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REMARKS ON SOME FIXED POINT THEOREMS

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The purpose of this short note is to compare some fixed point theorems of Khan [2],[3] with those previously proved by Kannan [1] and Reich [4].

Khan's results are:

Theorem A: ([2, p.227]) *Let* (X, d) *be a complete metric space and let* $T: X \to X$ *satisfy*

(1) $d(Tx, Ty) \leqq \alpha\{d(x, Tx) \cdot d(y, Ty)\}^{\frac{1}{2}}$

for all distinct $x, y \in X$ *and some* α, $0 < \alpha < 1$. *Then* T *has a unique fixed point and the sequence* $\{T^n x\}$ *converges to the fixed point for every* $x \in X$.

Theorem B: ([3, Theorem 1]) *Let* (X, d) *be a compact metric space and let* $T: X \to X$ *be continuous and satisfy*

(1) $d(Tx, Ty) < \{d(x, Tx) \cdot d(y, Ty)\}^{\frac{1}{2}}$

for all distinct $x, y \in X$. *Then* X *is a singleton and, hence,* T *has a unique fixed point.*

Theorem C: ([3, Theorem 4]) *Let* (X, d) *be a compact metric space and let* $T: X \to X$ *satisfy*

(1) $d(Tx, Ty) \leqq \{d(x, Tx) \cdot d(y, Ty)\}^{\frac{1}{2}}$

for all $x, y \in X$. *If* $d(x, Tx)$ *is not constant on any closed subset of* X *which contains more than one point and is invariant under* T, *then*

T *has a fixed point.*

Simply recalling that the arithmetic mean is greater than or equal to the geometric mean, we see that Theorem A is an immediate consequence of Kannan's result [1, Theorem 1] where in place of (1), Kannan used the property

(1') $d(Tx, Ty) < \beta\{d(x, Tx) + d(y, Ty)\}$

for all x, y X and some β, $0 < \beta < \frac{1}{2}$.

By a similar observation, in each case replacing (1) by the corresponding (1'), we see that Theorems B and C are a consequence of [4, Proposition 1.2] and [4, Proposition 1.1], respectively. Moreover, the continuity of T is superfluous in Theorem B and the fixed point is unique in Theorem C.

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