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The Cross-Euler Equation Approach to Testing for the Liquidity Constraint: Evidence from U.S. Macro Data

Shin-Ichi Nishiyama[a]

This paper tests the existence of liquidity constraint utilizing the concept of Cross-Euler equation proposed by Nishiyama (2005). We adopt standard two goods version of Life-Cycle model to study the consumption behavior of necessity goods and luxury goods. Based on the U.S. aggregate data, the test rejects the null of no liquidity constraints for necessity goods, while accepting the null for luxury goods. Since, by construction, large share of necessity goods are consumed by poor households, it is possible to interpret the results as evidence that poorer households are likely to be liquidity constrained.

Keywords Cross-Euler Equation, Liquidity Constraint, Luxury Goods, Necessity Goods

1 Introduction

In this paper, we employ the empirical method following Nishiyama (2020) in testing for the existence of liquidity constraint utilizing the concept of Cross-Euler equation. The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. It can be interpreted as the composite optimal condition that embeds both intertemporal and intratemporal optimal consumption relationships into one equation. Under addi-log type period-by-period utility function, Nishiyama (2005) showed that the Cross-Euler equation has an advantage over the standard Euler equation, in the sense that the cointegrating relationship is maintained even when the liquidity constraint is present in the agent's decision problem. Thus, by comparing the preference parameter estimates from the Cross-Euler equation to those from the standard Euler equation, it is possible to detect the existence

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of a liquidity constraint.

Reflecting the importance of liquidity constraints, considerable amount of research have been devoted in testing for the liquidity constraints based on aggregate data. Flavin (1981) and Campbell and Mankiw (1989, 1990), among others, have conducted an excess sensitivity test and found that consumption growth rate to be significantly correlated with lagged or predicted income growth, which can be interpreted as evidence of liquidity constraint. Turning to the excess sensitivity test based on panel data, Hall and Mishkin (1982), Shapiro (1986) and Hayashi (1985) all found some evidence that lagged income change or real disposable income change to be significantly correlated with consumption growth. Mariger (1987), Altonji and Siow (1987) and Zeldes (1989) specifically take into account for the Kuhn-Tucker condition emerging from the liquidity constraint.

In this paper, in order to explore the potential of the Cross-Euler equation approach in testing the liquidity constraints we adopt two goods version of Life-cycle model. Specifically, we study the consumption behaviors of necessity goods and luxury goods of the agents. Choice of necessity and luxury goods are, to some extent, arbitrary. However, this classification has its own motivation, especially in the context of aggregate data. In aggregate data, by construction, relatively larger share of luxury goods are consumed by "rich" agents, while relatively larger share of necessity goods are consumed by "poor" agents in the economy. Thus, by studying the behavior of standard Euler equations for both goods and also studying the behavior of the Cross-Euler equation linking both goods, there is a good possibility that we can infer which type of agents are more vulnerable to liquidity constraints even from the aggregate data. Naturally, since the poorer agents tend to be more vulnerable to the liquidity constraint, we expect that the Euler equation for the necessity goods to be misspecified, but the Euler equation for the luxury goods to be specified.

The rest of paper is organized as follows. Section 2 describes the standard two goods version of the life-cycle model to study the consumption behavior of necessity goods and luxury goods. Section 3 describes the U.S. aggregate data used in this paper and we apply the Cross-Euler equation approach in testing the liquidity constraints. Section 4 provides the concluding remark.

2 Model Description

This paper adopts the standard two-goods version of Life Cycle / Permanent Income Model (LCPIM) as in Ogaki (1992). Representative agent is assumed to maximize his expected lifetime utility under his lifetime budget constraint. Stating mathematically,

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U(N_t, L_t) \quad (1)$$

$$s.t.\ A_t = (1+r_t)A_{t-1} + Y_t - P_t^N N_t - P_t^L L_t \quad \text{for } \forall t \geq 0 \quad (2)$$

where N_t stands for necessity goods at period t, L_t stands for luxury goods, A_t stands for the asset holding of the agent, Y_t stands for the labor income of the agent, r_t stands for the real interest rate from period $t-1$ to t, P_t^N stands for the price of a necessity good, and P_t^L stands for the price of an luxury goods. Finally, we parameterize agent's subjective discount rate as constant β.

We have assumed that period-by-period utility is time separable for this agent and have implicitly assumed the additive separability between durable goods and non-durable goods. Solving above optimization problem yields the following first order conditions (FOC).

$$\frac{P_t^N}{P_t^L} = \frac{U_{N_t}}{U_{L_t}} \quad \text{for } \forall t \geq 0 \quad (3)$$

$$E_0\left[\beta \frac{U_{N_{t+1}}}{U_{N_t}}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N} - 1\right] = 0 \quad \text{for } \forall t \geq 0 \quad (4)$$

$$E_0\left[\beta \frac{U_{L_{t+1}}}{U_{L_t}}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L} - 1\right] = 0 \quad \text{for } \forall t \geq 0 \quad (5)$$

Eq. 3 represents the contemporaneous FOC for this representative agent. These FOC's follow if the agent is maximizing his utility given the contemporaneous price ratio of necessity and luxury goods. Eq. 4 represents the intertemporal FOC, the Euler equation, of necessity goods. The Euler equation for luxury goods (eq. 5) holds by the same logic.

Next, we are going to parameterize the utility function. We specify the utility function as a standard addi-log function following Houthakker(1960).

$$U(N_t, L_t) = \frac{(N_t)^{1-\alpha}}{1-\alpha} + K\frac{(L_t)^{1-\gamma}}{1-\gamma} \quad (6)$$

This addi-log specification was used in Ogaki (1992). Houthakker's addi-log specification reveals the non-homothetic preference of the agent in general, but contains the homothetic preference as a special case when $\alpha=\gamma$. This non-homothetic preference is crucial in our model since we try to capture intertemporal aspects of necessity goods (which by definition requires the income elasticity to be smaller than 1) and luxury goods (which requires the income elasticity to be greater than 1).

Under this specification, FOC will then be as follows.

$$\frac{P_t^N}{P_t^L} = \frac{1}{K}\frac{(N_t)^{-\alpha}}{(L_t)^{-\gamma}} \quad \text{for } \forall t \geq 0 \quad (7)$$

$$E_0\left[\beta\left(\frac{N_{t+1}}{N_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}-1\right]=0 \quad \text{for } \forall t\geq 0 \tag{8}$$

$$E_0\left[\beta\left(\frac{L_{t+1}}{L_t}\right)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}-1\right]=0 \quad \text{for } \forall t\geq 0 \tag{9}$$

Given these specifications, we are now ready to actually estimate and test the implication of the model.

3 Empirical Evidence from Macro Data

3.1 Data Description

The data we use in this paper are quarterly and seasonally adjusted U.S. non-durable goods consumption data covering the period from 1959 Q1 to 2000 Q2 (166 observations). Under the classification of National Income and Product Accounts (NIPA), we use real per adult personal consumption expenditure (PCE) for food and tobacco[1] and real per adult PCE for non-durable goods excluding food and tobacco in estimating and testing of the model's implications. PCE for food and tobacco is set to be a proxy for necessity goods and PCE of non-durables excluding food and tobacco is set to be a proxy for luxury goods. Table 1 summarizes how we categorized the components of non-durable goods into necessity goods and luxury goods in this paper.

Table 1: Categorizing Non-Durable Goods

Necessity Goods (N_t)	Luxury Goods (L_t)
Food ex. alcoholic beverages	Clothing and shoes
Tobacco	Alcoholic beverages
	News and magazine
	Entertainment
	Other goods

Note: To be accurate energy goods stands for "Gasoline, fuel oil and other energy goods". Definition of the type of consumption goods in this table follows NIPA's Table 2.6. "PCE by Type of Product".

For a price measure of each goods (i.e. P^N and P^L), we adopted chain-type price index (base year 1996) reported in NIPA Table 7.5. "Chain-Type Price Indexes for PCE by Type of Product".[2] In constructing real per adult consumption for food and tobacco (i.e. N_t), we deflated PCE for food and tobacco by the chain-type price index for PCE food and tobacco and further divided by U.S. population above 20 years old. It should be noted that by this manipulation, consumption of food and tobacco is now captured as the real *quantity* index rather than real expenditure.

This manipulation is crucial because what we try to capture in the model is quantity of consumed goods rather than expenditure. The real per adult consumption for non-durable goods excluding food and tobacco (i.e. L_t) was constructed in a similar fashion. Finally, real interest rate (i.e. r) was constructed based on quarterly average of 3-month U.S. Treasury Bill Rate subtracting inflation rate, where the inflation rate was calculated from quarterly average of Consumer Price Index.[3)]

As for the preliminary step for the cointegration analysis, we tested the null of difference stationarity against the null of (trend) stationarity for the variables included in the cointegrating regressions. To be specific, we tested the difference stationarity of following four variables: log of necessity goods (i.e. $\ln N_t$), log of luxury goods (i.e. $\ln L_t$), log opportunity cost of current necessity goods in terms of future luxury goods (i.e. $\ln[(1+r_{t+1})P_t^N/P_{t+1}^L]$) and log opportunity cost of future necessity goods in terms of current luxury goods (i.e. $\ln[(1+r_{t+1})^{-1}P_{t+1}^N/P_t^L]$). The results of the unit root tests are reported on Table 2.

Table 2: Unit Root Test

Variable	ADF test		PP test		J-test	
	cst.	cst. & trd.	cst.	cst. & trd.	J(0, 3)	J(1, 5)
$\ln N_t$	−1.616	−1.822	−1.525	−1.707	1.240	1.804
$\ln L_t$	0.378	−1.306	0.765	−0.637	67.771	3.063
$\ln[(1+r_{t+1})P_t^N/P_{t+1}^L]$	0.197	−3.047	0.084	−5.284**	25.865	0.670
$\ln[(1+r_{t+1})^{-1}P_{t+1}^N/P_t^L]$	−0.722	−2.999	−0.661	−3.076	8.864	0.406*

Note: Lag order used for ADF test and PP test was four. The 10% critical values of ADF test and PP test with a constant is −2.576 and with constant and trend is −3.143. The 5% critical values are −2.879 and −3.438, respectively. For J(0, 3) test and J(1, 5) test, 10% critical values are 0.577 and 0.452 and 5% critical values are 0.338 and 0.295, respectively. It should be noted under the J-test, the null of difference stationarity is rejected when the statistics are smaller than the critical value. * denotes the rejection of null hypothesis at the 10% level. ** denotes the rejection of null hypothesis at the 5% level.

We used Augmented Dickey-Fuller (ADF) test, PP test and J test in testing for the null of difference stationarity. As can be seen from Table 2, the tests do not reject the null of difference stationarity against the null of stationarity at 10% significance level for all variables. Further, the tests do not reject the null of difference stationarity against the null of trend stationarity at 10% significance level for both log necessity and luxury goods. Thus, log of necessity and luxury goods may well be thought of as stochastic processes containing unit root with possible drift. However, for the log opportunity costs, the test results were rather mixed. It is not clear whether the log opportunity cost follows a difference stationary process or trend stationary process from this result. Keeping in mind the possibility of trend stationarity in log opportunity cost, we now proceed to the cointegration analysis of the log-linearized cross-Euler equations.

3. 2 Estimation

3. 2. 1 Canonical Cointegration Regression

In this section we will explain the step in applying Park's (1992) Canonical Cointegration Regression (CCR) on log-linearized Cross-Euler equations. The model implies the following cointegrating restrictions:

$$\ln C_{t+1}^{L} - \frac{1}{\gamma}\ln\left[(1+r_{t+1})\frac{P_t^N}{P_{t+1}^L}\right] - \frac{\alpha}{\gamma}\ln C_t^N \sim I(0) \tag{10}$$

$$\ln C_t^{L} - \frac{1}{\gamma}\ln\left[\frac{1}{1+r_{t+1}}\frac{P_{t+1}^N}{P_t^L}\right] - \frac{\alpha}{\gamma}\ln C_{t+1}^N \sim I(0) \tag{11}$$

Following the result of unit root pretesting in the previous subsection, log necessity and luxury goods will be assumed to be a difference stationary process. However, for the log opportunity costs, the pretest results gave a mixed signal of difference stationary process and trend stationary process. Therefore, two cases must be considered in conducting the cointegration analysis: the case when deterministic trend is absent in the cointegrating system (i.e. log opportunity cost follows difference stationary process) and the case when deterministic trend is present in the cointegrating system (i.e. log opportunity cost follows the trend stationary process).

Case 1 (*Deterministic trend is not present inside the cointegrating system*)

Let y_t *be a scalar of difference stationary process and let* x_t *be the* $k\times 1$ *vector of difference stationary process whose components are not stochastically cointegrated. If* y_t *and* x_t *satisfies the deterministic cointegration restriction, then the cointegrated system can be expressed as*

$$y_t = \theta_c + v_x' \,\mathrm{x}_t + \varepsilon_t \tag{12}$$

where θ_c *is a scalar and* ε_t *is a stationary process with mean zero.*

In our model, y_t can be thought of as the log luxury goods and x_t can be thought of vector containing log opportunity cost and log necessity goods. Under the case that deterministic trend is absent in the log opportunity cost, the model implies the deterministic cointegration among the variables $\ln C^L$, $\ln(1+r)P^N/P^L$ and $\ln C^N$ with cointegrating vector $(1, 1/\gamma, \alpha/\gamma)'$. This sets the ground for applying the CCR in the above regression form.

Some remarks are in order regarding to the CCR estimator. As is the case for any cointegrating regression, CCR will yield a super-consistent estimate of the parameters. In addition, by the non-parametric correction for the long-run variance of $(\Delta x_t, \varepsilon_t)'$, CCR is known to be asymptotically efficient and does not require the strictly exogeneity assumption in $(\Delta x_t, \varepsilon_t)'$. This latter property is crucial for our purpose since the regressor x_t is constructed by the leads lags of $\ln P^N$, $\ln P^L$, and $\ln C^N$ and ε_t also consists of leads and lags of similar variables. For instance, applying OLS estimator to above regression form following Engle and Granger's (1987) method, which assumes the strict exogeneity, will yield asymptotically biased, though consistent estimates of $1/\gamma$ and α/γ. Thus, applying the CCR in the above regression form will yield a super-consistent estimate of the intertemporal substitution parameters.

By applying Park's (1990) G(p, q) test on the residuals, we can obtain the Park's H(p, q) statistics. Under the null of cointegration, Park showed that H(p, q) statistics is asymptotically χ^2 distributed with $q-p$ degrees of freedom. Since we are interested in both deterministic and stochastic cointegration relationship, we conducted H(0, q) and H(1, q) tests in this paper. The results of Park's CCR estimates are reported in Table 4. and in Table 3. In order to check the deterministic cointegration relationship, we have applied H(0, q) test for both equation. Also, to check for the stochastic cointegration relationship, H(1, q) test was also conducted.

Table 3: CCR Results

$$\ln L_{t+1} = const. + \frac{1}{\gamma}\ln[(1+r_{t+1})P_t^N/P_{t+1}^L] + \frac{\alpha}{\gamma}\ln N_t + I(0)$$

Estimates				Implied Estimates	
const.	$1/\gamma$	α/γ		α	γ
−2.345	1.304	1.295		0.992	0.766
(1.220)	(0.042)	(0.147)			
Test Statistics					
H(0, 1)	H(0, 2)	H(0, 3)	H(1, 2)	H(1, 3)	H(1, 4)
5.479*	6.073*	7.355	0.593	1.875	2.168
[0.019]	[0.047]	[0.061]	[0.441]	[0.391]	[0.538]

Note: Numbers in parenthesis stand for the estimated standard error. Numbers in square brackets stand for pvalue. * denotes the rejection of null of cointegration at the 5% level. ** denotes the rejection of null of cointegration at the 1% level.

Let us first turn to the estimation result of equation 10. Parameter estimate for α was 0.992 and γ was 0.766. Thus, IES for necessity goods (i.e. $1/\alpha$) was 1.007 and IES for luxury goods (i.e. $1/\gamma$) was 1.304. The test generally rejected the implication of the deterministic cointegration relationship, but was not able to reject the stochastic cointegration relationship.

Table 4: CCR Results

$\ln L_t = const. + \frac{1}{\gamma}\ln[(1+r_{t+1})^{-1}P^N_{t+1}/P^L_t] + \frac{\alpha}{\gamma}\ln N_{t+1} + I(0)$					
Estimates				Implied Estimates	
const.	$1/\gamma$	α/γ		α	γ
−2.096	1.332	1.264		0.949	0.750
(1.031)	(0.036)	(0.124)			
Test Statistics					
H(0, 1)	H(0, 2)	H(0, 3)	H(1, 2)	H(1, 3)	H(1, 4)
2.780	2.966	9.395*	0.185	6.614	7.470
[0.095]	[0.226]	[0.024]	[0.666]	[0.036]	[0.058]

Note: Numbers in parenthesis stand for the estimated standard error. Numbers in square brackets stand for p-value. * denotes the rejection of null of cointegration at the 5% level. ** denotes the rejection of null of cointegration at the 1% level.

Next, turning to the estimation result for equation 11, estimate for α was 0.949 and for γ was 0.750. Therefore the implied IES for necessity goods was 1.053 and for luxury goods was 1.332. We found that estimates of γ to be reasonably close between eq. 10 and 11. For the deterministic cointegration relationship, H(0, 1) and H(0, 3) test rejected the null hypothesis. For the stochastic cointegration relationship, only H(1, 3) test rejected the null hypothesis of stochastic cointegration.

3. 2. 2 GMM Estimation

In this section, we will conduct Hansen's (1982) GMM on eq. 8 and 9. Parameters α and γ will be estimated under single equation and system equation context. We will also discuss the choice of instrumental variables (IV) in this paper. Hansen's J test will also be reported.

As it was pointed out by Hall (1993) and Ogaki (1993), it is well known that the estimate of GMM is very sensitive to the choice of instrumental variables. To test for the robustness of the estimates against the choice of instruments, we estimated the parameters under several types of instruments with varying time lags. First family of the instrumental variables was chosen following the convention in applied GMM literature. As can be seen from the following table, six types of instrument sets were chosen.

The next issue in conducting GMM estimation is to choose the lag order of the error term when estimating the variance-covariance matrix of GMM disturbance terms. According to the rational expectation hypothesis, it is known that the forecast error will be serially uncorrelated.

Table 5: Set of Instrumental Variables

IV Type	Euler Equation 8	Euler Equation 9
IV1	const., $\frac{C^N_{t+1}}{C^N_t}$	const., $\frac{C^L_{t+1}}{C^L_t}$
IV2	const., $\frac{P^N_{t+1}}{P^N_t}$	const., $\frac{P^L_{t+1}}{P^L_t}$
IV3	const., r_{t+1}	const., r_{t+1}
IV4	const., $\frac{C^N_{t+1}}{C^N_t}$, r_{t+1}	const., $\frac{C^L_{t+1}}{C^L_t}$, r_{t+1}
IV5	const., $\frac{C^N_{t+1}}{C^N_t}$, $\frac{P^N_{t+1}}{P^N_t}$	const., $\frac{C^L_{t+1}}{C^L_t}$, $\frac{P^L_{t+1}}{P^L_t}$
IV6	const., $\frac{C^N_{t+1}}{C^N_t}$, $\frac{P^N_{t+1}}{P^N_t}$, r_{t+1}	const., $\frac{C^L_{t+1}}{C^L_t}$, $\frac{P^L_{t+1}}{P^L_t}$, r_{t+1}

Since our model is based on the representative agent with rational expectation, the economic theory suggests the lag order of zero. Nevertheless, taking into account for the time aggregation problem which was pointed out by Heaton (1995), we choose the lag order of one in estimating the variance-covariance matrix of GMM disturbance terms.[4] Also, to be consistent with the time aggregation issues, we have lagged the instrumental variables for two periods when conducting GMM estimations.

3.2.3 Result

GMM estimation was conducted using family of conventional instruments. The GMM estimation results for Euler equation 8 is summarized under Table 6. Similarly, the GMM estimation result for Euler equation 9 is summarized under Table 7. Hansen's J-statistics for each regression are also reported.

Let us first interpret the estimation result of Euler equation for necessity goods consumption. We first observe the large variance in the estimates of α. The estimates for α ranges from −11.917 to 15.136. This wide dispersion can also be confirmed from the estimated standard error for the estimator $\hat{\alpha}$. We can think of two possibilities that have contributed to these odd estimation results. First possibility is the weak instruments problem, i.e. if the instruments and the forcing variables in the regression are weakly correlated, the variance of the estimator will be large. It might be the case that in our GMM estimation, the conventional instruments were weakly correlated to the forcing variables.

Second possibility comes in when Euler equation is misspecified. The easiest way to check for the misspecification is to look at Hansen's J statistics. However, to our surprise, Hansen's

Table 6: GMM Results for Necessity Goods

$$E_t\left[\beta\left(\frac{N_{t+1}}{N_t}\right)^{-\alpha}(1+r_{t+1})\frac{P_t^N}{P_{t+1}^N}-1\right]=0$$

IV Type	β	α	J-statistics	D.F.
IV0	0.991 (0.007)	9.745 (16.023)	—	Just Identified
IV1	0.991 (0.009)	15.136 (7.346)	—	Just Identified
IV2	0.971 (0.035)	−11.917 (22.504)	—	Just Identified
IV3	0.991 (0.009)	15.485 (7.348)	0.066 [0.797]	1
IV4	0.984 (0.005)	−0.791 (3.957)	1.911 [0.588]	1
IV5	0.988 (0.008)	12.740 (5.934)	1.570 [0.456]	2
IV6	0.988 (0.007)	11.779 (4.606)	2.031 [0.730]	4

Note: All instruments are lagged for two periods. Numbers in parenthesis represent the estimated standard errors. Numbers in brackets represent the p-values. * denotes the rejection of null at the 5% level. ** denotes the rejection of null at the 1% level.

J test does not reject the null hypothesis that Euler equation 8 is specified for all cases. Does this mean that Euler equation 8 is correctly specified? Statistically speaking, we cannot deny this possibility. But then the odd estimates of α in Table 6. does not conform with the result of Hansen's J test. Or it might be the case that the low power of Hansen's J test resulted in the under-rejection of the null. As such, we propose to use the likelihood ratio type test proposed by Cooley and Ogaki (1996), which will be the topic of the next subsection.

To the sharp contrast to the estimation result of Euler equation for necessity goods, the estimation results of Euler equation for luxury goods have an intuitive result. As can be seen from Table 7, we can observe the relative tightness in the estimates of γ. The estimates of γ range from 0.368 to 3.778, with an exception of 16.780 under IV1. This observation is consistent with the conspicuously small estimated standard error of $\hat{\gamma}$ compared to that of $\hat{\alpha}$. However, turning to Hansen's J test, the test rejected the specification of the Euler equation 9 for 3 out of 4 cases, which is counter-intuitive given the stable estimates of $\hat{\gamma}$. Or it may well be the case that the rejection came from the size distortion of Hansen's J test. As such, we will rely on Cooley and

Table 7: GMM Results for Luxury Goods

$E_t\Big[\beta\Big(\frac{L_{t+1}}{L_t}\Big)^{-\gamma}(1+r_{t+1})\frac{P_t^L}{P_{t+1}^L}-1\Big]=0$				
IV Type	β	γ	J-statistics	D.F.
IV0	0.985 (0.007)	0.368 (1.114)	—	Just Identified
IV1	1.062 (0.065)	16.780 (15.111)	—	Just Identified
IV2	1.002 (0.009)	3.778 (1.864)	—	Just Identified
IV3	1.003 (0.006)	2.800 (1.037)	13.357 [0.000]	1
IV4	0.991 (0.006)	1.355 (0.960)	3.088 [0.588]	1
IV5	0.991 (0.006)	1.003 (0.969)	17.616 [0.000]	2
IV6	0.991 (0.006)	0.913 (0.966)	18.430 [0.001]	4

Note: All instruments are lagged for two periods. Numbers in parenthesis represent the estimated standard errors. Numbers in brackets represent the p-values. * denotes the rejection of null at the 5% level. ** denotes the rejection of null at the 1% level.

Ogaki's LR type test in testing the specification of the Euler eq. 9.

3. 3 Test of Liquidity Constraint

In this subsection, we will discuss why Cooley and Ogaki's (1996) test best suits for our purpose and also report the result of the test. Before we discuss Cooley and Ogaki's LR type test, it may be useful to review the standard LR type test in the GMM literature. For simplicity, we impose some linear restriction on the GMM estimator. In the most general linear form, the null hypothesis can be expressed as follow.

$$H_o : \mathbf{R}\hat{\theta}_{GMM}=\mathbf{q}$$

where $\mathbf{q}$ is $q\times 1$ vector of constant and $\mathbf{R}$ is some $q\times k$ matrix. Then the LR type statistics, denoted as QLR, is defined as follow and can be shown that it will be asymptotically χ^2 distributed with q degrees of freedom.

$$QLR=T\cdot J_{restricted}-T\cdot J_{unrestricted}\xrightarrow{d}\chi^2(q)$$

where T stands for the number of observations and J stands for the minimized objective function under GMM. Now, it should be noted that under the standard LR type test, **q** was simply a vector of *constants*.

The punch line of Cooley and Ogaki's LR type test is that they replaced **q** with the *estimator* of cointegrating vector $\hat{\mathbf{q}}_{coint}$. By exploiting the super-consistency of $\hat{\mathbf{q}}_{coint}$, they show that QLR will again be asymptotically χ^2 distributed with q degrees of freedom.[5)] Restating mathematically,

$$H_o : \mathbf{R}\hat{\theta}_{GMM} = \hat{\mathbf{q}}_{coint} \text{ and } QLR \xrightarrow{d} \chi^2(q).$$

Since our model involves the cointegration analysis and GMM in estimating the parameters α and γ, Cooley and Ogaki's LR type test seems to be the best candidate for our specification test.

We basically tested two types of null hypothesis. First null hypothesis is $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$ and results are reported under Table 8. Second null hypothesis is $H_0^1 : \hat{\gamma}_{GMM} = \hat{\gamma}_{coint}$ and results are reported in Table 9. Note again, if indeed eq.7, eq. 8 and eq. 9 are all well specified, then the test is likely to accept all of the above null hypotheses. We will interpret the results under three different nulls one by one.

First let us turn to the results under the null of $H_0^1 : \hat{\alpha}_{GMM} = \hat{\alpha}_{coint}$. As can be seen from Table 8, QLR statistics exceeds the critical value for most of the cases, which implies the rejection of the null hypothesis. Indeed, the test rejects 5 out of 7 cases. This evidence suggests that eq. 7 and/or eq. 8 are misspecified.

Next we will turn to the results under the null of $H_0^2 : \gamma_{GMM} = \gamma_{coint}$. To the sharp contrast to the former test, as one can see from Table 9, the test is not able to reject the null hypothesis, except for the case when we use the instrumental variable set, IV1. Except for this case, the QLR sta-

Table 8: LR-type Test Results: Necessity Goods

$H_0 : \hat{\alpha}_{CCR} = \hat{\alpha}_{GMM}$		
IV Type	QLR statistics	P-value
IV0	1.660	[0.197]
IV1	15.396**	[0.000]
IV2	4.214*	[0.040]
IV3	16.101**	[0.000]
IV4	2.267	[0.132]
IV5	14.590**	[0.000]
IV6	14.502**	[0.000]

Note: * denotes the rejection of null at the 5% level. ** denotes the rejection of null at the 1% level.

Table 9: LR-type Test Results: Luxury Goods

$H_0: \hat{\gamma}_{CCR}=\hat{\gamma}_{GMM}$		
IV Type	QLR statistics	P-value
IV0	0.130	[0.717]
IV1	15.302**	[0.000]
IV2	2.666	[0.102]
IV3	2.460	[0.116]
IV4	0.193	[0.660]
IV5	0.239	[0.624]
IV6	0.592	[0.441]

Note: * denotes the rejection of null at the 5% level. ** denotes the rejection of null at the 1% level.

tistics are below the critical values. According to this result, the test seems to support the hypothesis that both eq. 7 and eq. 9 are specified.

4 Conclusion

In this paper, we adopted standard two goods version of the life-cycle model to study the consumption behavior of necessity goods and luxury goods under addi-log utility function which allows for the non-homothetic preference. We employed the method following Nishiyama (2020) in testing for the existence of liquidity constraint utilizing the concept of Cross-Euler equation. The Cross-Euler equation represents the optimal consumption pattern of a good in the current period to another good at a future period. The Cross-Euler equation has an advantage over the standard Euler equation, in the sense that the cointegrating relationship is maintained even when the liquidity constraint is present in the agent's decision problem. Thus, by comparing the preference parameter estimates from the Cross-Euler equation to those from the standard Euler equation, it is possible to detect the existence of a liquidity constraint.

In testing for the existence of liquidity constraints, we applied the Cross-Euler equation approach to U.S. aggregate data. For the aggregate data, by construction, significant portion of the luxury goods expenditure comes from the richer agents in the economy, while significant portion of the necessity goods expenditure comes from the poorer agents. Since the poorer agents tend to be more vulnerable to the liquidity constraint, we expected that the Euler equation for the necessity goods to be misspecified, but the Euler equation for the luxury goods to be specified.

Indeed, the empirical results presented in this paper supported this view. We conducted LR type test on the null hypothesis that the IES parameter estimates from the Cross-Euler equations and the standard Euler equations are equal. We rejected null hypothesis for the necessity goods frequently, while that of luxury goods was not. This empirical results implies that the Euler equation for necessity goods is misspecified, but keeps the possibility open for the Euler equation of the luxury goods to be specified - empirical evidence that the liquidity constraint is a serious factor in rendering the Euler equation to be misspecified, but only for the poor agents. This result can be interpreted as empirical evidence from the aggregated data that supports the existence of liquidity constraint in the U.S. economy.

Notes

1) Following Ogaki (1992), we have excluded alcohol beverages from food consumption expenditure.

2) Chain-type price index of PCE for food and tobacco are published separately by BEA. In constructing the composite price index for food and tobacco, we simply computed the weighed average of two price index, where weight taken according to the food nominal expenditure and tobacco nominal expenditure.

3) Thus, real interest rate used in this paper is actually an ex-post real interest rate.

4) Since the lag order was explicitly chosen, we will use HAC estimator with truncated kernel when estimating the variance-covariance matrix of the GMM disturbance terms.

5) If, instead, the estimator $\hat{\mathbf{q}}$ were only consistent (i.e. $O(T^{-1/2})$ consistent), then one have to calculate the covariance of $\hat{\theta}_{GMM}$ and $\hat{\mathbf{q}}$ in order to conduct the statistical inference. For details, see Ogaki (1993).

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