11.394	−2.444	10.593	−0.039		
				Volatility				
BRL	75.533	3.660	0.288	0.354	20.138	0.025	24.467	−0.377
RUB	0.890	97.569	0.750	0.298	0.479	0.014	2.431	−0.006

INR	3.133	0.494	92.791	0.277	3.235	0.070	7.209	−0.234
CNH	3.679	0.213	2.713	85.576	7.123	0.696	14.424	0.678
ZAR	11.251	0.282	0.801	1.067	86.363	0.236	13.637	0.208
GASS	0.402	0.020	0.304	0.018	0.028	99.229	0.771	
To	19.356	4.669	4.856	2.015	31.003	1.041	**10.490**	
Net	−5.111	2.238	−2.354	−12.410	17.367	0.270		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASS: Henry Hub natural gas spot price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASS and exchange rates, which is calculated by the GASS to others minus the others to GASS. The number in bold means the total connectedness. All results are expressed as a percentage.

Table A2. Connectedness between the UK NBP and BRICS's exchange rates.

	BRL	RUB	INR	CNH	ZAR	NBP	From	GAS-FX
				Return				
BRL	69.761	7.791	1.352	3.139	17.707	0.249	30.239	−0.099
RUB	8.337	73.395	2.660	3.307	12.251	0.049	26.605	−0.014
INR	5.472	3.796	77.446	3.686	9.115	0.485	22.554	0.134
CNH	3.600	3.676	3.391	80.904	7.971	0.458	19.096	−0.088
ZAR	16.162	10.491	3.227	6.215	63.346	0.559	36.654	−0.286
NBP	0.348	0.062	0.352	0.546	0.845	97.847	2.153	
To	33.919	25.816	10.983	16.893	47.889	1.800	**22.883**	
Net	3.680	−0.788	−11.571	−2.204	11.236	−0.353		
				Volatility				
BRL	75.164	3.509	0.542	0.277	20.254	0.254	24.836	−0.041

RUB	0.947	96.034	0.952	0.188	0.549	1.329	3.966	1.213
INR	3.494	0.244	90.752	0.328	3.404	1.778	9.248	−1.243
CNH	3.879	0.269	2.684	85.577	6.804	0.788	14.423	0.611
ZAR	11.028	0.318	0.775	0.822	86.380	0.677	13.620	−0.742
NBP	0.295	0.116	3.021	0.177	1.419	94.971	5.029	
To	19.643	4.456	7.974	1.792	32.430	4.827	**11.854**	
Net	−5.193	0.491	−1.274	−12.631	18.810	−0.202		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; NBP: UK National Balancing Point. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the NBP and exchange rates, which is calculated by the NBP to others minus the others to NBP. The number in bold means the total connectedness. All results are expressed as a percentage.

Table A3. Connectedness between the natural gas spot price and exchange rates in the frequency domain (short term).

	BRL	**RUB**	**INR**	**CNH**	**ZAR**	**GASS**	**From**	**GAS-FX**
				Return				
BRL	58.953	6.079	1.107	2.772	14.530	0.118	24.606	−0.295
RUB	6.713	58.808	2.160	2.705	9.511	0.057	21.146	−0.060
INR	4.027	2.716	64.161	2.696	6.053	0.150	15.641	0.075
CNH	2.969	3.124	2.809	66.238	6.361	0.096	15.357	0.095
ZAR	13.475	8.616	2.769	5.042	51.620	0.089	29.991	0.081
GASS	0.414	0.118	0.075	0.001	0.008	81.932	0.615	
To	27.597	20.653	8.919	13.216	36.463	0.510	17.893	
Net	2.991	−0.494	−6.722	−2.141	6.472	−0.106		
				Volatility				

BRL	2.698	0.002	0.028	0.002	0.205	0.002	0.238	−0.003
RUB	0.001	1.256	0.011	0.002	0.026	0.001	0.041	−0.003
INR	0.016	0.009	1.099	0.006	0.030	0.000	0.061	−0.001
CNH	0.010	0.002	0.022	5.130	0.056	0.004	0.093	0.001
ZAR	0.090	0.011	0.014	0.006	1.193	0.000	0.121	−0.001
GASS	0.005	0.004	0.001	0.004	0.001	8.637	0.015	
To	0.123	0.028	0.076	0.019	0.318	0.007	**0.095**	
Net	−0.116	−0.014	0.015	−0.075	0.197	−0.008		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASS: Henry Hub natural gas spot price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASS and exchange rates, which is calculated by the GASS to others minus the others to GASS. The number in bold means the total connectedness. The frequency band of short term roughly corresponds to 1 day to 5 days. All results are expressed as a percentage.

Table A4. Connectedness between the natural gas spot price and exchange rates in the frequency domain (medium term).

	BRL	RUB	INR	CNH	ZAR	GASS	From	GAS-FX
				Return				
BRL	8.061	1.197	0.254	0.265	2.325	0.045	4.086	0.032
RUB	1.195	10.750	0.382	0.423	1.997	0.020	4.018	0.001
INR	1.143	0.821	10.068	0.705	2.216	0.003	4.888	−0.025
CNH	0.465	0.392	0.426	11.137	1.177	0.030	2.489	0.030
ZAR	2.045	1.423	0.359	0.856	8.868	0.014	4.697	0.010
GASS	0.013	0.019	0.028	0.000	0.004	13.017	0.065	
To	4.861	3.852	1.448	2.250	7.720	0.113	**3.374**	

Net	0.775	−0.166	−3.440	−0.240	3.023	0.048		
				Volatility				
BRL	9.181	0.005	0.052	0.020	1.331	0.005	1.413	−0.030
RUB	0.007	3.981	0.043	0.013	0.090	0.002	0.155	−0.003
INR	0.253	0.076	5.828	0.015	0.084	0.002	0.430	−0.012
CNH	0.090	0.003	0.086	14.160	0.385	0.093	0.657	0.087
ZAR	0.486	0.050	0.049	0.005	4.430	0.006	0.596	0.002
GASS	0.036	0.005	0.014	0.007	0.004	35.740	0.065	
To	0.872	0.139	0.244	0.059	1.894	0.108	**0.553**	
Net	−0.541	−0.016	−0.186	−0.598	1.298	0.044		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASS: Henry Hub natural gas spot price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASS and exchange rates, which is calculated by the GASS to others minus the others to GASS. The number in bold means the total connectedness. The frequency band of medium term roughly corresponds to 5 days to 21 days. All results are expressed as a percentage.

Table A5. Connectedness between the natural gas spot price and exchange rates in the frequency domain (long term).

	BRL	RUB	INR	CNH	ZAR	GASS	From	GAS-FX
				Return				
BRL	2.825	0.437	0.095	0.087	0.833	0.016	1.468	0.014
RUB	0.415	3.847	0.138	0.149	0.722	0.007	1.431	0.001
INR	0.401	0.296	3.498	0.250	0.797	0.000	1.744	−0.010
CNH	0.155	0.132	0.144	3.921	0.415	0.010	0.857	0.010
ZAR	0.713	0.520	0.124	0.308	3.155	0.005	1.669	0.003

GASS	0.001	0.006	0.010	0.000	0.002	4.352	0.019	
To	1.685	1.391	0.512	0.793	2.769	0.038	**1.198**	
Net	0.217	−0.040	−1.232	−0.063	1.099	0.019		
				Volatility				
BRL	63.654	3.654	0.208	0.333	18.603	0.018	22.815	−0.343
RUB	0.882	92.332	0.696	0.283	0.362	0.011	2.234	0.001
INR	2.864	0.409	85.864	0.257	3.121	0.068	6.719	−0.221
CNH	3.580	0.209	2.605	66.285	6.682	0.598	13.674	0.591
ZAR	10.675	0.220	0.738	1.056	80.740	0.230	12.919	0.207
GASS	0.361	0.011	0.289	0.008	0.023	54.851	0.691	
To	18.362	4.502	4.535	1.937	28.790	0.926	**9.842**	
Net	−4.454	2.268	−2.183	−11.737	15.872	0.235		

Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South African Rand; GASS: Henry Hub natural gas spot price. From column: the total directional connectedness from others to x_i; To row: the total directional connectedness from x_i to others; Net row: the net total directional connectedness; GAS-FX column: the net pairwise connectedness between the GASS and exchange rates, which is calculated by the GASS to others minus the others to GASS. The number in bold means the total connectedness. The frequency band of long term roughly corresponds to more than 21 days. All results are expressed as a percentage.

Chapter 3

Can BRICS's currency be a hedge or a safe haven for energy portfolio? An evidence from vine copula approach

3.1 Introduction

With the development of emerging economies, developed economies no longer dominate and developing economies have also become major players under intense international competition. Developing economies constitute most of the global population, and account for almost half of the gross world product (GWP) (see Figure 1, which is based on data from the International Monetary Fund (IMF)). The gap between gross domestic product (GDP) for developed and developing economies has narrowed, especially after the financial crisis in 2008. In the meantime, BRICS, which is the association of five emerging economies (Brazil, Russia, India, China, and South Africa) is growing rapidly and these economies are likely to be a new center of gravity in the global economic system. Not only is it a major contributor to global economic growth, but BRICS countries have also accounted for a large proportion of world trade. Even when world total imports and exports are shrinking, BRICS's imports and exports still continue to grow. On the other hand, energy is a key factor for development. Based on the BP Statistical Review of World Energy (2019), consumption of primary energy (commercially-traded fuels, including modern renewables used to generate electricity) in BRICS countries increased from 3750.2 million tons oil equivalent (Mtoe) in 2008 to 5222.5 Mtoe in 2018, an increase of 39.2% over the past decade, accounting for 37.7% of total global primary energy consumption. Therefore, it is reasonable to assume that there is a relationship between the economic expansion of BRICS countries and energy markets.

The connection between energy and economy (Kraft and Kraft, 1978; Yu and Hwang, 1984; Yu and Choi, 1985) means that the energy market is easily affected by economic volatilities, and investing in energy portfolios carries an increased risk. Baur and McDermott (2010) provide definitions of a hedge and a safe haven. An asset can serve as a strong or weak hedge if it has no or a negative relationship with other assets or portfolios on average, while a strong or weak safe haven plays the same role as the hedge but can only be effective during extreme events such as a market crash. Thus, for investors in the energy market, choosing the right hedge asset or safe haven asset is crucial for risk management. Against this background, this study considers currency as a hedge or a safe haven asset for energy investment because exchange rate is important, it can affect international trade and help to determine a nation's economic health. In addition, since most energy resources are settled in US dollar, it is reasonable to think that there is a connection between currency market and energy market. In this study, we examine whether BRICS's currencies could be a hedge or a safe haven for energy market investment. There are several reasons behind this aim. First, with the ever-increasing importance of BRICS countries, we expect reserves of BRICS's currencies to increase, especially Chinese yuan. According to the IMF, official foreign exchange reserves of the Chinese yuan increased from 90.2 billion US dollars in 2016 Q3 to 219.6 billion US dollars in 2019 Q3 with a growth rate of about 50% per year. Second, BRICS countries established the Contingent Reserve Arrangement (CRA) in 2015, a 100 billion US dollar reserve pool, to deal with any short-run balance of payments pressures, which provides security to stabilize individual economies in the long run. Furthermore, most BRICS's currencies are stable, which makes us confident that BRICS's currencies could be a hedge or a safe haven for energy investors (Figure 2). Although the Russian ruble experienced significant fluctuations during 2014 to 2016, it could act as a hedge against risk in energy investment given its reliance on the energy market.

This study investigates the relationship between the energy market and BRICS's currencies to determine whether BRICS's currencies could hedge against risk in energy market investment. To this end, we choose crude oil and natural gas as representatives of energy markets. Compared with other energy resources, crude oil is plentiful and widely used, it can be refined into many other forms as a power source, more importantly, it has multiple uses such as plastics or chemical production. As Figure 3 reveals, although crude oil's share of global primary energy consumption is decreasing year by year, it still dominates the energy market. On the other hand, as a more environmentally friendly energy resource than other fuels such as coal, cross-regional trade of natural gas became viable thanks to the expansion of transmission pipeline networks and development of liquefied natural gas (LNG) storage. Furthermore, with technological innovation in horizontal drilling and hydraulic fracturing, shale gas has become an important source of natural gas, which is expected to raise the status of natural gas in the energy market. Crude oil and natural gas are also two important energy sources in BRICS countries. In terms of crude oil, in 2018, China consumed 641.2 Mtoe and accounts for 13.5% of global total consumption. After the United States, China is the second largest consumer of crude oil, followed by India. Russia is the second largest producer and consumer of natural gas, consuming 390.8 Mtoe and producing 669.5 Mtoe, which accounts for 11.8% and 17.3% of total global consumption and production, respectively. Details about consumption and production of crude oil and natural gas in BRICS countries are presented in Table 1.

A significant body of literature investigates the relationship between crude oil and exchange rates (Table 2 Panel A). However, there is relatively little research on the connection between natural gas and exchange rates (Table 2 Panel B). Our contribution to the literature is threefold.

First, we apply the vine copula approach, which is a novel approach introduced by Joe (1996), to investigate the relationship between the energy market and BRICS currencies. The copula model is a suitable approach for modeling multivariate data. Different marginal distributions and nonsymmetric dependencies between two variables are very common for multivariate data, which cannot be accommodated by conditional parametric distribution such as multivariate normal or student's t-distribution. Since a multivariate copula model still leaves many questions to be solved, vine copula provides a flexible way to construct a high-dimensional copula model using bivariate copula building blocks. The vine copula approach has already been widely used in the investigation of dependence structure, risk management, and so on (Table 2 Panel D).

Second, as a traditional hedge asset, the role of gold in the global financial system has already been widely examined. In recent years, numerous researchers have also considered Bitcoin as a new hedge asset instead of gold (Table 2 Panel C). However, to the best of our knowledge, the hedge ability of BRICS's currencies against energy market movement still remains understudied. In our previous research, He and Hamori (2019) and He et al. (2019), we used a copula model and connectedness methodology to investigate the relationship between the crude oil market and BRICS's currencies, and the correlation between the natural gas market and BRICS's currencies, respectively. We found that there is a significant negative relationship between crude oil and BRICS's exchange rates while the connection between natural gas and BRICS's exchange rates is modest, which provides evidence that BRICS's currencies could hedge crude oil risk and act as a weak hedge asset in natural gas investment. In this research, in order to rule out the influence of different methods to the results, we aim to use the alternative approach to reexamine the results.

Third, for investors, traders, and market makers, risk management is a necessary process. An appropriate estimation of risk measures can not only hedge risk, but also optimize trading strategies and improve investment decisions. Thus, we calculate two popular risk measurements, value-at-risk (VaR) and expected shortfall (ES), in two groups, a portfolio of energy price and a portfolio of energy price with five BRICS's exchange rates, and compare the difference. We also apply a moving-window approach to determine the time-varying VaR and ES in these two groups.

The rest of this paper is organized as follows. In section 2, we introduce the vine copula methodology, and the definitions of VaR and ES. We present our empirical results in section 3 and section 4 concludes. The results from an ARMA-GARCH model are reported in Appendix A. Appendix B outlines the functions of commonly used copulas, and Appendix C provides a brief introduction to graph and tree theory.

3.2 Methodology

3.2.1 Copula

A copula is a multivariate distribution with uniformly distributed marginals. Sklar (1959) first introduce the fundamental representation for multivariate distribution using copula conception. Let $\mathbf{X}\equiv[X_1,X_2,\ldots,X_n]'$ be a n-dimensional random vector. $\mathbf{X}$ has joint distribution function F and univariate marginal distribution F_i, i=1, 2, …, n. Then, the joint cumulative distribution function can be written as

$$F(x_1, x_2, \dots, x_n) = C(F_1(x_1), F_2(x_2), \dots F_n(x_n)) \tag{1}$$

with associated probability density function

$$f(x_1, x_2, \dots, x_n) = c\big(F_1(x_1), F_2(x_2), \dots F_n(x_n)\big) \times f_1(x_1) \times f_2(x_2) \times \dots \times f_n(x_n) \tag{2}$$

Casella and Berger (1990) indicate that when F_i is continuous, the variable of probability integral transforms $U_i\equiv F_i(X_i)$ will following uniform distribution. Thus, Briefly, a multivariate distribution can be decomposed into two parts, one is the uniform marginal distribution, the other is the dependence structure copula.

Depending on how we handle the marginal distributions, the multivariate distribution can be divided into two cases, parametric case and semiparametric case. If we model marginal distributions with parametric models, the multivariate distribution is parametric. Otherwise, if we model marginal distributions with non-parametric models, the multivariate distribution is semiparametric. In this paper, we consider both parametric and semiparametric cases using skewed t distribution function and empirical distribution function respectively.

As for copulas, up to now, there exists rich variety of bivariate copula families, they are usually divided into three classes, elliptical copulas (e.g. Normal copula, Student's t copula), Archimedean copulas (e.g. Clayton copula, Gumbel copula, Frank copula, Joe copula, BB copulas) and extreme-value copulas (e.g. Marshall-Olkin copula). The most commonly used bivariate copula functions and its contour plots are presented in Appendix B Table B1 and Figure B1 respectively.

3.2.2 Regular vine

Although copulas have already widely used in many filed, most applications are applied only in bivariate case, the higher dimensional case still leave many questions. On the other hand, regular vine provides a great flexible way to solve multivariate dependency problems and construct high dimensional copula model by combing bivariate copula building blocks. We follow Czado (2019) to give a brief introduction of regular vine.

3.2.2.1 Pair copula construction (PCC)

Conditioning is an appropriate way to construct multivariate distribution using bivariate building blocks. Considering a three random variables (X_1, X_2 and X_3) example.

The recursive factorization of the joint density given by

$$f(x_1, x_2, x_3) = f_{3|12}(x_3|x_1, x_2) \times f_{2|1}(x_2|x_1) \times f_1(x_1) \tag{3}$$

The conditional density of $f_{2|1}(x_2|x_1)$ can be expressed as

$$f_{2|1}(x_2|x_1) = c_{12}(F_1(x_1), F_2(x_2)) \times f_2(x_2) \tag{4}$$

Because $f_{3|12}(x_3|x_1,x_2)$ is the conditioning density of X_3 given X_1=x_1, X_2=x_2, it can be rewritten as

$$f_{3|12}(x_3|x_1,x_2) = f_{13|2}(x_1,x_3|x_2)/f_{1|2}(x_1|x_2) \tag{5}$$

By applying Sklar's theorem, we have

$$f_{13|2}(x_1,x_3|x_2) = c_{13;2}(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2);x_2) \times f_{1|2}(x_1|x_2) \times f_{3|2}(x_3|x_2) \tag{6}$$

The conditional density of $f_{3|2}(x_3|x_2)$ can be rewritten as

$$f_{3|2}(x_2|x_1) = c_{23}(F_2(x_2),F_3(x_3)) \times f_3(x_3) \tag{7}$$

Taking Equation (6) and Equation (7) into Equation (5), we have

$$\begin{aligned} f_{3|12}(x_3|x_1,x_2) = {} & c_{13;2}(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2);x_2) \times c_{23}(F_2(x_2),F_3(x_3)) \\ & \times f_3(x_3) \end{aligned} \tag{8}$$

Thus, a pair copula decomposition of an arbitrary three dimensional density is given as

$$\begin{aligned} f(x_1,x_2,x_3) = {} & c_{13;2}(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2);x_2) \times c_{23}(F_2(x_2),F_3(x_3)) \\ & \times c_{12}(F_1(x_1),F_2(x_2)) \times f_1(x_1) \times f_2(x_2) \times f_3(x_3) \end{aligned} \tag{9}$$

This decomposition is not unique, for example, it also can be written as

$$\begin{aligned} f(x_1,x_2,x_3) = {} & c_{12;3}(F_{1|3}(x_1|x_3),F_{2|3}(x_2|x_3);x_3) \times c_{13}(F_1(x_1),F_3(x_3)) \\ & \times c_{23}(F_2(x_2),F_3(x_3)) \times f_1(x_1) \times f_2(x_2) \times f_3(x_3) \end{aligned} \tag{10}$$

Or

$$\begin{aligned} f(x_1,x_2,x_3) = {} & c_{23;1}(F_{2|1}(x_2|x_1),F_{3|1}(x_3|x_1);x_1) \times c_{13}(F_1(x_1),F_3(x_3)) \\ & \times c_{12}(F_1(x_1),F_2(x_2)) \times f_1(x_1) \times f_2(x_2) \times f_3(x_3) \end{aligned} \tag{11}$$

Using Equation (9) with parameters $\theta_{13;2}$, θ_{23} and θ_{12} for copula density $c_{12;3}$, c_{23} and c_{12} respectively, a pair copula construction in three dimensions given as follows:

$$\begin{aligned} f(x_1,x_2,x_3;\boldsymbol{\theta}) = {} & c_{13;2}(F_{1|2}(x_1|x_2),F_{3|2}(x_3|x_2);\theta_{13;2}) \times c_{23}(F_2(x_2),F_3(x_3);\theta_{23}) \\ & \times c_{12}(F_1(x_1),F_2(x_2);\theta_{12}) \times f_1(x_1) \times f_2(x_2) \times f_3(x_3) \end{aligned} \tag{12}$$

Therefore, a pair copula construction for three variates copula density can be defined as follows:

$$\begin{aligned} c(u_1,u_2,u_3;\boldsymbol{\theta}) = {} & c_{13;2}(F_{1|2}(u_1|u_2),F_{3|2}(u_3|u_2);\theta_{13;2}) \times c_{23}(F_2(u_2),F_3(u_3);\theta_{23}) \\ & \times c_{12}(F_1(u_1),F_2(u_2);\theta_{12}) \end{aligned} \tag{13}$$

3.2.2.2 Regular vine tree sequence and regular vine distribution

With increasing of the number of variables, the pair copula construction will become more and more complex. For this reason, we usually use graph and tree sequence theory to illustrate the structure of regular vine. The concepts of graph and tree sequence are introduced in Appendix C.

Definition of regular vine tree sequence $V=(T_1,T_2,\ldots T_{n-1})$ is given as follows

i. Each tree $T_j=(N_j,E_j)$ is connected.

ii. T_1 is a tree with node set $N_1=\{1,2,\ldots,n\}$ and edge set E_1.

iii. For $j\geq2$, T_j is a tree with node set $N_j=E_{j-1}$ and edge set E_j.

iv. For $j=2,\ldots,n-1$ and $\{a,b\}\in Ej$, it must hold that $|a\cap b|=1$.

Figure 4 gives an example of regular vine tree sequence. It is easily to find that the edge labels of regular vine tree sequence ((1,2), (2,3) and (13;2)) match with the notations of pair copula construction (c_{12}, c_{23} and $c_{13;2}$) from the three dimensional example given before. Thus, we can express regular vine structure in a simple way.

In this backdrop, regular vine distribution can be expressed as a triplet (F,V,B), $F=(F_1,F_2,\ldots F_n)$ is a set of marginal distribution functions, V is an regular vine tree sequence, which represent the structure of regular vine copula, $B=\{C_e|e\in E_i; i=1,2,\ldots,n-1\}$ is a set of bivariate copula functions C_e, where E_i is the edge set of tree T_i in the regular vine tree sequence V.

3.2.2.3 Selection of regular vine tree sequence

As mentioned before, for multivariate copula, pair copula construction is not unique, so it is important to decide how to construct a high dimensional copula, in other words, it is crucial to decide the structure of regular vine tree sequence. A top-down approach proposed by Diβmann et al. (2013) is the most often used model selection algorithm for regular vine copulas. This approach selects the regular vine tree sequence start with the first tree T_1 and continue tree by tree up to the last tree T_{d-1}. Each tree decided by maximum a general weight ω. There are several possible choices for general weight, such as Akaike Information Criterion (AIC) of each pair copula, the absolute empirical Kendall's τ and so on. In this paper, we choose AIC as the weight ω.

We use a five-dimension example to illustrate the algorithm of top-down approach. Assume there are five variables (x_1, x_2, x_3, x_4 and x_5), for each tree, we first calculate the weight for each pair, then choose the pair with maximum weight value for each variable. Suppose in the first tree, the weight values for pair (x_1,x_2), (x_1,x_3), (x_1,x_4), (x_1,x_5), (x_2,x_3), (x_2,x_4), (x_2,x_5), (x_3,x_4), (x_3,x_5), (x_4,x_5) are $\omega_{12}=5$, $\omega_{13}=7$, $\omega_{14}=4$, $\omega_{15}=5$, $\omega_{23}=6$, $\omega_{24}=1$, $\omega_{25}=3$, $\omega_{34}=2$, $\omega_{35}=3$, $\omega_{45}=1$, respectively. For variable x_1, ω_{13} has the maximum value, thus we choose pair (x_1,x_3). Similarly, we choose pair (x_2,x_3), (x_1,x_4), (x_1,x_5). In the second tree, under the definition of tree sequence, we will have

4 pairs, {(x_1,x_3),(x_1,x_4)}, {(x_1,x_3),(x_1,x_5)}, {(x_1,x_3),(x_2,x_3)} and {(x_1,x_4),(x_1,x_5)}. Supposing the weight values are $\omega_{3,4;1}$=5, $\omega_{3,5;1}$=6, $\omega_{1,2;3}$=4, $\omega_{4,5;1}$=2, respectively. In the same way, we choose pair {(x_1,x_3),(x_1,x_4)}, {(x_1,x_3),(x_1,x_5)} and {(x_1,x_3),(x_2,x_3)} for the second tree. Therefore, the structure of T_1 and T_2 are decided, the plot are presented in Figure 5. For the leftover tree, we use the same way to decide the structure.

3.2.3 Value-at-risk and expected shortfall

Value-at-risk (VaR) and expected shortfall (ES) are two popular method for estimating the risk in investment. VaR used to calculate the probability of investment loss over a specified period with a certain level of confidence, while ES, which is also called conditional value-at-risk, measure the expected losses that are greater or equal to VaR. Both VaR and ES tell investors the level of potential risk in an asset or a portfolio investment, help them to do a better decision. For example, a one-day 95% VaR of 3% means that with 95% confidence, we expect that our worst daily loss will not exceed 3%. A one-day 95% ES of 10 US dollars means that the daily expected loss of the worst 5% scenario is 10 US dollars.

According to Artzner et al. (1997; 1999), the mathematical expression of VaR and ES at 100(1−p) percent confidence level can be defined as

$$VaR_p(X) = inf\ \{x|P(X \leq x) > p\} \tag{14}$$

$$ES_p(X) = E[X|X \geq VaR_p(X)] \tag{15}$$

Where X is a random variable, inf{x | A} is the lower limit of x given A.

There are several methods to calculate VaR such as historical method, gaussian method. In this paper, we calculate VaR and the corresponding ES by using modified method, which adjust the standard deviation to account for higher-order moments, skewness and kurtosis, of the asset or portfolio distribution. The modified VaR (MVaR) at 100(1−p) percent confidence level is defined as

$$\begin{gathered} Z = (Z_c + 1/6\ (Z_c^2 - 1)S + 1/24\ (Z_c^3 - 3Z_c)K - 1/36\ (2Z_c^3 - 5Z_c)S^2) \\ MVaR_p(X) = \mu - Z\sigma \end{gathered} \tag{16}$$

Where Z is the Cornish-Fisher expansion for the quantile of the probability distribution of X, Z_c is the quantile of the distribution of X, S is the skewness of X, K is the kurtosis of X, μ is the mean of X and σ is the standard deviation of X.

3.3 Empirical results

3.3.1 Data sets

We use daily data over period 24 August 2010 to 29 November 2019. For exchange rates, we use data from BRICS countries, which are Brazilian Real (BRL), Russia Ruble (RUB), India Rupee (INR), Chinese offshore Yuan (CNH) and South Africa Rand (ZAR) against US dollar, respectively. Specially we use Chinese offshore Yuan instead of Chinese Yuan (CNY) in this study. Although China had adopted a floating exchange rate regime in 2015, its exchange rate fluctuated in a narrow range for a long period, such as before 2005 or the period from 2008 to 2010. This longstanding steadiness of exchange rate may have influence on the results, thus we use a more volatile exchange rate—Chinese offshore Yuan—to conduct our analysis. For energy market, we use crude oil and natural gas future prices. Crude oil prices contain West Texas Intermediate (WTI), which is the benchmark for North America, and Brent North Sea Crude (BRENT), which is the benchmark for African, European, and Middle Eastern crude oil, for robustness analysis. Natural gas prices contain the United States Henry Hub future (HH)—the world's biggest natural gas hub—and United Kingdom National Balancing Point future (NBP), which is one of the main natural gas hub in Europe, for robustness analysis. All data series are obtained from Bloomberg.

We use Equation (17) to obtain the stationary return series, which are expressed in percentage:

$$r_{i,t} = 100 \times \ln{(p_{i,t}/p_{i,t-1})} \tag{17}$$

The return series are plotted in Figure 6. Figure 6 shows obvious volatilities around 2015 in WTI, BRENT and RUB return series, when crude oil price crashed, which leaded to the financial turmoil in Russia. HH return series fluctuate most in winter season, we consider that Henry Hub natural gas price is easily affected by demand, when temperature is lower, the demand for natural gas is higher. NBP return series had drastic fluctuation at the latter half of 2019, when natural gas price in Europe reached its historical low level as a natural gas war between the United States and Russia happened—Russia has been Europe's main gas supplier, nevertheless the US also want to expand the natural gas market share in Europe. CNH return series are steady compare to other currencies which is likely to attribute to the government regulation in China. The statistics for daily returns are presented in Table 3. Standard deviations are consisted with Figure 6, CHN has the lowest standard deviation while HH has the highest one. Skewness, kurtosis and the results from Jarque-Bera test show all return series apparently deviate from normal distributions. In addition, we apply a test from Bai and Perron (1998; 2003) to eliminate the influence of structural breaks. The p-value of structural breaks test on return series are presented in Table 4. All results show no breakpoint in our data period.

3.3.2 Vine copula results

Because the copula model request independent and identically distributed (i.i.d) uniform data, we first take the return series into autoregressive moving average Glosten-Jagannathan-Runkle GARCH (ARMA-GJR-GARCH) models to obtain standard residuals, which is i.i.d data (the results of GJR-GARCH model are presented in Appendix A), then take the standard residuals into skewed t distribution (parametric case) and empirical distribution (semi-parametric case) respectively to get copula data which follow uniform distributions. Finally, we take the transformed copula data into vine copula models.

Different types of copula models have different properties and different range of parameters, for example, Clayton copula can only capture negative dependence structure while Gumbel copula can only capture positive one. Student's t copula, which have two parameters, one is the degree of freedom parameter, the other is the dependence parameter, taking value in $[-1,1]$, negative values show negative dependence and visa verse. In addition, Student's t copula can also capture symmetric tail dependence. Therefore, we choose Student's t copula models for all pair copula construction in vine copula tree sequence.

The structure of vine copula is plotted in Figure 7. Table 5 shows the vine copula estimation results of parametric case while table 6 shows the results of semi-parametric case. The values of loglikelihood, AIC and Bayesian Information Criterion (BIC) indicate that semi-parametric case fit the data better than parametric case. In both case, most WTI-exchange rate pairs show negative dependence, except WTI-INR pair in parametric case, WTI-INR and WTI-CNH pairs in semi-parametric case. Although the strength of dependence for WTI-RUB pair is strong, that for other pairs are weak. We consider the reason of this difference is because Russian economy heavily rely on the export of energy resource more than other BRICS countries. In the main, the general weak negative dependence structure between WTI and BRICS's exchange rate gives us an evidence that

BRICS's currency can provide a weak hedge against oil price movement. There are little difference between our results and the previous literatures, most of them have indicated that negative relationship is existed between crude oil price and exchange rate. Our previous research, He and Hamori (2019), also show strong negative dependence structure among all crude oil-BRICS's exchange rate pairs by using independent bivariate copula models. For HH-exchange rate pairs, most indicate positive dependence structures, but the strength are weak, which mean the connection between natural gas price and BRICS's exchange rate is modest, BRICS's currency cannot serve as a hedge against natural gas price movement. This result is consisted with He et al. (2019).

Tail dependence describes the co-movement in the tail of bivariate distribution. Loosely speaking, tail dependence measures when one margin has exceeded a certain threshold, the limiting proportion of the other margin exceed that threshold. In both case and each pair, the value of tail dependence are about to zero, which indicates that there are no co-movement during extreme events. This results reveal that BRICS's currency can serve as a safe haven to energy price movement.

3.3.3 Value-at-risk and Expected shortfall

Based on the vine copula estimation, we calculate the VaR and ES of equally weighted portfolios. The portfolios are divided into two groups, one group is as the benchmark, which composed of WTI and HH, the other group add BRICS's exchange rate into the benchmark portfolio. The results are presented in Table 7. Both VaR and ES indicate that the mixed portfolios, which consist of BRICS's exchange rate and two energy prices, reduce the risk and perform better than the benchmark portfolio which only consist of WTI and HH. This results underline the risk reduction effectiveness of BRICS's currency, confirm the results from vine copula estimation.

Rolling-window method is applied to obtain the time-varying VaR and ES. Following previous researches, we choose the window size of 250 (almost one year). We take the following steps to obtain the time-varying VaR and ES.

1. Generate simulations from fitted vine copula model.
2. Transform the simulated uniform margin from step 1 to standard residuals.
3. Convert the standard residuals from step 2 to return series using conditional mean and conditional variation from ARMA-GARCH models.
4. Generate an equally weighted portfolio of return series from step 3, calculate

the 1% and 5% VaR and ES.

5. Repeats step 1 to 4 for the next moving window.

The time-varying VaR and ES are plotted in Figure 8 and Figure 9, respectively. The values of VaR and ES of portfolio composed of BRICS's exchange rate and two energy prices in each time t are lower than that of the benchmark portfolio. Although this difference is not obvious in the plot, we can see the volatility of equally weighted return is significantly reduced when adding the BRICS's exchange rate into the benchmark portfolio. VaR and ES in semi-parametric case have more fluctuations than that in parametric case. The fluctuations happened around 2012, 2015 and the latter half of 2018, when crude oil price or natural gas price have sudden up and down. For example, in 2012, the average wholesale price for Henry Hub natural gas price dropped 31% compare to the average price in 2011, reached its lowest average annual price since 1999. In 2015, the crude oil price crash happened and at the latter half of 2018, crude oil dropped sharply while natural gas price reached a peak.

3.3.4 robustness

In order to check the effectiveness of our results, we use BRENT and NBP future price, which are two famous energy price in European market to conduct a robustness analysis.

The structure of vine copula is plotted in Figure 10. This structure is totally the same with the WTI-HH case. The estimation results of vine copula are summarized in Table 8 (parametric case) and Table 9 (semi-parametric case). Although the negative or positive relationship in some pairs have opposite direction with the WTI-HH case, generally, the two results are similar, all reveal that BRICS's currency can hedge risk in oil market but cannot in natural gas market. The results of tail dependence are also the same, indicate that BRICS's currency can serve as a safe haven to energy market.

As we have done before, we divide the equally weighted portfolios into two groups, benchmark portfolio only consist of BRENT and NBP prices, the other portfolio add BRICS's exchange rate into the benchmark portfolio. The results of VaR and ES in two groups are shown in Table 10. Both VaR and ES are significantly reduced by adding the BRICS's exchange rate. The time-varying VaR and ES are plotted in Figure 11 and Figure 12, respectively. Both VaR and ES are similar to the results from WTI-HH case, several fluctuations happened around 2012, 2015 and the second half of 2018, when energy prices changed suddenly.

3.4 Conclusion

This study examines the role of BRICS's currencies in the energy market. To this end, the vine copula method is applied. To conduct our analysis, we choose BRICS's exchange rates for the currency market, and WTI crude oil and Henry Hub natural gas future prices for the energy market. BRENT crude oil and NBP natural gas future prices are used to check the robustness of our research. In addition, two commonly used risk measurements, VaR and ES, are calculated. The data period is from August 24, 2010 to November 29, 2019.

The results of vine copula show that a negative dependence structure exists in most crude oil-exchange rate pairs while the results are the opposite in natural gas-exchange rate pairs. In each pair, the strength of dependence is weak except pairs with the ruble, which is considered to be caused by the Russian economy's high reliance on the energy market. The results from crude oil-exchange rate show little difference from our previous research, which found a strong negative dependence between crude oil price and exchange rates. However, the results from natural gas-exchange rates are consistent with our previous study that there are weak connections between natural gas price and exchange rate. Tail dependence based on vine copula suggests that there are almost no co-movements between energy price and exchange rate in extreme events. According to the definition of hedge and safe haven, our empirical results provide evidence that BRICS's currencies can provide a hedge to crude oil movement but not to natural gas movement. BRICS's currencies can serve as a safe haven asset for both crude oil and natural gas and reduce the investment risk when faced with a market crash.

We calculate the VaR and ES of two portfolios. One consists of only crude oil and natural gas prices, which is the benchmark portfolio, the other is composed of two energy prices and BRICS's exchange rates. The results show adding BRICS's exchange rates into the benchmark portfolio is highly effective in terms of risk reduction, which confirms the conclusion from vine copula estimation.

Although the roles of gold and Bitcoin in financial systems are widely examined, there is still a lack of research on the hedge ability of BRICS's currencies in the energy market. According to our results, it is advisable to pay more attention to the BRICS's currency market, especially when investors consider investing in the energy market. Our findings provide a new viewpoint for researchers and participants in the energy market, which may assist them in making better investments and risk management.

This research has at least two limitations. First, considering crude oil and natural gas are two principle energy resources, we chose WTI, BRENT, HH, and NBP to represent the energy market. However, both crude oil and natural gas are unrenewable fossil fuels. With increasing environmental awareness, renewable resources such as wind power, and solar energy will become more important and also need to be considered. Second, we only used the vine copula method to calculate VaR and ES but did not compare it with other methods. Therefore, for our future research, we first want to add renewable energy data into our analysis to compare its relationship with BRICS's currencies and the relationship among crude oil, natural gas, and BRICS's currencies. Second, to measure the accuracy of our results, we want to compare the results of VaR and ES from vine copula and other methods such as the GARCH model.

Appendix A Results of marginal distribution

Table A1 reports the results of ARMA-GJR-GARCH models. the lags of ARMA model are decided by AIC criterion. Almost all estimated parameters from GJR-GARCH model are significant at the 1% level. The results of the 20 lags Ljung-Box test and Lagrange Multiplier are presented in Table A2. All value are insignificant which indicate that the standardized residuals of GJR-GARCH model have no serial correlation and autoregressive conditional heteroscedasticity.

Appendix B Bivariate copula models

Table B1 summarizes common bivariate copula functions, which contain Normal, Student's t , Frank, Clayton, Gumbel and Joe copula, Figure B1 presents its contour plots.

Appendix C Graph and tree theory

According to Diestel (2006), a graph G=(N,E) is a pair of sets such that $E \subseteq \{\{x,y\}: x,y \in N\}$. E is called edged, while N is called node of the graph G. Figure C1 gives 4 examples of graphs.

Definition of tree T=(N,E) is given as follow

i. Any two nodes of T are connected by a unique path in T (T is undirected).
ii. T is minimally connected (T is connected).
iii. T is maximally acyclic (T contains no cycle).

Under the definition of tree , in Figure C1, (1), (2), (3) are not tree, (4) is a tree

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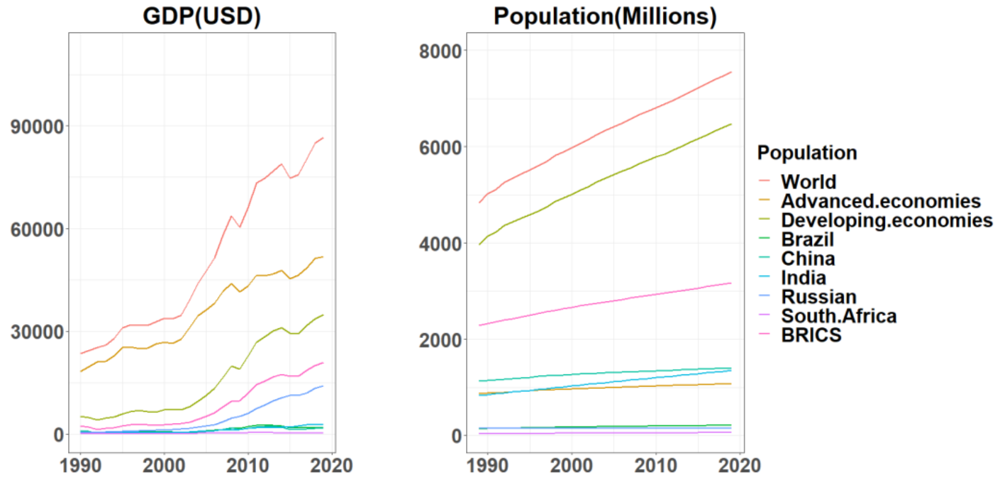


Figure 1. GDP and population by regions. Notes: BRICS: Brazil, Russia, India, China and South Africa. Data source: International Monetary Fund (IMF).

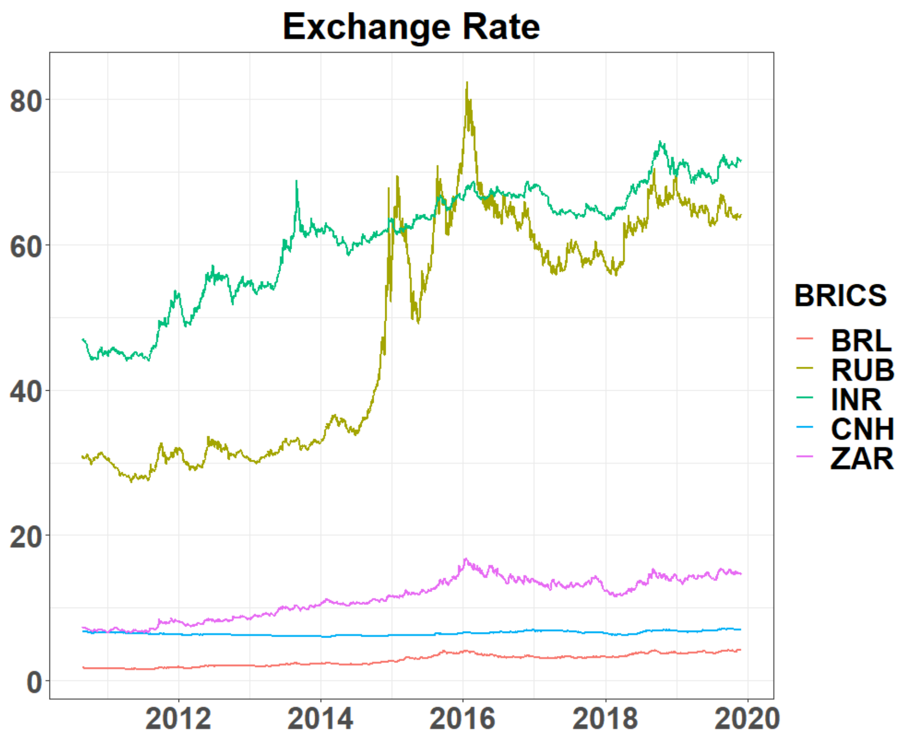


Figure 2. daily price of BRICS exchange rate. Notes: BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. Data source: Bloomberg.

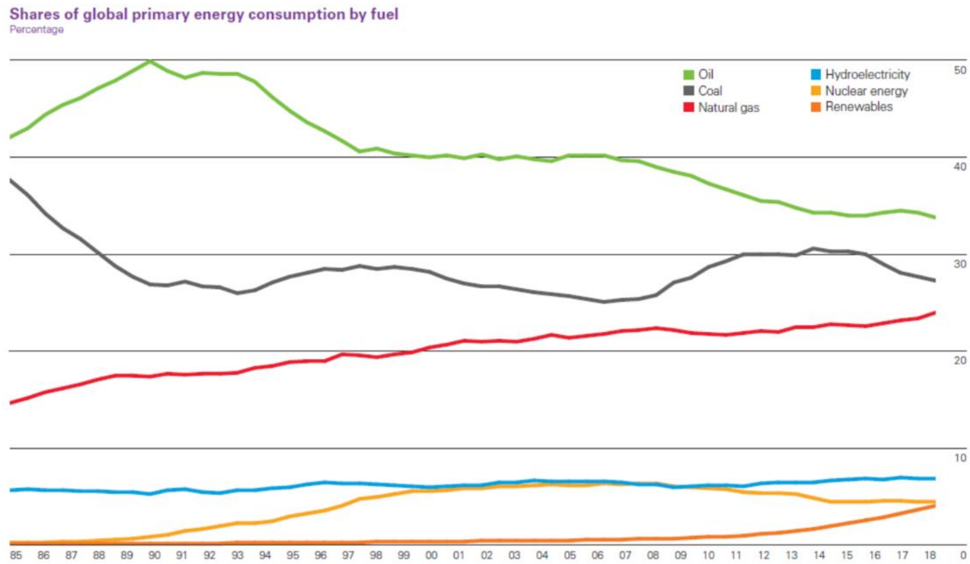


Figure 3. Share of global primary energy consumption by fuel. Source: BP statistical review of world energy (2019).

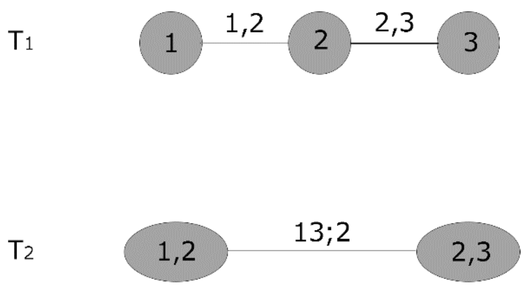


Figure 4. example of regular vine tree sequence.

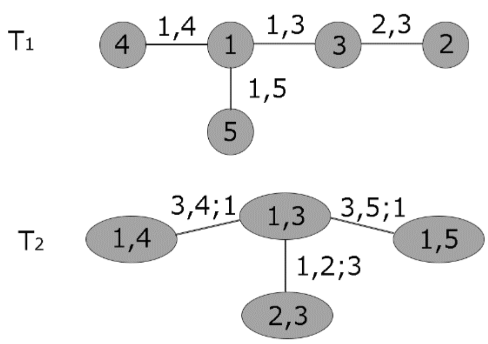


Figure 5. example of regular vine tree sequence selection.

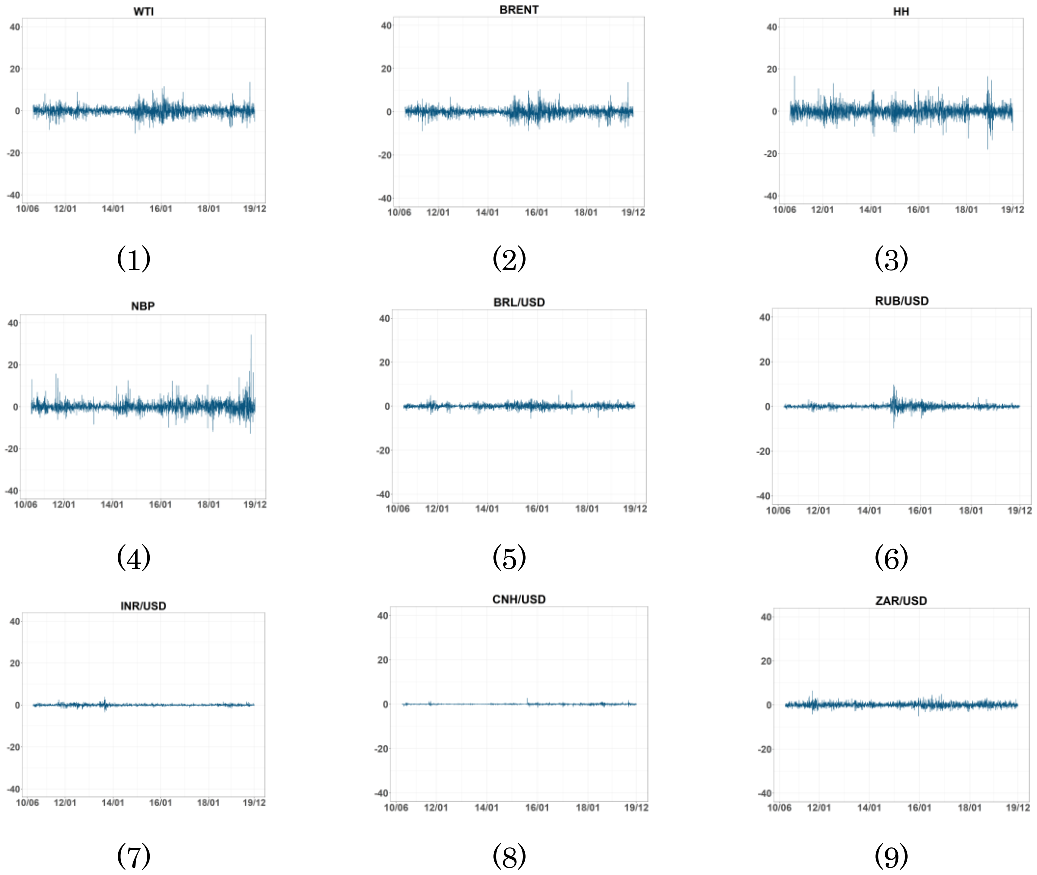
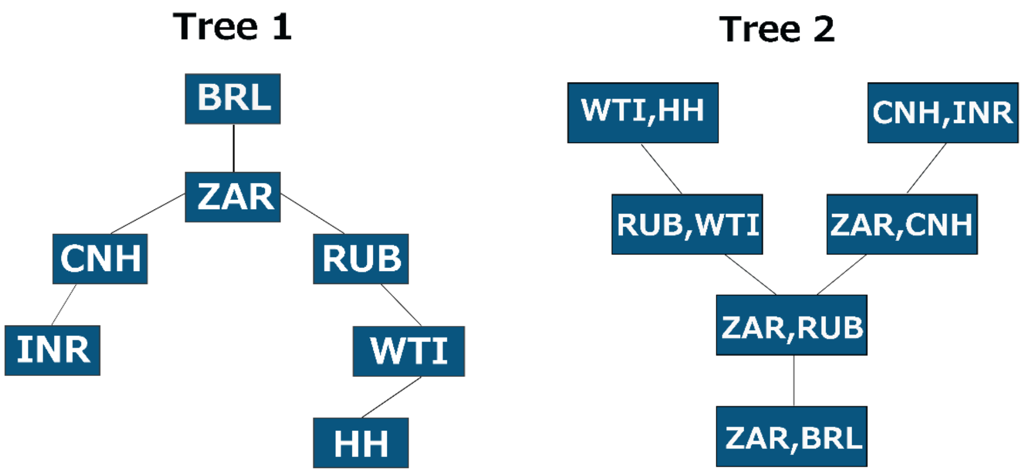


Figure 6. Daily return. Notes: WTI: West Texas Intermediate crude oil price future; BRENT: Brent crude oil price future; HH: Henry Hub natural gas price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. (1–9) refer to BRL, RUB, INR, CNH, ZAR, and GASF return series, respectively.



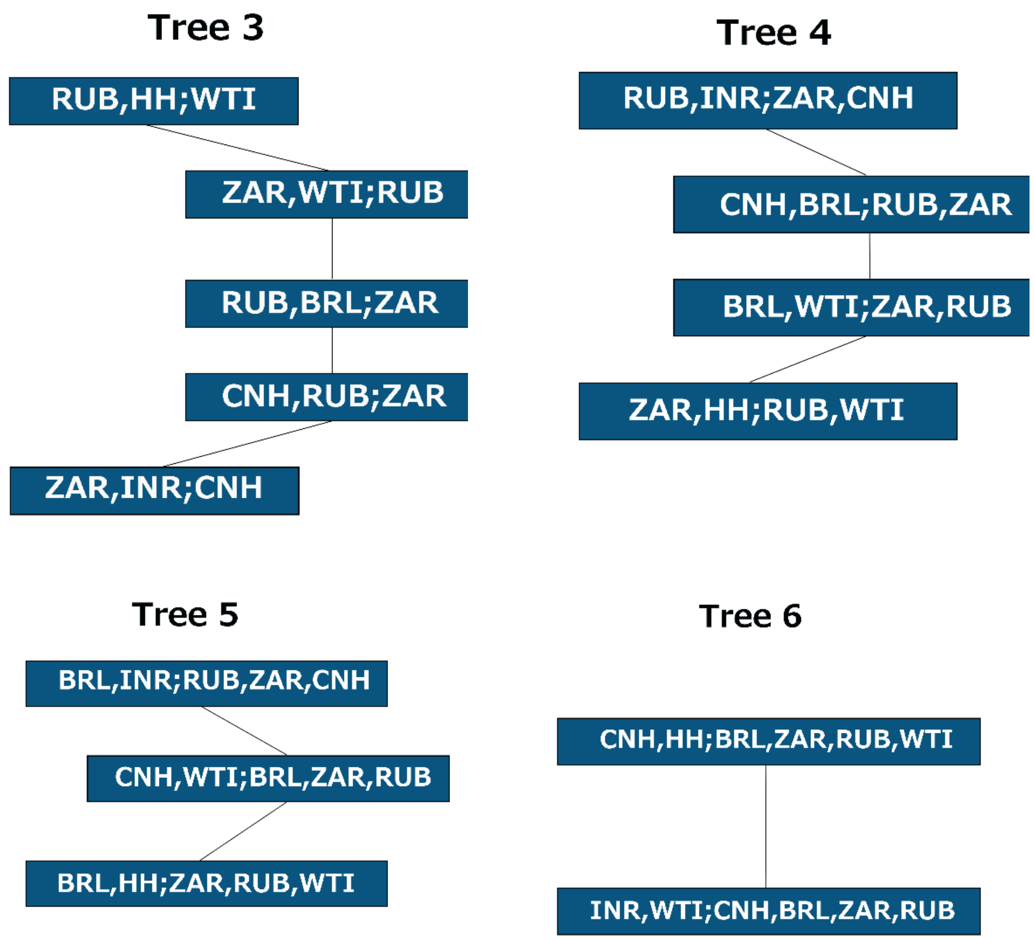
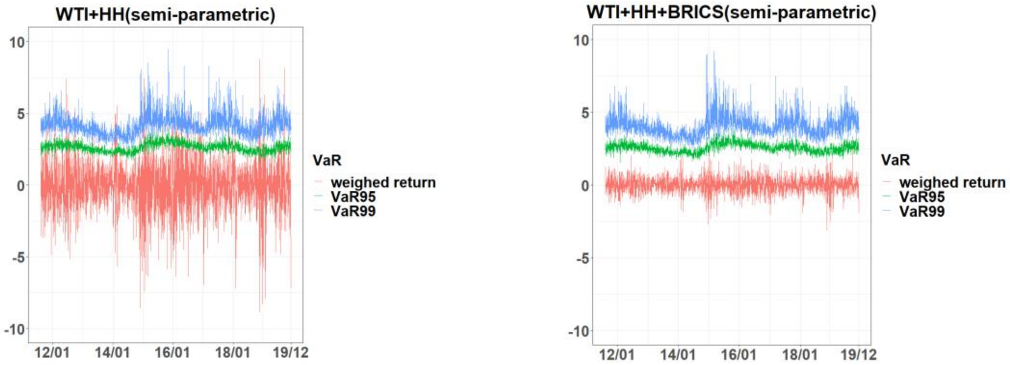


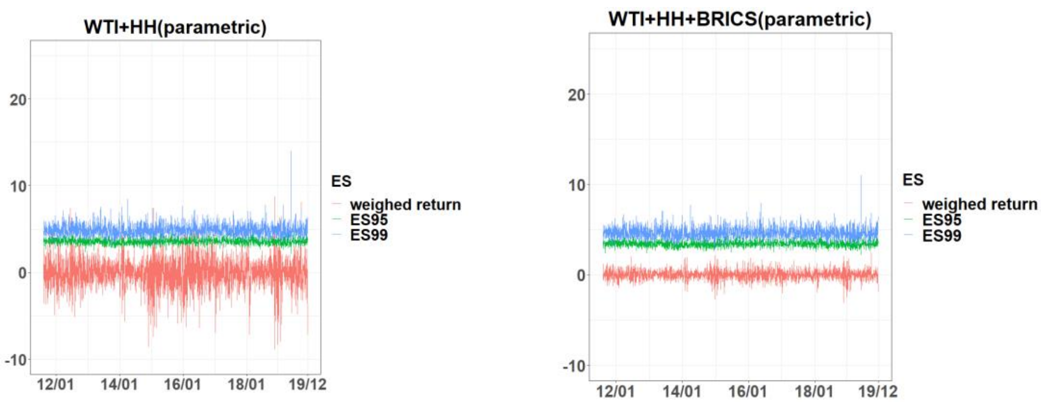
Figure 7. Regular vine tree structure (WTI+HH+BRICS). Notes: WTI: West Texas Intermediate crude oil price future; HH: Henry Hub natural gas price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

(1) (2)



(3) (4)

Figure 8. Time-varying value-at-risk (window size=250). Notes: WTI: West Texas Intermediate crude oil price future; HH: Henry Hub natural gas price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. (1): Parametric case (WTI+HH); (2): Parametric case (WTI+HH+BRICS); (3): Semi-parametric case (WTI+HH); (4): Semi-parametric case (WTI+HH+BRICS).



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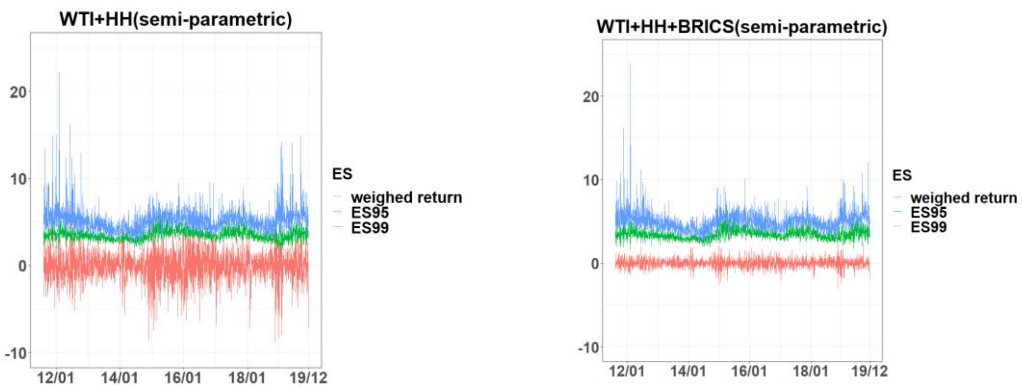
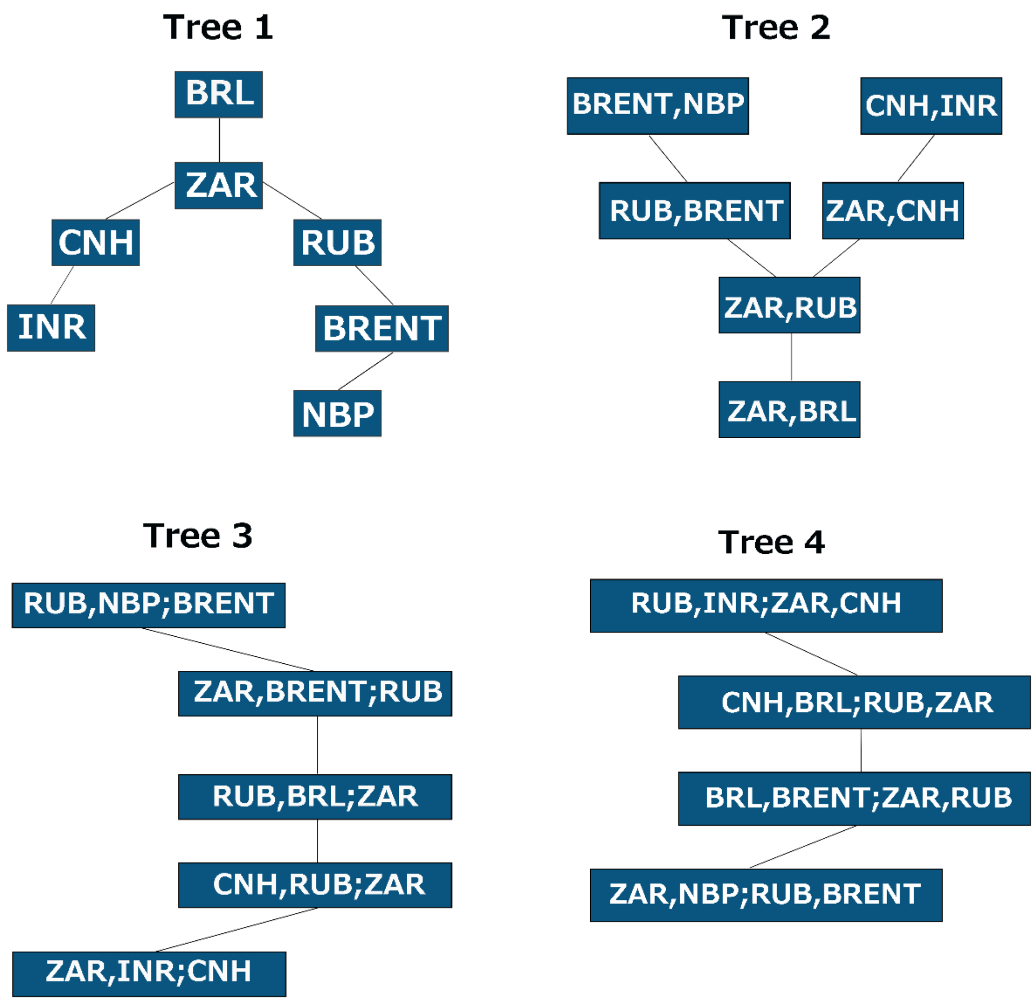


(3) (4)

Figure 9. Time-varying expected shortfall (window size=250). Notes: WTI: West Texas Intermediate crude oil price future; HH: Henry Hub natural gas price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. (1): Expected shortfall of parametric case (WTI+HH); (2): Expected shortfall of parametric case (WTI+HH+BRICS); (3): Expected shortfall of semi-parametric case (WTI+HH); (4): Expected shortfall of semi-parametric case (WTI+HH+BRICS).



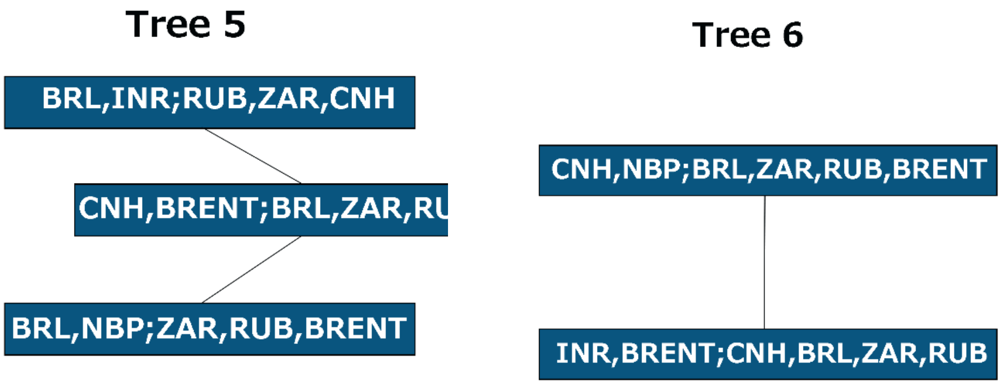
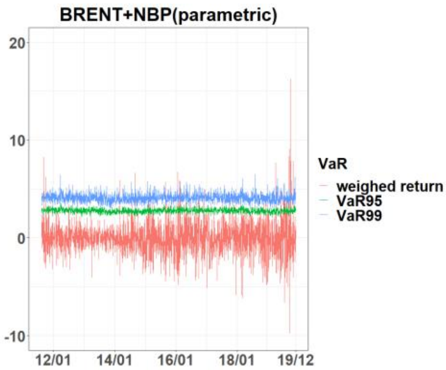
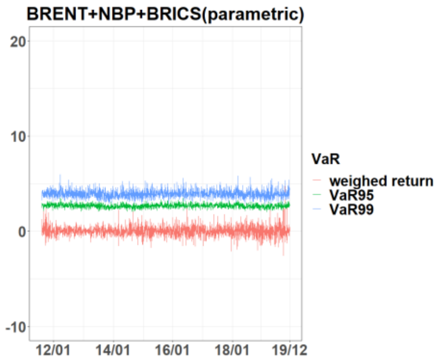


Figure 10. Regular vine tree structure (BRENT+NBP+BRICS). Notes: BRENT: Brent crude oil price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.



(1)



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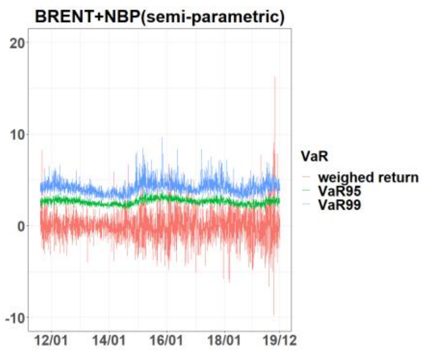

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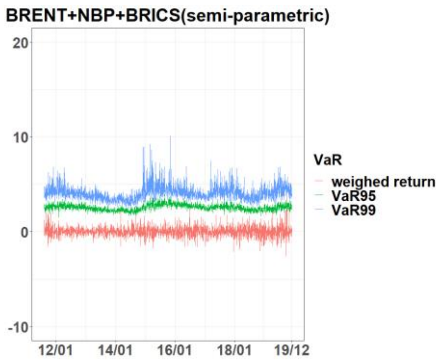
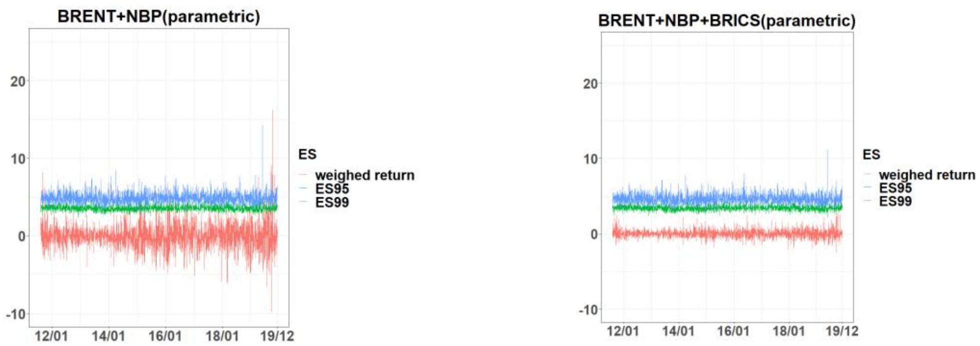


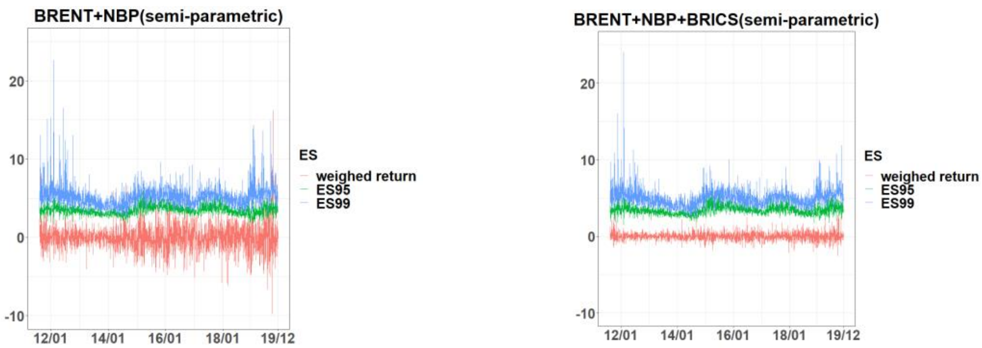
(4)

Figure 11. Time-varying value-at-risk (window size=250). Notes: BRENT: Brent crude oil price future; NBP: National Balancing Point natural gas

price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. (1): Parametric case (BRENT+NBP); (2): Parametric case (BRENT+NBP+BRICS); (3): Semi-parametric case (BRENT+NBP); (4): Semi-parametric case (BRENT+NBP+BRICS).

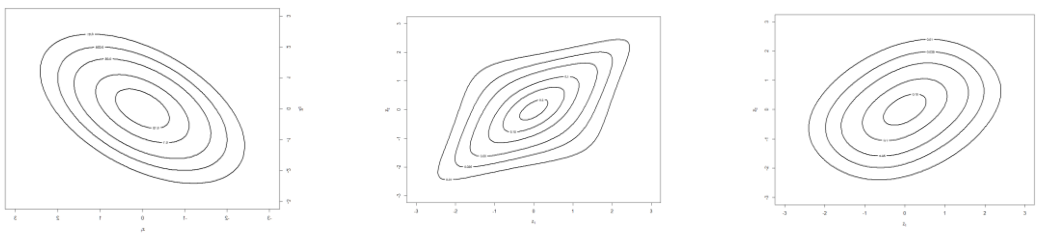


(1) (2)



(3) (4)

Figure 12. Time-varying expected shortfall (window size=250). Notes: BRENT: Brent crude oil price future; NBP: National Balancing Point natural gas price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. (1): Expected shortfall of parametric case (BRENT+NBP); (2): Expected shortfall of semi-parametric case (BRENT+NBP); (3): Expected shortfall of parametric case (BRENT+NBP+BRICS); (2): Expected shortfall of semi-parametric case (BRENT+NBP+BRICS).



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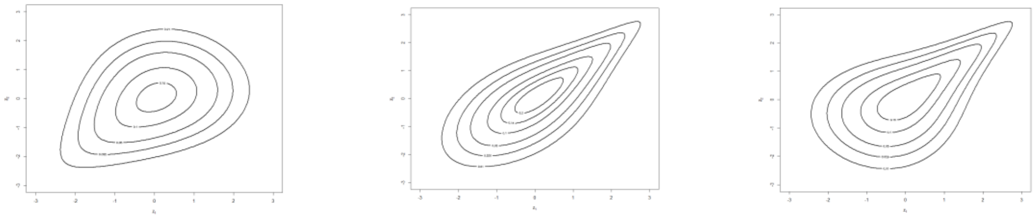
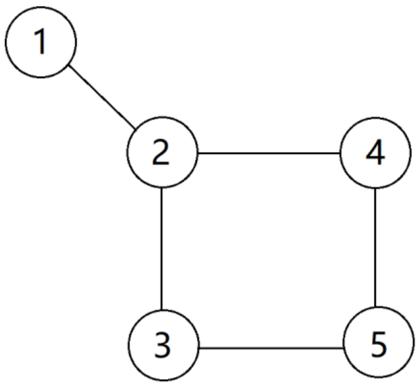
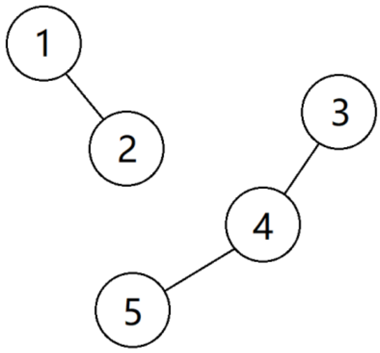


(4) (5) (6)

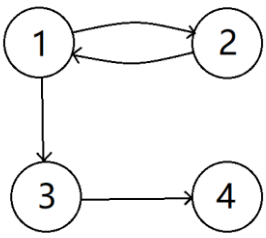
Figure B1. Contour plots of common bivariate copula models. Notes: (1): Normal copula (θ=0.5); (2): Student t copula (θ1=0.5, θ2=3); (3): Frank copula (θ=2); (4): Clayton copula (θ=0.5); (5): Gumbel copula (θ=2); (6): Joe copula (θ=2).



(1)



(2)



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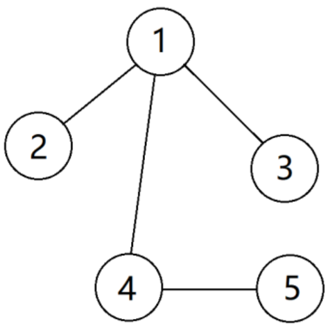

(4)

Figure C1. Example graphs. Notes: (1): graph with a cycle (2-4-3-5-2); (2): disconnected graph; (3): directed graph; (4): graph.

Table 1. Consumption and production of crude oil and natural gas in BRICS.

	Brazil	Russia	India	China	South Africa
	Consumption (million tonnes oil equivalent)				
Crude oil	135.9	152.3	239.1	641.2	26.3
Share	2.9%	3.3%	5.1%	13.8%	0.6%
Growth rate(2008-2018)	20.7%	10.3%	60.1%	66.7%	4.0%
Natural gas	30.9	390.8	49.9	243.3	3.7
Share	0.9%	11.8%	1.5%	7.4%	0.1%
Growth rate(2008-2018)	39.8%	7.5%	45%	245.6%	8.8%
	Production (million tonnes oil equivalent)				
Crude oil	140.3	563.3	39.5	189.1	-
Share	3.1%	12.6%	0.9%	4.2%	-
Growth rate(2008-2018)	42%	14.0%	4.5%	−0.7%	-
Natural gas	21.6	669.5	27.5	161.5	-
Share	0.7%	17.3%	0.7%	4.2%	-
Growth rate(2008-2018)	74.2%	9.5%	−6.7%	99.6%	-

Notes: This table is based on BP Statistical Review of World Energy (2019). the production of crude oil and natural gas in South Africa are too small such that no details.

Table 2. Literature summary.

Authors	Data	Period	Methodology	Principle results
Panel A: Relationship between crude oil and exchange rate				
Krugman (1983)	-	-	Theoretical analysis with three models: (1) trade balance model; (2) model with capital flows; (3) model with speculation	1. From model (1), after an oil shock, oil-dependent countries will tend to have depreciation while countries which are relatively successful at selling to OPEC tend to have appreciation. 2. From model (2), in the short term, after an oil shock, currency will initially depreciate if share of that currency in OPEC's portfolio is less than share of the country with that currency in world oil imports. The long term effects is opposite. 3. From model (3), currency appreciate initially after oil price increase. In the long term, currency depreciate.
Golub (1983)	US Dollar; UK Sterling; Japanese Yen; Deutsche Mark	1972~1980	Stock/flow model of the effect of oil price increase on exchange rates	If oil price increases lead to reallocate the world wealth in the manner of increasing the demand for currency A and lower the demand for currency B, then currency A will appreciate against the currency B.
Chen and Chen (2007)	Nominal exchange rates (G7 countries);	January 1972~October 2005	Panel analysis: cointegration test; prediction regression	1. Cointegrating relationship exists between oil prices and exchange rates. 2. Oil price has significant forecasting power for

	world average crude price; the United Arab Emirates price of oil; Brent; West Texas Intermediate		test	exchange rates.
Lizardo and Mollick (2010)	Nominal exchange rates of Canada, Denmark, Euro zone, Japan, Norway, Mexico, Russia, Sweden and the UK; WTI	Canada, Denmark, Euro zone, Sweden, the UK: January 1975~December 2007; Japan: January 1980~December 2007; Mexico: January 1995~December 2007	Cointegration test; vector error-correction models (VECM)	1. For oil-exporter countries, increase of oil price cause appreciation of its currency. 2. For oil-importer, increase of oil price cause depreciation of its currency. 3. For country neither exporter nor importer, currency depreciate.
Wen et al. (2018)	Trade Weighted U.S.	7 January 2000~25 July	Hiemstra and Jones test; the Diks	1. From crude oil to exchange rate, nonlinear granger causality is existed but not vice versa.

	Dollar Index; WTI;	2014	and Panchenko test; time-varying parameter structural vector autoregression model; Dynamic Conditional Correlation-General Autoregressive Conditional Heteroscedasticity model (DCC-GARCH)	2. In the short run, exchange rate has negative influence on crude oil, but this influence gradually weakens after 2012. 3. The negative dependence increases greatly during the economic crash.
Nusair and Olson (2019)	Real exchange rates of Indonesia, Japan, Korea, Malaysia, Philippines, Singapore and Thailand; Dubai price of oil	1973 Q2~ 2016 Q4	Quantile model	Crude oil shock has asymmetric impact on exchange rate. This impact vary in countries, market conditions. When domestic currency in bullish market, increase of oil prices leads to appreciation of the currency of Indonesia, Korea, Philippines and Thailand. However, When domestic currency in bearish market, increase (decrease) of oil price leads to depreciation of the currency of Indonesia (Malaysia).
Panel B: Relationship between natural gas and exchange rate				
Wang and Wu (2012)	Trade-weighted US	2 January 2003~3 June	Linear and nonlinear Granger	1. Exchange rate and natural gas are not cointegrated.

	exchange rate; crude oil; gasoline; heating oil; natural gas	2011	causality test	2. Before financial turmoil, weak nonlinear causality exists from exchange rate to natural gas.
Panel C: Risk management				
Baur and McDermott (2010)	Gold; world stock indices; G7 stock market indices; BRIC stock indices; Australia stock indices; Switzerland stock indices	2 March 1979~2 March 2009	Regression analysis	1. For most developed countries, gold has safe haven effect while for some emerging countries, gold is only a weak safe haven. 2. During the financial crisis, gold is a strong safe haven for most developed countries.
Selmi et al. (2018)	Oil; gold; Bitcoin	13 September 2011~29 August 2017	Quantile on quantile regression	1. Bitcoin and gold can serve as safe havens, hedges, and diversifiers. But this abilities are sensitive to difference degrees of oil price movements and market conditions. 2. The connection between oil and gold is weaker than it and Bitcoin.
Panel D: Application of vine copula				
Hernández (2014)	Coal-uranium stocks;	January 2005~July 2012	Vine copula	During financial crisis period, oil stocks have higher risk than uranium, the coal and gas stocks.

	oil-gas stocks			
Aloui and Aïssa (2016)	WTI; Dow Jones Industrial Average Index; trade weighted US dollar	4 January 2000~31 May 2013	Vine copula	Crude oil price rise, currency depreciate.
Yu et al. (2018)	WTI; Brent; Dubai crude oil; Cinta crude oil	20 June 2014~30 September 2016	GARCH model; vine copula; extreme value theory (EVT)	The combination of GARCH-type-EVT with vine copula methods can produce accurate risk measure of the oil portfolio.

Table 3. Summary statistics for daily returns.

	Min	Max	Mean	Std Dev	Skewness	Kurtosis	JB-Test
WTI	−10.794	13.694	−0.011	2.051	0.068	3.767	1435.779***
BRENT	−8.963	13.639	−0.006	1.906	0.120	3.911	1551.745***
HH	−18.055	16.691	−0.024	2.748	0.105	3.978	1603.198***
NBP	−12.815	34.299	0.012	2.476	1.696	20.401	43174.403***
BRL	−5.601	7.270	0.036	0.941	0.141	3.745	1425.083***
RUB	−9.771	9.731	0.030	1.005	0.446	14.369	20923.565***
INR	−3.294	3.904	0.018	0.447	0.311	7.550	5794.669***
CNH	−1.471	2.747	0.002	0.233	0.516	13.967	19799.746***
ZAR	−5.081	6.444	0.028	0.977	0.273	2.005	436.788***

Notes: WTI: West Texas Intermediate crude oil price future; BRENT: Brent crude oil price future; HH: Henry Hub natural gas price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. The sample period is from 24 August 2010 to 29 November 2019. JB-Test: Jarque-Bera test for normality. *** indicates rejection of the null hypothesis that the data are normally distributed at the 1% level of significance.

Table 4. Bai–Perron breakpoint test on return series.

	WTI	BRENT	HH	NBP	BRL	RUB	INR	CNH	ZAR
p-value	0.747	0.335	0.908	0.557	0.887	0.437	0.233	0.190	0.791

Notes: WTI: West Texas Intermediate crude oil price future; BRENT: Brent crude oil price future; HH: Henry Hub natural gas price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. Each number indicates the p-value of the Bai-Perron breakpoint test.

Table 5. Regular vine estimation of parametric case (WTI+HH+BRICS).

	WTI	HH	BRL	RUB	INR	CNH	ZAR
				Parameter1			
WTI	-	-	-	-	-	-	-
HH	0.117	-	-	-	-	-	-

BRL	−0.053	−0.040	-	-	-	-	-
RUB	−0.444	0.014	0.167	-	-		-
INR	0.015	0.008	0.011	0.137	-	-	-
CNH	−0.003	−0.010	0.067	0.150	0.280	-	-
ZAR	−0.070	0.004	0.516	0.511	0.157	0.361	-
				Parameter2			
WTI	-	-	-	-	-	-	-
HH	30.000	-	-	-	-	-	-
BRL	30.000	30.000	-	-	-	-	-
RUB	13.796	30.000	30.000	-	-		-
INR	30.000	30.000	30.000	30.000	-	-	-
CNH	30.000	30.000	30.000	30.000	26.696	-	-
ZAR	30.000	30.000	17.261	13.670	30.000	18.780	-
				Upper (lower) tail dependence			
WTI	-	-	-	-	-	-	-
HH	0.000	-	-	-	-	-	-
BRL	0.000	0.000	-	-	-	-	-
RUB	0.000	0.000	0.000	-	-		-
INR	0.000	0.000	0.000	0.000	-	-	-
CNH	0.000	0.000	0.000	0.000	0.000	-	-
ZAR	0.000	0.000	0.026	0.046	0.000	0.006	-
Loglikelihood				AIC			BIC
1340.115				−2596.231			−2353.022

Notes: WTI: West Texas Intermediate crude oil price future; HH: Henry Hub natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

Table 6. Regular vine estimation of semi-parametric case (WTI+HH+BRICS).

	WTI	**HH**	**BRL**	**RUB**	**INR**	**CNH**	**ZAR**
				Parameter1			
WTI	-	-	-	-	-	-	-
HH	0.113	-	-	-	-	-	-
BRL	−0.054	−0.039	-	-	-	-	-
RUB	−0.441	0.009	0.163	-	-	-	-
INR	0.020	0.005	0.007	0.132	-	-	-
CNH	0.001	−0.013	0.061	0.146	0.263	-	-
ZAR	−0.063	0.007	0.511	0.511	0.149	0.363	-
				Parameter2			
WTI	-	-	-	-	-	-	-
HH	30.000	-	-	-	-	-	-
BRL	30.000	30.000	-	-	-	-	-
RUB	9.191	30.000	30.000	-	-	-	-
INR	30.000	30.000	30.000	30.000	-	-	-
CNH	30.000	30.000	30.000	30.000	24.450	-	-
ZAR	30.000	30.000	13.416	11.283	30.000	11.434	-
				Upper (lower) tail dependence			
WTI	-	-	-	-	-	-	-
HH	0.000	-	-	-	-	-	-
BRL	0.000	0.000	-	-	-	-	-

RUB	0.000	0.000	0.000	-	-	-	-
INR	0.000	0.000	0.000	0.000	-	-	-
CNH	0.000	0.000	0.000	0.000	0.001	-	-
ZAR	0.000	0.000	0.048	0.069	0.000	0.032	-

Loglikelihood	AIC	BIC
1384.048	−2684.096	−2440.887

Notes: WTI: West Texas Intermediate crude oil price future; HH: Henry Hub natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

Table 7. Value-at-risk and expected shortfall.

	WTI+HH	**WTI+HH+BRICS**
	Parametric	
VaR 1%	6.461	2.792
VaR 5%	3.987	1.798
ES 1%	8.173	3.587
ES 5%	5.587	2.387
	Semi-parametric	
VaR 1%	6.566	1.523
VaR 5%	3.981	0.869
ES 1%	8.325	2.100
ES 5%	5.655	1.237

Notes: VaR: Value-at-risk; ES: expected shortfall. WTI: West Texas Intermediate crude oil price future; HH: Henry Hub natural gas price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

Table 8. Regular vine estimation of parametric case (BRENT+NBP+BRICS)

	BRENT	**NBP**	**BRL**	**RUB**	**INR**	**CNH**	**ZAR**
			Parameter1				
BRENT	-	-	-	-	-	-	-
NBP	0.070	-	-	-	-	-	-
BRL	−0.037	−0.002	-	-	-	-	-
RUB	−0.435	0.091	0.167	-	-	-	
INR	0.004	0.048	0.011	0.137	-	-	-
CNH	0.017	0.027	0.067	0.150	0.280	-	-
ZAR	−0.051	0.063	0.516	0.511	0.157	0.361	-
			Parameter2				
BRENT	-	-	-	-	-	-	-
NBP	30.000	-	-	-	-	-	-
BRL	30.000	30.000	-	-	-	-	-

RUB	14.117	30.000	30.000	-	-	-	
INR	30.000	30.000	30.000	30.000	-	-	-
CNH	30.000	30.000	30.000	30.000	26.696	-	-
ZAR	30.000	30.000	17.261	13.670	30.000	18.780	-
				Upper (lower) tail dependence			
BRENT	-	-	-	-	-	-	-
NBP	0.000	-	-	-	-	-	-
BRL	0.000	0.000	-	-	-	-	-
RUB	0.000	0.000	0.000	-	-	-	
INR	0.000	0.000	0.000	0.000	-	-	-
CNH	0.000	0.000	0.000	0.000	0.000	-	-
ZAR	0.000	0.000	0.026	0.046	0.000	0.006	-

Loglikelihood	AIC	BIC
1319.351	−2554.702	−2311.493

Notes: BRENT: Brent crude oil price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

Table 9. Regular vine estimation of semi-parametric case (BRENT+NBP+BRICS).

	BRENT	**NBP**	**BRL**	**RUB**	**INR**	**CNH**	**ZAR**
				Parameter1			
BRENT	-	-	-	-	-	-	-
NBP	0.075	-	-	-	-	-	-
BRL	−0.037	−0.002	-	-	-	-	-
RUB	−0.434	0.088	0.163	-	-	-	-
INR	0.003	0.048	0.007	0.132	-	-	-
CNH	0.020	0.021	0.061	0.146	0.263	-	-
ZAR	−0.046	0.066	0.511	0.511	0.149	0.363	-
				Parameter2			
BRENT	-	-	-	-	-	-	-
NBP	30.000	-	-	-	-	-	-
BRL	30.000	30.000	-	-	-	-	-
RUB	30.000	30.000	30.000	-	-	-	-
INR	9.414	30.000	30.000	30.000	-	-	-
CNH	30.000	30.000	30.000	30.000	24.450	-	-
ZAR	30.000	30.000	13.416	11.283	30.000	11.434	-
				Upper (lower) tail dependence			
BRENT	-	-	-	-	-	-	-
NBP	0.000	-	-	-	-	-	-
BRL	0.000	0.000	-	-	-	-	-
RUB	0.000	0.000	0.000	-	-	-	-
INR	0.000	0.000	0.000	0.000	-	-	-
CNH	0.000	0.000	0.000	0.000	0.001	-	-
ZAR	0.000	0.000	0.048	0.069	0.000	0.032	-

Loglikelihood	AIC	BIC
1373.836	−2663.671	−2420.462

Notes: BRENT: Brent crude oil price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

Table 10. Value-at-risk and expected shortfall.

	BRENT+NBP	BRENT+NBP+BRICS
	Parametric	
VaR 1%	6.717	2.743
VaR 5%	3.556	1.791
ES 1%	8.487	3.429
ES 5%	5.273	2.374
	Semi-parametric	
VaR 1%	6.062	1.367
VaR 5%	3.723	0.736
ES 1%	8.007	2.578
ES 5%	5.145	0.882

Notes: VaR: Value-at-risk; ES: expected shortfall. BRENT: Brent crude oil price future; NBP: National Balancing Point natural gas price future. BRICS: BRICS's exchange rate, which consist of BRL, RUB, INR, CNH and ZAR. BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand.

Table A1. Marginal distribution parameter estimation.

Variables	φ	AR (1)	AR (2)	MA (1)	MA (2)	ω	α	β	γ	λ	ν	Persistence
WTI	−0.022	-	-	−0.047**	-	0.019***	0.011*	0.955***	0.060***	0.897***	6.150***	0.996
Std.err	0.031	-	-	0.020	-	0.007	0.006	0.004	0.011	0.025	0.752	-
BRENT	−0.013	-	-	−0.074**	-	0.006***	0.017*	0.956***	0.052***	0.914***	5.400***	0.999
Std.err	0.028	-	-	0.020	-	0.004	0.006	0.004	0.011	0.025	0.601	
HH	−0.027	−0.040	-	-	-	0.161***	0.078***	0.917***	−0.033*	0.990***	6.624***	0.978
Std.err	0.046	0.020	-	-	-	0.045	0.013	0.013	0.018	0.027	0.830	-
NBP	−0.061	-	-	-	-	0.061***	0.075***	0.910***	0.026	0.990***	4.425***	0.998
Std.err	0.037	-	-	-	-	0.024	0.020	0.017	0.021	0.025	0.406	-
BRL	0.025*	-	-	−0.075***	-	0.007**	0.106***	0.924***	−0.070***	0.990***	7.047***	0.995
Std.err	0.014	-	-	0.021	-	0.003	0.017	0.012	0.016	0.026	0.946	-
RUB	0.013	-	-	0.024	-	0.006***	0.109***	0.920***	−0.076***	0.990***	8.079***	0.991
Std.err	0.013	-	-	0.021	-	0.002	0.022	0.019	0.017	0.028	1.223	-
INR	−0.001	0.665***	-0.935***	−0.685***	0.939***	0.002**	0.086***	0.926***	−0.038**	0.990***	4.158***	0.993
Std.err	0.007	0.026	0.018	0.031	0.023	0.001	0.020	0.018	0.016	0.023	0.405	-
CNH	−0.008	-	-	-	-	0.000***	0.101***	0.897***	0.000	0.981***	3.977***	0.998
Std.err	0.003	-	-	-	-	0.000	0.022	0.015	0.021	0.027	0.327	-
ZAR	0.032*	-	-	-	-	0.008***	0.058***	0.960***	−0.056***	0.990***	11.709***	0.990
Std.err	0.018	-	-	-	-	0.003	0.007	0.003	0.011	0.028	2.432	-

Notes: WTI: West Texas Intermediate crude oil price future; BRENT: Brent crude oil price future; HH: Henry Hub natural gas price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. Std.err: standard error; φ, AR (p), MA (q): parameters from ARMA(p, q) model; ω, α, β, γ: parameters from the GJR-GARCH model; λ, ν: parameters from skewed t model for the distribution of the error term; Persistence: $\alpha + \beta + \gamma/2$. *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

Table A2. Ljung-Box test and Lagrange Multiplier test.

Variable	Q (20)	Q^2 (20)	ARCH (20)
WTI	11.637	17.053	17.531
p-value	0.928	0.650	0.618
BRENT	22.651	14.000	13.251
p-value	0.306	0.831	0.866
HH	16.287	14.124	14.016
p-value	0.699	0.824	0.830
NBP	16.564	14.160	15.758
p-value	0.681	0.822	0.732
BRL	20.174	10.531	10.491
p-value	0.447	0.957	0.958
RUB	17.374	27.744	26.099
p-value	0.629	0.116	0.163
INR	27.011	16.201	16.186
p-value	0.135	0.704	0.705
CNH	23.079	0.175	0.173
p-value	0.285	1.000	1.000
ZAR	28.184	28.201	27.397
p-value	0.105	0.105	0.124

Notes: WTI: West Texas Intermediate crude oil price future; BRENT: Brent crude oil price future; HH: Henry Hub natural gas price future; NBP: National Balancing Point natural gas price future; BRL: Brazilian Real; RUB: Russian Ruble; INR: Indian Rupee; CNH: offshore Chinese Yuan; ZAR: South Africa Rand. Q (20): Ljung-Box test statistics for serial correlation of order 20 for the standardized residuals; Q^2 (20): Ljung-Box test statistics for serial correlation of order 20 for the squared standardized residuals; ARCH (20): Lagrange Multiplier test statistics of order 20 for autoregressive conditional heteroscedasticity.

Table B1. Common bivariate copula models.

Name	Parameter(s)	Probability density function
Normal	$\theta \in (-1,1)$	$\int_{-\infty}^{\Phi^{-1}(u_1)} \int_{-\infty}^{\Phi^{-1}(u_2)} (1/2\pi\sqrt{1-\theta^2}) \exp\{-(s^2 - 2\theta st + t^2)/[2(1$
Student's t	$\theta_1 \in (-1,1)$ $\theta_2 \in (2,\infty)$	$\int_{-\infty}^{t^{-1}(u_1)} \int_{-\infty}^{t^{-1}(u_2)} (1/2\pi\sqrt{1-\theta_1{}^2}) \exp\{-(s^2 - 2\theta_1 st + t^2)/[2($
Frank	$\theta \in (-\infty,\infty) \setminus \{0\}$	$-\frac{1}{\theta} \ln\{1 + [(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)]/(e^{-\theta} - 1)\}$
Clayton	$\theta \in [-1,\infty) \setminus \{0\}$	$[\max(u_1{}^{-\theta} + u_2{}^{-\theta} - 1, 0)]^{-(1/\theta)}$
Gumbel	$\theta \in [1,\infty)$	$\exp\{-[(-\ln u_1)^\theta + (-\ln u_2)^\theta]^{1/\theta}\}$
Joe	$\theta \in [1,\infty)$	$1 - [(1-u_1)^\theta + (1-u_2)^\theta - (1-u_1)^\theta 1 - u_2{}^\theta]^{1/\theta}$

Notes: u_1, u_2: uniform variates. $\Phi^{-1}(\cdot)$: inverse cumulative distribution function of the univariate standard normal distribution; $t^{-1}(\cdot)$: inverse cumulative distribution function of the univariate Student's t distribution. This table is based on Aloui et al. (2013), Patton (2013) and Nelsen (2006).

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