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LINK DIAGRAMS REALIZING PRESCRIBED SUBDIAGRAM PARTITIONS

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Abstract

Suppose $D_1, \dots, D_n$ are link diagrams. Given a link L which is partitioned into sublinks $L_1, \dots, L_n$ admitting diagrams $D_1, \dots, D_n$, respectively, there is a diagram D of L whose restrictions to $L_1, \dots, L_n$ are equivalent to $D_1, \dots, D_n$, respectively.

A *link* is a union of mutually disjoint finitely many piecewise smooth simple closed curves in $\mathbb{R}^3$. Two links L and L' are said to be *equivalent*, denoted $L \sim L'$, if there is an orientation preserving autohomeomorphism of $\mathbb{R}^3$ mapping one onto the other. A link is said to be *trivial* if it is the boundary of a collection of mutually disjoint embedded disks.

A *diagram* D of a link L is the planar image of a link equivalent to L , under the projection $\mathbb{R}^3 \rightarrow \mathbb{R}^2 \equiv \mathbb{R}^2 \times 0$ having only finitely many transverse double point singularities at each of which there is an extra information specifying which strand is above the other, by cutting the lower one. Two diagrams D and D' are said to be *equivalent*, denoted $D \sim D'$, if there is an orientation preserving autohomeomorphism of $\mathbb{R}^2$ mapping one onto the other.

Every connected component of a link is called a *component* and a nonempty collection of components is called a *sublink*. A link is said to be *brunnian* if every proper sublink is trivial.

Given a diagram D of a link L and a sublink L_0 of L , the diagram of L_0 obtained from D by removing the components not belonging to L_0 will be called the restriction of D to L_0 and denoted by $D(L_0)$.

LEMMA 1. *Suppose D is a diagram of a link L . For any sublink L_0 of L and a diagram D_0 of L_0 , there is a diagram D' of L such that $D'(L_0) \sim D_0$ and $D'(L \setminus L_0) = D(L \setminus L_0)$.*

PROOF. There is a sequence of diagrams $D(L_0) = D_1, \dots, D_n \sim D_0$ such that D_{i+1} is obtained from D_i by one application of the Reidemeister moves, $i = 1, \dots, n-1$, for some positive integer n [BZ, Proposition 1.14].

In Figures 2–4, shaded regions indicate the regions containing the arcs of $L \setminus L_0$, thick solid curves indicate the arcs of L_0 which overpass the arcs of $L \setminus L_0$, thick dotted curves indicate the arcs of L_0 which underpass the arcs of $L \setminus L_0$ and the thin solid curves indicate the strands of L_0 which may underpass and may overpass the arcs of $L \setminus L_0$. These figures show how the successive Reidemeister moves on $D(L_0)$ can be achieved with $L \setminus L_0$ kept fixed. $\square$

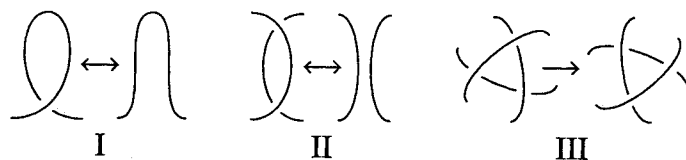


Fig. 1. Reidemeister moves

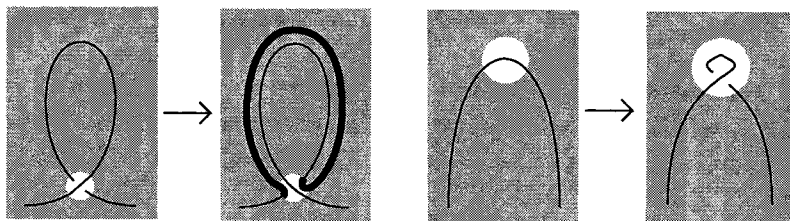


Fig. 2. Reidemeister move I

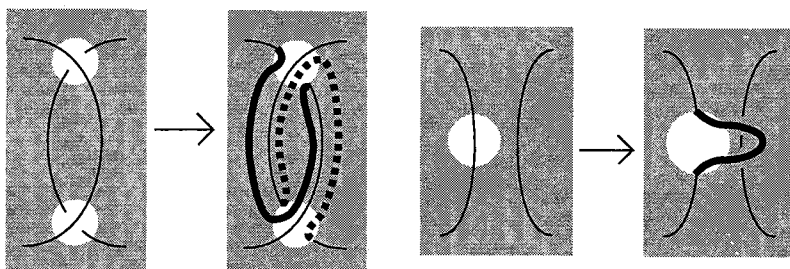


Fig. 3. Reidemeister move II

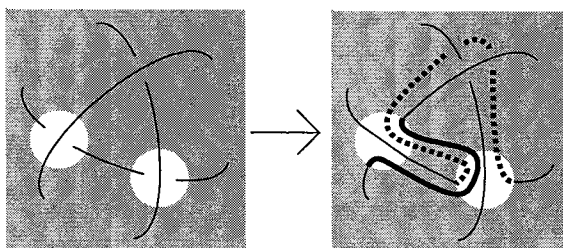


Fig. 4. Reidemeister move III

THEOREM 2. *Suppose $D_1, \dots, D_n$ are diagrams. Given a link L which is partitioned into sublinks $L_1, \dots, L_n$ admitting diagrams $D_1, \dots, D_n$, respectively, there is a diagram D of L whose restrictions to $L_1, \dots, L_n$ are equivalent to $D_1, \dots, D_n$, respectively.*

PROOF. Let $D^{(0)}$ be a diagram of L . By Lemma 1, there is a finite sequence $D^{(0)}, D^{(1)}, \dots, D^{(n)}$ of diagrams of L such that $D^{(i)}(L_i) \sim D_i$ and $D^{(i)}(L \setminus L_i) = D^{(i-1)}(L \setminus L_i)$ for $i = 1, \dots, n$, for some positive integer n . Then $D^{(n)}$ has the required property. $\square$

COROLLARY 3. *Every link has a diagram which restricts to a minimal crossing diagram for each component.*

COROLLARY 4. *Every link whose components are trivial has a diagram in which no components have self-crossings.*

COROLLARY 5. *Let L be a brunnian link and K a component of L . Then L has a diagram D such that the subdiagrams $D(K)$ and $D(L \setminus K)$ have no crossings.*

References

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