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Characterization of Certain Classes of Semigroups I

By Sandor LAJOS*

Let S be a semigroup. A subsemigroup A of S is called an (m, n)-*ideal* of S if the inclusion

$$A^m S A^n \subset A \tag{1}$$

holds, where m, n are nonnegative integers. This concept was introduced by the author in 1959 (cf. the bibliography of [4], part I). For (m, n)-ideals of associative rings, see [8].

In this short note, certain classes of semigroups will be characterized in terms of (m, n)-ideals. For the notation and notions not defined here we refer to [2] and [10].

First we give a characterization of completely regualr semigroups.

Theorem 1. *A semigroup S is a union of groups if and only if the condition*

$$A \cap B = A^2 B \tag{2}$$

holds for every $(2, 0)$*-ideal* A *and every* $(0, 2)$*-ideal* B *of* S.

Proof. Necessity: Let S be a completely regular semigroup. Then every

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(2, 0)-ideal of S is a right ideal and every (0, 2)-ideal of S is a left ideal of S (see [7]), Thus we have to show the relation

$$L \cap R = R^2 L \tag{3}$$

for every left ideal L and every right ideal R of S. But this holds because of S is regular and thus every right ideal of S is globally idempotent.

Sufficiency: Suppose that S is a semigroup having property (2) for every (2, 0)-ideal A and every (0, 2)-ideal B of S. Then, for any element a of S, we have

$$a \in \langle a \rangle_{(2,0)} \cap \langle a \rangle_{(0,2)} = (\{a, a^2\} \cup a^2 S)^2 (\{a, a^2\} \cup Sa^2) \subset \{a^3\} \cup a^2 Sa. \tag{4}$$

Hence it follows that $a \in a^2 Sa$ for every element a of S. Therefore S is a completely regular semigroup. The proof of Theorem 1 is complete.

The following more general criterion can be proved similarly.

Theorem 2. *A semigroup S is completely regular if and only if the condition* (2) *holds for every* $(m, 0)$*-ideal A and every* $(0, n)$*-ideal B of S* (m, n *are positive integers and* $m > 1$).

Theorem 3. *A semigroup S is a semilattice of left groups if and only if the condition*

$$B \cap L = B^2 L \tag{5}$$

holds for every bi-ideal B and every left ideal L of S.

Proof. If S is a semilattice of left groups, then $B \cap L = BL$ and $B^2 = B$ holds for every bi-ideal B and every left ideal L of S (cf. [5]). Hence we obtain (5).

Conversely, if S is a semigroup with property (5) for every (1, 1)-

ideal B and every left ideal L of S, then (5) implies

(6) $$B \cap L \subset BL$$

for any bi-ideal B and left ideal L of S. Hence, by the left-right dual of Theorem 9 in [9], it follows that S is regular. On the other hand, (5) implies $L = L^2S$ for any left ideal L, whence L is a right ideal of S. Thus S is a regular left duo semigroup which is a semilattice of left groups (see [5]).

Remark. Theorem 3 shows that a statement by I. A. Amin [1], p. 11, is not quite correct. (He claimed that condition (5) characterizes the class of completely regular semigroups.) Theorem 3 is a special case of a result in the author's paper [6].

Corollary (to Theorem 1). *Let S be a completely regular semigroup. Then every $(0, 2)$-ideal of S is a left ideal and every $(2, 0)$-ideal is a right ideal of S. More generally, every $(0, n)$-ideal of S is a left ideal and every $(m, 0)$-ideal is a right ideal of S.*

This Corollary was obtained in [7] by different proof.

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