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Coequalizer in the Category of BCK-Algebras II

Dedicated to Professor Kiyoshi Iséki on his 60th birthday

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In this Note, we shall show that *notions of coequalizer and onto homomorphism coincide in the category* BCK *of BCK-algebras.*

In the previous Note [2], we showed that there always exists a coequalizer of any pair of morphisms in BCK and that it is given by an onto homomorphism. Since any isomorphism in BCK is a bijective homomorphism and any coequalizer of a given pair of morphisms is unique up to isomorphism, any coequalizer is an onto homomorphism.

Now we shall prove the converse by a routine method. For an onto homomorphism $f:A \to B$, we define $Z \subset A \times A$ by $Z = \{(a_1, a_2) \mid f(a_1) = f(a_2)\}$. Let p_1 and p_2 be projections from Z onto A. Then it is easily verified that Z is a subalgebra of the product BCK-algebra $A \times A$, and that p_1 and p_2 are homomorphisms. By the definition, we have $fp_1 = fp_2$. We choose, for proof of the universal property, a homomorphism $g:A \to C$ such that $g \cdot p_1 = g \cdot p_2$. For any b in B, there exists an a in A with $f(a) = b$, since f is onto. So

$$\begin{tikzcd} Z \arrow[r, shift left, "p_1"] \arrow[r, shift right, "p_2"'] & A \arrow[r, "f"] \arrow[dr, "g"'] & B \arrow[d, dashed, "h"] \\ & & C \end{tikzcd}$$

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we may define a mapping $h: B \to C$ by letting $h(b) = g(a)$. This h is surely well-defined, because $f(a_1) = f(a_2)$ implies $g(a_1) = g(a_2)$ for any a_1, a_2 in A. It is evident that h is a homomorphism satisfying $h \cdot f = g$. The uniquness of h which satisfies $h \cdot f = g$ follows from the fact that f is an onto mapping. Thus h is a coequalizer of p_1, p_2. Hence notions of coequalizer and onto homomorphism coincide in the category of BCK-algebras.

Remark. The category BCK has zero object $\{0\}$ and the kernel $(f^{-1}(0), i)$ of any morphism $f: A \to B$, where i is an inclusion mapping. Generally speaking, the notion of kernels coincides with that of the pairs of ideals and their inclusion mappings (see K. Isêki [1]).

We show that *there also exists the cokernel of any given morphism* $f: A \to B$.

Let I be the minimum ideal of B containing $f(A)$, Z be the quotient BCK-algebra B/I, and $p: B \to Z$ be the canonical onto mapping. By the similar way as in [1], we may easily verify that the Diagram 1 is a pushout. Consequently (Z, p) is the cokernel of f.

$$\begin{array}{ccc} A & \xrightarrow{f} & B \\ \downarrow & & \downarrow p \\ \{0\} & \longrightarrow & Z \end{array}$$

Diagram 1

References

[1] K. Isêki: Some topics from the category of BCK-algebras, these NOTES, 5 (1977), 465-468.

[2] H. Yutani: Coequalizer in the category of BCK-algebras, these NOTES, 6 (1978), 187-188.